

# Equivalence Principle and Cosmic Acceleration

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Based on collaboration with:

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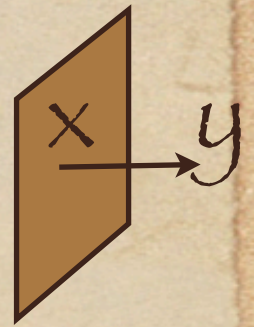
# Outline

- Long distance modification of gravity - the generic nature of scalar-tensor theory.
- 2 screening mechanisms - mandatory suppression of scalar on small scales.
- The problem of motion - how do things move?  
Do they really all fall at the same rate under gravity (i.e. equivalence principle)?
- Observational tests - look for  $O(1)$  violations.



## Examples of IR modification of GR - relation to scalar-tensor theories:

- $f(R)$  and generalizations - scalar-tensor (Chiba).
- DGP - brane bending mode (Luty, Porrati, Rattazzi).
- massive gravity - Stueckelberg (Arkani-Hamed, Georgi, Schwartz, de Rahm, Gabadadze, Tolley).
- resonance gravity/filtering/degravitation - Stueckelberg (Arkani-Hamed, Dimopoulos, Dvali, Gabadadze; Dvali, Hofmann, Khoury).
- ghost condensate (Arkani-Hamed, Cheng, Luty, Mukohyama; Dubovsky).
- galileon (Nicolis, Rattazzi, Trincherini) & generalizations (de Rahm, Tolley).
- cucuston (Afshordi, Chung, Geshnizjani).
- extrinsic curvature (Gabadadze).





Weinberg's theorem: at low energy, a Lorentz invariant theory of massless spin-2 particle must be GR (see also Deser).

Therefore, to modify gravity, either add new degrees of freedom (e.g. scalar) or make the graviton massive (which via Stueckelberg also contains scalar) or violate Lorentz invariance (e.g. ghost condensate).

Some form of scalar-tensor theory seems generic.



**Also:** modified gravity is in a sense no more exotic than quintessence. Absent symmetries, quintessence should be coupled to matter at gravitational strength i.e. scalar-tensor theory yet again.



## Screening

No matter what the precise theory is, we generally want the scalar to be alive on large scales i.e. induce  $O(1)$  modification on Hubble scale.

But the scalar must be screened on small scales to match solar system tests (recover GR).

Two known screening mechanisms:

chameleon (also symmetron) and Vainshtein.

Both make use of scalar self-interactions, one uses potential, the other uses derivatives.

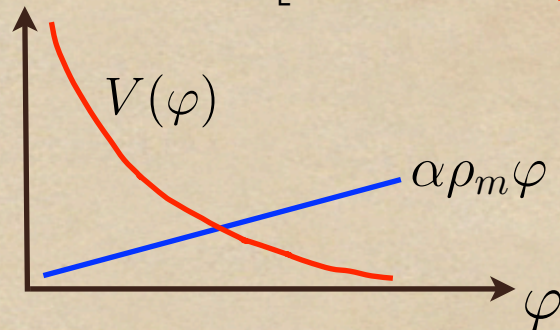


## Chameleon screening:

Khoury & Weltman

(Einstein frame)

$$S_{\text{scalar}} \sim \int d^4x \left[ -\frac{1}{2}(\partial\varphi)^2 - \underline{V(\varphi)} + \alpha\varphi T_m^{\mu}{}_{\mu} \right]$$



e.o.m.:

$$\square\varphi \sim [V + \alpha\rho_m\varphi]_{,\varphi} \quad (T_m^{\mu}{}_{\mu} \sim -\rho_m)$$

( $\varphi$  dimensionless,  $M_P \approx 1$ )

## Vainshtein screening:

e.g. DGP

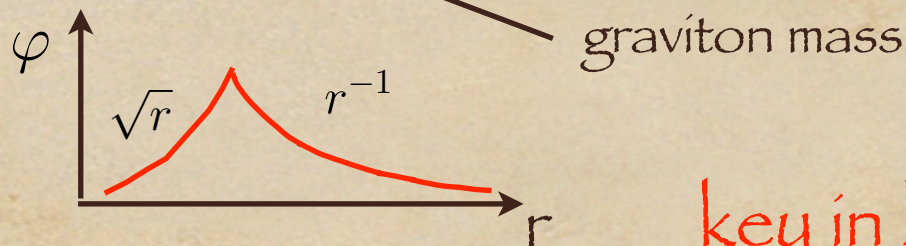
$$S_{\text{scalar}} \sim \int d^4x \left[ -\frac{1}{2}(\partial\varphi)^2 - \underline{\frac{1}{m^2}(\partial\varphi)^2\square\varphi} + \alpha\varphi T_m^{\mu}{}_{\mu} \right] \quad (\text{Einstein frame})$$

$$\text{e.o.m.:} \quad \square\varphi + \frac{1}{m^2} [(\square\varphi)^2 - \partial^\mu\partial^\nu\varphi\partial_\mu\partial_\nu\varphi] \sim \alpha\rho_m$$

$$\varphi \propto \frac{1}{r} \quad \text{large } r$$

$$\varphi \propto \sqrt{r} \quad \text{small } r$$

point mass solution



key in both: nonlinear interaction

$\alpha$  = universal scalar-matter coupling  $\approx O(1)$  generically



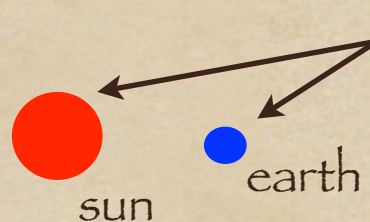
## Constrasting chameleon and Vainshtein screening:

- Consider an object in the presence of a long wavelength external  $\varphi$  (i.e. ignoring tides).



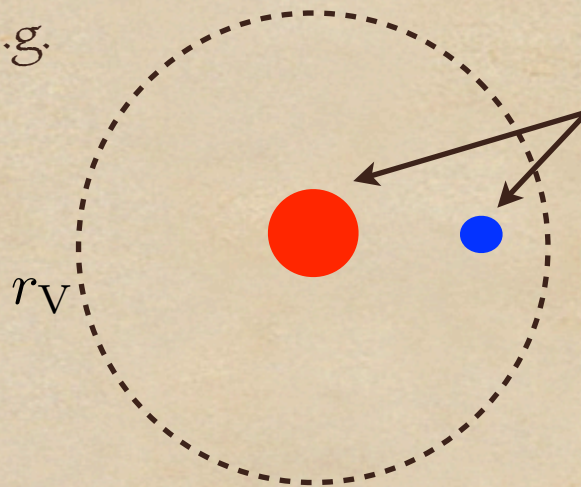
The object-scalar interaction is described by  $S_{\text{int}} \sim -\alpha Q \int d\tau \varphi$   
 where  $Q$  is the object's scalar charge i.e. scalar force  $F = -\alpha Q \nabla \varphi$ .

- Chameleon: e.g.



both have  $Q \ll M$ ,  
 because  $\nabla^2 \varphi = V_{,\varphi} + \alpha \rho_m \sim 0$

- Vainshtein: e.g.

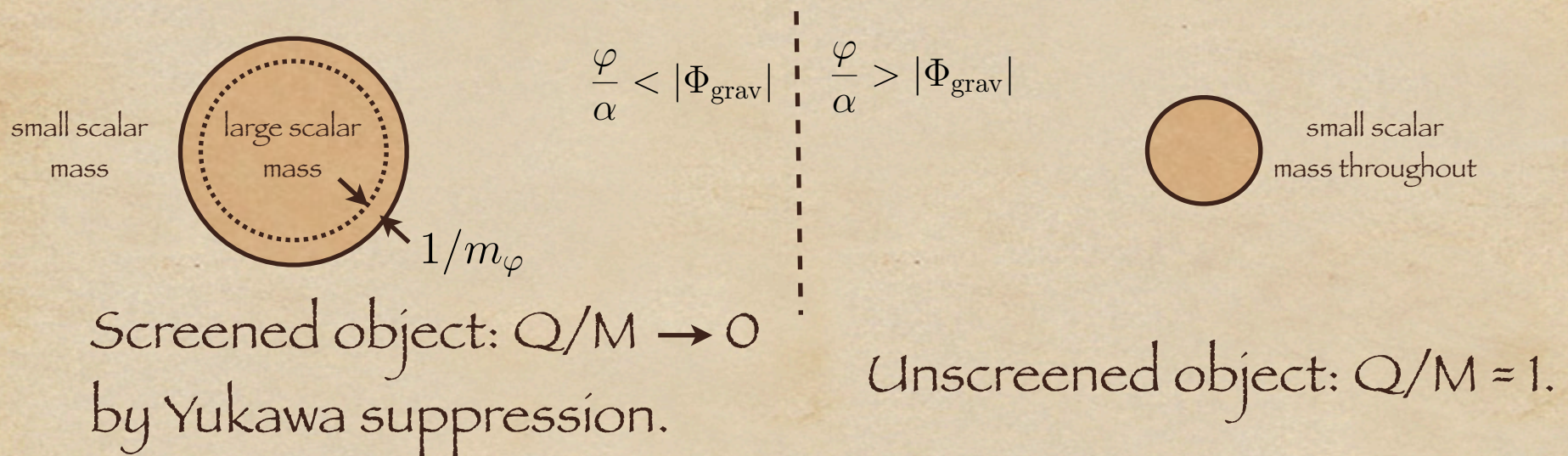


both have  $Q \approx M$ ,  
 because shift symmetry implies  
 e.o.m. takes the Gauss-law form  
 $\partial \cdot J = \alpha \rho_m$

A large scalar force on the earth is avoided by having the sun source a very suppressed scalar profile within the Vainshtein radius.



- Chameleon screening: an  $O(1)$  equivalence principle violation, from classical renormalization of  $Q$ .



- Screened and unscreened objects have  $O(1)$  difference in  $Q/M$ , and therefore  $O(1)$  equivalence principle violation.



# A rigorous way to deduce the motion of an object:



momentum  $P_i = \int d^3x t_i^0$

momentum flux

$$\dot{P}_i = \int d^3x \partial_0 t_i^0 = - \int d^3x \partial_j t_i^j = - \oint dS \hat{x}_j t_i^j$$



where  $t_\mu^\nu$  = pseudo energy-momentum

Einstein, Hofmann, Infeld; Damour

- Here,  $t_{ij} \sim \partial_i \varphi \partial_j \varphi + \dots$

Object-bgd. split:

$$\varphi = \varphi_{\text{obj}} + \varphi_{\text{ext}} \Rightarrow \dot{P}_i \sim -\partial_i \varphi_{\text{ext}} f(\partial \varphi_{\text{obj}}, \varphi_{\text{obj}})$$

long wavelength ext. bgd.

- Key here is that on scale of object,  $\varphi_{\text{ext}}$  can be treated as a linear gradient. Note that with galileon symmetry, a linear gradient can always be added to a solution (such as  $\varphi_{\text{obj}}$ ) to obtain another solution with the desired boundary condition.



# Jordan frame summary for chameleon:

$$M\ddot{X}_i = -M \left[ \frac{1 + 2\epsilon\alpha^2}{1 + 2\alpha^2} \right] \partial_i \Phi_{\text{ext}} \quad \leftarrow \text{eff. G variation}$$

Milky way & Sun has  $|\Phi_{\text{object}}| \sim 10^{-6}$   
 $\rightarrow \varphi/\alpha \lesssim 10^{-6}$

$\epsilon \sim 1$  for unscreened objects    and     $\epsilon \sim 0$  for screened objects

$(\varphi/\alpha > |\Phi_{\text{object}}|)$

$(\varphi/\alpha < |\Phi_{\text{object}}|)$

grav. mass = inertial mass

grav. mass  $\neq$  inertial mass

Generically  $\alpha \sim 1$ , so expect O(1) violation of equivalence principle between screened and unscreened objects.

Only unscreened objects move on Jordan frame geodesics.

E.g.  $f(R)$ :  $\alpha = 1/\sqrt{6}$ , unscreened/screened grav. mass = 4/3.

Note:  $f(R)$ 's special  $\alpha$  is not protected against quantum corrections.

Important parameters:

$\alpha$  &

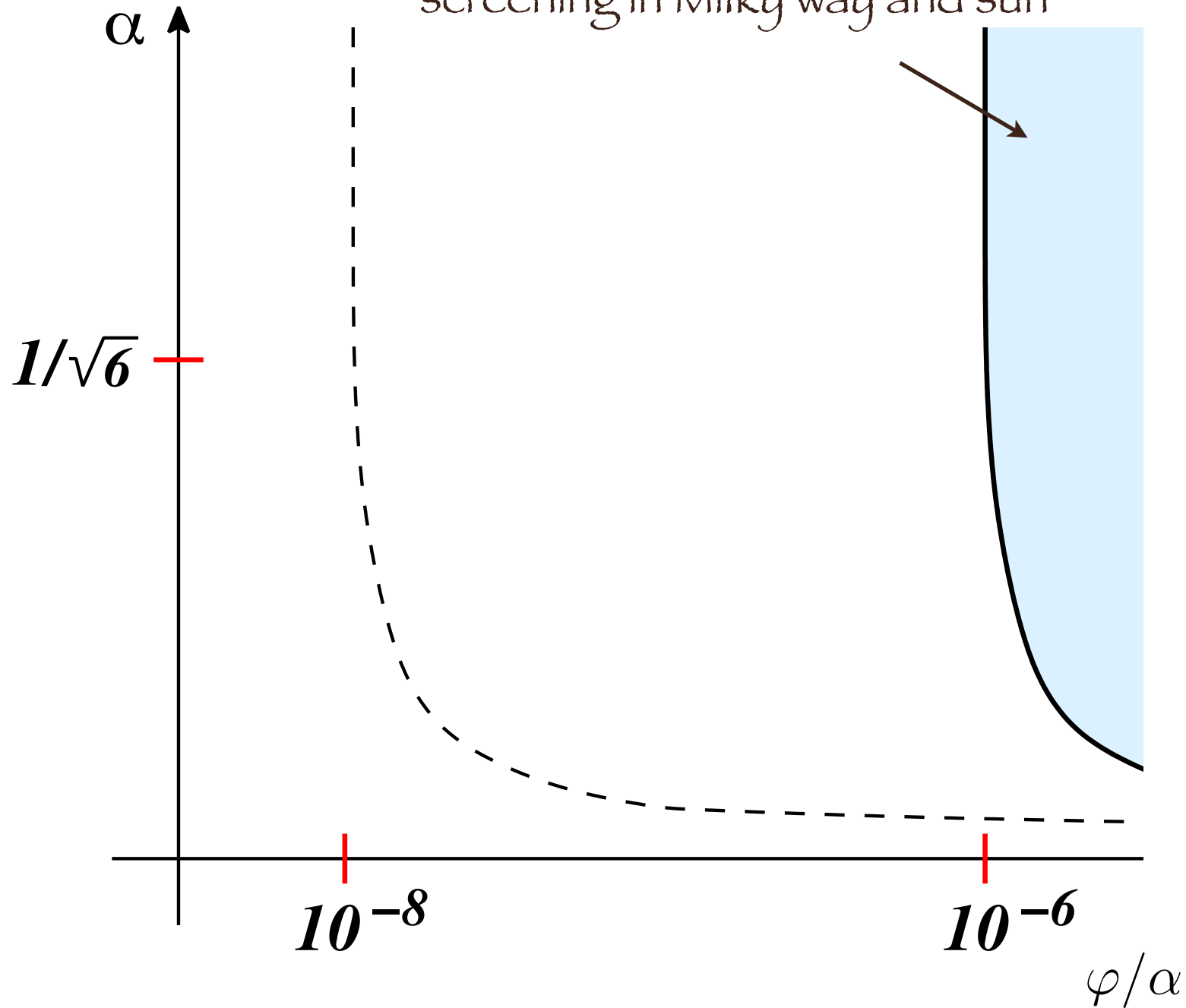
$\frac{\varphi}{\alpha}$

scalar-matter coupling:  
controls e.p. violation level

controls screening



Ruled out by demanding  
screening in Milky way and sun





## Observational tests of chameleon screening

An object is chameleon screened ( $Q \ll M$ ) if the -grav. potential ( $GM/R$ ) is deeper than  $\varphi_{\text{ext}}/\alpha$ , and unscreened ( $Q=M$ ) otherwise. Observationally, we know any object with -grav. pot. deeper than  $10^{-6}$  should be screened (from Milky way).

- A screened object does not experience scalar force, while an unscreened object does. They therefore fall at rates that are  $O(1)$  different (violation of equivalence principle).



## Bulk motion tests:

Idea - unscreened small galaxies, screened large galaxies.

1. Small galaxies should move faster than large galaxies (i.e. an effective velocity bias - redshift distortion needs to be reworked) in unscreened environments. Beware: Yukawa suppression.
2. Small galaxies should stream out of voids faster than large galaxies creating larger than expected voids defined by small galaxies (see Peebles).



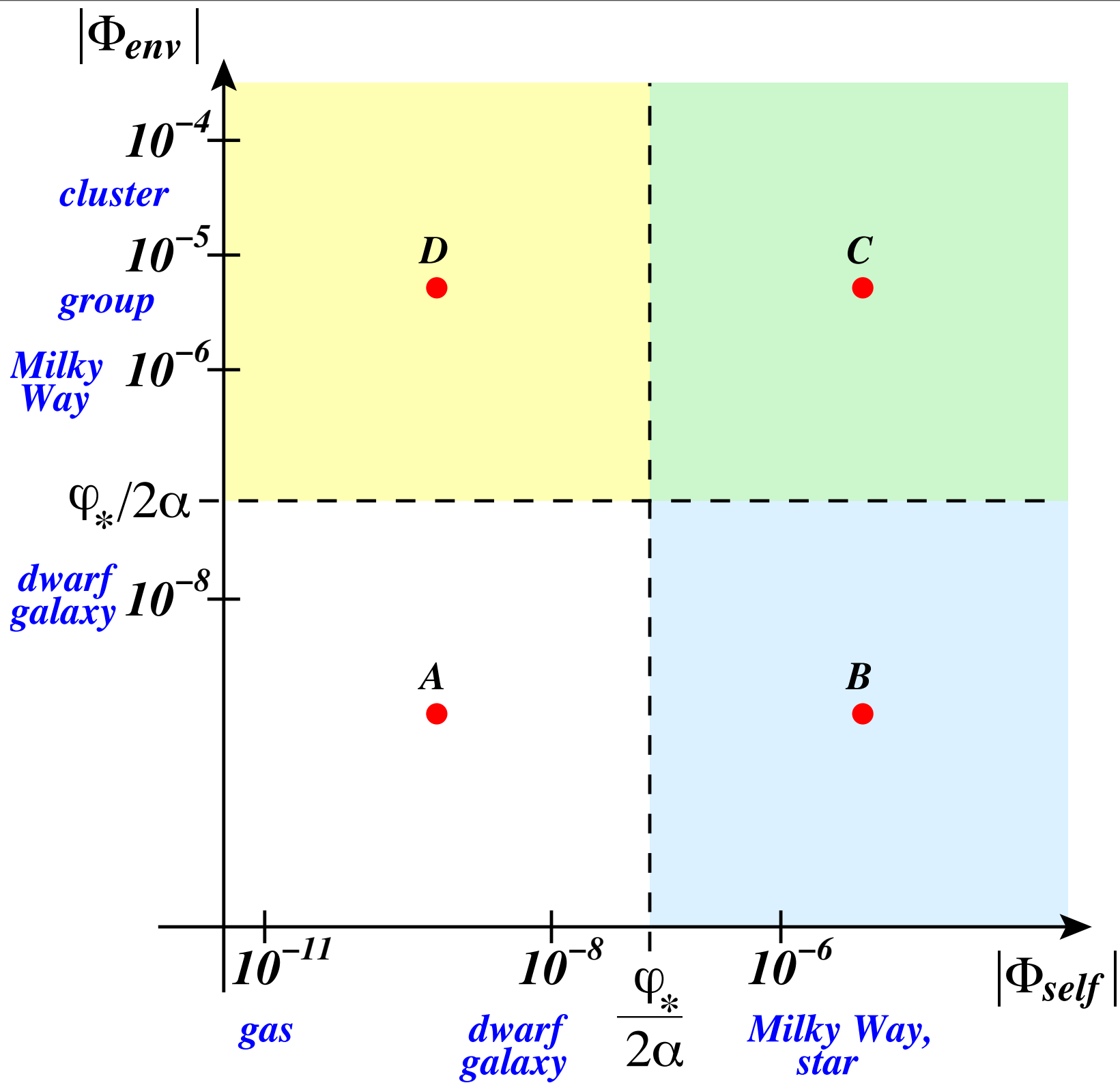
## Internal motion tests:

Idea - unscreened HI gas clouds, screened stars.

3. Diffuse gas (e.g. HI) should move faster than stars in small galaxies even if they are on the same orbit. Beware: asymmetric drift.
4. Gravitational lensing mass should agree with dynamical mass from stars, but disagree with that from HI in small galaxies.

Key: avoid blanket screening.







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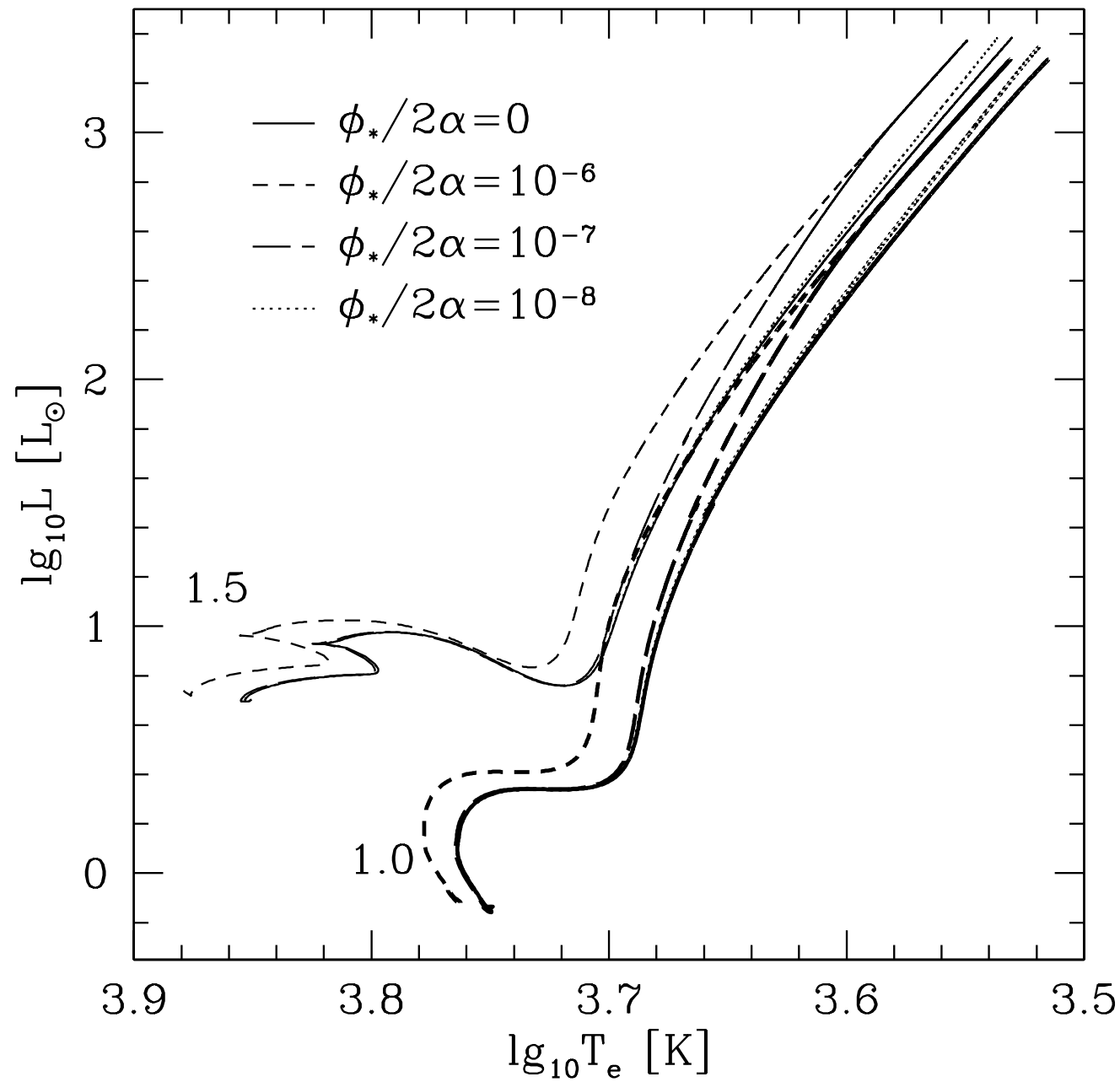


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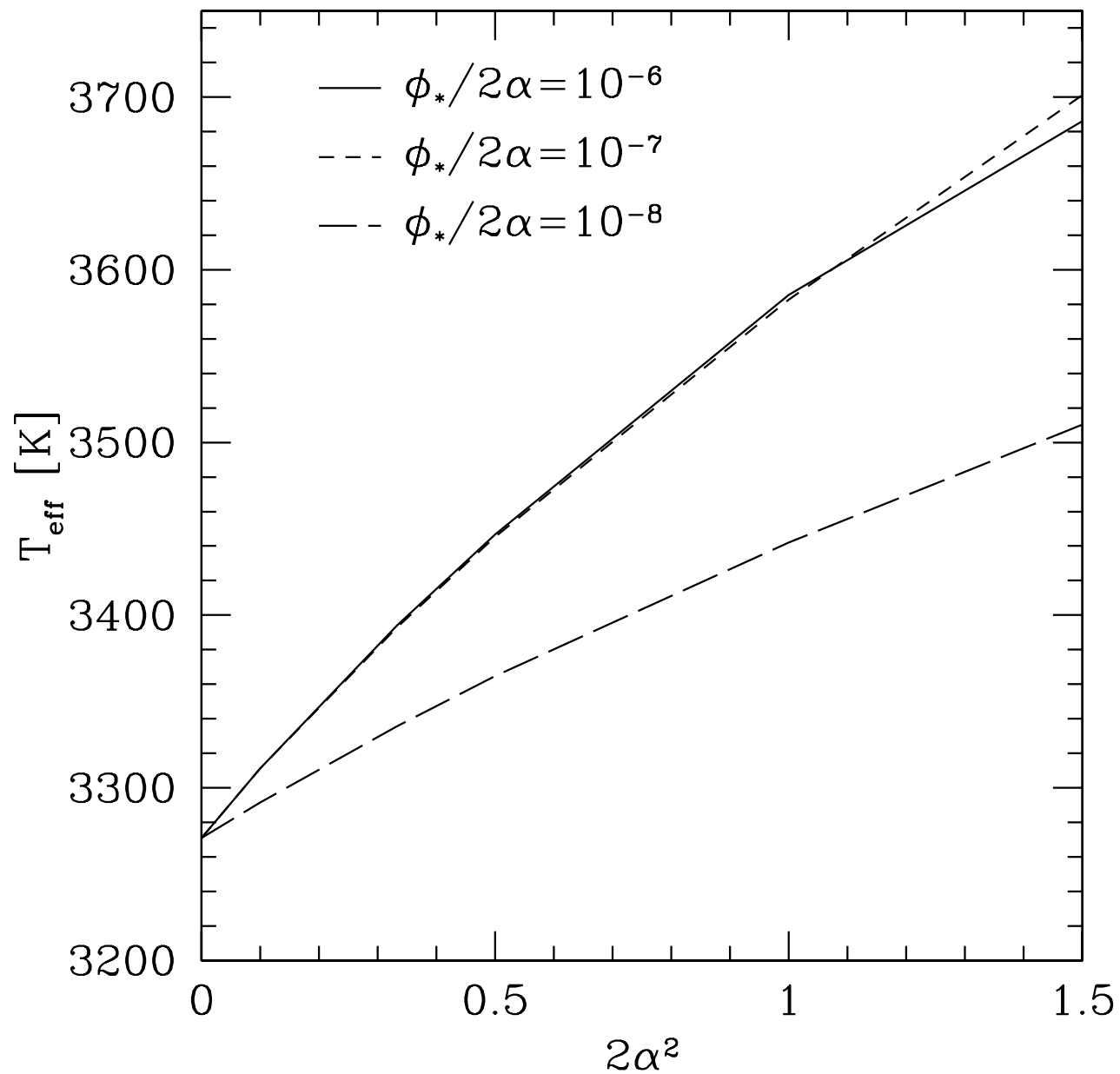
- A screened object does not experience scalar force, while an unscreened object does. They therefore fall at rates that are  $O(1)$  different (violation of equivalence principle).
- Red giants would have a compact screened core, and a diffuse unscreened envelope. Thus, effectively Newton's  $G$  changes value in the star. This affects the observed temperature at the 100 K level.





Chameleon effects on red giants (Chang, LH).  
See also Davis, Lim, Sakstein, Shaw.





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## Side-remark: condition for true self-acceleration

$$\begin{array}{ccc} & \nearrow & \nwarrow \\ & \tilde{g}_{\mu\nu} = e^{\alpha\varphi} g_{\mu\nu} & \\ \text{Jordan frame metric} & & \text{Einstein frame metric} \end{array}$$

- Want no acceleration in Einstein frame, but acceleration in Jordan frame i.e. do not want acceleration to be caused by some form of dark energy, but rather by the non-minimal scalar coupling itself.
- This suggests  $\alpha\varphi$  cannot be too small.
- Since observations constrain  $\varphi/\alpha \lesssim 10^{-6}$  for chameleon screening, it cannot support self-acceleration whatever the actual model is (assuming  $\alpha \sim 1$ ).

(with Junpu Wang & Justin Khoury)



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Issue 2, a more serious problem: black holes and stars are generally found inside galaxies. Wouldn't the fact that they are both inside the Vainshtein radius of the galaxy mean the effect is very small?



The key is to recognize that there are regions in the universe where the scalar  $\varphi$  is in the linear regime - in and around voids (see sim. by Chan & Scoccimarro). Rewriting the scalar e.o.m.:

$$H^{-2}\partial^2\varphi + (H^{-2}\partial^2\varphi)^2 \sim \alpha \frac{\rho_m}{\bar{\rho}_m}$$

in regions of sufficiently low density, the linear term dominates over the nonlinear term i.e.  $\varphi$  is unsuppressed by interactions.



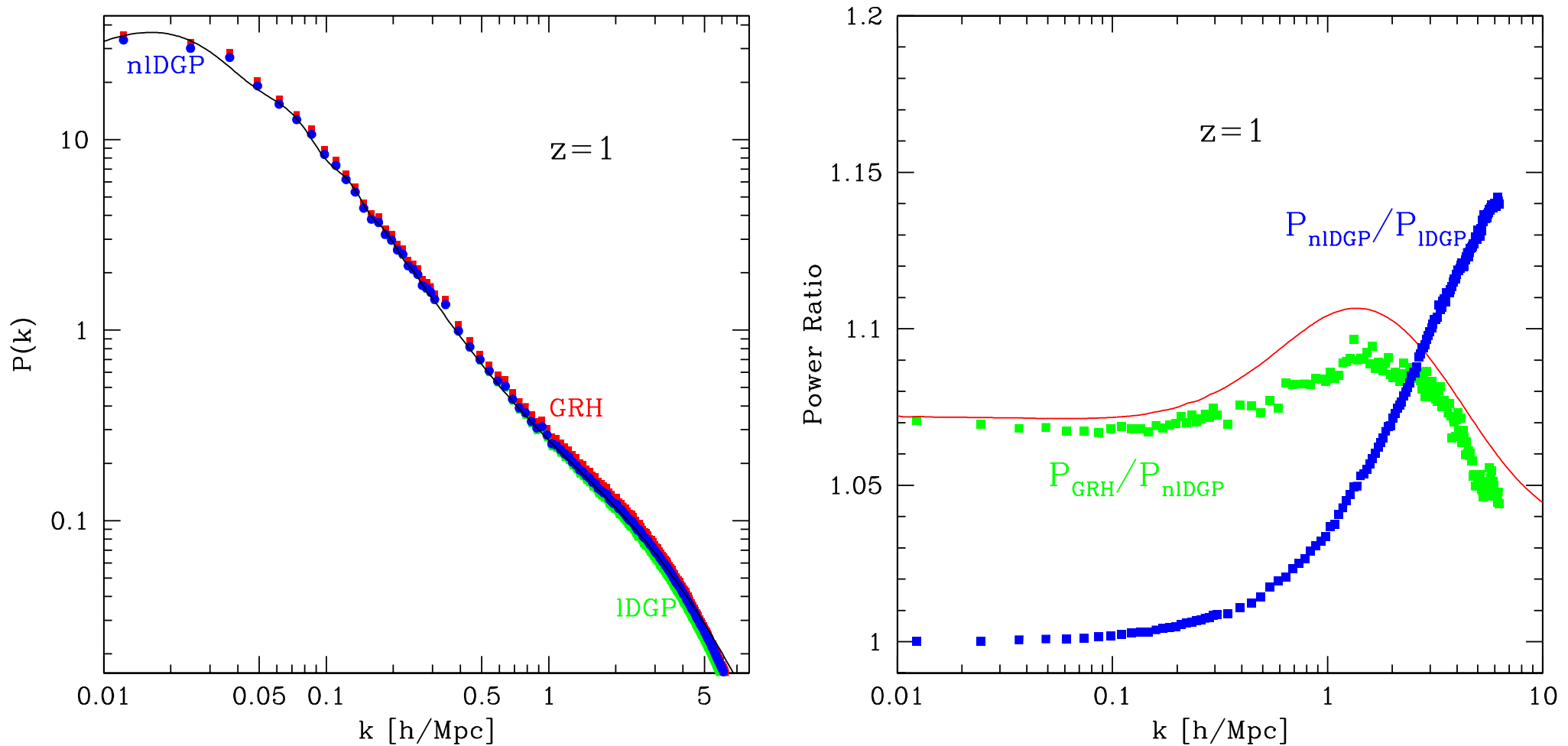


FIG. 5: Dark matter power spectra from the nonlinear DGP model (nIDGP) , linear DGP (IDGP), and GR perturbations with the same expansion history (GRH) at  $z = 1$ . The left panels show the power spectra, and the right panels shows ratios to better see the differences. Two sets of computational boxes are shown for each case, covering a different range in  $k$  (see text). The solid line denotes the predictions from paper I for  $P_{\text{nIDGP}}$  (left panel) and  $P_{\text{GRH}}/P_{\text{nIDGP}}$  (right panel).



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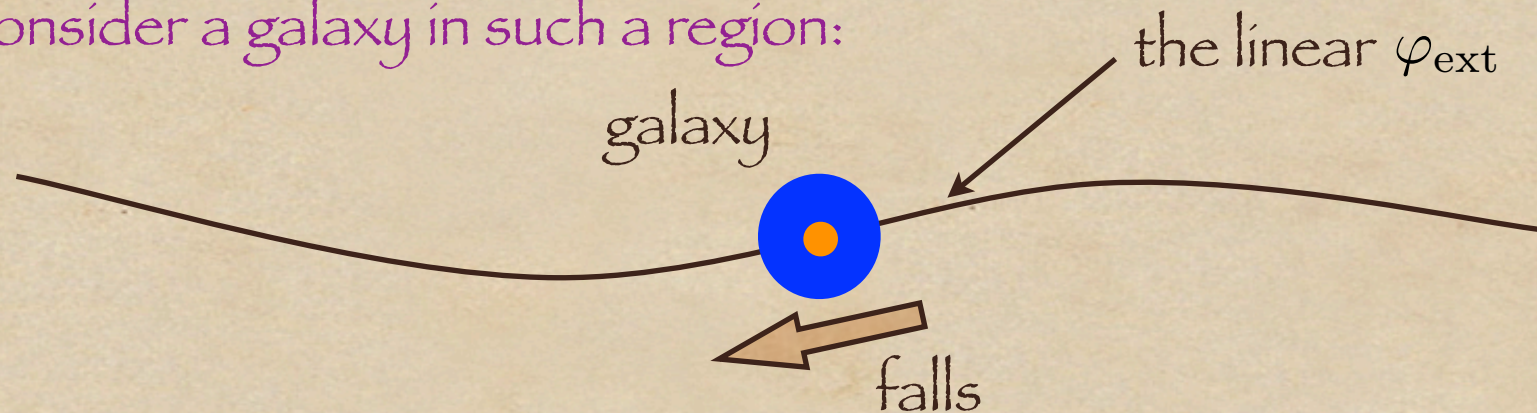


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Consider a galaxy in such a region:



The galaxy (with its stars and dark matter) would fall under this external scalar field. The black hole won't. Both of course still respond in the same way to the Einstein part of gravity.

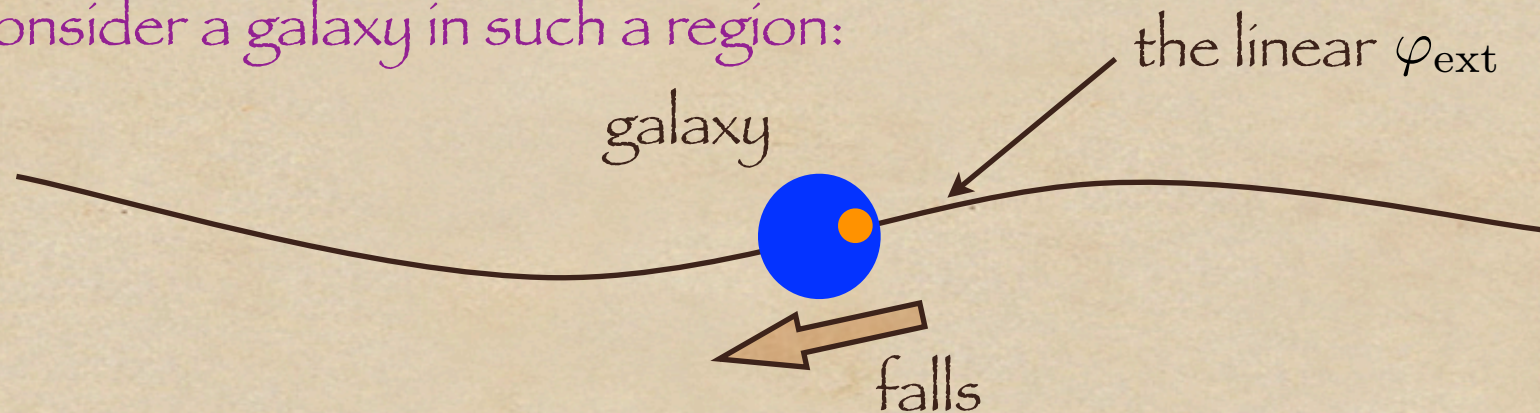


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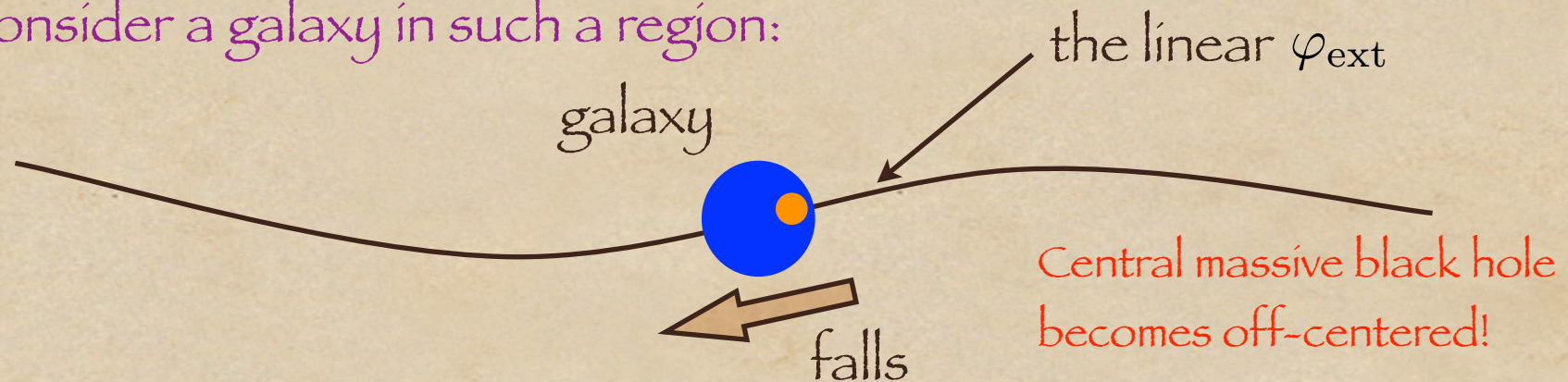


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The idea is to look for the offset of massive black holes from the centers of galaxies which are streaming out of voids.

The offset should be correlated with the direction of the streaming motion. The massive black holes can take the form of quasars or low luminosity galactic nuclei i.e. Seyferts.

The offset is estimated to be up to 1 - 100 pc, depending on galaxy.

In this test, one should avoid clusters, where the (external) scalar is in the nonlinear regime. Note however the galaxy (which contains the black hole) can be as massive as we wish.



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A natural question: can one observe this effect for chameleon screening as well? No, because both stars and black holes have zero scalar charge, but one can compare motions of stars versus dark matter (Jain & Vanderplas).



## A more general viewpoint:

Is a universally coupled scalar stable against classical and quantum corrections? Answer: partial yes, for corrections in the matter sector, i.e. a scalar equivalence principle.

But, first caveat: equivalence principle can be violated if graviton self interactions are important, e.g. black holes (Nordvedt effect).

Second caveat: equivalence principle can be violated if the scalar self interactions are important (e.g. chameleon) unless protected by galileon/shift symmetry (e.g. DGP).

LH, Nicolás



## Summary:

- Observational tests of chameleon screening: compare the motions of screened (stars or massive galaxies) and unscreened objects (gas clouds or dwarf galaxies). Voids/low density regions are particularly good places to look.
- Observational tests of Vainshtein screening: compare the motions of stars and black holes.