Non-Linear Neutrino Effects in Large Scale Structure

DEREK INMAN

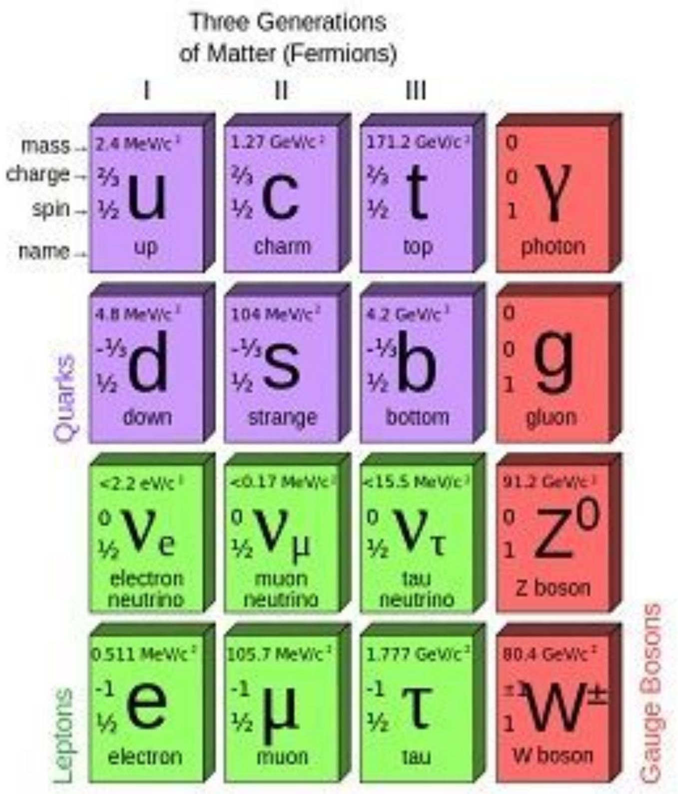
CANADIAN INSTITUTE FOR THEORETICAL ASTROPHYSICS

COLLABORATORS: UE-LI PEN, JD EMBERSON, HAO-RAN YU, TONG-JIE ZHANG, JOACHIM HARNOIS-DÉRAPS, HONG-MING ZHU (AND MORE!)

Outline

- Introduction to Neutrinos
- Simulations with Neutrinos
- Non-Linear Effects
  1. Neutrino Power Spectrum
  2. The Relative Velocity Effect
  3. Differential Neutrino Condensation
- Conclusion

Neutrino Mass – from below



Credit: CERN

- Oscillations:

$$\delta m^2 = 7.4 \times 10^{-5}\ \mathrm{eV}^2$$

$$\Delta m^2 = 2.5 \times 10^{-3}\ \mathrm{eV}^2$$

(Capozzi et al. 2016)

- Two allowed hierarchies

$$\sum_{i=1}^{3} m_{\nu_i} \gtrsim 0.06\ \mathrm{eV}\ \text{[Normal]}$$

$$\sum_{i=1}^{3} m_{\nu_i} \gtrsim 0.10\ \mathrm{eV}\ \text{[Inverted]}$$

Neutrino Mass – from above

- Ground based detectors
  - Troitsk: m_ν<2.2 eV (95%) (Aseev et al. 2011)
  - Future
    - KATRIN: m_ν~0.2 eV
    - PTOLEMY: m_ν~0.15 eV
- Cosmology
  - Planck (arXiv:1502.01589)

$$\sum_{i=1}^{3} m_{\nu_i} < 0.17 \text{ eV}$$

  - CMB S4: 2+σ detection (Science Book) in 2020s

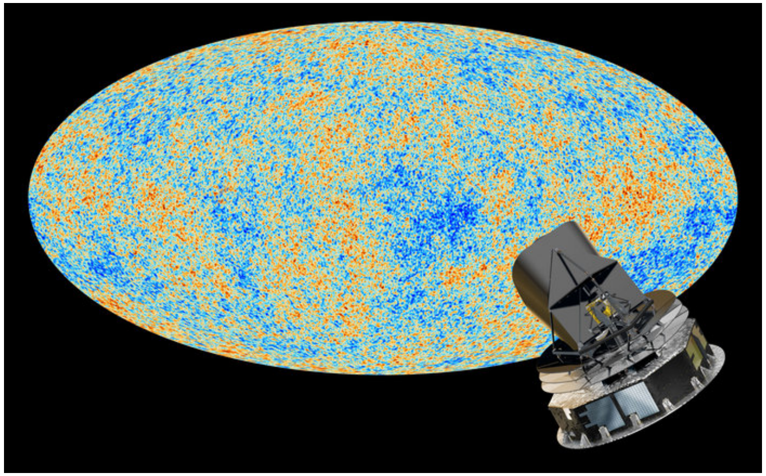


Image: NSA and the Planck Collaboration

Neutrino Mass – from Wikipedia

https://en.wikipedia.org/wiki/Neutrino

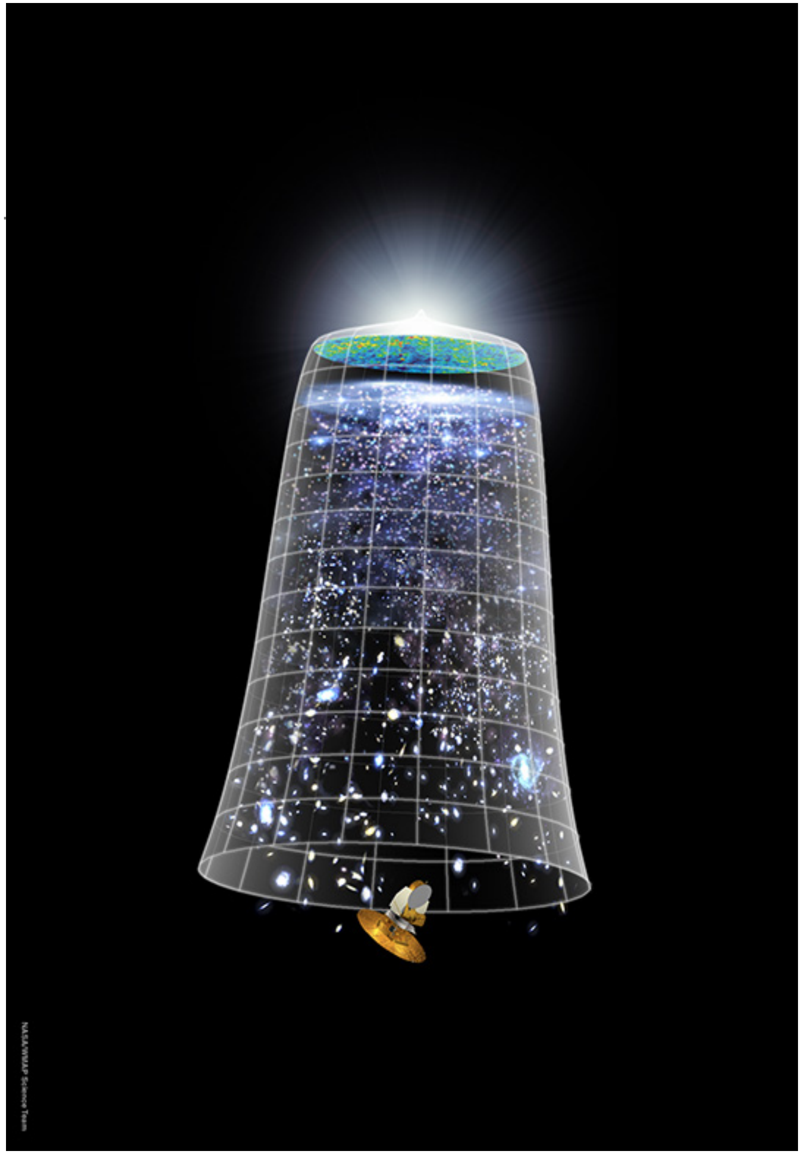
Discovered	ν_e: Clyde Cowan, Frederick Reines (1956) ν_μ: Leon Lederman, Melvin Schwartz and Jack Steinberger (1962) ν_τ: DONUT collaboration (2000)
Types	3 – electron neutrino, muon neutrino and tau neutrino
Mass	0.320 ± 0.081 eV/c^2 (sum of 3 flavors)[1][2][3]
Electric charge	0 *e*
Spin	$\frac{1}{2}$
Weak hypercharge	−1
B* − *L	−1
X	−3

Mass	0.320 ± 0.081 eV/c^2 (sum of 3 flavors)[1][2][3]

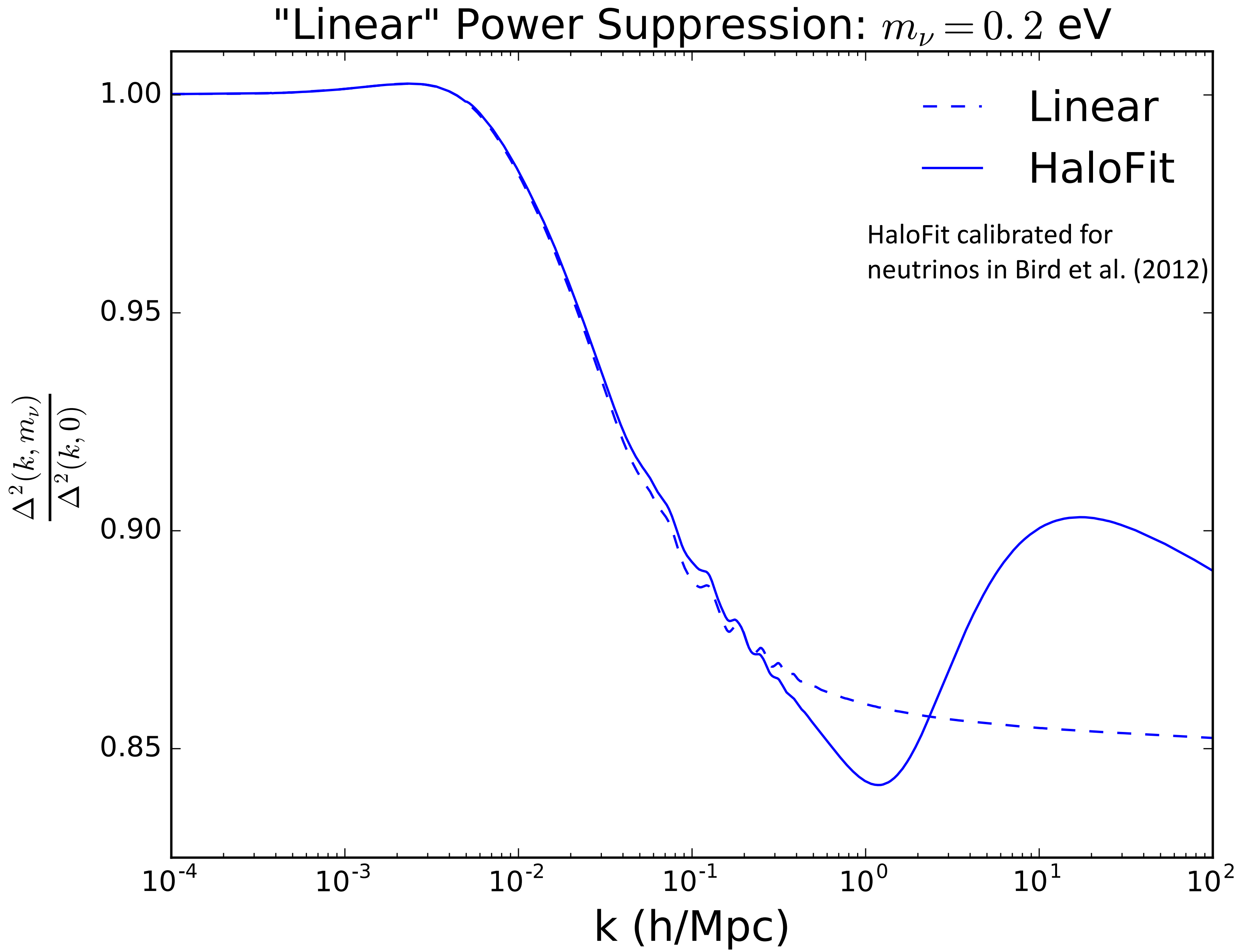
From Battye & Moss (2014), as well as similar papers, dealing with reconciling CMB and local measurements.

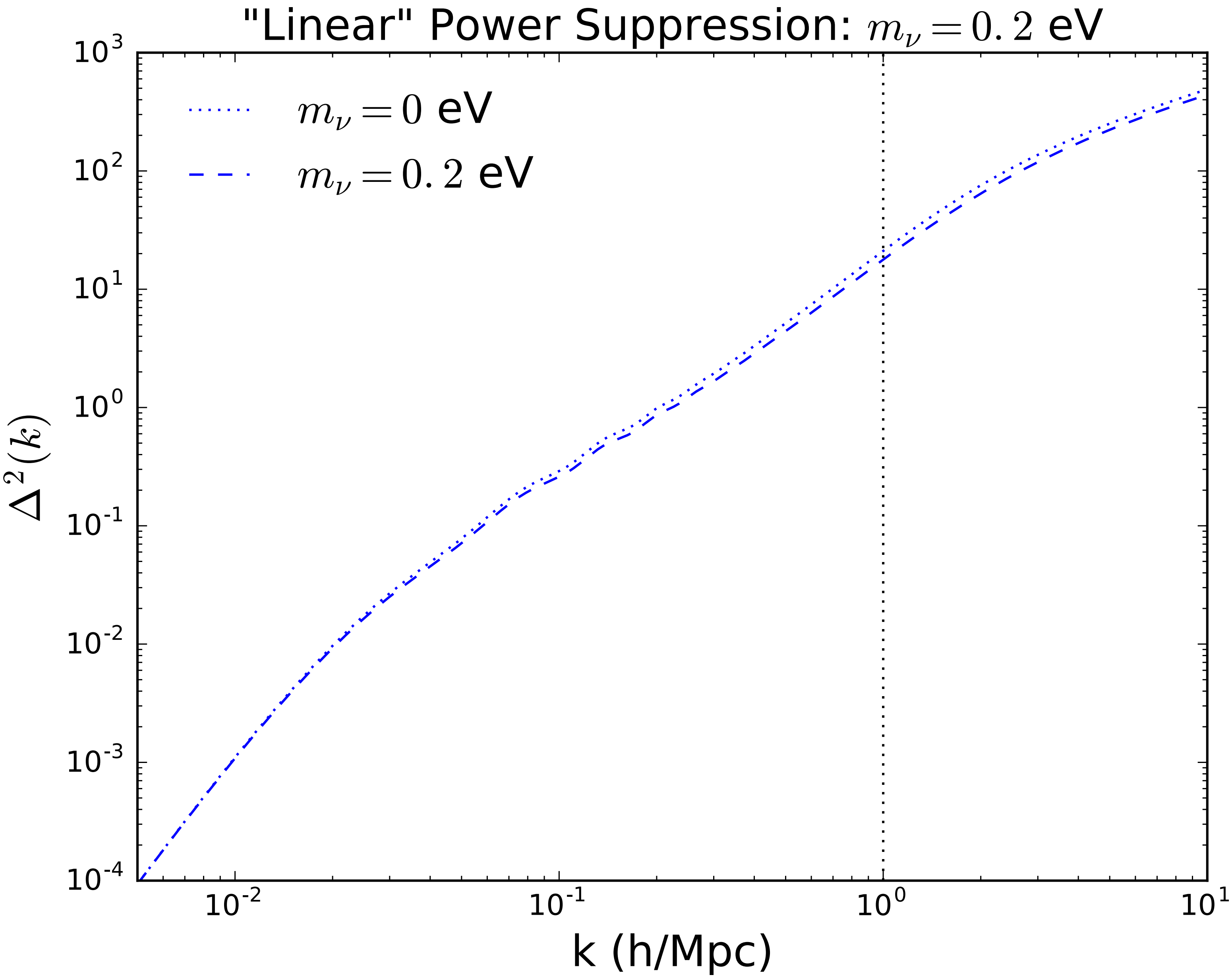
Neutrinos:

- Decouple while relativistic (around 1 MeV)
- Slowly cool and become non-relativistic (z ~ 100s)
- Velocities still large today (1000s of km/s)



Credit: NASA / WMAP Science Team


"Linear" Power Suppression: $m_\nu = 0.2$ eV
Linear
HaloFit
HaloFit calibrated for
neutrinos in Bird et al. (2012)
$\frac{\Delta^2(k, m_\nu)}{\Delta^2(k,0)}$
1.00
0.95
0.90
0.85
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
k (h/Mpc)


"Linear" Power Suppression: $m_\nu = 0.2$ eV
$m_\nu = 0$ eV
$m_\nu = 0.2$ eV
$\Delta^2(k)$
k (h/Mpc)
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}

Neutrino Perturbations

- At late times, neutrinos follow the Vlasov equation:

$$f_s + \vec{v} \cdot \vec{\nabla}_x f - a^2 \vec{\nabla}_x \phi \cdot \vec{\nabla}_v f = 0$$

- which is linearized via:

$$\vec{\nabla}_v f \to \vec{\nabla}_v f^0(v)$$

$$f^0(v) = (e^{\beta v} + 1)^{-1} \ ; \ \beta = m_\nu / T_\nu$$

which can be integrated over velocity:

$$\delta_\nu(s,k) = \int_{-\infty}^{s} ds' a^2(-k^2\phi)(s-s') \frac{\int u^2 (e^u+1)^{-1} j_0(ku(s-s')/\beta) du}{\int u^2 (e^u+1)^{-1} du}$$

"Non-Linear" Neutrinos

- Non-linear: "neutrino^2"

$$\text{Acceleration}: \; -a^2\vec{\nabla}_x\phi\vec{\nabla}_v f$$

$$\text{Poisson}: \; \nabla_x^2\phi = \frac{3}{2}H_0^2\Omega_m\frac{\delta_m}{a}$$

$$\text{Matter}: \; \delta_m = f_c\delta_c + f_\nu\delta_\nu$$

$$\text{But:} \quad f_\nu = \frac{\Omega_\nu}{\Omega_m} \ll 1$$

- Non-linear: "CDM is non-linear"
  - Captured by linear response calculation
- Non-linear: "not first order" (in cosmological perturbations)
- Require simulations containing CDM and neutrinos!

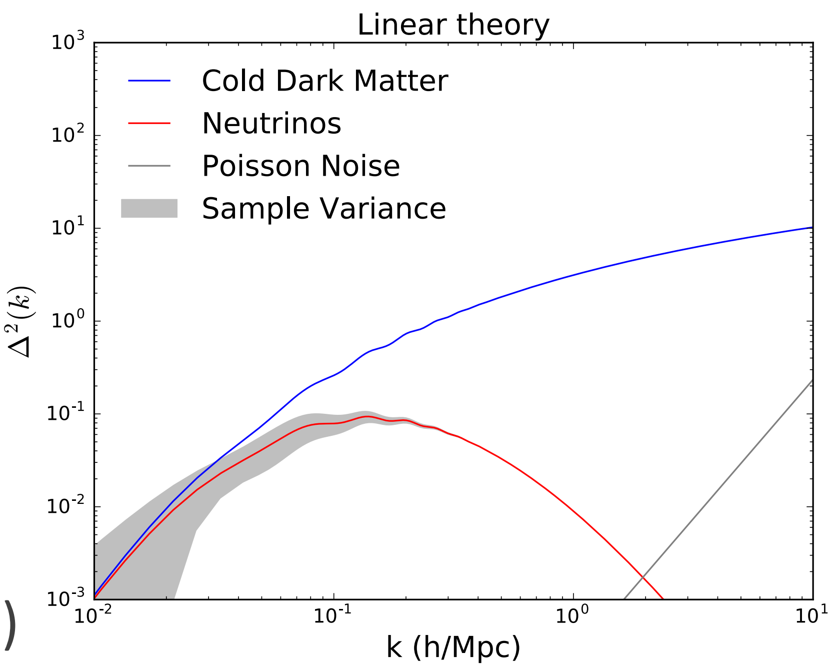
Simulation Approaches

- Not feasible to solve full Vlasov equation: N^6
  - Linear Response to CDM
    - Brandbyge & Hannestad (2009)
    - Ali-Haimoud & Bird (2012)
  - Fluid Approach
    - Hannestad, Haugbølle and Schultz (2012)
  - N-body particles
    - Brandbyge, Hannestad, Haugbølle, and Thomsen (2008)
    - Viel, Haehnelt, and Springel (2010)
    - Bird, Viel, Haenhelt (2012)
    - & many more!

Efficiency	Accuracy
Low memory usage, little additional CPU time	Get power suppression correct
Neutrinos use 90% of memory, takes 50% longer	Non-linear effects: clustering, bulk motions, etc.

N-body Neutrinos

- Neutrinos: additional particles
  - Particle mass proportional to fraction of energy density
  - Fermi-Dirac thermal velocity
    - Poisson noise
    - Late start
- Scales:
  - Poisson noise (small volume, many particles, heavy neutrino)
  - Sample variance (large volume)



Noise estimates based on
Neutrino Mass: 0.4 eV
Volume: $(1\ \mathrm{Gpc})^3$
Number of particles: 3000^3

Our Simulations

- CUBEP^{3}M code: particle mesh, pp-force
- Four simulations with neutrino mass: 0.4, 0.2, 0.1, 0.05 eV
  - Number of particles:
    - 1536^3 cold dark matter
    - 3072^3 neutrinos
  - Volume: $(500\ \mathrm{Mpc/h})^3$
  - Starting redshift:
    - CDM starts at z=100
    - Neutrinos injected at z=10 (5 for 0.05 eV simulation)
- More details: arXiv 1503.07480

Simulating the Neutrino Sky

- The TianNu Simulation was run on Tianhe-2 (#1 at the time)
- World's largest N-body simulation (by particle number)
  - 13824 nodes (86% of machine)
  - 6912^3 cold dark matter, 13824^3 neutrinos, ~3 trillion total
  - Volume: $(1.2\ \text{Gpc/h})^3$
  - ~27 million halos
- Corresponding simulation without neutrinos (TianZero)
- More information: arXiv: 1609.08968
  http://cita.utoronto.ca/~haoran/thnu/thnu.html

Simulations at Extreme Scale

- Optimizations:
  - FFT decomposition
  - Nested OpenMP
  - Data Compression
- Some stories:
  - Logging in!
  - MPI!
  - File IO!
  - Temperature!
  - Moving data!

Webpage with Movie:

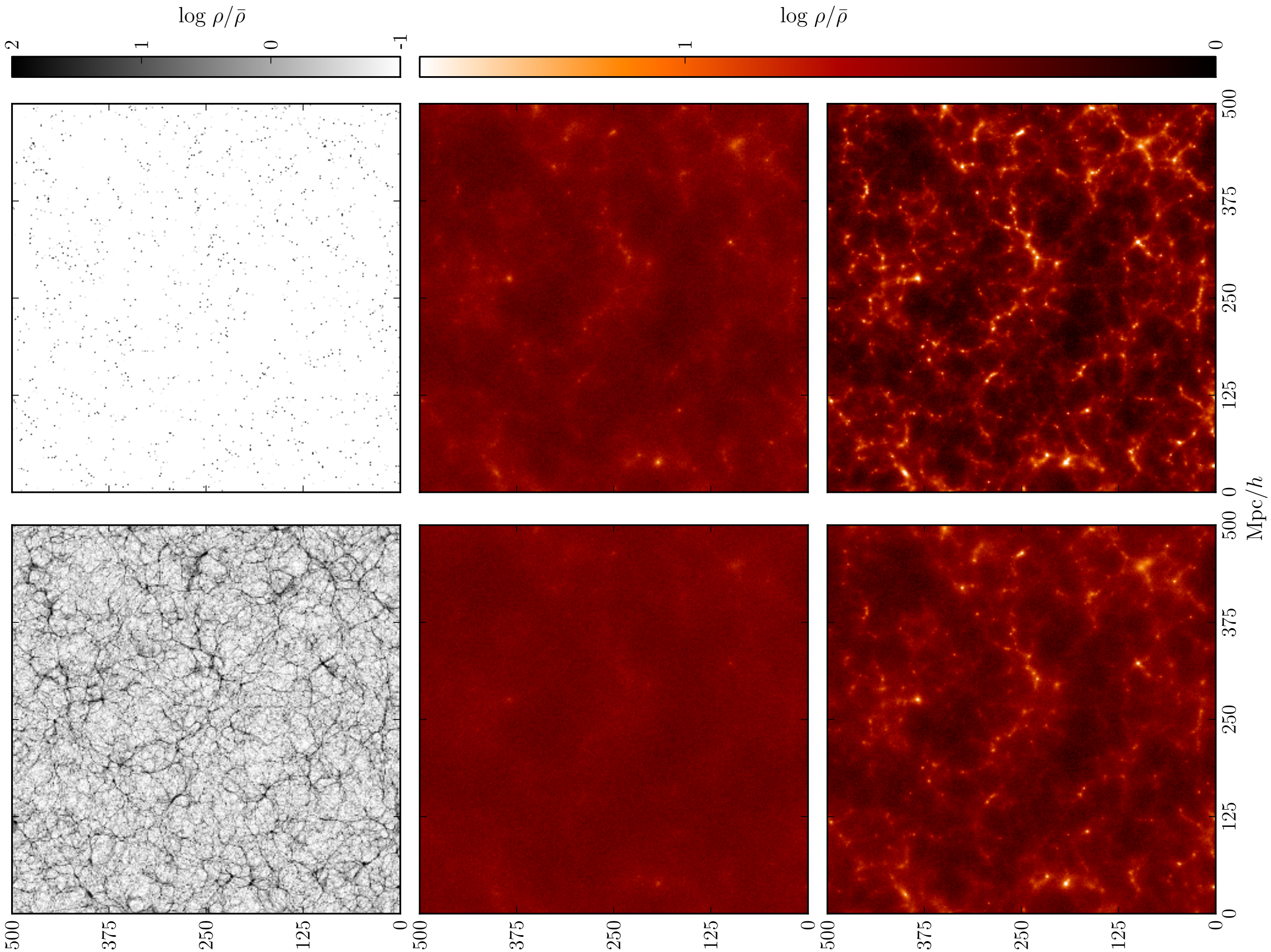
http://cita.utoronto.ca/~haoran/thnu/thnu.html

Non-Linearity #1:
Neutrino Power Spectrum

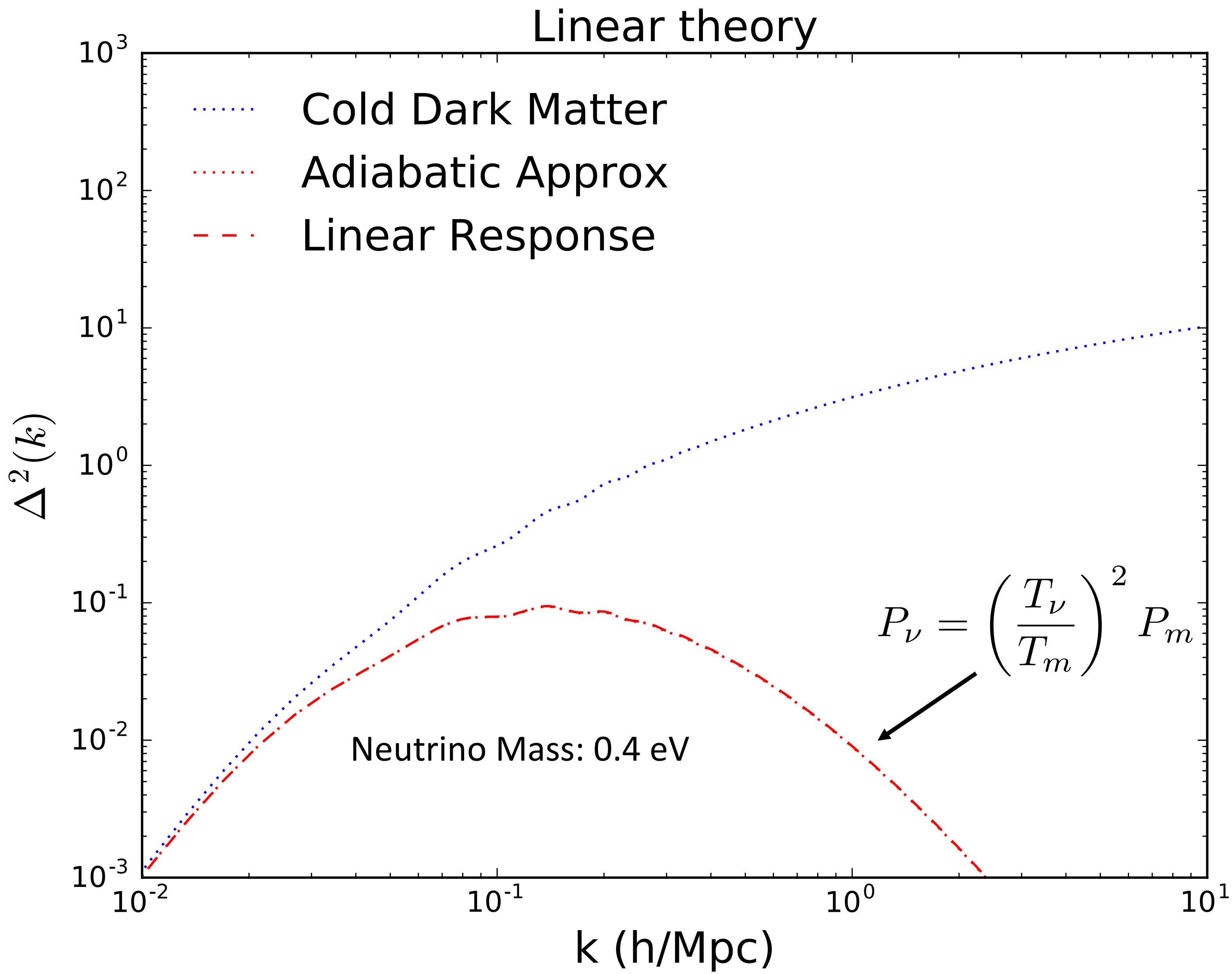
Density Fields

Halos

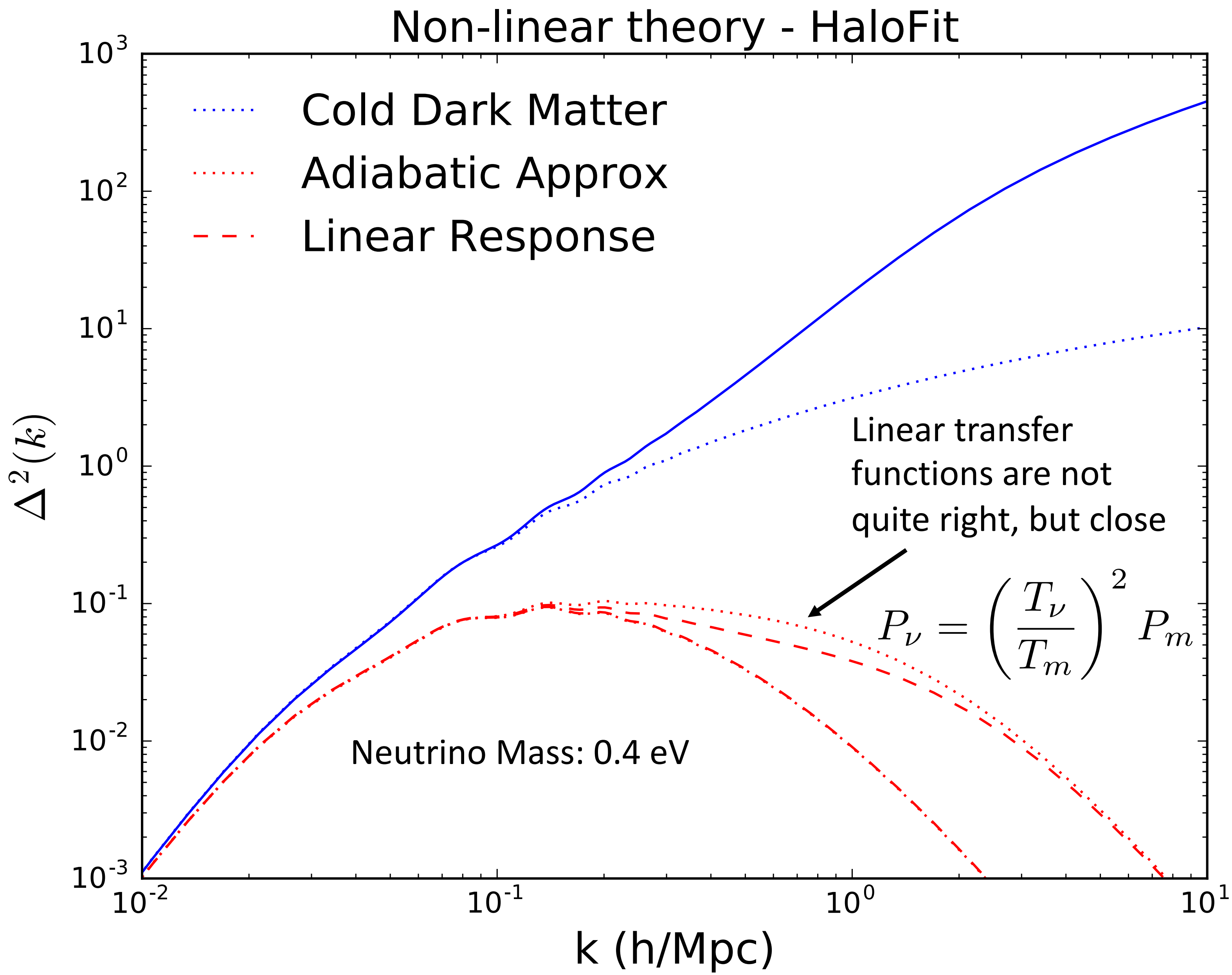
CDM

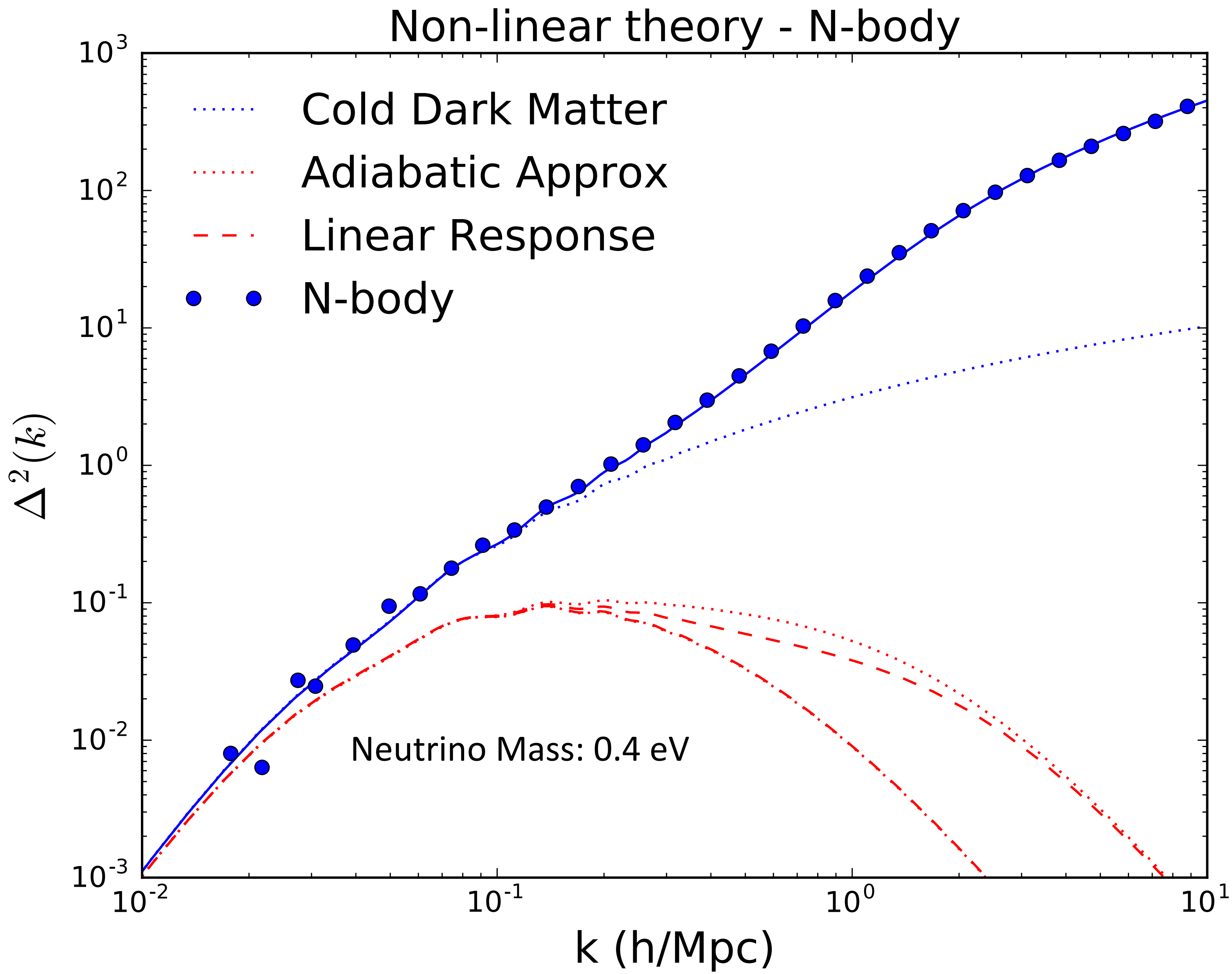


Neutrinos

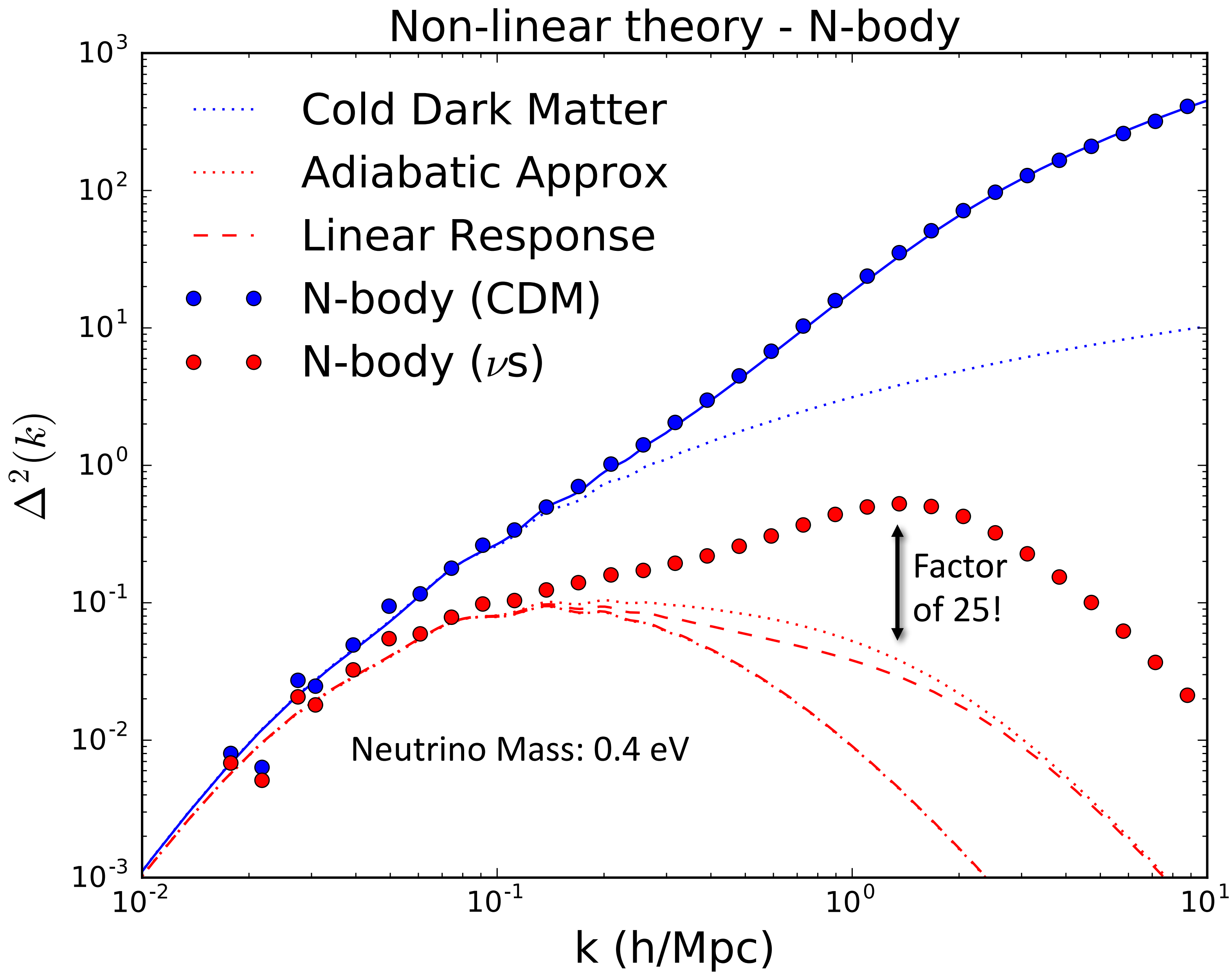


Linear theory
Cold Dark Matter
Adiabatic Approx
Linear Response
$P_\nu = \left(\frac{T_\nu}{T_m}\right)^2 P_m$
Neutrino Mass: 0.4 eV
$\Delta^2(k)$
k (h/Mpc)
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}

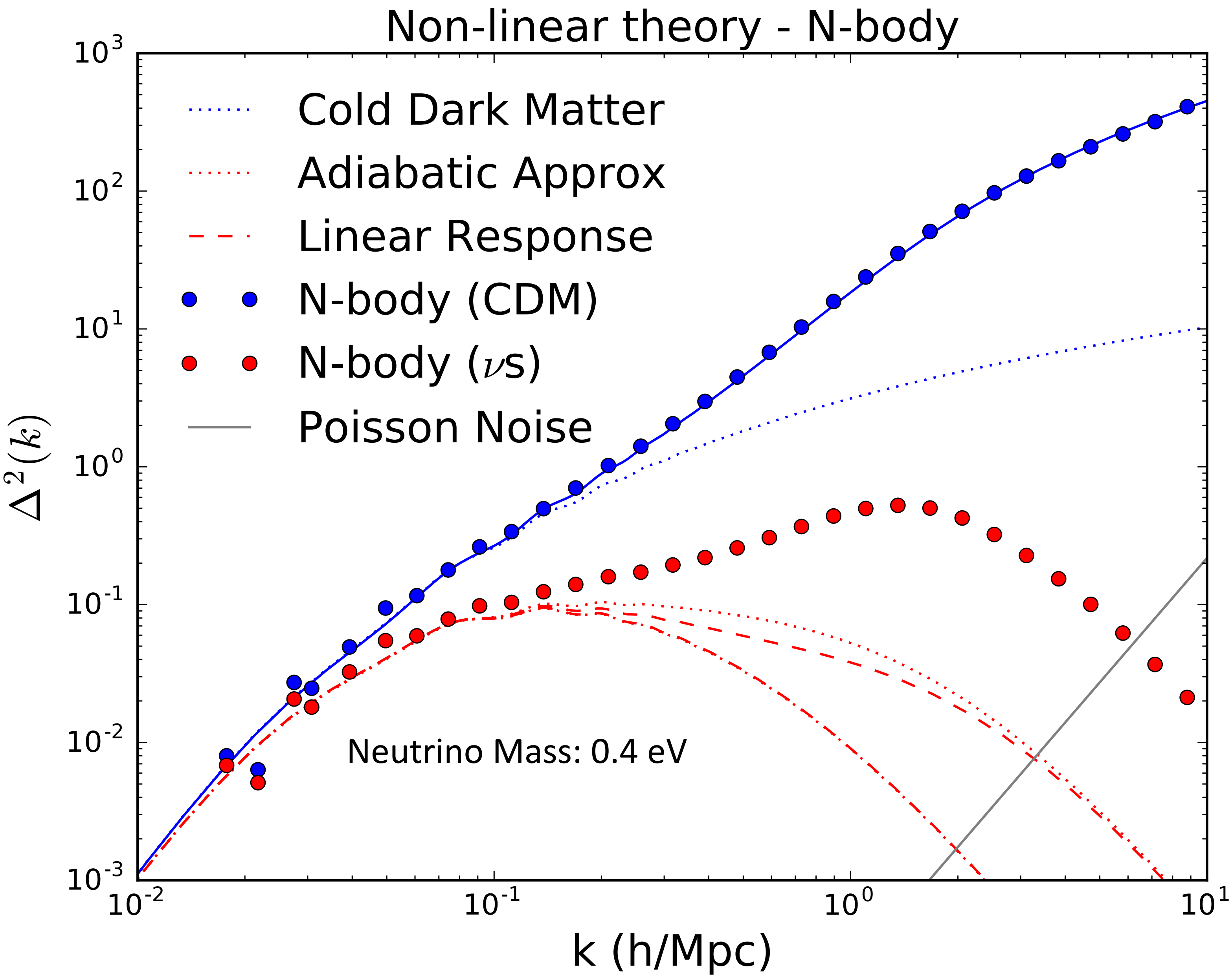

Non-linear theory - HaloFit
Cold Dark Matter
Adiabatic Approx
Linear Response
$\Delta^2(k)$
Linear transfer functions are not quite right, but close
$P_\nu = \left(\frac{T_\nu}{T_m}\right)^2 P_m$
Neutrino Mass: 0.4 eV
k (h/Mpc)
10^{-2}
10^{-1}
10^0
10^1
10^{-3}
10^2
10^3



Non-linear theory - N-body
Cold Dark Matter
Adiabatic Approx
Linear Response
N-body
Neutrino Mass: 0.4 eV
$\Delta^2(k)$
k (h/Mpc)
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}



Non-linear theory - N-body
Cold Dark Matter
Adiabatic Approx
Linear Response
N-body (CDM)
N-body (νs)
$\Delta^2(k)$
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-2}
10^{-1}
10^0
10^1
k (h/Mpc)
Factor of 25!
Neutrino Mass: 0.4 eV



Non-linear theory - N-body
Cold Dark Matter
Adiabatic Approx
Linear Response
N-body (CDM)
N-body (νs)
Poisson Noise
Neutrino Mass: 0.4 eV
$\Delta^2(k)$
k (h/Mpc)
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}

Fluid Approach

- Solve hydrodynamic equations instead of kinetic ones but require a sound speed to close the system
- Can compute the (linear) sound speed directly from the linear solution to the Vlasov equation:

$$\beta^2 c_s^2 = \frac{\int_{-\infty}^{s} ds' a^2(-k^2\phi)(s-s')\langle u^2 j_0(ku(s-s')/\beta)\rangle_0}{\int_{-\infty}^{s} ds' a^2(-k^2\phi)(s-s')\langle j_0(ku(s-s')/\beta)\rangle_0}$$

$\beta = m_\nu / T_\nu$

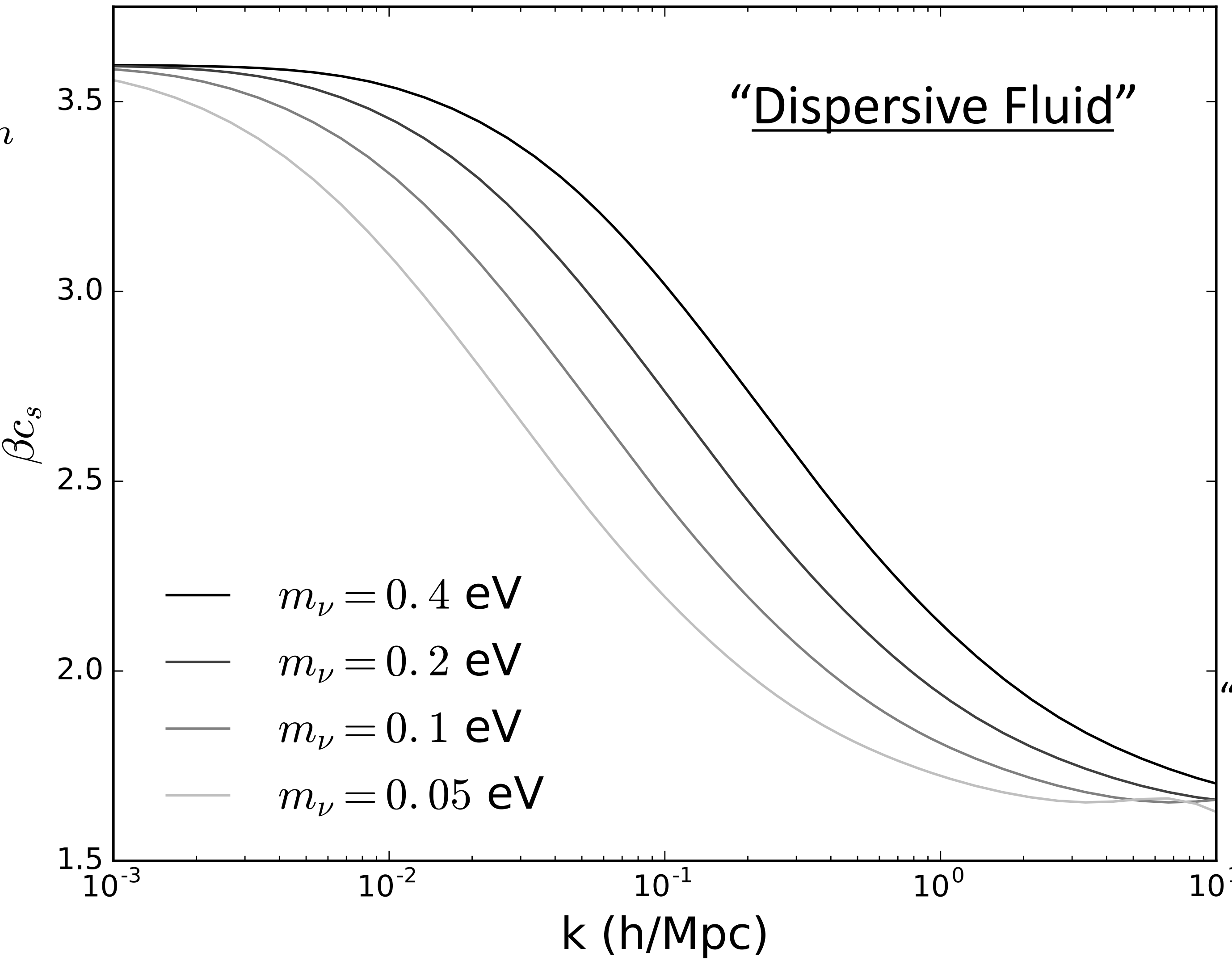
$$\langle A(u)\rangle_0 = \frac{\int u^2 A(u)(e^u+1)^{-1} du}{\int u^2 (e^u+1)^{-1} du}$$

$$\delta_{ss} + k^2 c_s^2 \delta = a^2(-k^2)\phi$$

"Linear"

$\delta_\nu = \delta_m$

β=m_ν/T_ν



"Instantaneous"

$\delta_\nu \propto \frac{1}{c_s^2}\frac{\delta_m}{k^2}$

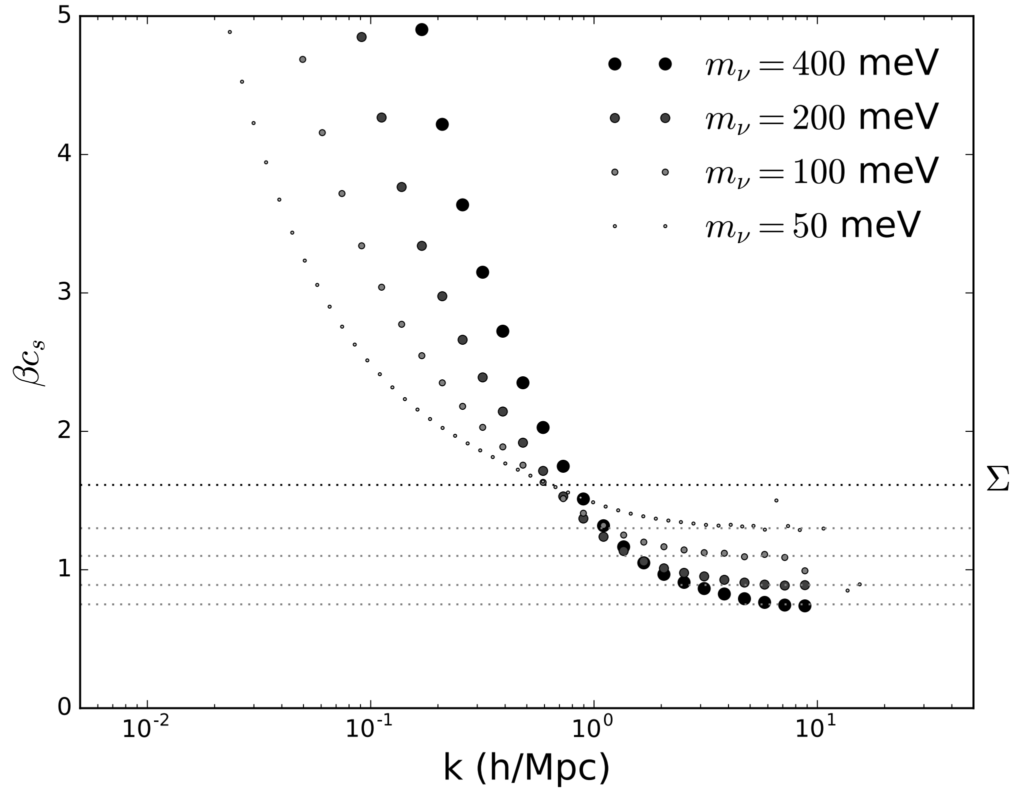
- On small scales, can measure the sound speed if we know CDM and neutrino density fields:

$$c_s^2 = \frac{3}{2} H_0^2 \Omega_m a \frac{\delta_m}{k^2 \delta_\nu}$$

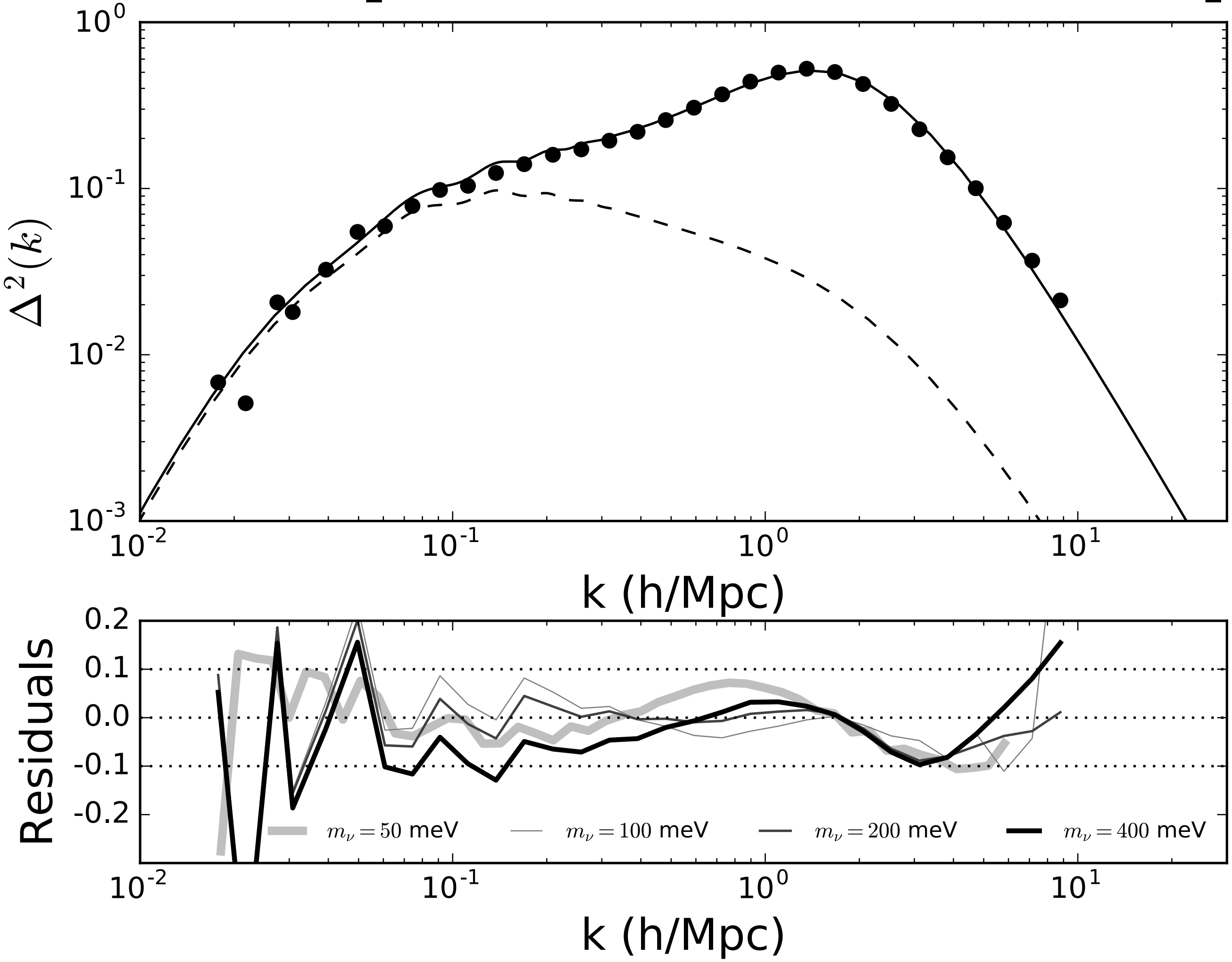
- We can measure this!

m_ν/eV	$\varsigma = \beta c_s$
0.4	0.75
0.2	0.89
0.1	1.10
0.05	1.30
Σ	1.61

- Sound speed drops from linear value!



$$\Delta_\nu^2 = \Delta_m^2 \left[\frac{T_\nu}{T_m} + \left(\frac{k_\beta}{k}\right)^2 \left(\frac{1}{\varsigma^2} - \frac{1}{\Sigma^2}\right) W(k/k_\varsigma) \right]^2$$



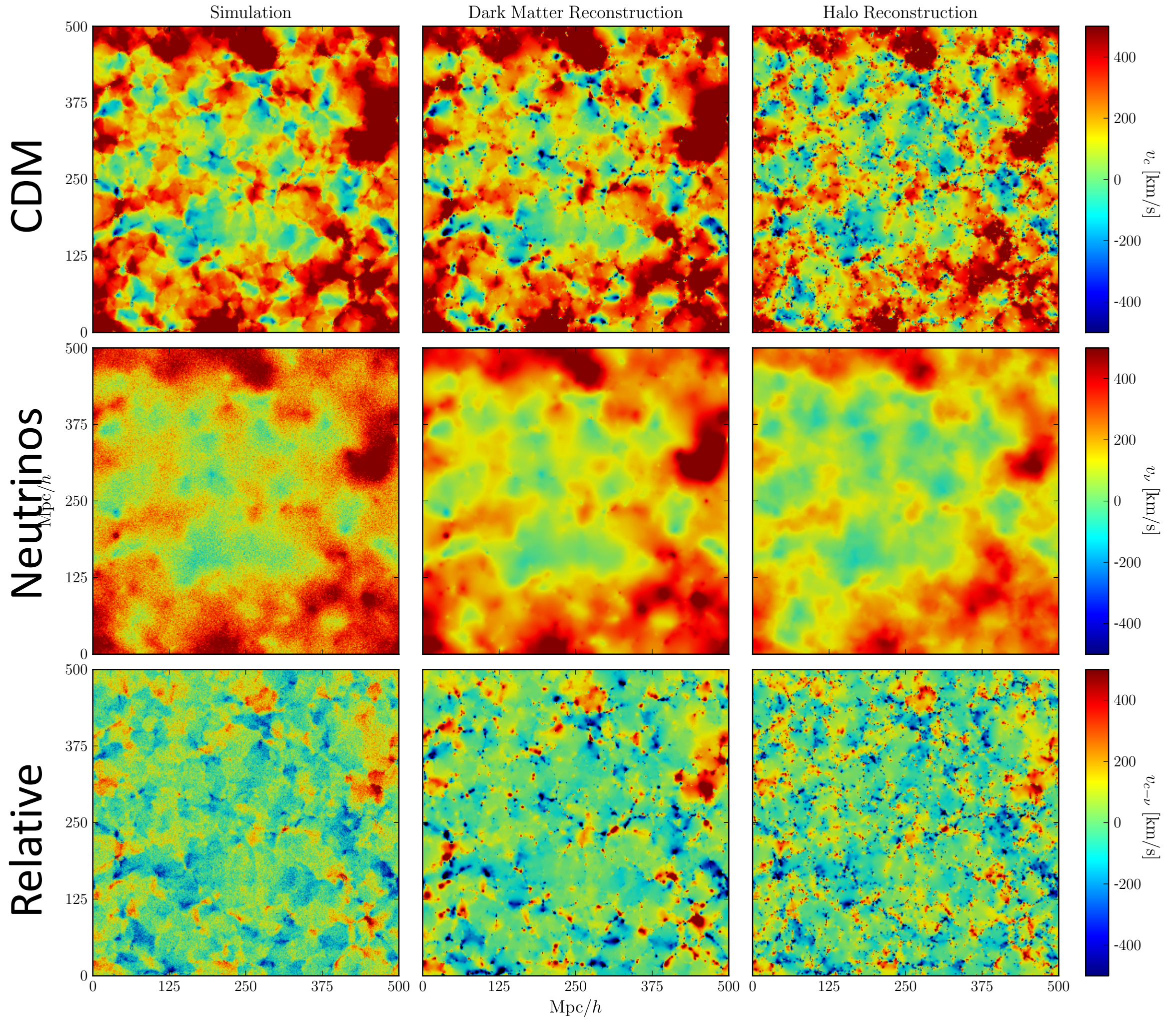
More information:
arXiv:1609.09469

Non-Linearity #2:
The Relative Velocity Effect

Velocity Fields

- Requires some care (unlike density or momentum)
- Use "Nearest N-particles" for velocity of cell
- N=1 for CDM/halos
- N=64 for neutrinos

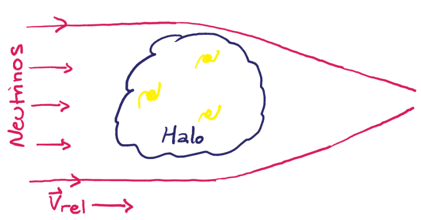
Slices of v_z in the xy plane



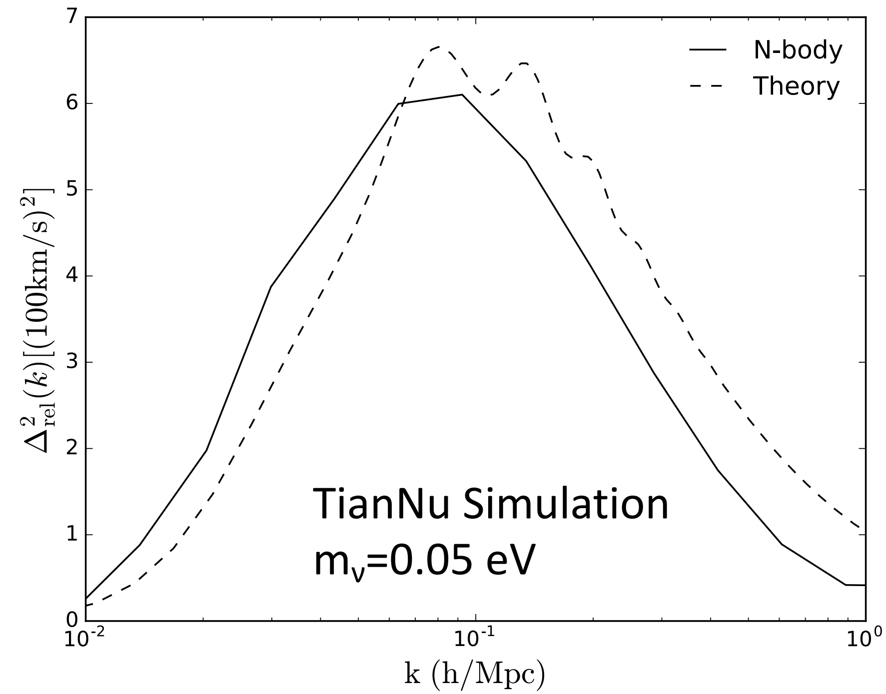
Predictable given a density field! $\vec{v}(\vec{k}) = \frac{T_v}{T_\delta}\delta(\vec{k})\hat{k}$

Relative Velocity Effect

- Analogous to baryon-CDM relative velocity effect (Tseliakhovich and Hirata 2010)
- Key differences:
  - Neutrino-CDM relative velocity is generated at late times
  - Large scale ~ 10 Mpc
- Effects:
  - Wake (arXiv:1412.1660)



  - Dipole correlation (arXiv:1311.3422)



$$v_{\rm rel} = \sqrt{\int \Delta^2_{\rm rel}(k)\frac{dk}{k}} \simeq 390{\rm km/s}$$

Moving Background Perturbation Theory

A paper on the next few slides is in preparation.

- If one fluid is moving relative to another, it becomes displaced:

$$\delta_c(\vec{x}) \to \delta_c(\vec{x} + \vec{v}_{\rm rel}\Delta\eta)$$

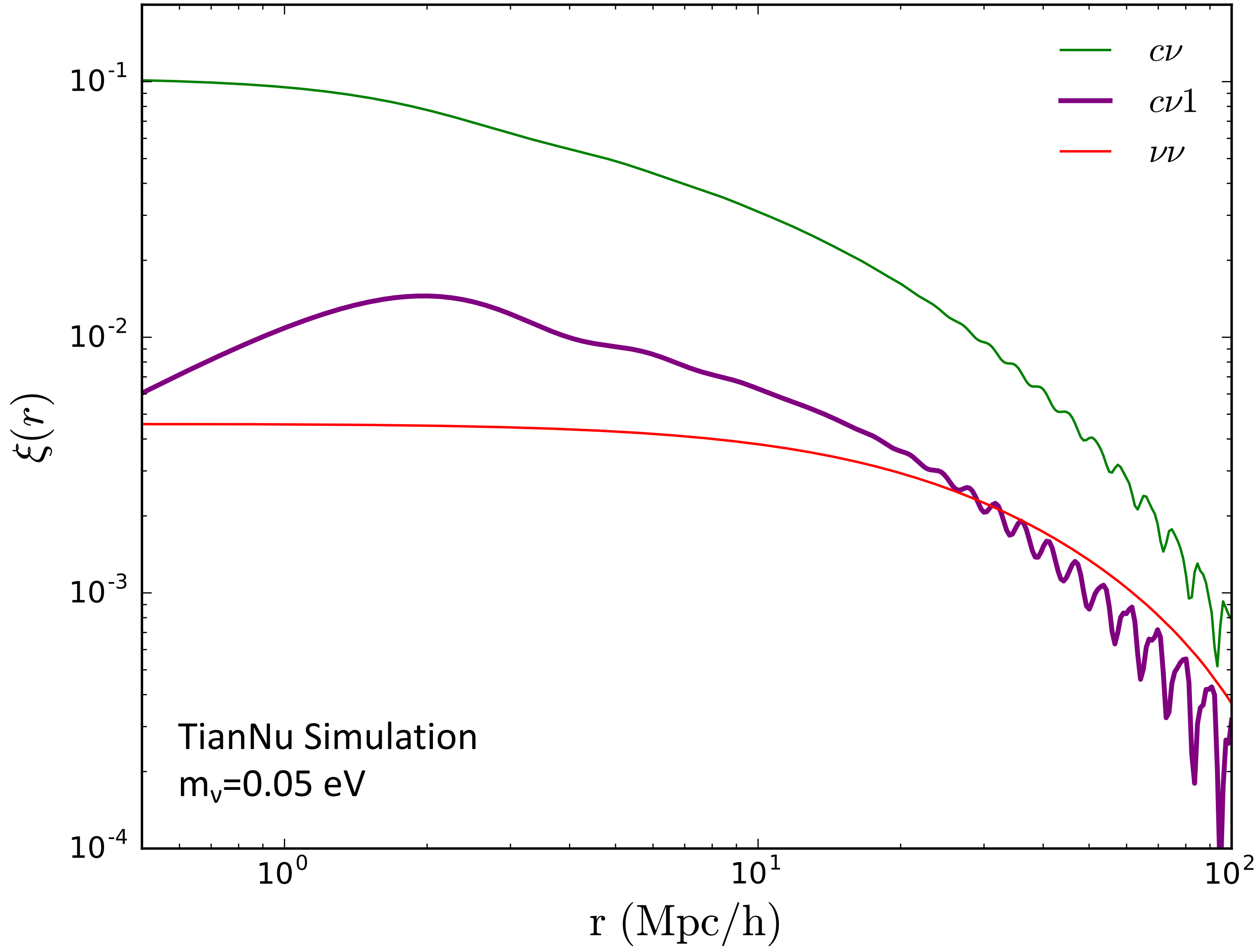
which is a phase in Fourier space:

$$\delta_c(\vec{k}) \to \delta_c(\vec{k})e^{i\mu k v_{\rm rel}\Delta\eta} \simeq \delta_c(\vec{k})(1 + i\mu k v_{\rm rel}\Delta\eta)$$

Dipole!

- Can use this in line of sight solution for neutrino density to get the dipole correlation function!

Dipole Correlation Function



Dipole from Simulations

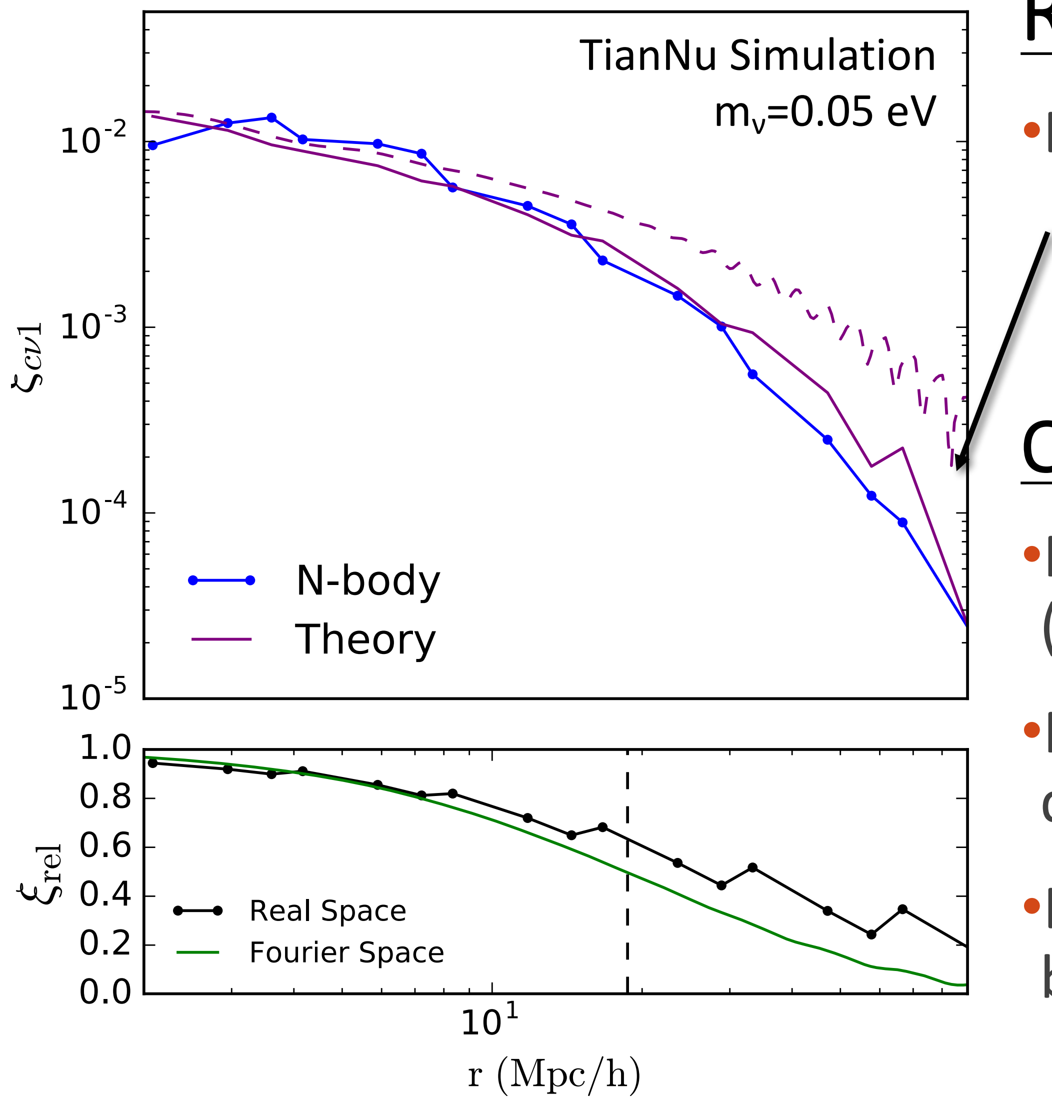
- Three point function:

$$\xi_{c\nu 1}(r) = \langle [\delta_c(\vec{x})\delta_\nu(\vec{x}+\vec{r}) - \delta_\nu(\vec{x})\delta_c(\vec{x}+\vec{r})]\, \hat{v}_{\rm avg}(\vec{x},\vec{r}) \cdot \hat{r} \rangle$$

$$\vec{v}_{\rm avg}(\vec{x},\vec{r}) = \frac{1}{2}\left(\vec{v}_{\rm rel}(\vec{x}) + \vec{v}_{\rm rel}(\vec{x}+\vec{r})\right)$$

- Anti-symmetric combination of density fields
  → cancel generic effects

- Computed in real space using the TianNu simulation



Results

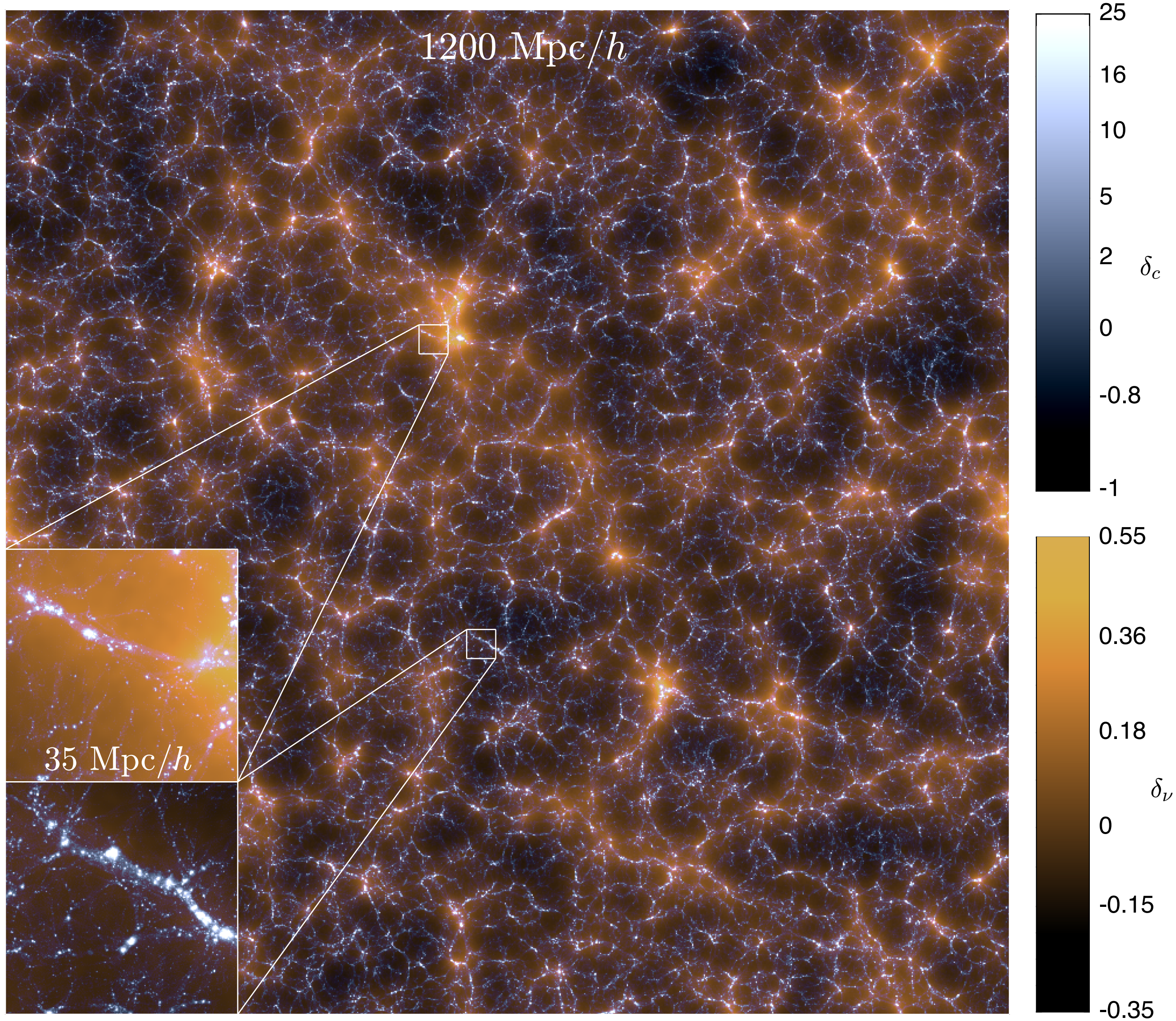
- Dipole correlation: exists
  - Matches theory after correction to account for MBPT
  - Mass Dependent

Observability:

- Predict relative velocity (direction and magnitude)
- Need two density tracers that depend differently on neutrinos
- But: using tracers leads to backgrounds

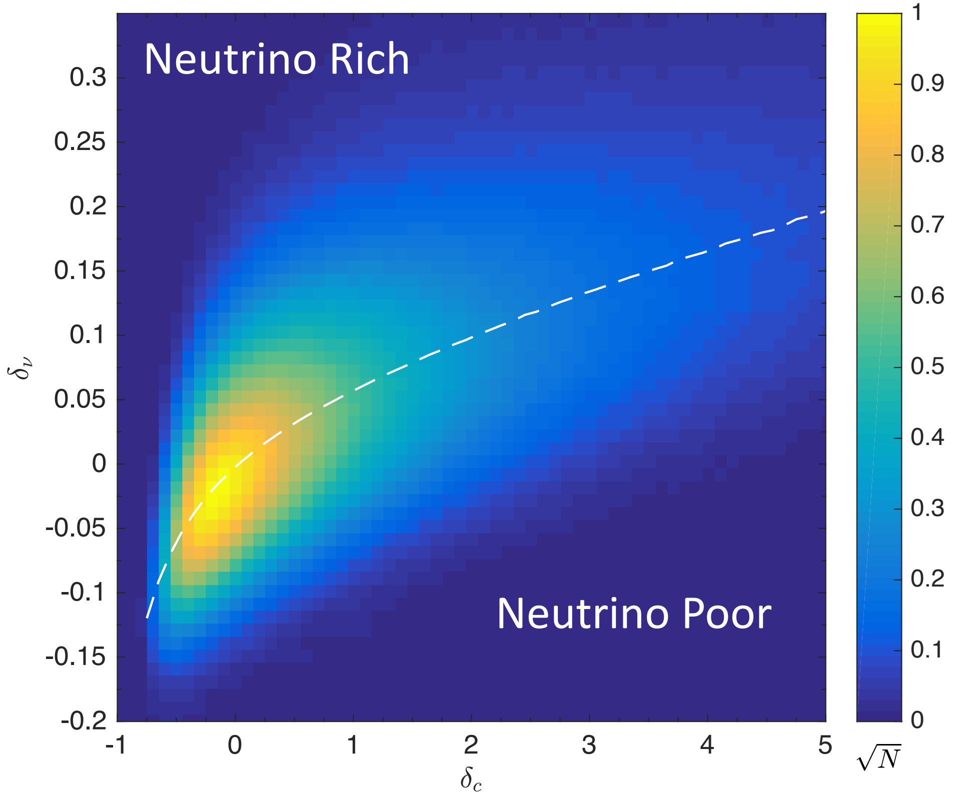
Non-Linearity #3:
Differential Neutrino Condensation

Differential Neutrino Condensation



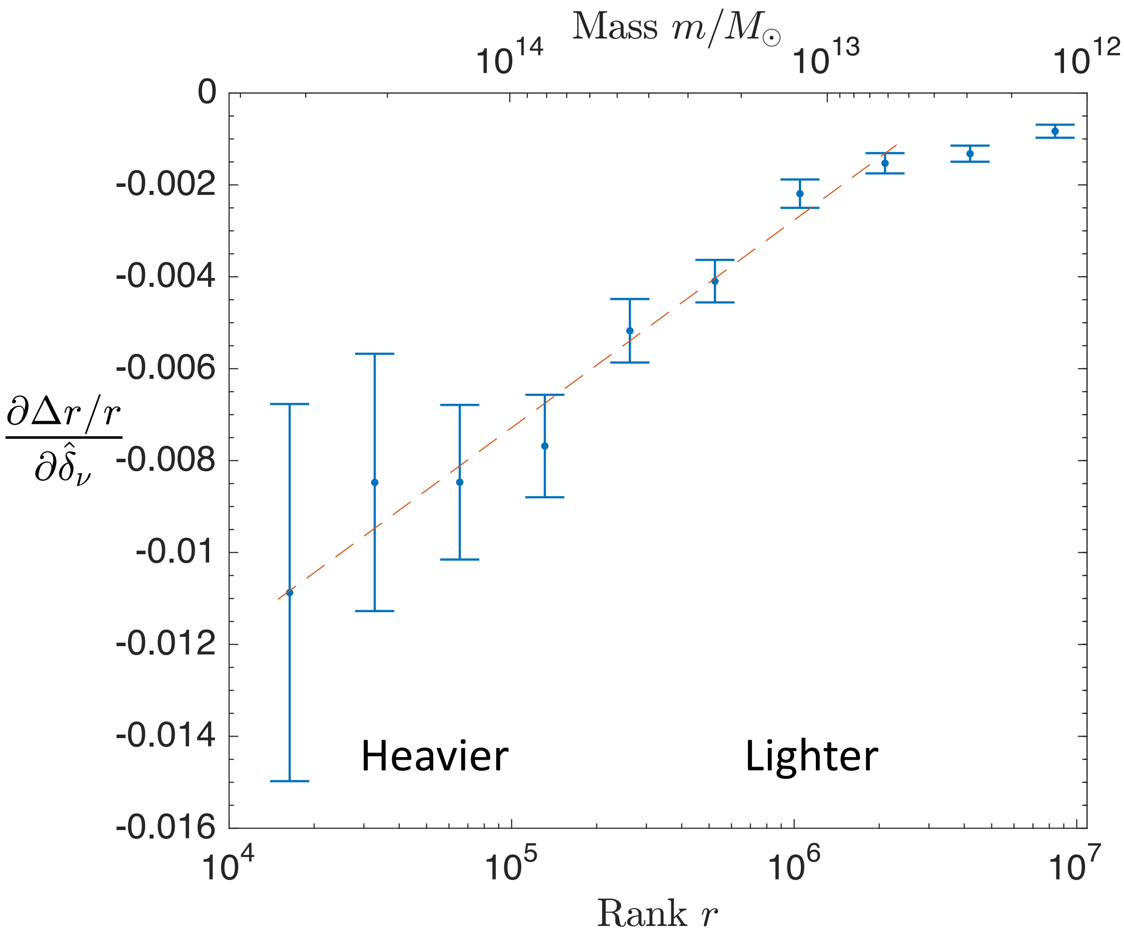
Regions with similar CDM content can have different neutrino content

- Local variations in δ_ν/δ_c lead to "neutrino rich" and "neutrino poor" sectors
- A halo in a rich region will be heavier than if it evolved in a poor region
- Sensitive to the *difference* between neutrinos and CDM rather than the *sum*



Dependency on Halo Mass

- Compare the *same* halos in TianNu and TianZero
- Heavier halos are more affected by condensation!
- Observability:
  - 4.5 sigma detection when comparing between simulations
  - 1 sigma detection in single simulations
  - Need larger volume!



Conclusions

- Linear theory does not provide a complete picture of neutrino dynamics, need simulations
  - Calibrated model of the neutrino power spectrum
- Neutrinos affect LSS in ways beyond two point statistics
- Higher order effects can be sensitive to neutrino mass and can be designed to cancel generic CDM effects
  - Confirmed the relative velocity effect exists in simulations and matches predictions. Potential to provide new constraints on neutrino mass.
  - Discovered differential neutrino condensation: at fixed CDM density, halo masses depend on local neutrino environment