Large-Scale Structure with non-Gaussian initial conditions

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Berkeley, April 2011

Smith & LoVerde, 1010.0055
LoVerde & Smith, 1102.1439
Smith, Ferraro & LoVerde, to appear

Outline

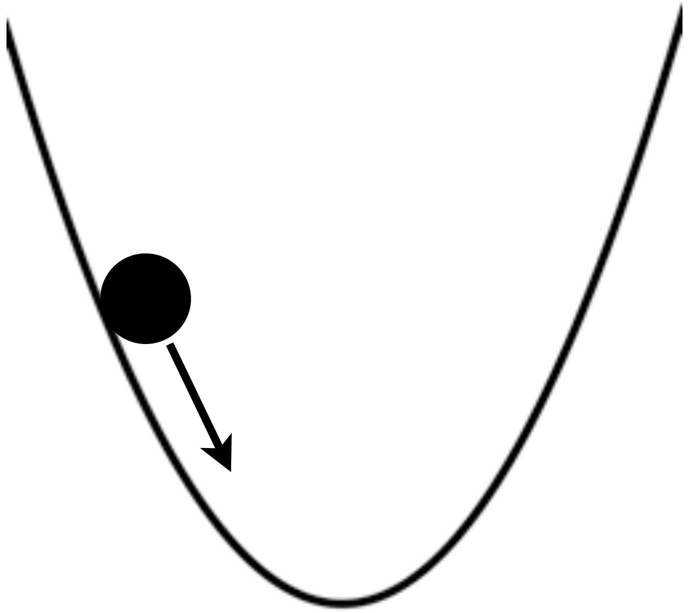
1. Introduction and motivation

2. Halo mass function

3. Large-scale halo clustering

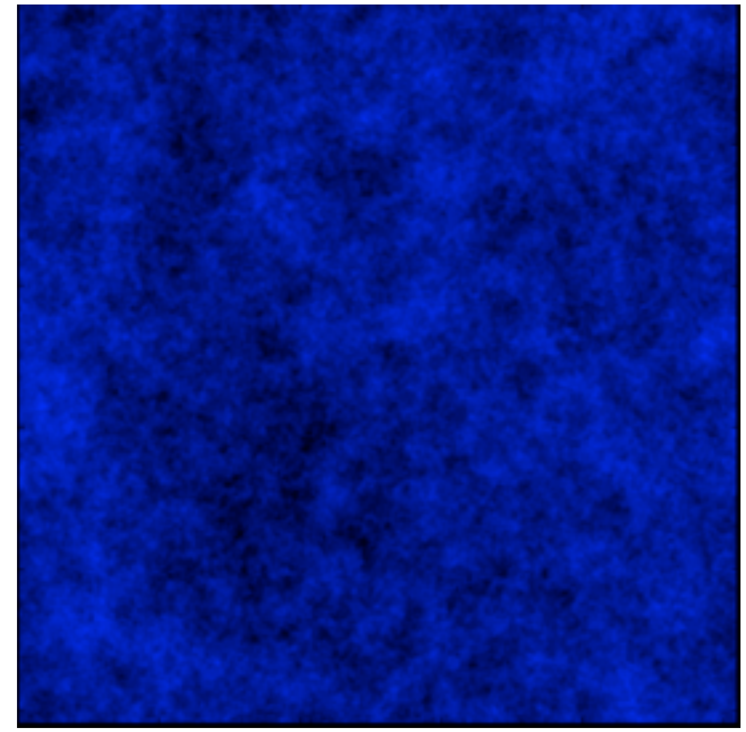
From inflation to observations

$$\mathcal{L} = \frac{M_{\rm pl}^2}{2} R - \frac{1}{2}(\partial\phi)^2 - V(\phi)$$



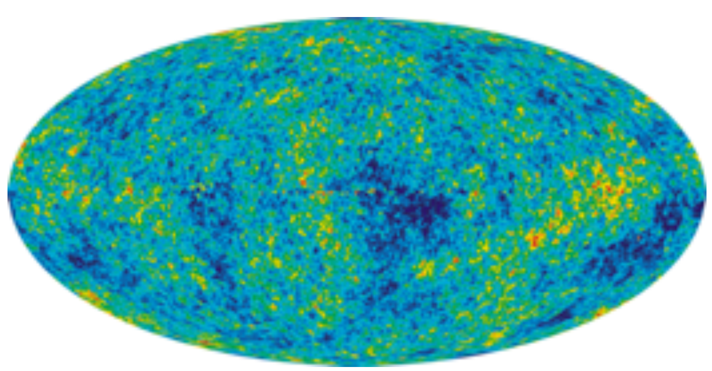
Inflation

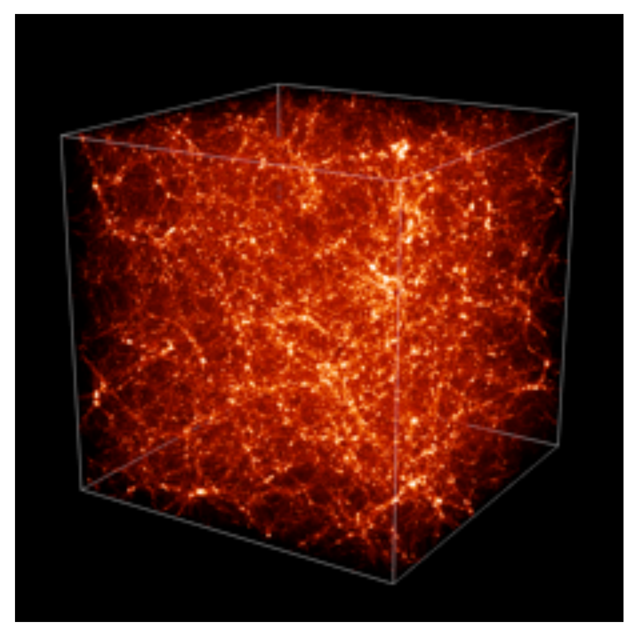
$$\langle |\zeta(k)|^2 \rangle \propto (k_0/k)^{4-n_s}$$



Initial curvature fluctuation

$$\Omega_b h^2,\ \Omega_m h^2,$$
$$\Omega_\Lambda,\ \tau, \cdots$$





cosmological observables

Constraining inflation

In the simplest models of inflation, the initial fluctuations are.....

- nearly scale invariant $(P(k) \propto k^{n_s - 4})$
- scalar
- adiabatic
- Gaussian

Constraining inflation

In the simplest models of inflation, the initial fluctuations are.....

- nearly scale invariant $(P(k) \propto k^{n_s - 4})$

  "running"? $(P(k) \propto k^{n_s - 4 + \alpha \log(k/k_0)})$
  features/glitches?

- scalar

  tensor modes ("r")?

- adiabatic

  isocurvature modes?

- Gaussian

  primordial non-Gaussianity?

A non-Gaussian model: curvaton scenario

- Light scalar field σ ("curvaton") is subdominant during inflation (spectator field)

- After inflation ends, inflaton decays to radiation before the curvaton
  $\Rightarrow \rho_{\rm infl} \propto a^{-4}, \;\; \rho_{\rm curv} \propto a^{-3}$

- Suppose curvaton dominates the energy density, and oscillates near the minimum of a quadratic potential $(V(\sigma) = V_0 + m_\sigma^2 \sigma^2)$ before decaying to SM particles

- Induced curvature perturbation will contain a term proportional to the *square* of the Gaussian field perturbation that was generated during inflation:

$$\Phi(\mathbf{x}) = \Phi_G(\mathbf{x}) + f_{NL}(\Phi_G(\mathbf{x})^2 - \langle \Phi_G^2 \rangle)$$

where f_{NL} is a free parameter.

(Notation: $\Phi = -\frac{3}{5}\zeta$)

“Local non-Gaussianity”

Primordial non-Gaussianity defined by:

$$\Phi(\mathbf{x}) = \Phi_G(\mathbf{x}) + f_{NL}(\Phi_G(\mathbf{x})^2 - \langle\Phi_G^2\rangle)$$

Possible mechanisms:

- curvaton scenario (spectator field during inflation subsequently dominates energy density)
- models with variable inflaton decay rate
- models with modulated reheating
- multifield ekpyrotic models (e.g. “New Ekpyrosis”)

WMAP constraint: $f_{NL} = 32 \pm 21 \ (1\sigma)$

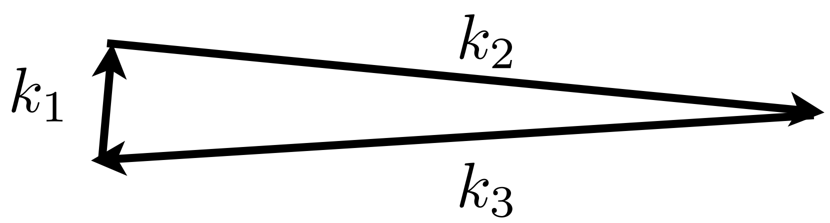
(Smith, Senatore & Zaldarriaga 2009; Komatsu, Smith et al 2010)

Single-field slow-roll inflation predicts $f_{NL} = \frac{5}{12}(1-n_s) \approx 0.017$ (Maldacena 2002)

Conversely, detection of $f_{NL} \gtrsim \mathcal{O}(10^{-2})$ would rule out all single-field models of inflation (Maldacena 2002; Creminelli et al 2004)

Single-field consistency relation

In an f_{NL} cosmology, the three-point function is large in the "squeezed" limit $k_1 \ll \min(k_2, k_3)$



$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle \to \frac{12}{5} f_{NL} \frac{1}{k_1^3 k_2^3}$$

Creminelli & Zaldarriaga (2004): Simple, general formula for the bispectrum in the squeezed limit, valid in all models of single field inflation

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle \to (1 - n_s) \frac{1}{k_1^3 k_2^3}$$

Interpretation: single field => $f_{NL} = \mathcal{O}(10^{-2})$

Physical intuition: in single field inflation, value of the inflaton field is the only "clock"

After a long-wavelength mode exits the horizon, evolution is indistinguishable from case where inflaton evolves along the same classical trajectory, but all k's have been slightly rescaled

=> When short-wavelength mode crosses the horizon, its power spectrum gets rescaled by a factor which is proportional to the deviation from scale invariance

“Generalized local non-Gaussianity”

Cubic term in potential:

$$\Phi(\mathbf{x}) = \Phi_G(\mathbf{x}) + g_{NL}(\Phi_G(\mathbf{x})^3 - 3\langle\Phi_G^2\rangle\Phi_G(\mathbf{x}))$$

Generically arises if non-quadratic corrections to curvaton potential $V(\sigma)$ are important

Two-field models in which initial potential is sum of Gaussian and non-Gaussian fields:

$$\Phi(\mathbf{x}) = \alpha\Phi_G^{(i)}(\mathbf{x}) + \beta\Phi_G^{(c)}(\mathbf{x}) + \frac{f_{NL}}{\beta^2}(\Phi_G^{(c)}(\mathbf{x})^2 - \langle\Phi_G^{(c)2}\rangle)$$

where $\Phi_G^{(i)}, \Phi_G^{(c)}$ are uncorrelated Gaussian fields with the same power spectra

$$(P_{\Phi_G^{(i)}}(k) = P_{\Phi_G^{(c)}}(k) = A(k/k_0)^{n_s-4}, \qquad P_{\Phi_G^{(i)}\Phi_G^{(c)}}(k) = 0)$$

and $\alpha^2 + \beta^2 = 1$

Summary: generalized local non-Gaussianity

"f_{NL} cosmology"

$$\Phi(\mathbf{x}) = \Phi_G(\mathbf{x}) + f_{NL}(\Phi_G(\mathbf{x})^2 - \langle\Phi_G^2\rangle)$$

"g_{NL} cosmology"

$$\Phi(\mathbf{x}) = \Phi_G(\mathbf{x}) + g_{NL}(\Phi_G(\mathbf{x})^3 - 3\langle\Phi_G^2\rangle\Phi_G(\mathbf{x}))$$

"τ_{NL} cosmology"

$$\Phi(\mathbf{x}) = \alpha\Phi_G^{(i)}(\mathbf{x}) + \beta\Phi_G^{(c)}(\mathbf{x}) + \frac{f_{NL}}{\beta^2}(\Phi_G^{(c)}(\mathbf{x})^2 - \langle\Phi_G^{(c)2}\rangle) \qquad (\text{where } \alpha^2 + \beta^2 = 1)$$

$$\left[\text{Note: } \tau_{NL} = \left(\frac{6f_{NL}}{5\beta}\right)^2\right]$$

f_{NL} cosmology corresponds to special case: $\tau_{NL} = \left(\frac{6}{5}f_{NL}\right)^2$ or $(\alpha,\beta) = (0,1)$

Scope of talk: study halo statistics in these models, specifically

- halo mass function
- large-scale clustering

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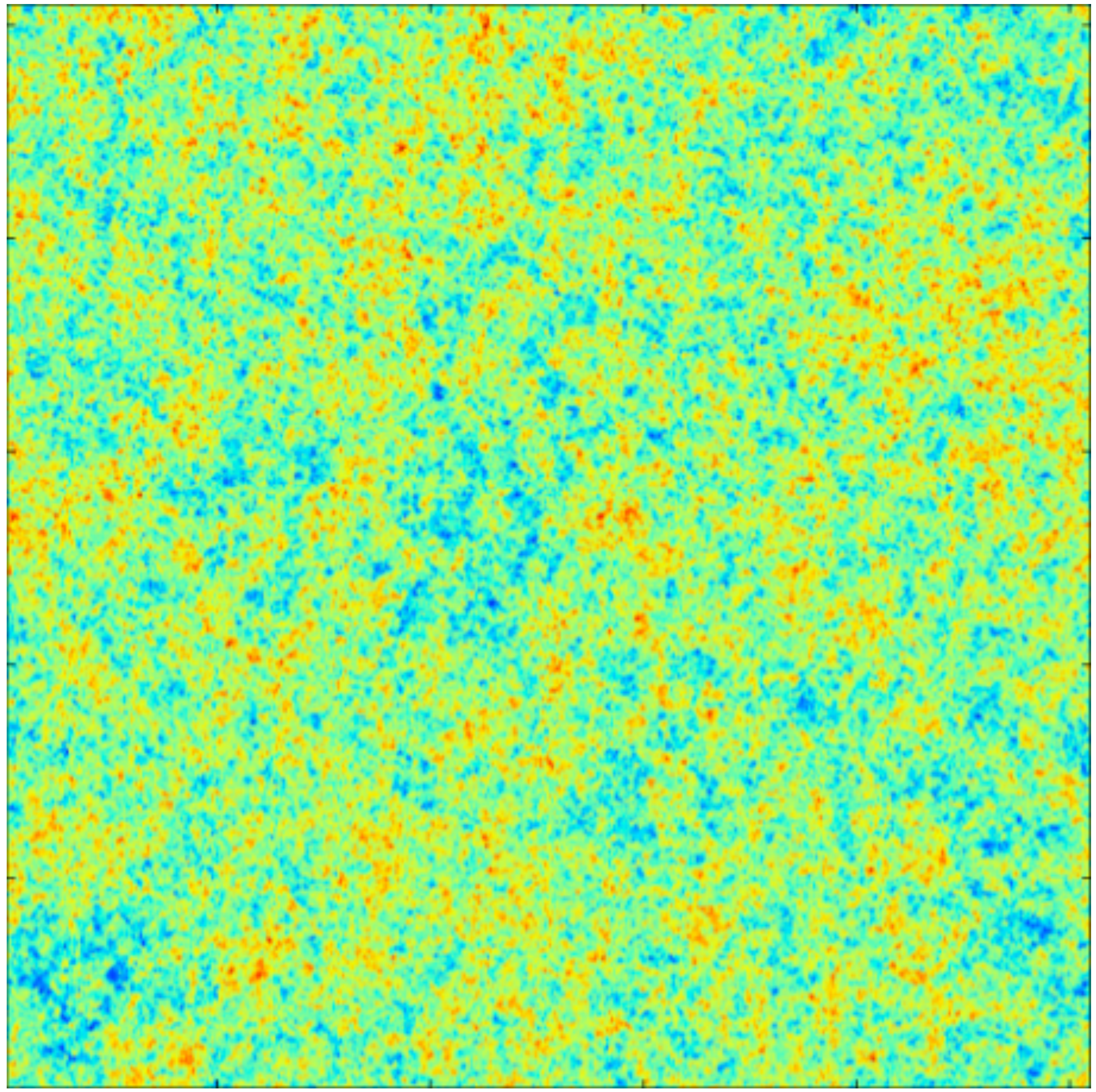
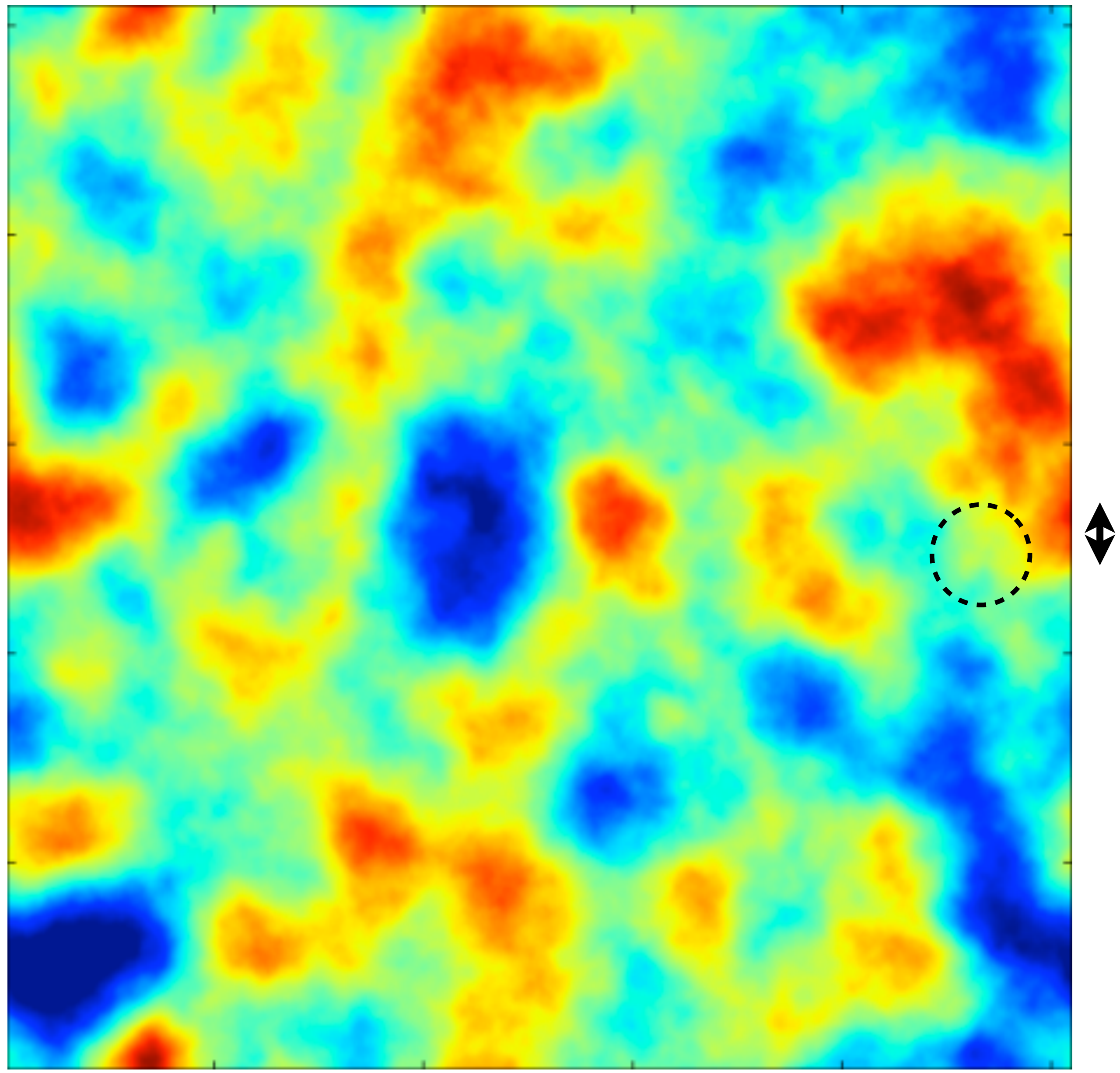
Press-Schechter Model



Start with *linear* density field $\delta_{\rm lin}(\mathbf{x}, z)$

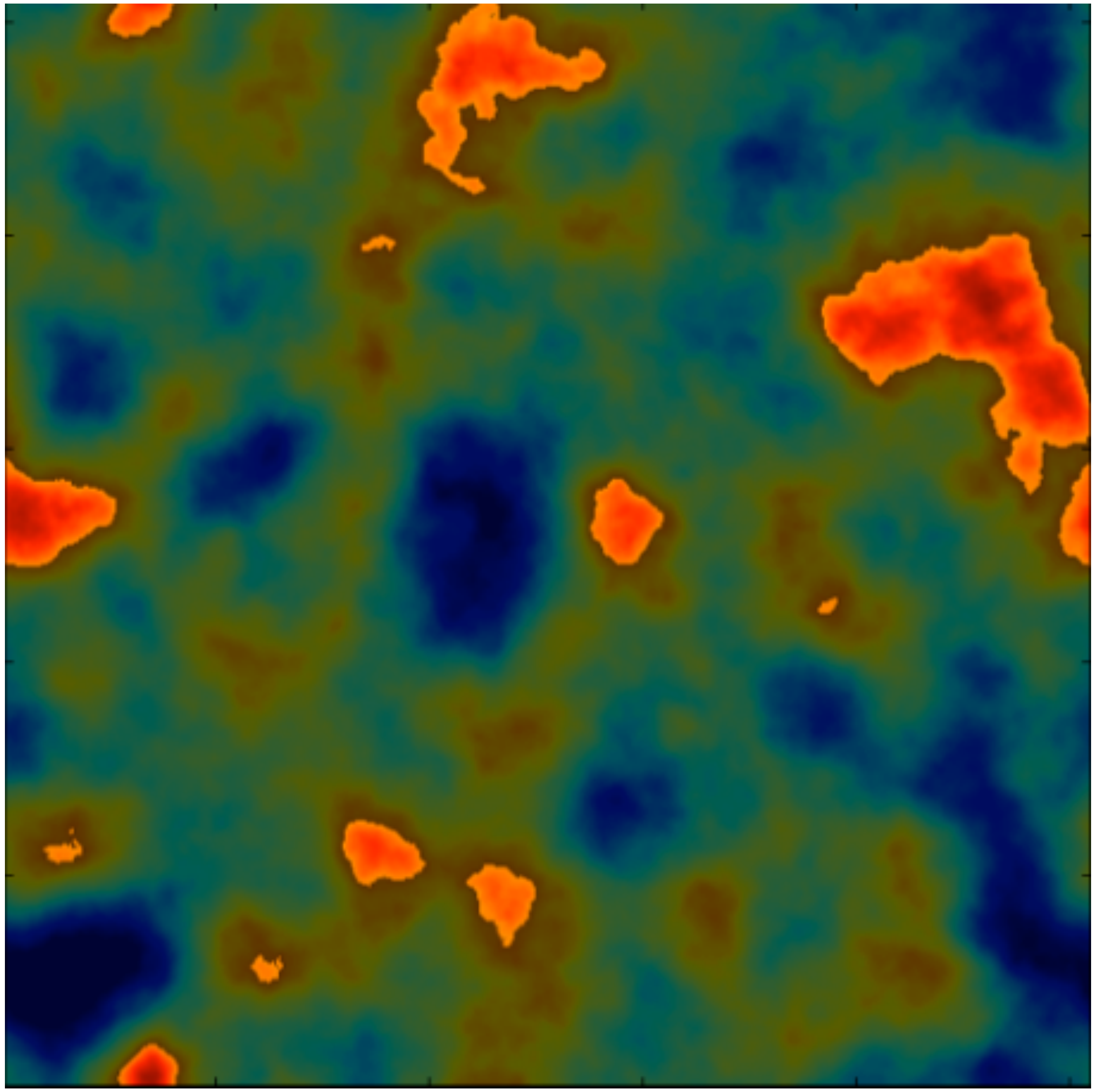
Press-Schechter Model



$$R = \left(\frac{3M}{4\pi\rho_m}\right)^{1/3}$$

Apply tophat smoothing on mass scale M to obtain smoothed linear density $\delta_M(\mathbf{x}, z)$

Press-Schechter Model



Apply threshhold: (halos of mass $\geq M$) $\Leftrightarrow$ (regions where $\delta_M(\mathbf{x}, z) \geq \delta_c$)

$\delta_c = 1.68$ motivated by analytic spherical collapse model

$\delta_c = 1.42$ gives better agreement with N-body simulations

Press-Schechter Model

$$n_h = \begin{cases} \rho_m/M & \text{if } \delta_M(\mathbf{x}, z) \geq \delta_c \\ 0 & \text{if } \delta_M(\mathbf{x}, z) < \delta_c \end{cases}$$

This description omits some ingredients which will be important for clustering but not the mass function:

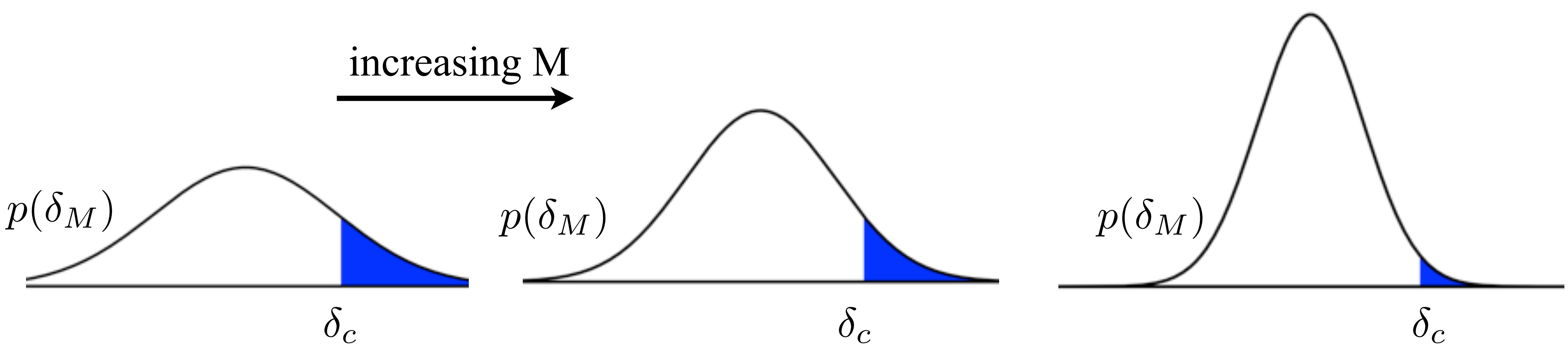
1) Lagrangian to Eulerian mapping
2) Poisson noise

Press-Schechter Model: Mass Function

In the Press-Schechter model, the halo mass function $n(M)$ is directly related to the 1-point PDF $p(\delta_M)$ of the smoothed linear density field

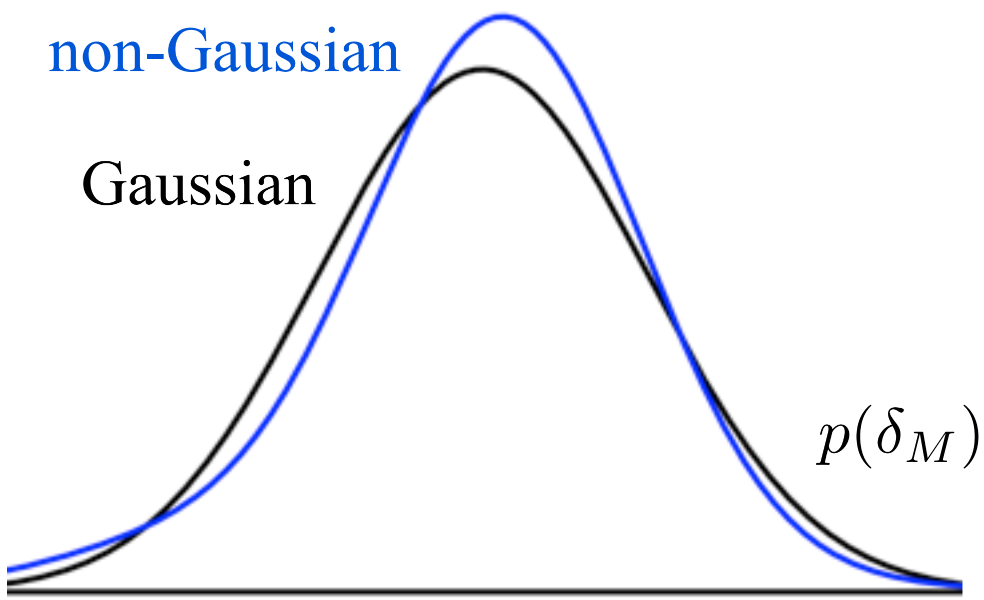
$$\int_M^\infty n(M')dM' = \frac{\rho_m}{M}\int_{\delta_c}^\infty p(\delta_M)\,d\delta_M$$

$$n(M) = -\frac{d}{dM}\left[\frac{\rho_m}{M}\int_{\delta_c}^\infty p(\delta_M)\,d\delta_M\right]$$

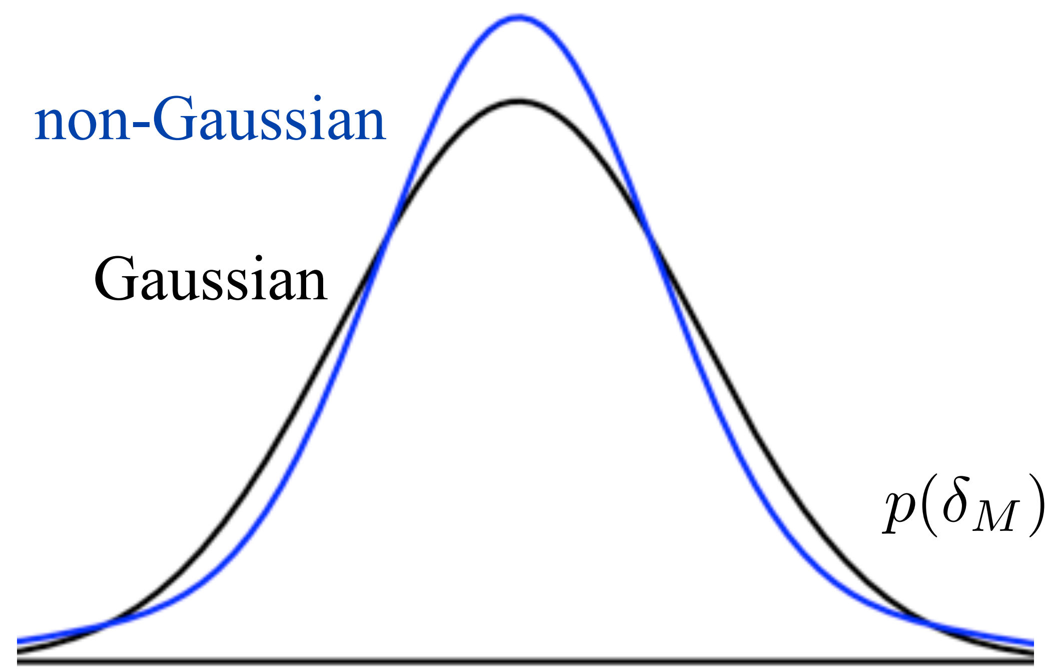


Non-Gaussian 1-point PDF

Primordial non-Gaussianity perturbs the 1-point PDF $p(\delta_M)$ from a Gaussian distribution



f_{NL} cosmology



g_{NL} cosmology

Skewness $\propto f_{NL}$

Kurtosis $\propto \tau_{NL}$

Skewness $= 0$

Kurtosis $\propto g_{NL}$

Non-Gaussian 1-point PDF: Edgeworth expansion

Technical tool for describing perturbation of 1-point PDF due to non-Gaussianity

Gives series representation of $p(\delta_M)$ parameterized by cumulants $\kappa_n(M) = \frac{\langle \delta_M^n \rangle_{\rm conn.}}{\langle \delta_M^2 \rangle^{n/2}}$

$$p(\delta) = \int \frac{dk}{2\pi} e^{-ik\delta} \exp\left(-\frac{\langle \delta_M^2 \rangle}{2} k^2 + \sum_{n \geq 3} \kappa_n(M) \frac{(ik\langle \delta_M^2 \rangle^{1/2})^n}{n!} \right)$$

Plugging into Press-Schechter expression for mass function, can calculate derivatives

$$\frac{\partial \log n(M)}{\partial f_{NL}} = \frac{F_1'(M)}{F_0'(M)}$$

$$\frac{\partial \log n(M)}{\partial g_{NL}} = \frac{F_2'(M)}{F_0'(M)}$$

$$\begin{aligned}
F_0(M) &= \frac{1}{2}\mathrm{erfc}\left(\frac{\nu_c(M)}{\sqrt{2}}\right) \\
F_1(M) &= \frac{1}{(2\pi)^{1/2}} e^{-\nu_c(M)^2/2} \left(\frac{\kappa_3(M)}{6} H_2(\nu_c(M)) \right) \\
F_2(M) &= \frac{1}{(2\pi)^{1/2}} e^{-\nu_c(M)^2/2} \left(\frac{\kappa_2(M)}{2} H_1(\nu_c(M)) + \right. \\
&\quad \left. + \frac{\kappa_4(M)}{24} H_3(\nu_c(M)) + \frac{\kappa_3(M)^2}{72} H_5(\nu_c(M)) \right)
\end{aligned}$$

Loverde & Smith 2011

N-body simulations

Collisionless N-body simulations, GADGET-2 TreePM code.

Unless otherwise specified:

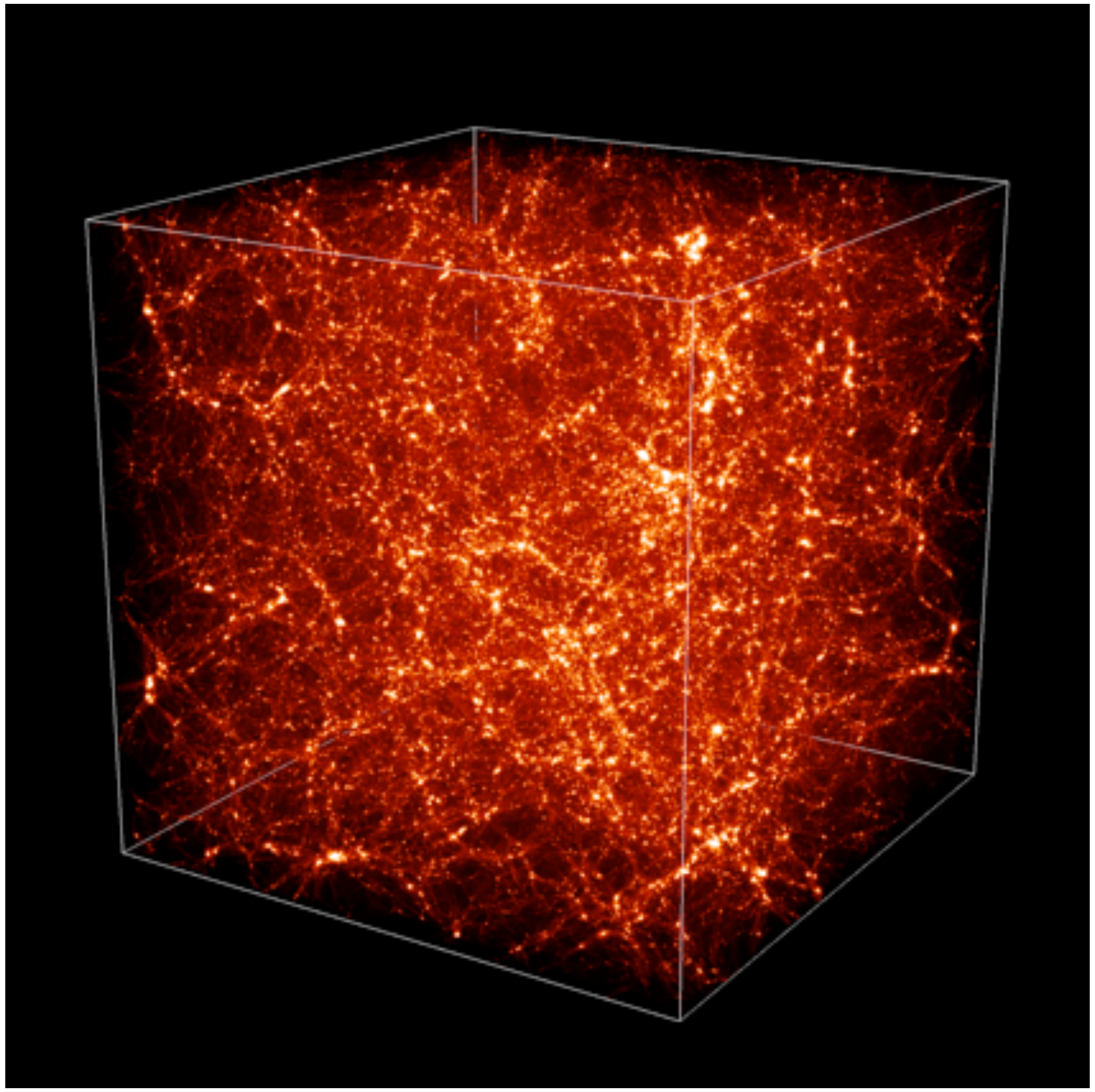
- periodic boundary conditions,
  $L_{\rm box} = 1600\ h^{-1}\ {\rm Mpc}$

- particle count $N = 1024^3$

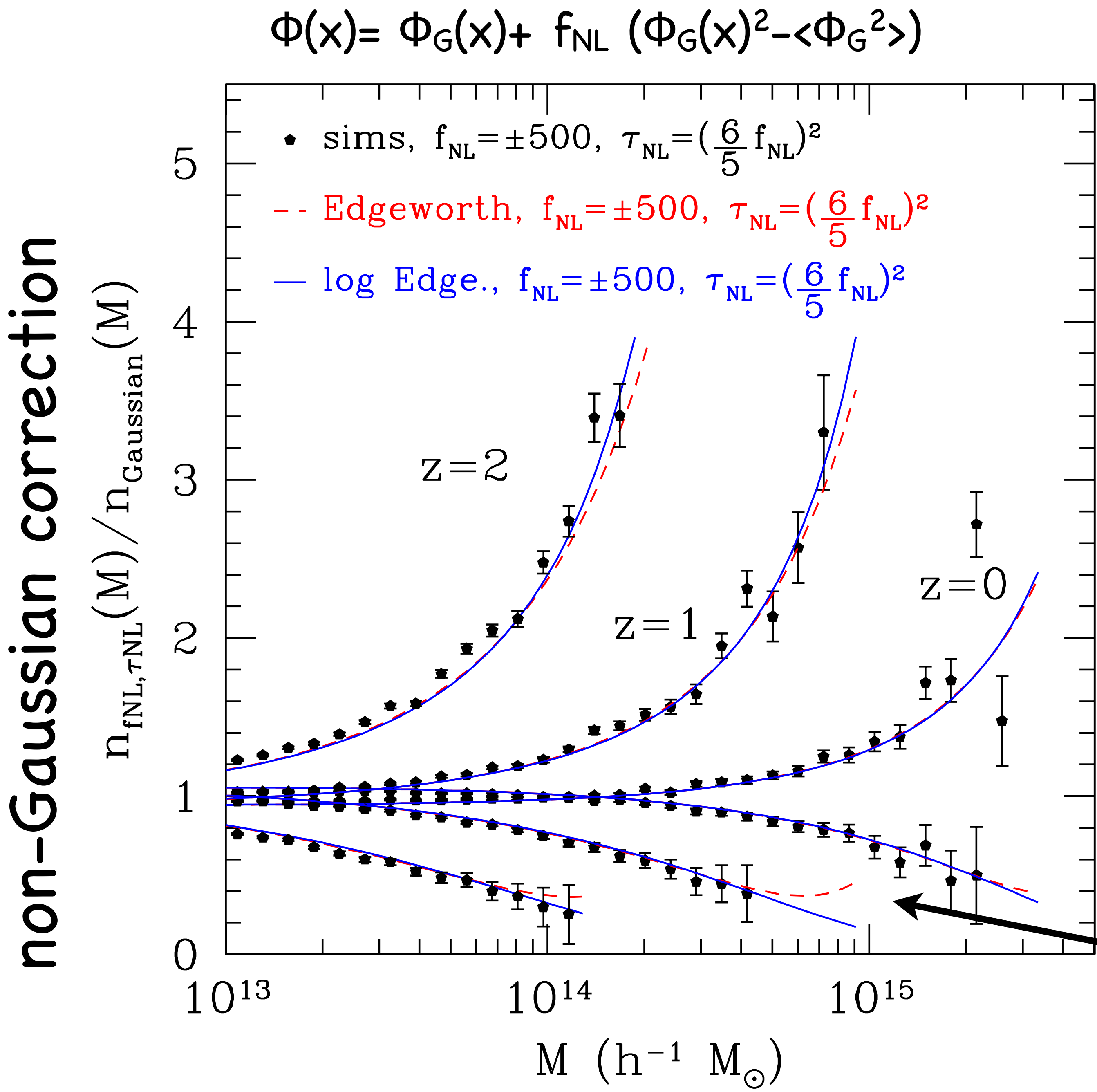
- force softening length
  $R_s = 0.05\,(L_{\rm box}/N^{1/3})$

- initial conditions simulated at $z_{\rm ini} = 100$
  using Zeldovich approximation

- FOF halo finder, link length
  $L_{\rm FOF} = 0.2\,(L_{\rm box}/N^{1/3})$



Mass function: f_{NL} simulations



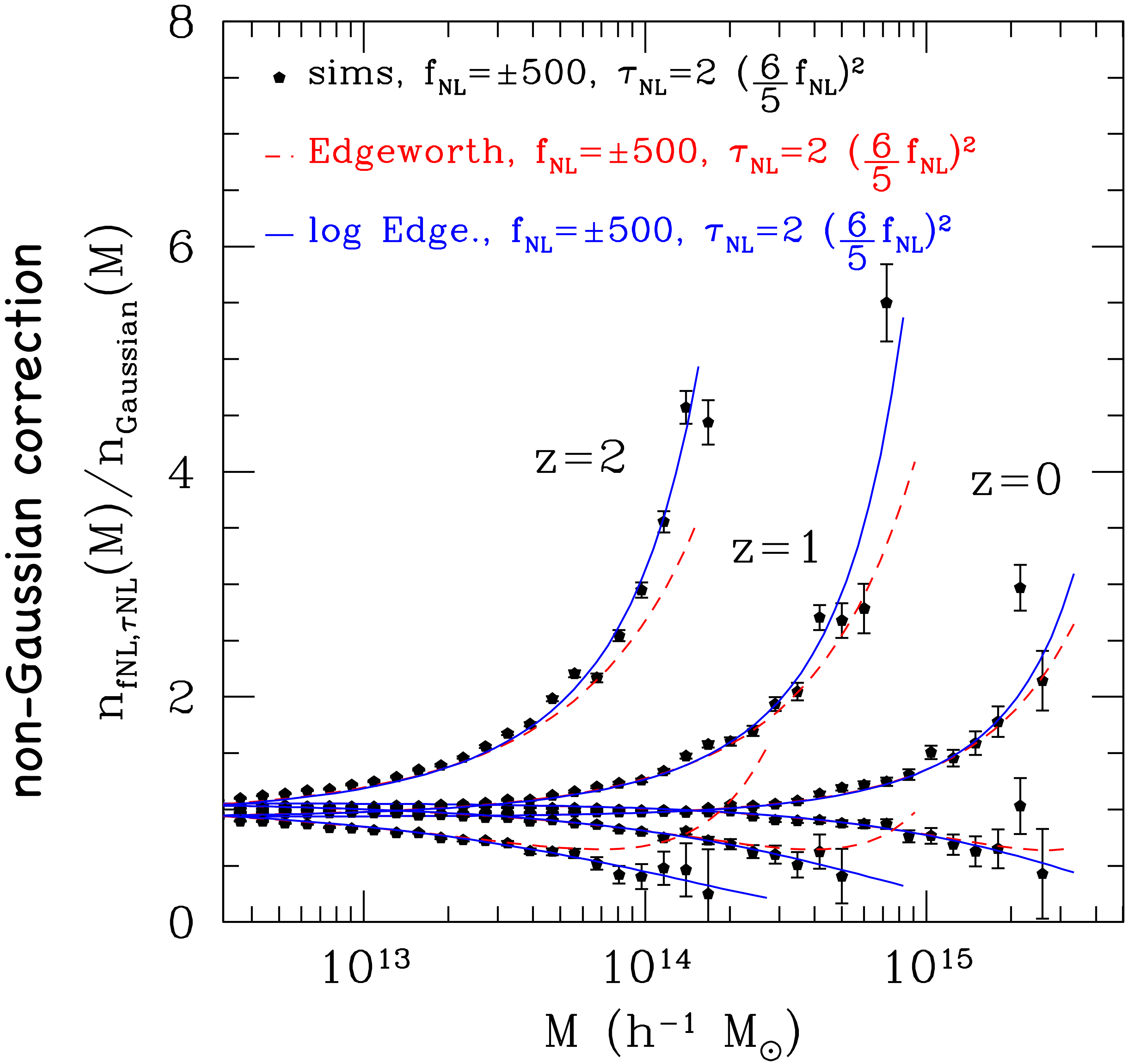
log-Edgeworth has correct asymptotic behavior at high M

"Edgeworth" mass function: $n(M) \approx n_G(M) + \left(\frac{\partial n}{\partial f_{NL}}\right) f_{NL} + \left(\frac{\partial n}{\partial \tau_{NL}}\right) \tau_{NL} + \left(\frac{\partial n}{\partial g_{NL}}\right) g_{NL}$

"Log-Edgeworth" mass function: $n(M) \approx n_G(M) \exp\left(\frac{\partial \log n}{\partial f_{NL}} f_{NL} + \frac{\partial \log n}{\partial \tau_{NL}} \tau_{NL} + \frac{\partial \log n}{\partial g_{NL}} g_{NL}\right)$

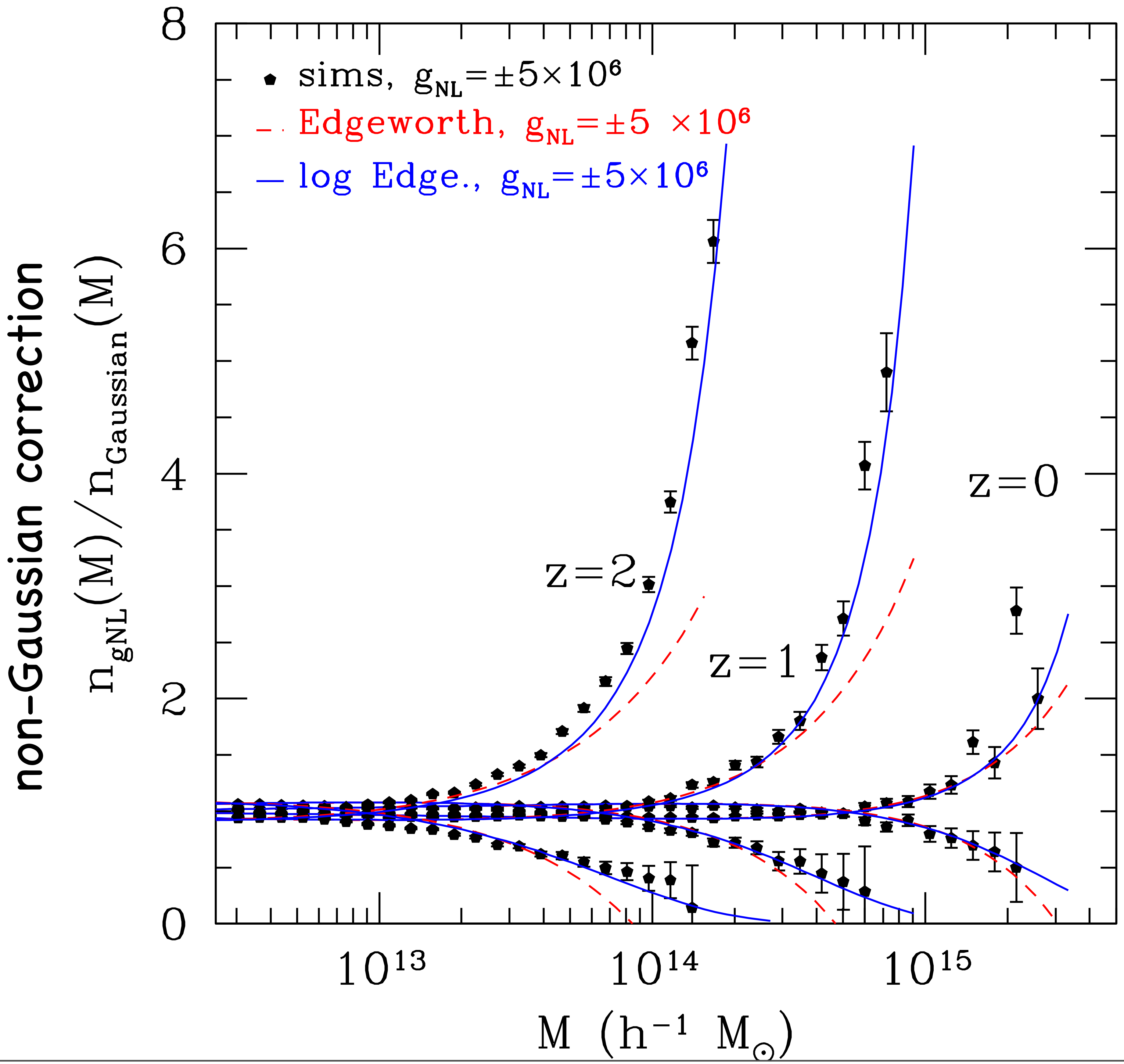
Mass function: τ_{NL} simulations

[log-Edgeworth mass function looks better here!]



Mass function: g_{NL} simulations

[log-Edgeworth mass function looks better here too!]



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Local non-Gaussianity: large-scale clustering

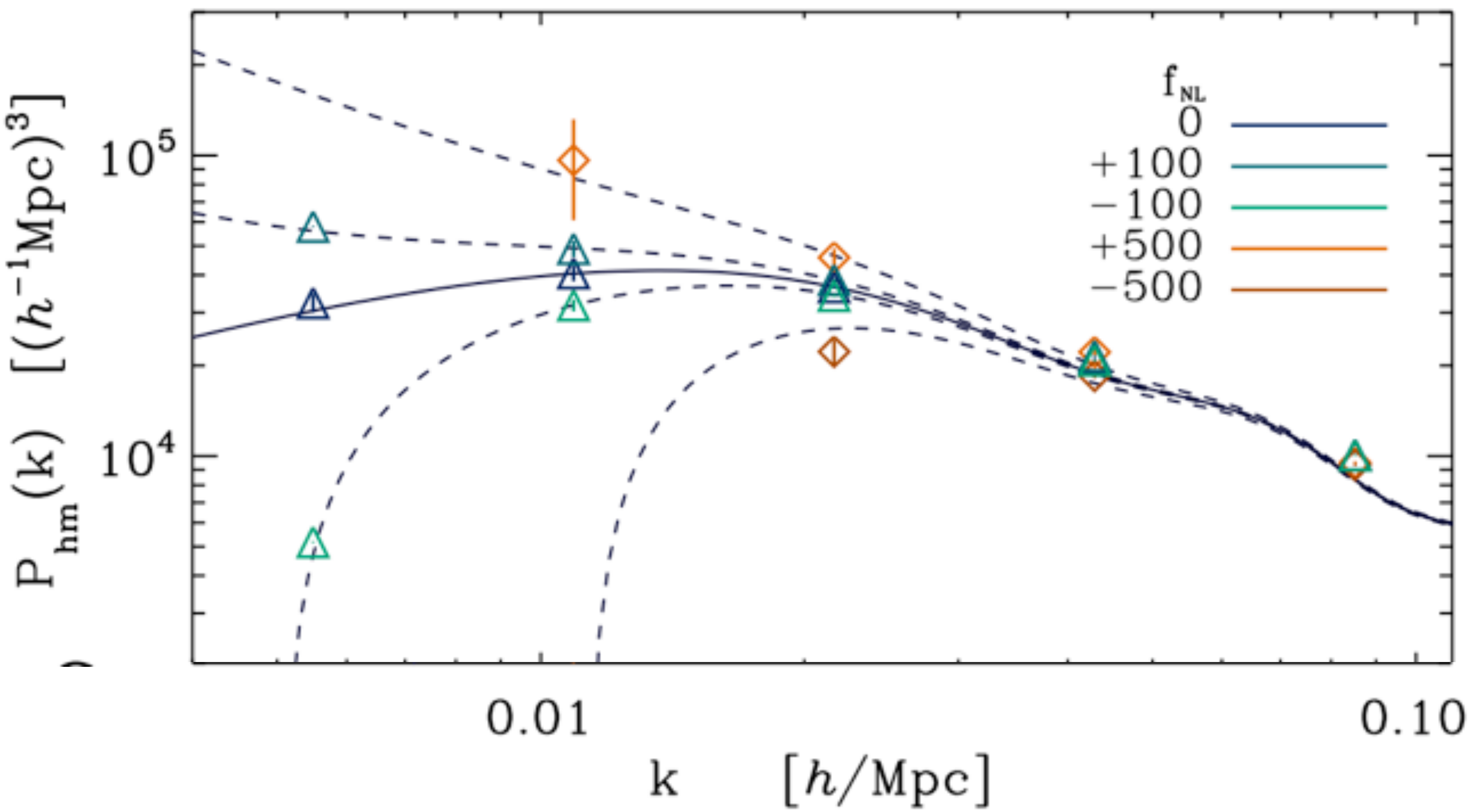
Dalal et al (2007): extra halo clustering on large scales in an f_{NL} cosmology

Clustering $\propto 1/\alpha(k)$, where

$$\alpha(k,z) = \frac{2}{3}\frac{k^2 T(k) D(z)}{\Omega_m H_0^2}$$

satisfies

$$\delta_{\rm lin}(\mathbf{k},z) = \alpha(k,z)\Phi(\mathbf{k})$$



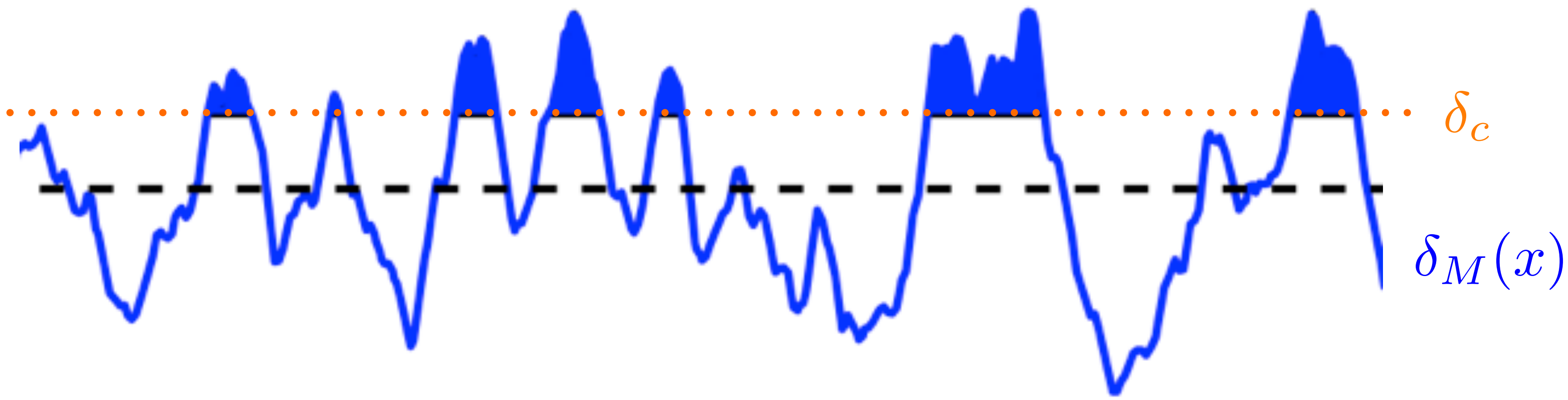
Dalal, Dore, Huterer & Shirokoff (2007)

Large-scale structure constraints are competitive with the CMB
Slosar et al (2008): $f_{NL} = 20 \pm 25\ (1\sigma)$ from SDSS-II

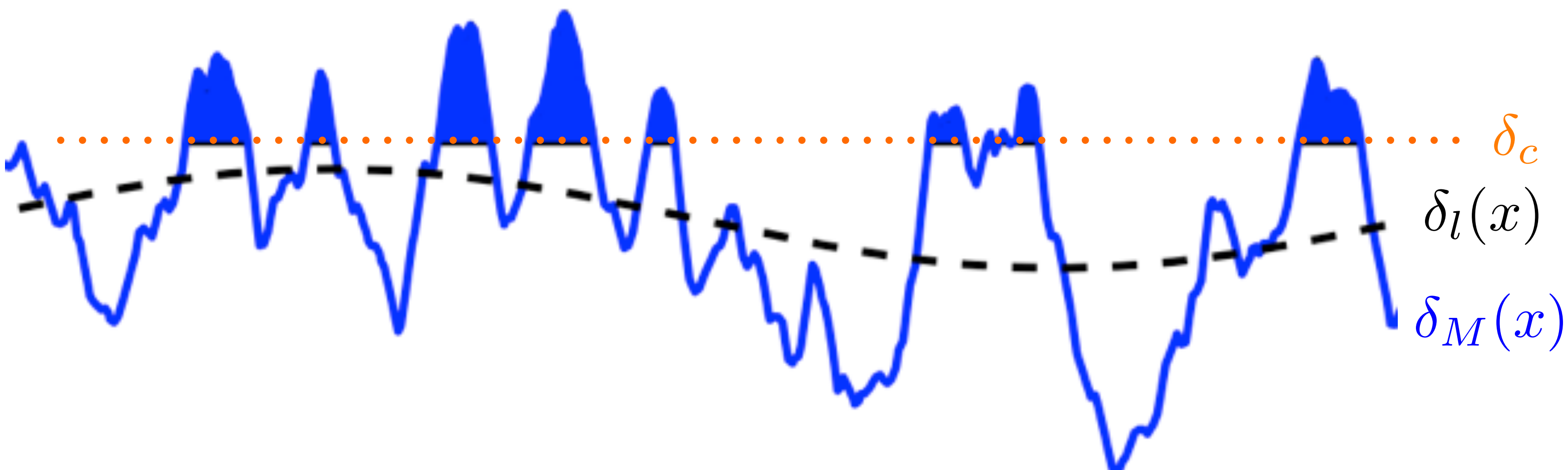
What happens in a g_{NL} or τ_{NL} cosmology?

Large-scale halo bias: Gaussian case

Barrier crossing model: (halos of mass $\geq M$) $\Leftrightarrow$ (regions where $\delta_M \geq \delta_c$)



How is halo abundance affected by the presence of a long-wavelength overdensity $\delta_l(x)$?



Local halo overdensity $\delta_h \approx b_0 \delta_l$ (where $b_0 = \dfrac{\partial \log n}{\partial \delta_l}$)

Define halo bias $b(k) = \dfrac{P_{mh}(k)}{P_{mm}(k)}$

$b(k) \rightarrow b_0$ (as $k \rightarrow 0$) ("weak" form of prediction)

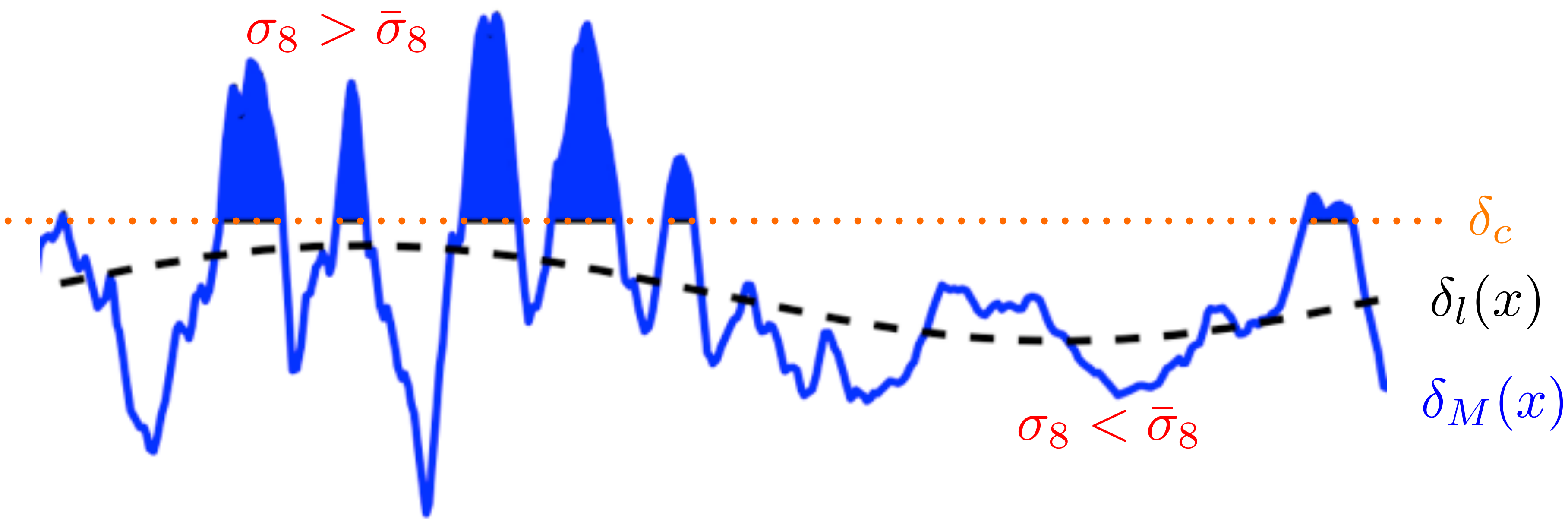
$b_0 = \dfrac{\partial \log n}{\partial \delta_l}$ ("strong" prediction)

Large-scale bias: f_{NL} cosmology

$$\Phi(\mathbf{x}) = \Phi_G(\mathbf{x}) + f_{NL}(\Phi_G(\mathbf{x})^2 - \langle \Phi_G^2 \rangle)$$

Write $\Phi_G = \Phi_l + \Phi_s$

$$\Phi = \Phi_l + \underbrace{f_{NL}(\Phi_l^2 + \Phi_s^2 - \langle \Phi^2 \rangle)}_{\text{irrelevant for large-scale bias}} + \underbrace{(1 + 2 f_{NL} \Phi_l)\Phi_s}_{\text{Modulates "local" } \sigma_8:\ \sigma_8(x) = \bar{\sigma}_8(1 + 2 f_{NL} \Phi_l(x))}$$



Local halo overdensity $\delta_h \approx b_0 \delta_l + f_{NL} b_1 \Phi_l$ $\left(b_0 = \frac{\partial \log n}{\partial \delta_l}, \quad b_1 = 2 \frac{\partial \log n}{\partial \log \sigma_8} \right)$

Halo bias $b(k) \to b_0 + f_{NL} \frac{b_1}{\alpha(k)}$ $(\text{as } k \to 0)$ ("weak" prediction)

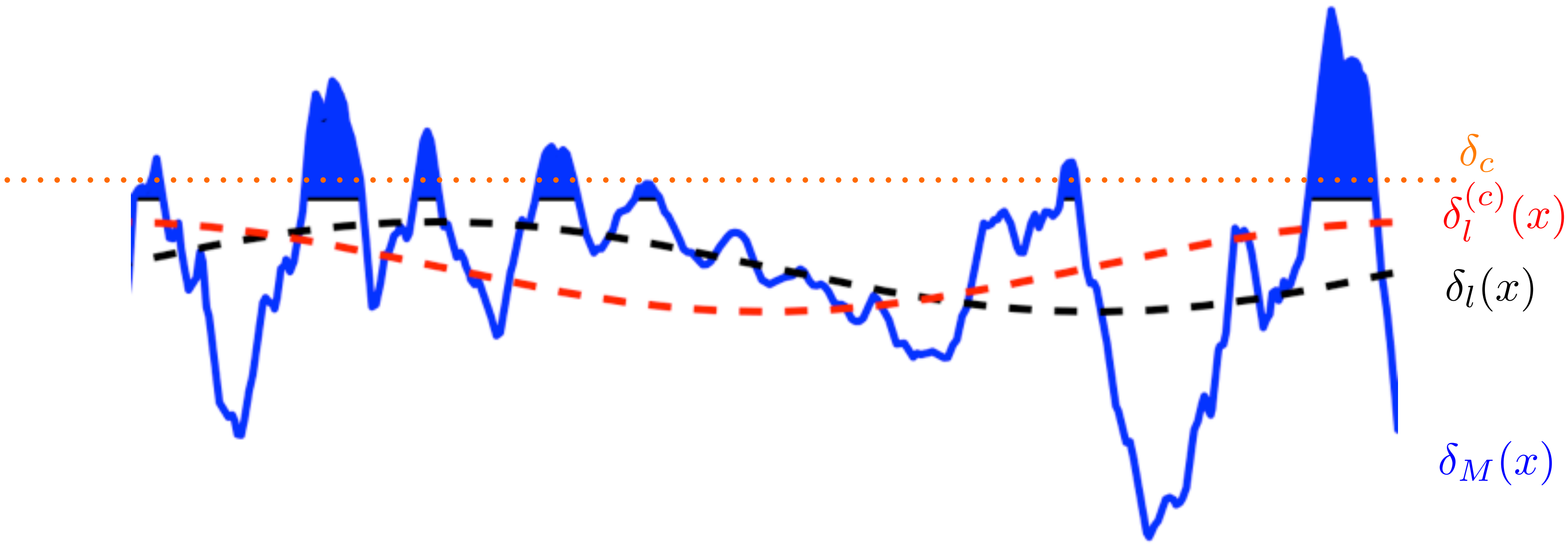
$b_1 = 2\delta_c(b_0 - 1)$ ("strong" prediction)

Large-scale bias: τ_{NL} cosmology

$$\Phi(\mathbf{x}) = \alpha\Phi_G^{(i)}(\mathbf{x}) + \beta\Phi_G^{(c)}(\mathbf{x}) + \frac{f_{NL}}{\beta^2}(\Phi_G^{(c)}(\mathbf{x})^2 - \langle\Phi_G^{(c)2}\rangle)$$

$$\Phi = \Phi_l + \underbrace{\frac{f_{NL}}{\beta^2}(\Phi_l^{(c)2} + \Phi_s^{(c)2} - \langle\Phi^2\rangle)}_{\text{irrelevant for large-scale bias}} + \underbrace{\left(\Phi_s + 2\frac{f_{NL}}{\beta^2}\Phi_l^{(c)}\Phi_s^{(c)}\right)}_{\text{Looks like spatially varying } \sigma_8:}$$

$$\sigma_8(x) = \bar\sigma_8\left(1 + 2\frac{f_{NL}}{\beta}\Phi_l^{(c)}\right)$$



Local halo overdensity $\delta_h \approx b_0\delta_l + \frac{f_{NL}}{\beta}b_1\Phi_l^{(c)}$ $\left(b_0 = \frac{\partial \log n}{\partial \delta_l}, \quad b_1 = 2\frac{\partial \log n}{\partial \log \sigma_8}\right)$

Gaussian and non-Gaussian bias terms are not 100% correlated

Stochastic halo bias

f_{NL} cosmology

Local halo overdensity $\delta_h \approx b_0 \delta_l + f_{NL} b_1 \Phi_l$

Halo bias $b(k) \to b_0 + f_{NL} \dfrac{b_1}{\alpha(k)}$

$$P_{mh}(k) = b(k) P_{mm}(k)$$

$$P_{hh}(k) = b(k)^2 P_{mm}(k) + \frac{1}{n}$$

τ_{NL} cosmology

Local halo overdensity $\delta_h \approx b_0 \delta_l + \dfrac{f_{NL}}{\beta} b_1 \Phi_l^{(c)}$

Halo bias $b(k) \to b_0 + f_{NL} \dfrac{b_1}{\alpha(k)}$

$$P_{mh}(k) = b(k) P_{mm}(k)$$

$$P_{hh}(k) = b(k)^2 P_{mm}(k) + \frac{\alpha^2 f_{NL}^2}{\beta^2} \frac{b_1^2 P_{mm}(k)}{\alpha(k)^2} + \frac{1}{n}$$

Halos and matter not 100% correlated
("stochastic bias")

Different halo samples not 100% correlated

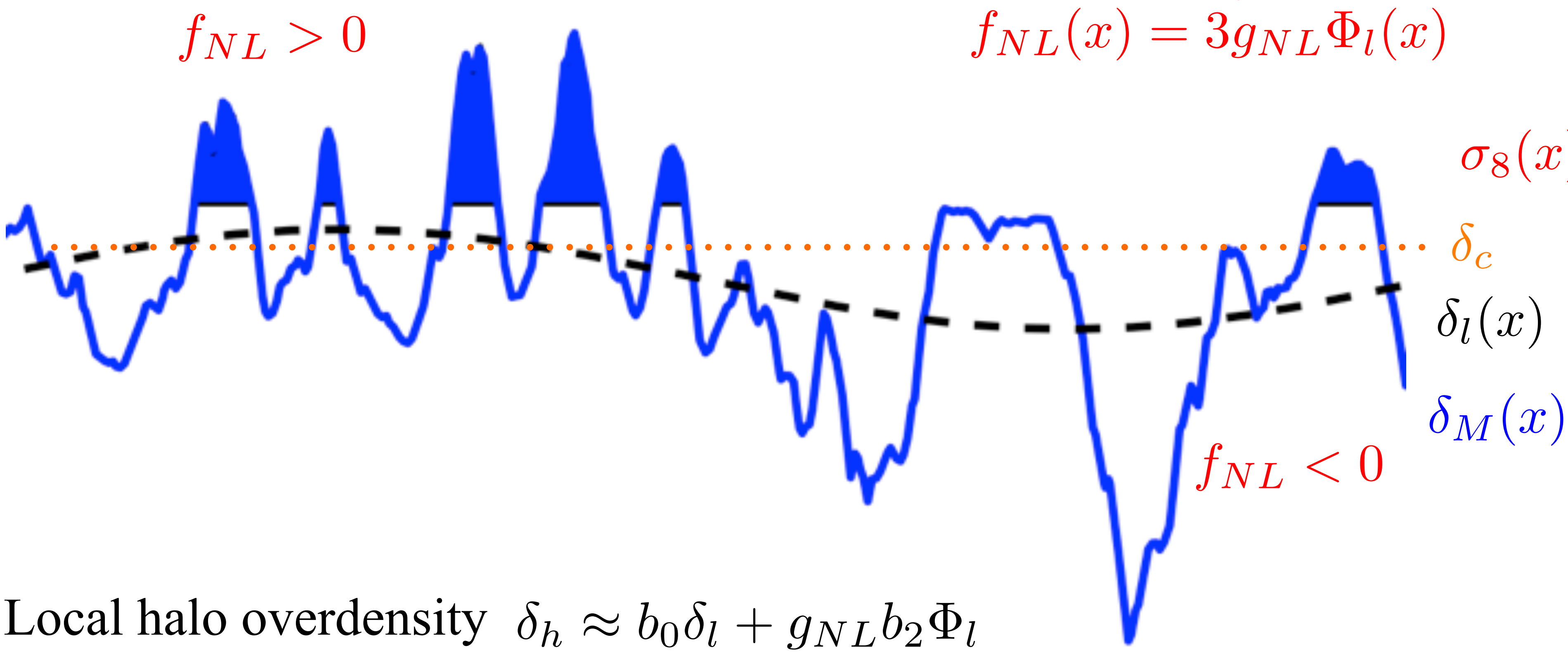
Large-scale bias: g_{NL} cosmology

$$\Phi(\mathbf{x}) = \Phi_G(\mathbf{x}) + g_{NL}(\Phi_G(\mathbf{x})^3 - 3\langle\Phi_G^2\rangle\Phi_G(\mathbf{x}))$$

$$\Phi = \Phi_l + \Phi_s + \underbrace{g_{NL}(\Phi_l^3 + \Phi_s^3 - 3\langle\Phi_l^2\rangle\Phi_l - 3\langle\Phi_s^2\rangle\Phi_s)}_{\text{irrelevant for large-scale bias}} + \underbrace{3g_{NL}\Phi_l(\Phi_s^2 - \langle\Phi_s^2\rangle)} + \underbrace{3g_{NL}(\Phi_l^2 - \langle\Phi_l^2\rangle)\Phi_s}$$

Looks like spatially varying f_{NL}: $f_{NL}(x) = 3g_{NL}\Phi_l(x)$

Looks like spatially varying σ_8: $\sigma_8(x) = 3g_{NL}(\Phi_l(x)^2 - \langle\Phi_l^2\rangle)\bar{\sigma}_8$



Local halo overdensity $\delta_h \approx b_0\delta_l + g_{NL}b_2\Phi_l$

Halo bias $b(k) \to b_0 + g_{NL}\dfrac{b_2}{\alpha(k)}$ (as $k \to 0$) ("weak" prediction)

$$b_2 = 3\left(\frac{\partial \log n}{\partial f_{NL}}\right) \qquad \text{(stronger)}$$

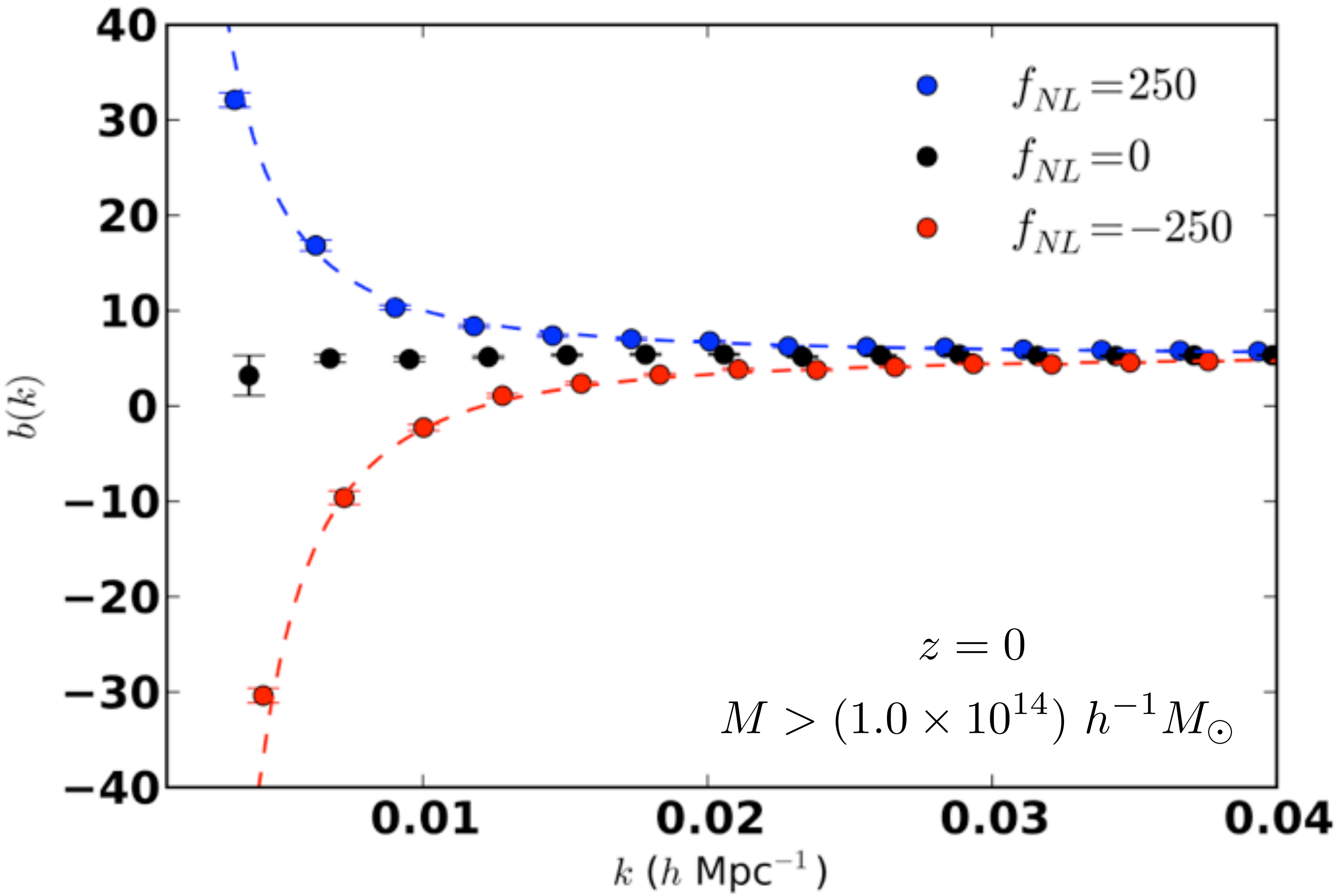
$$= \frac{\kappa_3(M)}{2}H_3\left(\frac{\delta_c}{\sigma(M)}\right) - \frac{d\kappa_3/dM}{d\sigma/dM}\frac{\sigma(M)^2}{2\delta_c}H_2\left(\frac{\delta_c}{\sigma(M)}\right) \qquad \text{(strongest)}$$

Halo bias: f_{NL} simulations

Prediction from barrier crossing model:

$$b(k) \to b_0 + f_{NL} \frac{b_1}{\alpha(k)} \qquad\qquad b_1 = 2\delta_c(b_0 - 1)$$

Agreement with simulations: perfect!



Stochastic halo bias: τ_{NL} simulations

Define stochasticity $r(k)$ by:

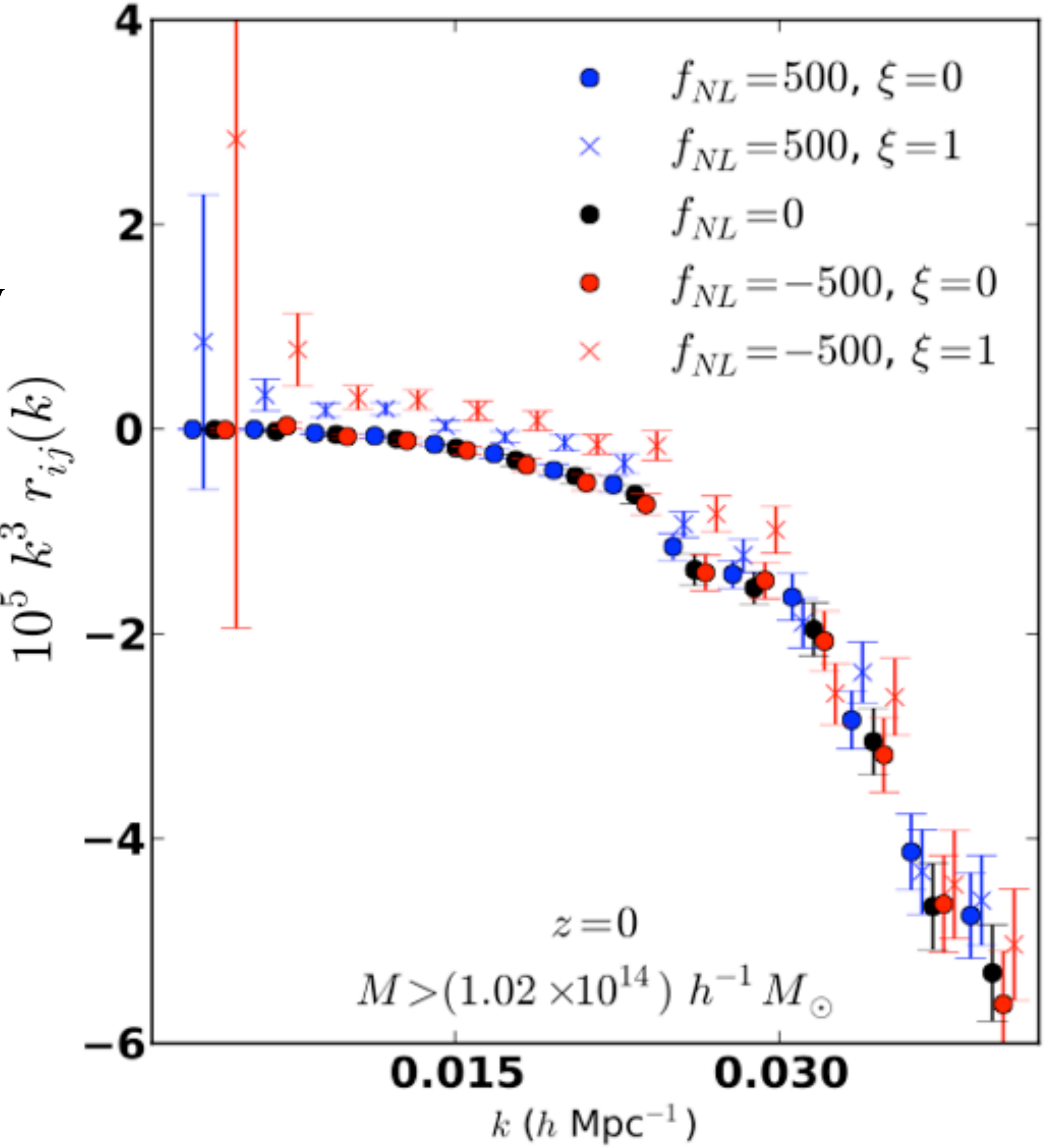
$$r(k) = \frac{P_{hh}(k) - 1/n}{P_{mm}(k)} - \left(\frac{P_{mh}(k)}{P_{mm}(k)}\right)^2$$

Prediction from barrier crossing model:

$$r(k) = \frac{\alpha^2 f_{NL}^2}{\beta^2} \frac{b_1^2}{\alpha(k)^2} \quad \text{in } \tau_{NL} \text{ cosmology}$$

Results from simulations:

- significant stochasticity in Gaussian cosmology
- no change to stochasticity in f_{NL} cosmology
- boosted stochasticity in τ_{NL} cosmology



Stochastic halo bias: Gaussian simulations

Can we use the halo model to explain the Gaussian stochasticity seen in simulations?

$$r(k) = \frac{P_{hh}(k) - 1/n}{P_{mm}(k)} - \left(\frac{P_{mh}(k)}{P_{mm}(k)}\right)^2$$

Leading halo model contribution:

$$r(k) \approx -\left(\frac{2}{P_{mm}(k)}\right)\frac{f}{n}$$

f ⟵ fraction of total mass in halos

n ⟵ halo number density

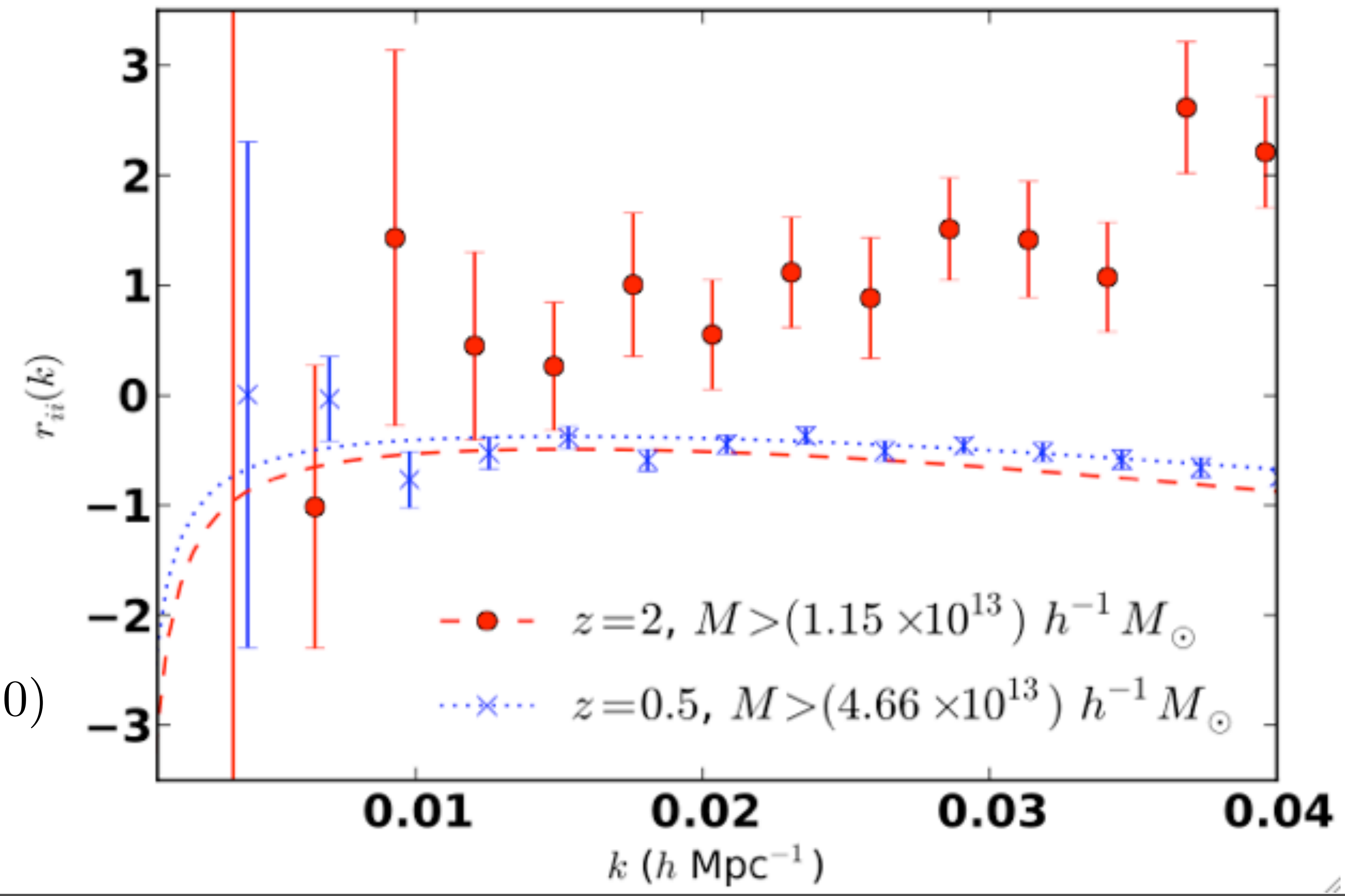
Does this agree with simulations?
Answer: sometimes

Halo model does not seem to give complete description of large-scale stochasticity in a Gaussian cosmology

Empirical observation:

$$r(k) \to \frac{r_0}{P_{mm}(k)} \qquad (\text{as } k \to 0)$$

$$r_0 = \,?$$



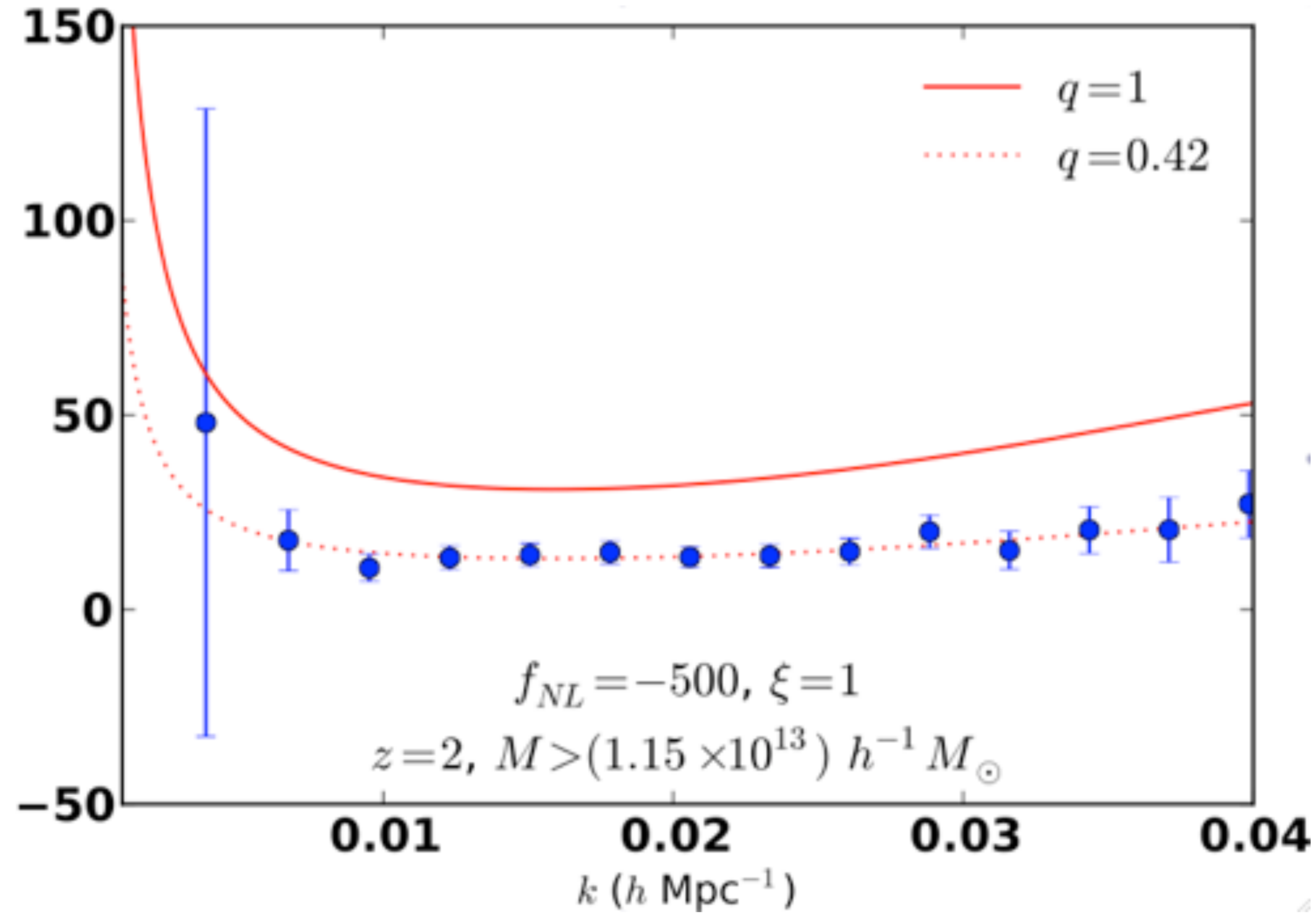
Stochastic halo bias: τ_{NL} simulations

Interpret barrier crossing result as prediction for $r_{NG}(k) - r_G(k)$, i.e. non-Gaussian contribution

$$r_{NG}(k) - r_G(k) = \frac{\alpha^2 f_{NL}^2}{\beta^2} \frac{b_1^2}{\alpha(k)^2}$$

Comparison with simulations: shape is correct, amplitude is not!

$$r_{NG}(k) - r_G(k) = q \left(\frac{\alpha^2 f_{NL}^2}{\beta^2} \frac{b_1^2}{\alpha(k)^2} \right)$$



	Mass range ($h^{-1}M_\odot$)	$f_{NL}=500$	$f_{NL}=250$	$f_{NL}=-250$	$f_{NL}=-500$
$z=2$	$M > 1.15\times10^{13}$	0.98 ± 0.07	0.88 ± 0.08	0.62 ± 0.06	0.42 ± 0.03
$z=1$	$1.15\times10^{13} < M < 2.32\times10^{13}$	0.79 ± 0.09	0.83 ± 0.12	0.67 ± 0.09	0.46 ± 0.04
	$M > 2.32\times10^{13}$	0.83 ± 0.07	0.70 ± 0.08	0.66 ± 0.07	0.51 ± 0.04
$z=0.5$	$1.15\times10^{13} < M < 2.32\times10^{13}$	1.01 ± 0.18	0.92 ± 0.29	0.45 ± 0.19	0.57 ± 0.10
	$2.32\times10^{13} < M < 4.66\times10^{13}$	0.80 ± 0.15	0.58 ± 0.22	0.73 ± 0.19	0.48 ± 0.08
	$M > 4.66\times10^{13}$	0.81 ± 0.09	0.79 ± 0.12	0.80 ± 0.10	0.51 ± 0.05
$z=0$	$1.15\times10^{13} < M < 2.32\times10^{13}$	1.37 ± 0.80	1.06 ± 1.12	1.00 ± 1.41	0.90 ± 0.51
	$2.32\times10^{13} < M < 4.66\times10^{13}$	1.35 ± 0.44	1.57 ± 0.77	0.82 ± 0.59	0.58 ± 0.25
	$4.66\times10^{13} < M < 1.02\times10^{14}$	0.71 ± 0.26	0.90 ± 0.49	1.12 ± 0.41	0.63 ± 0.17
	$M > 1.02\times10^{14}$	0.79 ± 0.13	0.93 ± 0.21	0.73 ± 0.15	0.53 ± 0.07

Table 3: Values of the q-parameter, defined in Eq. (35), obtained from N-body simulations for various values of f_{NL}, redshift, and mass bin. (We take $\xi=1$ throughout)

Halo bias: g_{NL} simulations

Predictions from barrier crossing model:

$$b(k) \to b_0 + g_{NL} \frac{b_2}{\alpha(k)}$$

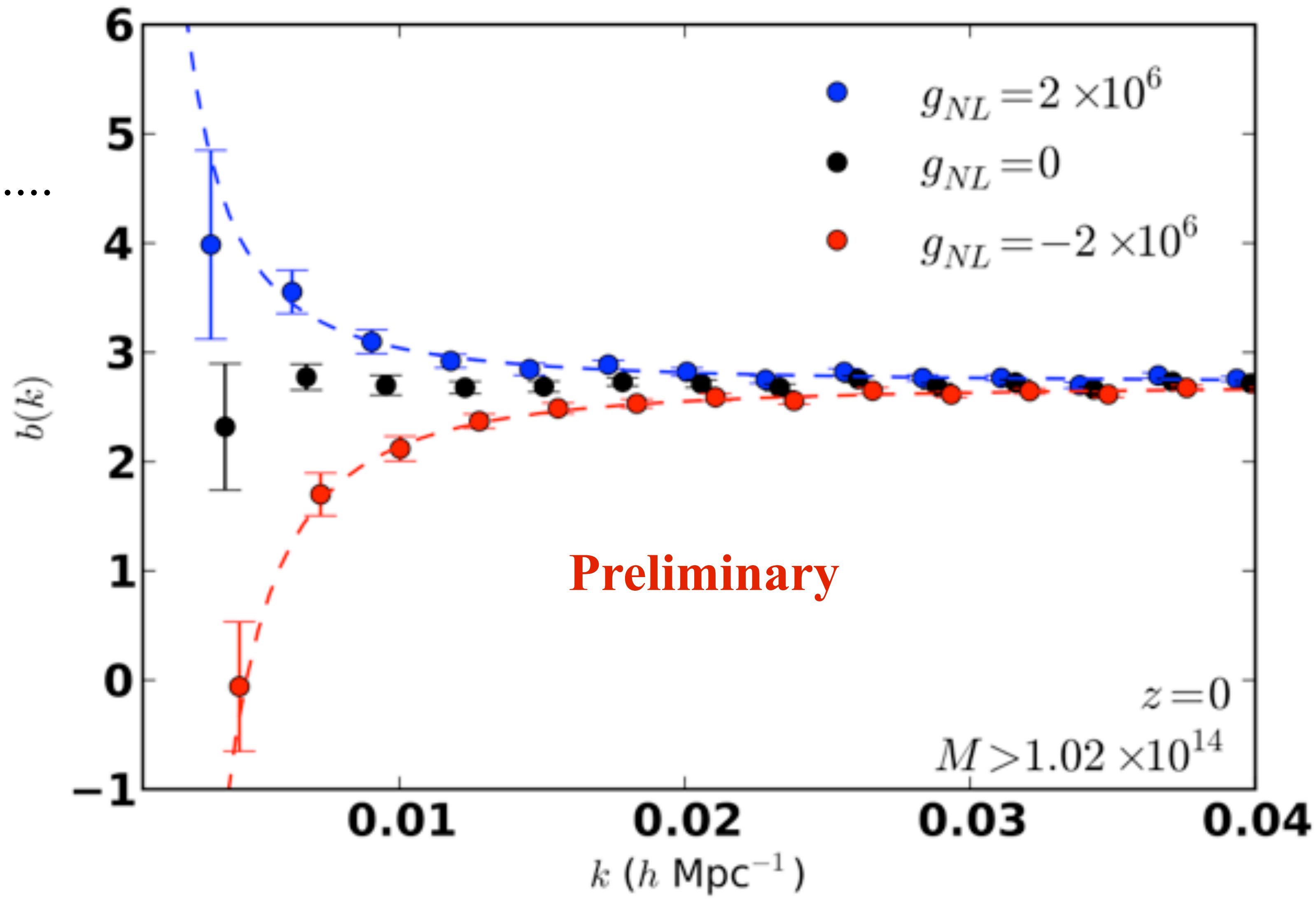
$$\begin{aligned} b_2 &= 3\left(\frac{\partial \log n}{\partial f_{NL}}\right) \\ &= \frac{\kappa_3(M)}{2} H_3\left(\frac{\delta_c}{\sigma(M)}\right) - \frac{d\kappa_3/dM}{d\sigma/dM} \frac{\sigma(M)^2}{2\delta_c} H_2\left(\frac{\delta_c}{\sigma(M)}\right) \end{aligned}$$

Let's test this prediction in several steps.....

First: is $b(k) = b_0 + g_{NL} \frac{b_2}{\alpha(k)}$ a good fit, treating b_0, b_2 as free parameters?

Answer: yes!

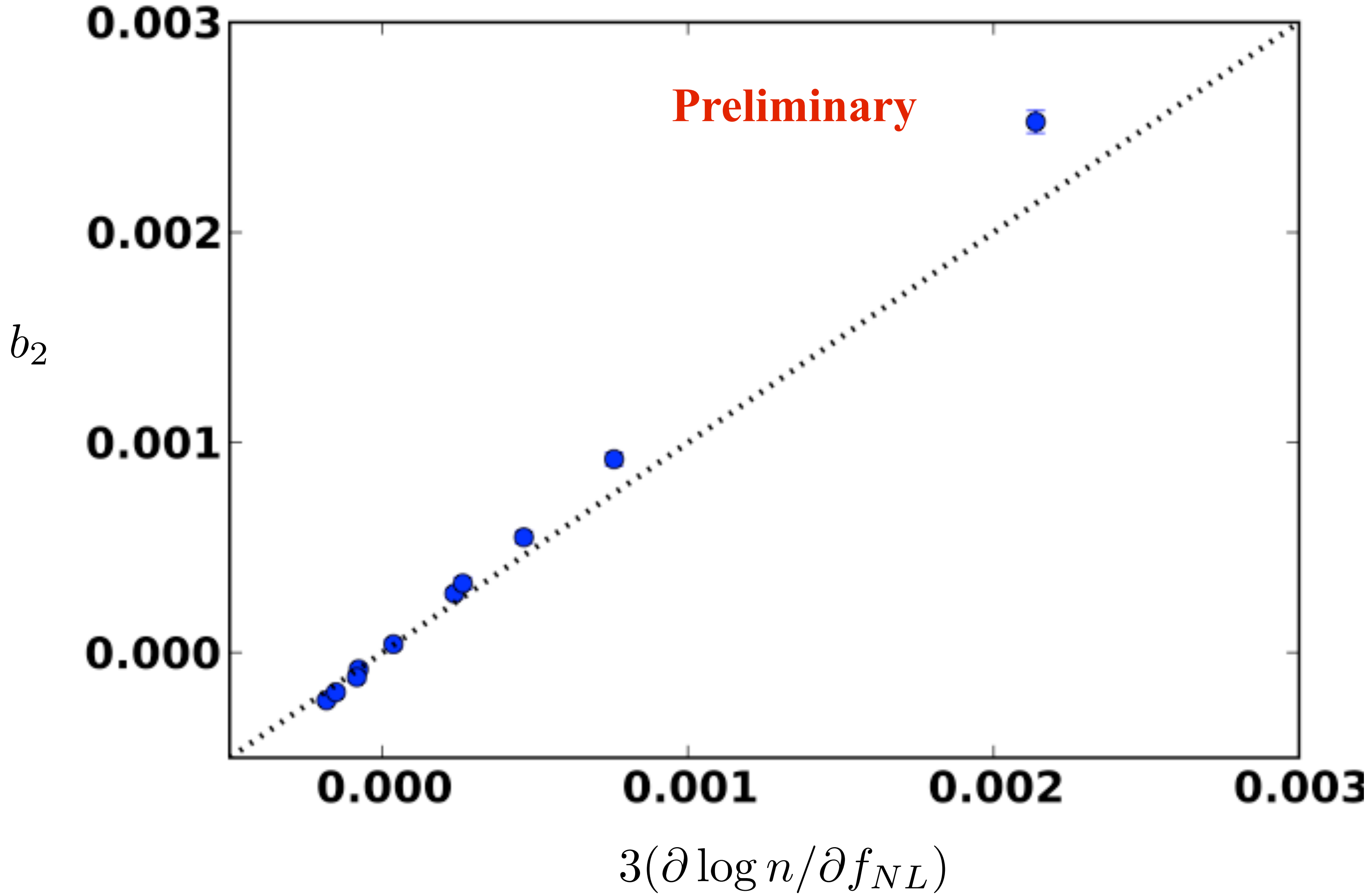
(see also Desjacques & Seljak 2010)



Halo bias: g_{NL} simulations

Second: general relation between g_{NL} dependence of bias and f_{NL} dependence of mass function

$$b_2 = 3\left(\frac{\partial \log n}{\partial f_{NL}}\right)$$



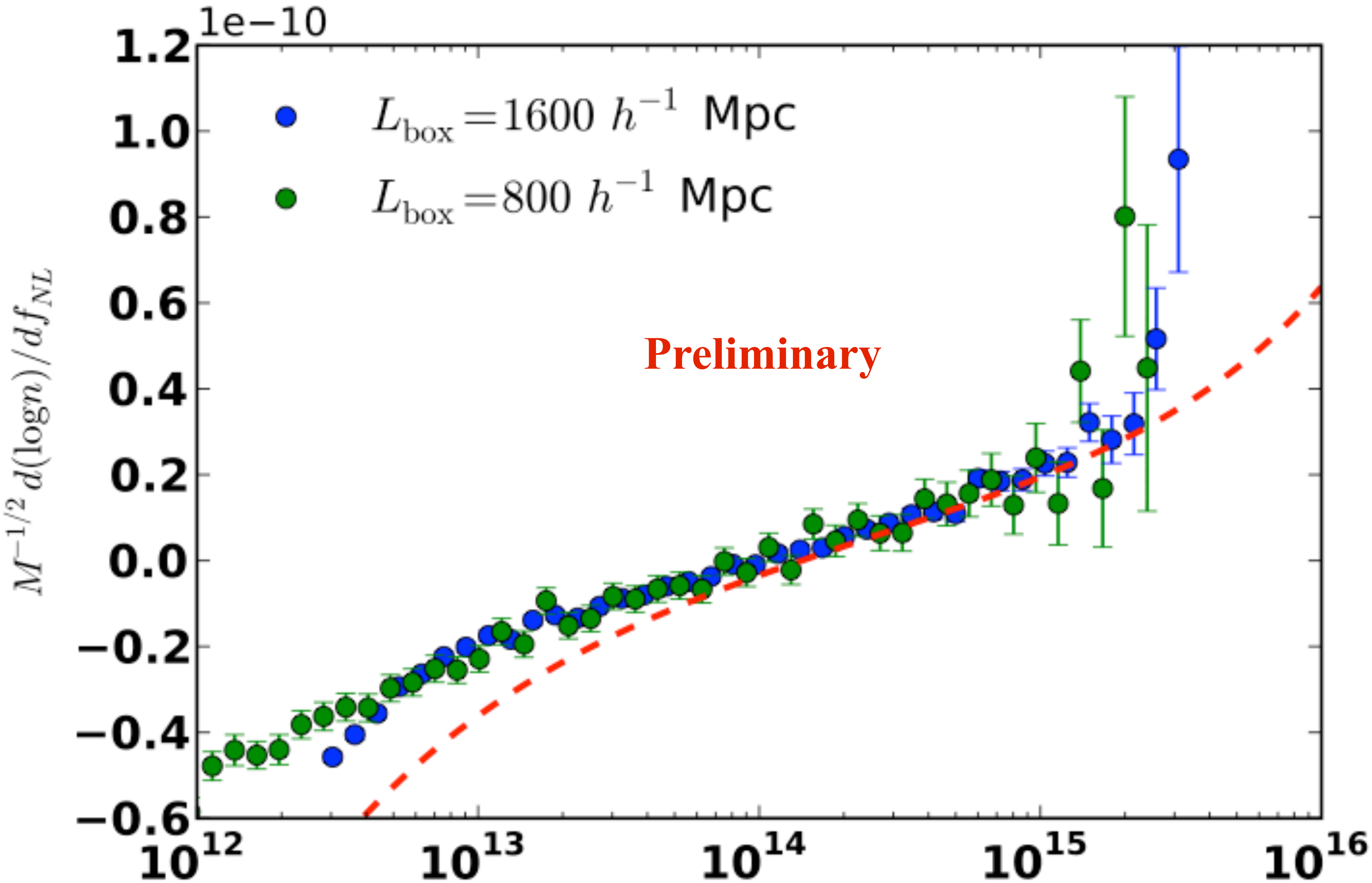
Simulations disagree by ~20%

very puzzling since derivation of this general relation seems to make few assumptions!!

Halo bias: g_{NL} simulations

Third: comparison between $\dfrac{\partial \log n}{\partial f_{NL}}$ and barrier crossing prediction

$$\frac{\kappa_3(M)}{6} H_3\left(\frac{\delta_c}{\sigma(M)}\right) - \frac{d\kappa_3/dM}{d\sigma/dM}\frac{\sigma(M)^2}{6\delta_c} H_2\left(\frac{\delta_c}{\sigma(M)}\right)$$



Summary

- Mass function: Victory! Log-Edgeworth form works well everywhere

- Clustering in f_{NL} cosmology: barrier crossing model predicts non-stochastic bias of the form

$$b(k) \to b_0 + f_{NL} \frac{b_1}{\alpha(k)} \qquad\qquad b_1 = 2\delta_c(b_0 - 1)$$

  N-body simulations agree!

- Clustering in τ_{NL} cosmology: predict stochastic bias of the form

$$r(k) = \frac{\alpha^2 f_{NL}^2}{\beta^2} \frac{b_1^2}{\alpha(k)^2}$$

In N-body simulations, find qualitative agreement: shape is correct, but find correction to the amplitude that we don't currently understand semianalytically. Stochasticity not completely understood even for Gaussian initial conditions!

- Clustering in g_{NL} cosmology: predict bias of the form

$$b(k) \to b_0 + g_{NL} \frac{b_2}{\alpha(k)} \qquad\qquad b_2 = 3\left(\frac{\partial \log n}{\partial f_{NL}}\right)$$

In N-body simulations, find small correction ($3 \to 3.6$); can we understand this semianalytically?

Halo stochasticity: g_{NL} simulations