Recent Developments in Cosmological Recombination

Christopher Hirata

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With special thanks to:
E. Switzer (Chicago)
D. Grin, J. Forbes, Y. Ali-Haïmoud (Caltech)

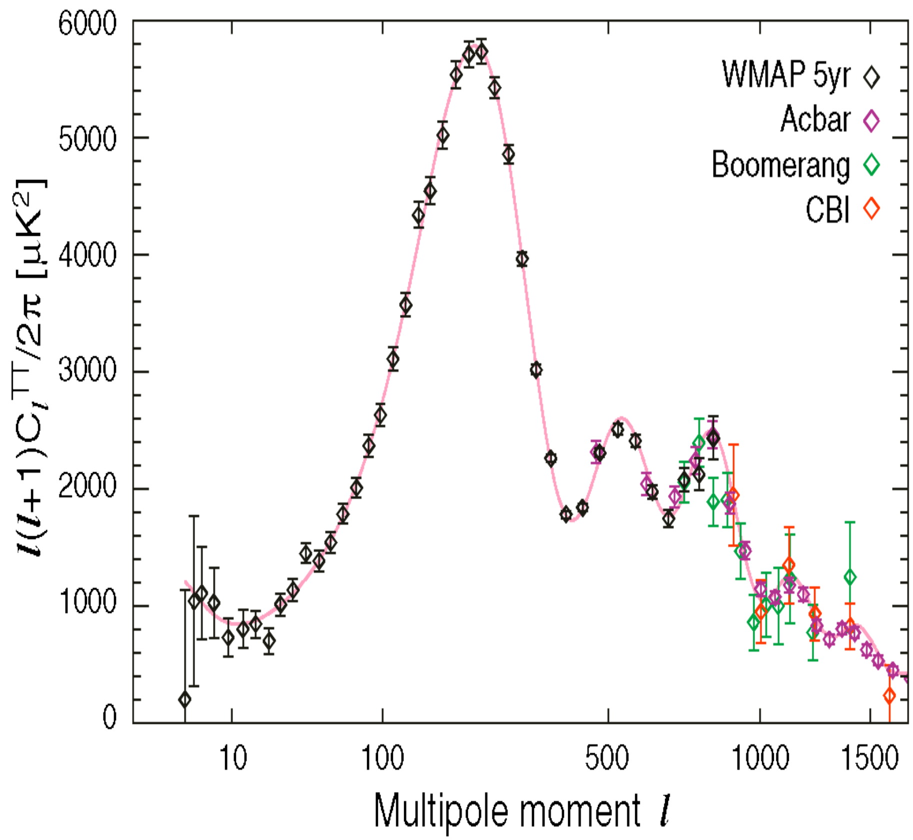
Outline

1. Motivation, CMB
2. Standard picture
3. He recombination
4. Two-photon decays in H
5. Lyman-α diffusion
6. What remains to be done?

Cosmic microwave background

The CMB has revolutionized cosmology:

- Tight parameter constraints (in combination with other data sets)
- Stringent test of standard assumptions: Gaussianity, adiabatic initial conditions
- Physically robust: understood from first principles. (Linear perturbation theory.)



WMAP Science Team (2008)

CMB and inflation

- Primordial scalar power spectrum: n_s, α_s

$$\ln k^3 P_\xi(k) = \text{const} + (n_s - 1)\ln\frac{k}{k_*} + \frac{1}{2}\alpha_s\left(\ln\frac{k}{k_*}\right)^2 + \ldots$$

  - measured by broad-band shape of CMB power spectrum
  - the damping tail will play a key role in the future (Planck, ACT, SPT, …)
  - probe of inflationary slow-roll parameters:

$$n_s = 1 - 6\varepsilon + 2\eta \qquad \alpha_s = -16\varepsilon\eta + 24\varepsilon^2 + 2\xi^2$$

(for single field inflation; but measurements key for all models)

This is the CMB theory!

$$\Psi = -\frac{4\pi G a^2}{k^2}\left(\delta\rho + \frac{3\mathcal{H}J}{k}\right); \quad \dot{\Psi} + \mathcal{H}\Phi = \frac{4\pi G a^2}{k}J; \quad \Phi - \Psi = \frac{8\pi G a^2 \hat{\Sigma}}{k^2}.$$

(One of these is redundant in the sense that it is implied by energy-momentum conservation.) The CDM evolution equations are

$$\dot{\delta}_c = -kv_c + 3\dot{\Psi}; \quad \dot{v}_c = -\mathcal{H}v_c + k\Phi.$$

For the baryons,

$$\dot{\delta}_b = -kv_b + 3\dot{\Psi}; \quad \dot{v}_b = -\mathcal{H}v_b + c_s^2 k\delta\rho_b + k\Phi + \frac{4\bar{\rho}_\gamma}{3\bar{\rho}_b}|\dot{\kappa}|(\Theta_{I,1}^{(m)} - v_b^{(m)}).$$

For the scalar fields,

$$\delta\ddot{\phi} = -2\mathcal{H}\delta\dot{\phi} - (k^2 + a^2 V'')\delta\phi + (\dot{\Phi} + 3\Psi)\dot{\phi} - 2a^2 V'\Phi.$$

For the massive neutrinos,

$$\delta\dot{f}_l = \frac{k\mathsf{q}}{a\mathcal{E}}\left(\frac{l}{2l-1}\delta f_{l-1} - \frac{l+1}{2l+3}\delta f_{l+1}\right) - \frac{df_0}{d\ln\mathsf{q}}\left(\dot{\Psi}\delta_{l,0} + k\Phi\frac{\mathsf{q}}{a\mathcal{E}}\delta_{l,1}\right);$$

for the photons,

$$\begin{aligned}
\dot{\Theta}_{I,l} &= k\left(\frac{l}{2l-1}\Theta_{I,l-1} - \frac{l+1}{2l+3}\Theta_{I,l+1}\right) + \dot{\Psi}\delta_{l,0} + k\Phi\delta_{l,1} \\
&\quad + |\dot{\kappa}|\left(\delta_{l,1}v_b - \Theta_{I,l} + \delta_{l,0}\Theta_{I,0} + \delta_{l,2}\frac{\Theta_{I,2} - \sqrt{6}\Theta_{E,2}}{10}\right); \\
\dot{\Theta}_{E,l} &= k\left(\frac{\sqrt{l^2-4}}{2l-1}\Theta_{E,l-1} - \frac{\sqrt{(l-1)(l+3)}}{2l+3}\Theta_{E,l+1}\right) + |\dot{\kappa}|\left(-\Theta_{E,l} + \delta_{l,2}\frac{6\Theta_{E,2} - \sqrt{6}\Theta_{I,2}}{10}\right)
\end{aligned}$$

and again $\Theta_{B,l} = \Theta_{V,l} = 0$. The metric sources are

$$\begin{aligned}
\delta\rho &= \bar{\rho}_c\delta_c + \bar{\rho}_b\delta_b + \frac{\dot{\phi}\delta\dot{\phi} - A\dot{\phi}^2}{a^2} + V'\delta\phi + \frac{4\pi g_\nu}{a^3}\int \mathsf{q}^2\mathcal{E}\delta f_0\, d\mathsf{q} + 4\bar{\rho}_\gamma\Theta_{I,0}; \\
J &= \rho_c v_c + \rho_b v_b + \frac{k\dot{\phi}\delta\phi}{a^2} + \frac{4\pi g_\nu}{3a^3}\int \mathsf{q}^3\delta f_1\, d\mathsf{q} + \frac{4}{3}\bar{\rho}_\gamma\Theta_{I,1}; \\
\hat{\Sigma} &= \frac{4\pi g_\nu}{5a^5}\int\frac{\mathsf{q}^4}{\mathcal{E}}\delta f_2\, d\mathsf{q} + \frac{4}{5}\bar{\rho}_\gamma\Theta_{I,2}.
\end{aligned}$$

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\end{aligned}$$

$$|\dot{\kappa}| = a\sigma_T n_e$$

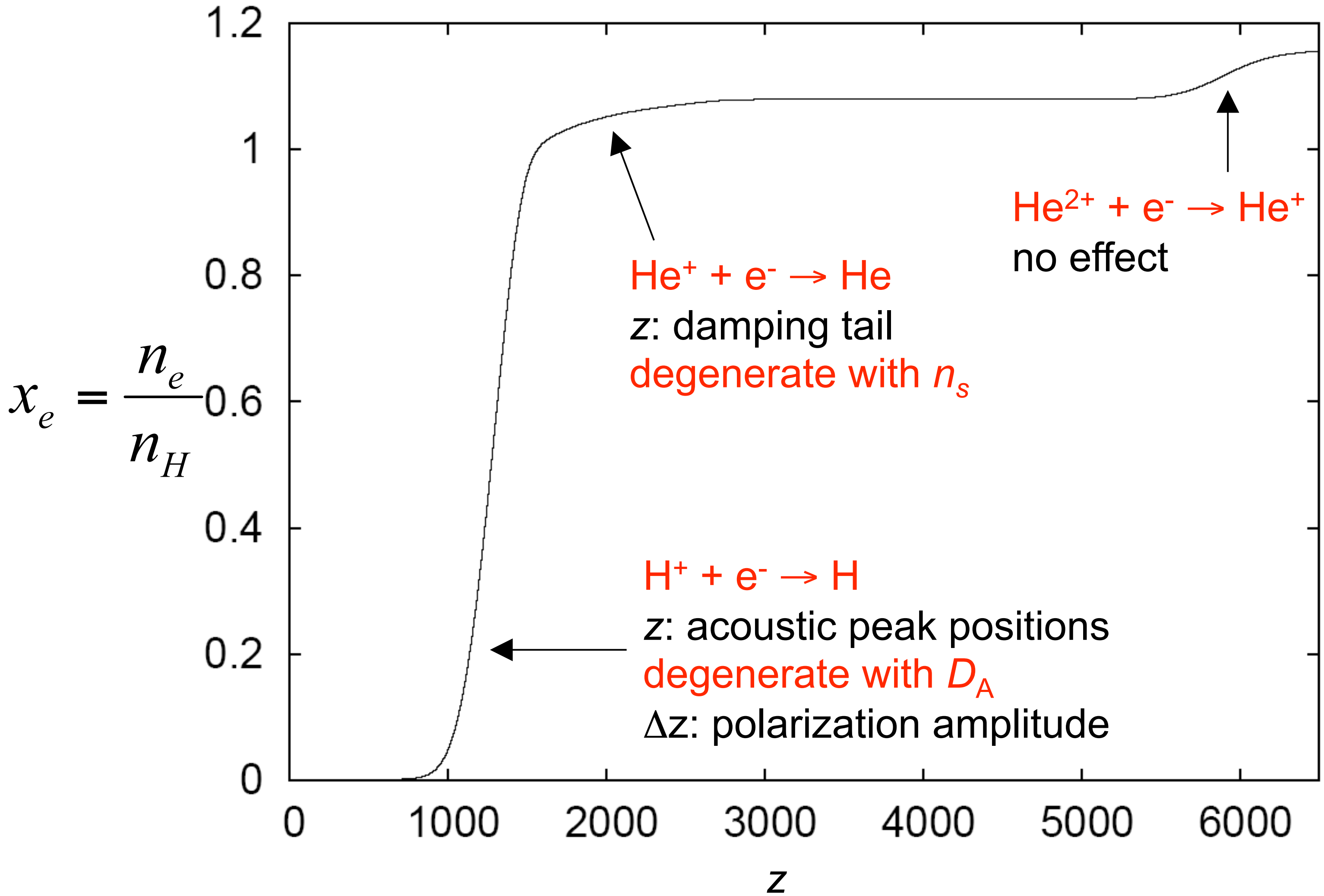
n_e = electron density (depends on recombination)

The Current Situation

- Different recombination histories disagree, sometimes at several percent level (e.g. Dubrovich & Grachev 2005).

- Not yet a problem for WMAP. But discrepancies are many sigmas for Planck.
  - e.g. 7σ from 2-photon decay corrections.

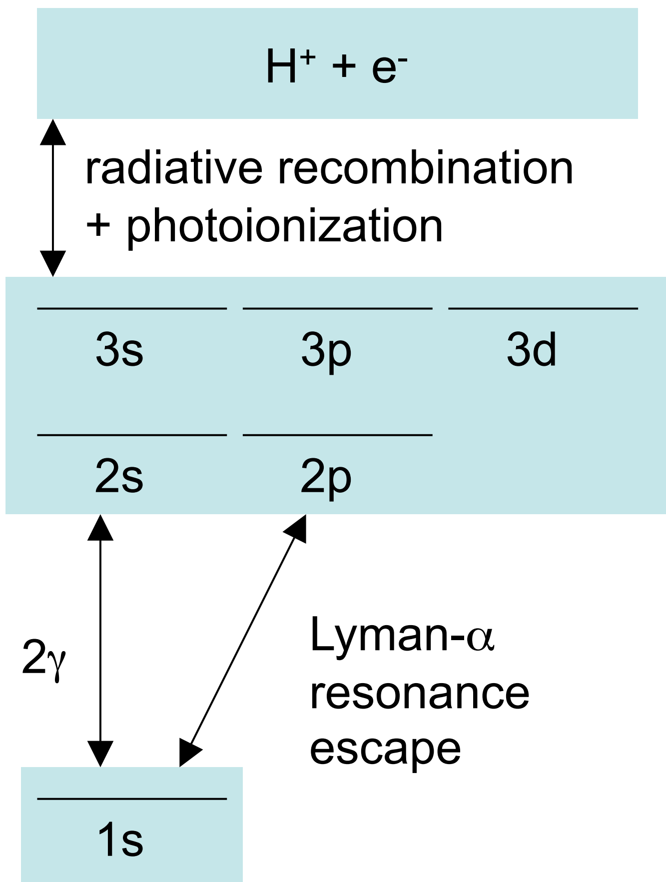
Recombination history



… as computed by RECFAST1.3 (Seager, Sasselov, Scott 2000)
The "standard" recombination code until Feb. 2008.

Standard theory of H recombination

(Peebles 1968, Zel'dovich et al 1968)

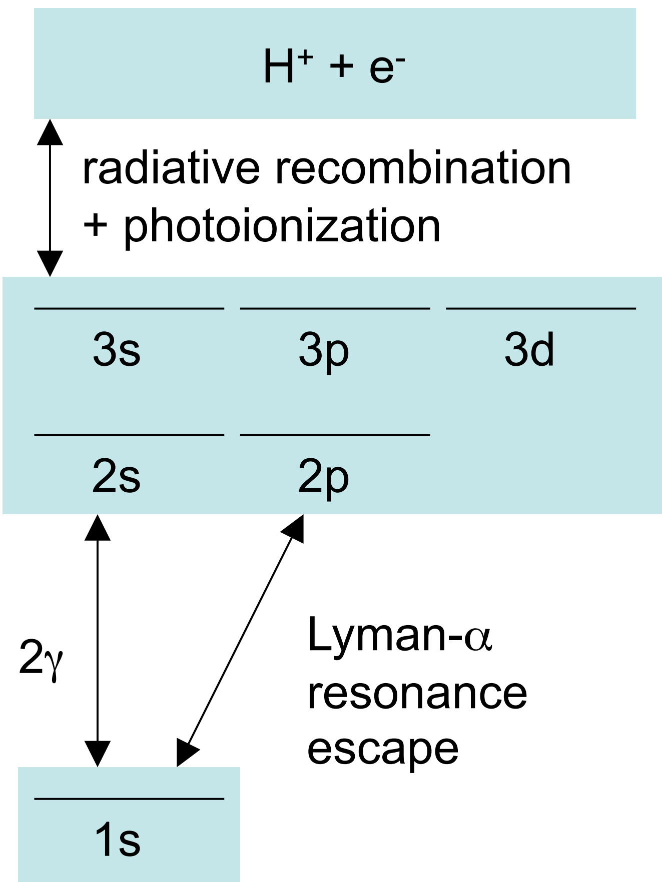


- Effective "three level atom": H ground state, H excited states, and continuum
- Direct recombination to ground state ineffective.
- Excited states originally assumed in equilibrium. (Seager et al followed each level individually and found a slightly faster recombination.)

2. Standard picture

Standard theory of H recombination

(Peebles 1968, Zel'dovich et al 1968)



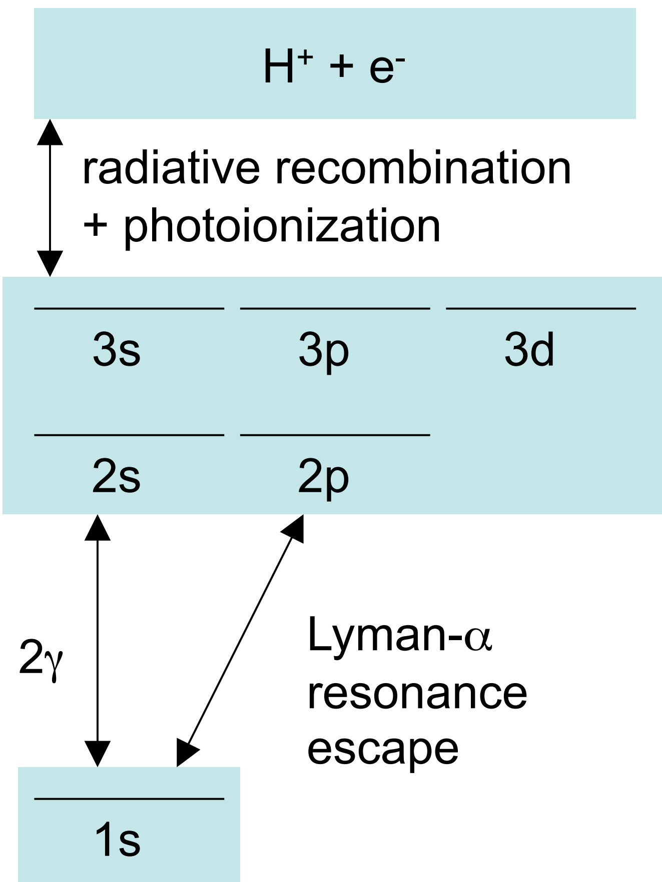
For H atom in excited level, 3 possible fates:

- 2γ decay to ground state ($\propto 2\Lambda$)
- Lyman-α resonance escape* ($\propto 6A_{Ly\alpha}P_{esc}$)
- photoionization ($\propto \sum_i g_i e^{-(E_i-E_2)/kT}\beta_i$)

* $P_{esc} \sim 1/\tau \sim 8\pi H/3n_{HI}A_{Ly\alpha}\lambda_{Ly\alpha}^3$.

Standard theory of H recombination

(Peebles 1968, Zel'dovich et al 1968)



- Effective recombination rate is recombination coefficient to excited states times branching fraction to ground state:

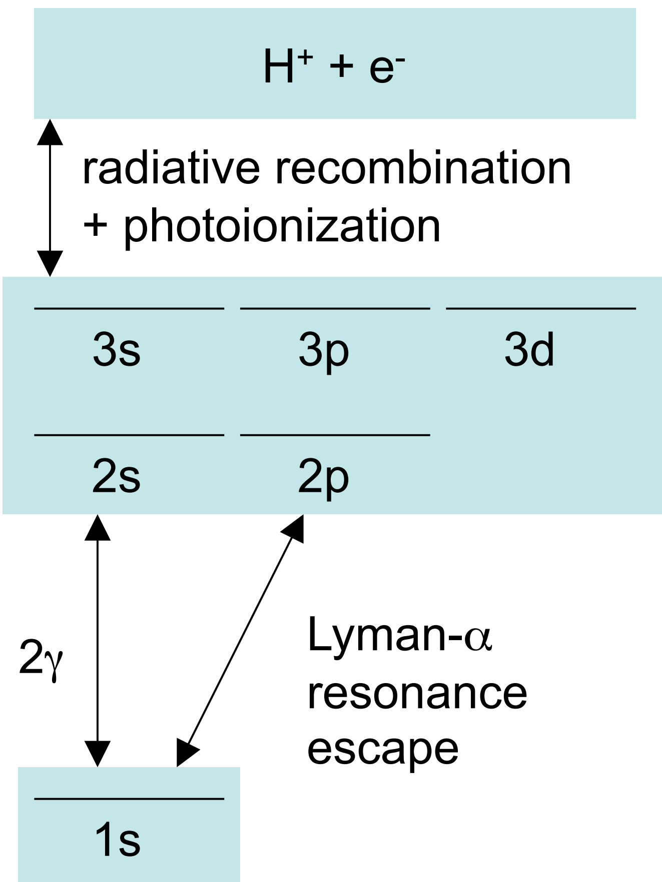
$$\frac{\#\mathrm{rec}}{\Delta \mathrm{V}\ \Delta \mathrm{t}} = \frac{2\Lambda + 6A_{Ly\alpha}P_{esc}}{2\Lambda + 6A_{Ly\alpha}P_{esc} + \sum_i g_i e^{-(E_i-E_2)/kT}\beta_i}\alpha_e n_e n_p$$

$$\alpha_e = \sum_{nl,n\geq 2}\alpha_{nl}$$

Standard theory of H recombination

(Peebles 1968, Zel'dovich et al 1968)

$$\frac{dx_{HI}}{dt} = \frac{2\Lambda + 6A_{Ly\alpha}P_{esc}}{2\Lambda + 6A_{Ly\alpha}P_{esc} + \sum_i g_i e^{-(E_i - E_2)/kT}\beta_i}\alpha_e\left[x_e x_p n_H - \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-R/kT} x_{HI}\right]$$



Λ = 2-photon decay rate from 2s

P_{esc} = escape probability from Lyman-α line

$A_{Ly\alpha}$ = Lyman-α decay rate

α_e = recombination rate to excited states

g_i = degeneracy of level i

β_i = photoionization rate from level i

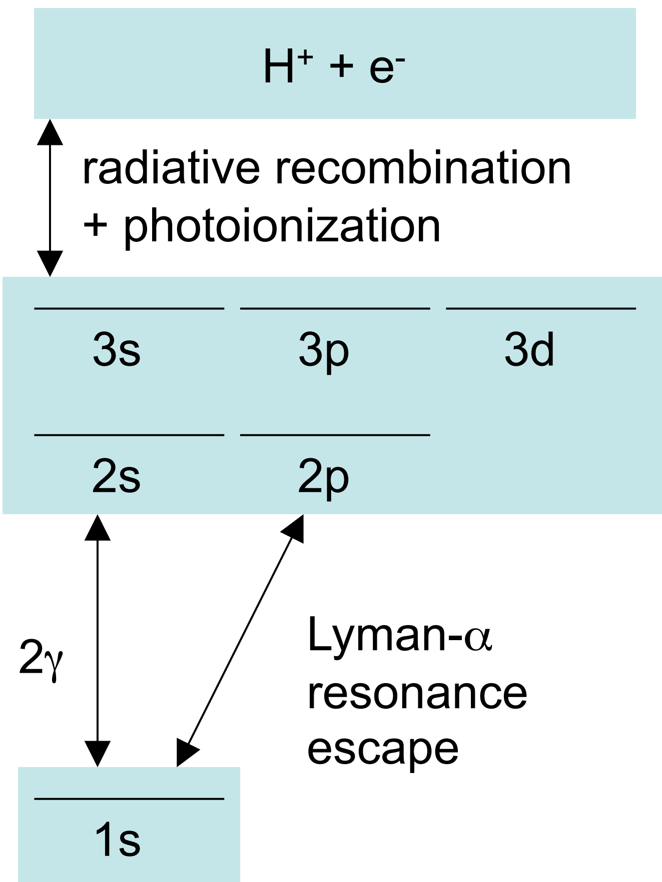
R = Rydberg

2. Standard picture

Standard theory of H recombination

(Peebles 1968, Zel'dovich et al 1968)

$$\frac{dx_{HI}}{dt} = \frac{2\Lambda + 6A_{Ly\alpha}P_{esc}}{2\Lambda + 6A_{Ly\alpha}P_{esc} + \sum_i g_i e^{-(E_i - E_2)/kT}\beta_i}\alpha_e\left[x_e x_p n_H - \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-R/kT} x_{HI}\right]$$



Λ = 2-photon decay rate from 2s

P_{esc} = escape probability from Lyman-α line = probability that Lyman-α photon will not re-excite another H atom.

Higher Λ or P_{esc} $\rightarrow$ faster recombination. If Λ or P_{esc} is large we have approximate Saha recombination.

Recent updates

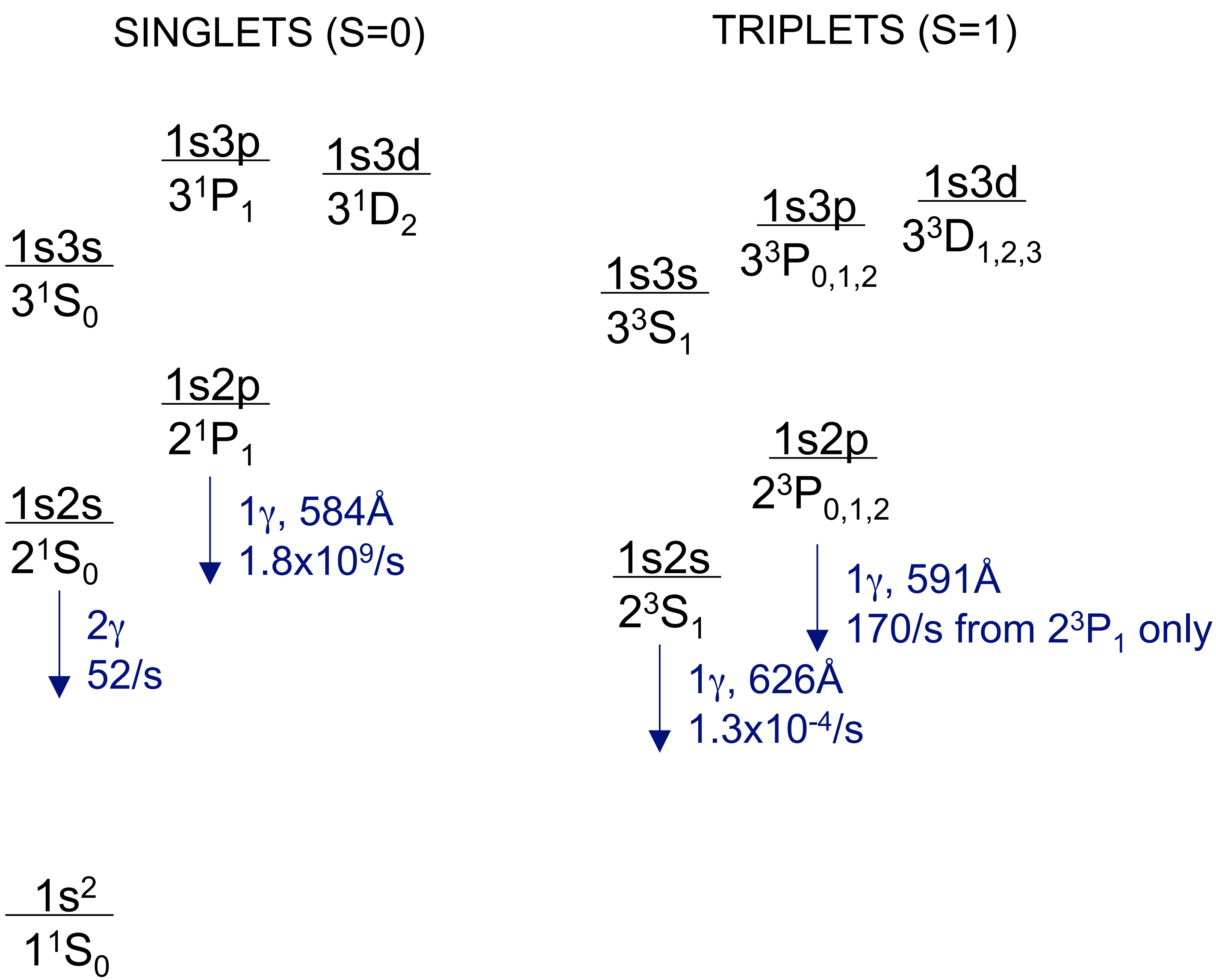
- Subject largely ignored for ~5 years, revived by new developments:

1. Paper by Dubrovich & Grachev (2005) claimed that recombination was significantly faster than Seager et al.
   - 2γ decays from highly excited levels:
     $H(3d) \rightarrow H(1s) + \gamma + \gamma$
     $He(3^1D_2) \rightarrow He(1^1S_0) + \gamma + \gamma$
   - semiforbidden decays:
     $He(2^3P_1) \rightarrow He(1^1S_0) + \gamma$.
2. Success of WMAP made percent-level CMB physics "real"; Planck coming soon!

- At least 3 groups working on comprehensive solution to the recombination problem (Wong & Scott @ UBC, Chluba & Sunyaev @ MPA, us).

3. He recombination

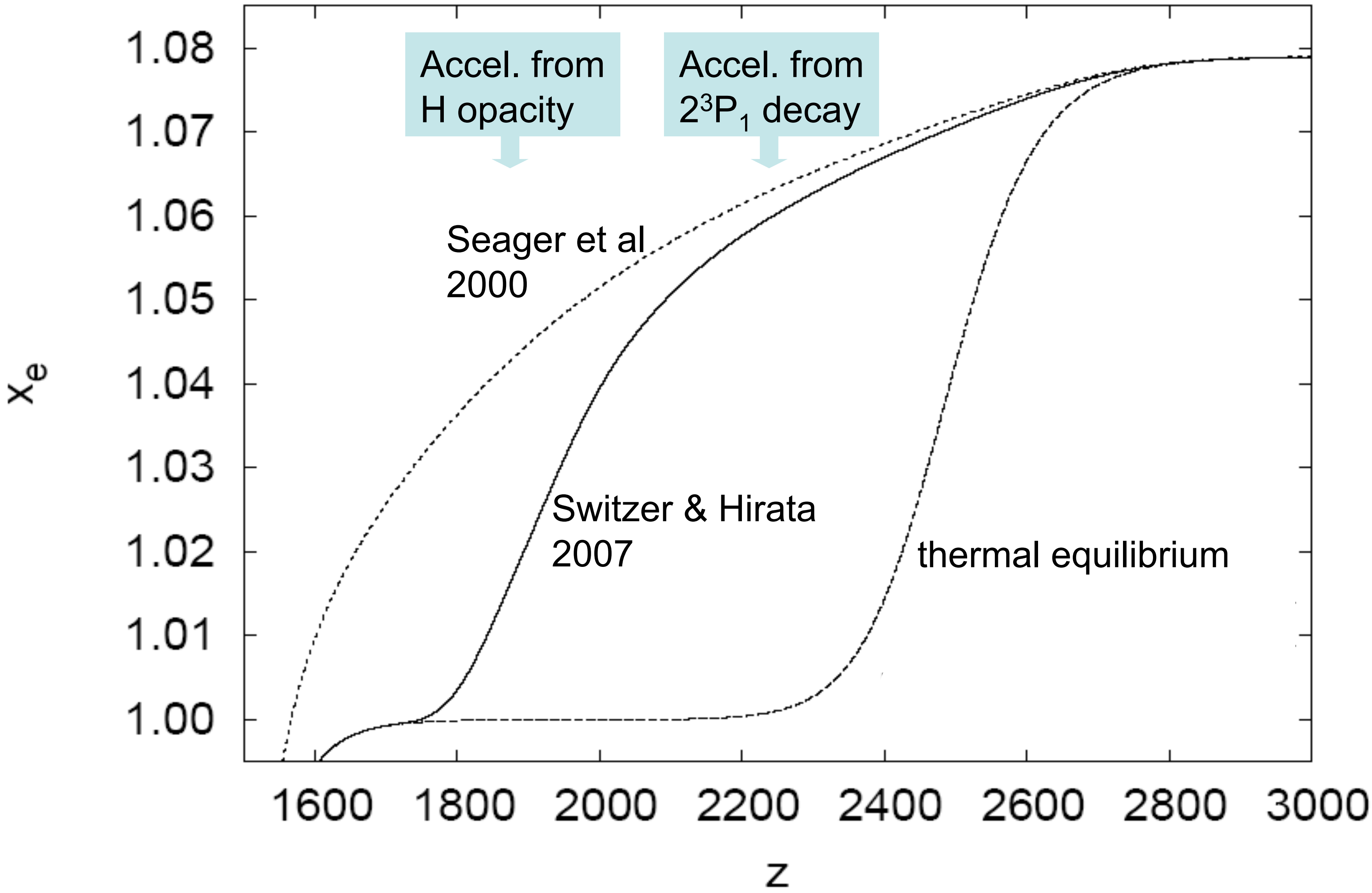
Helium level diagram



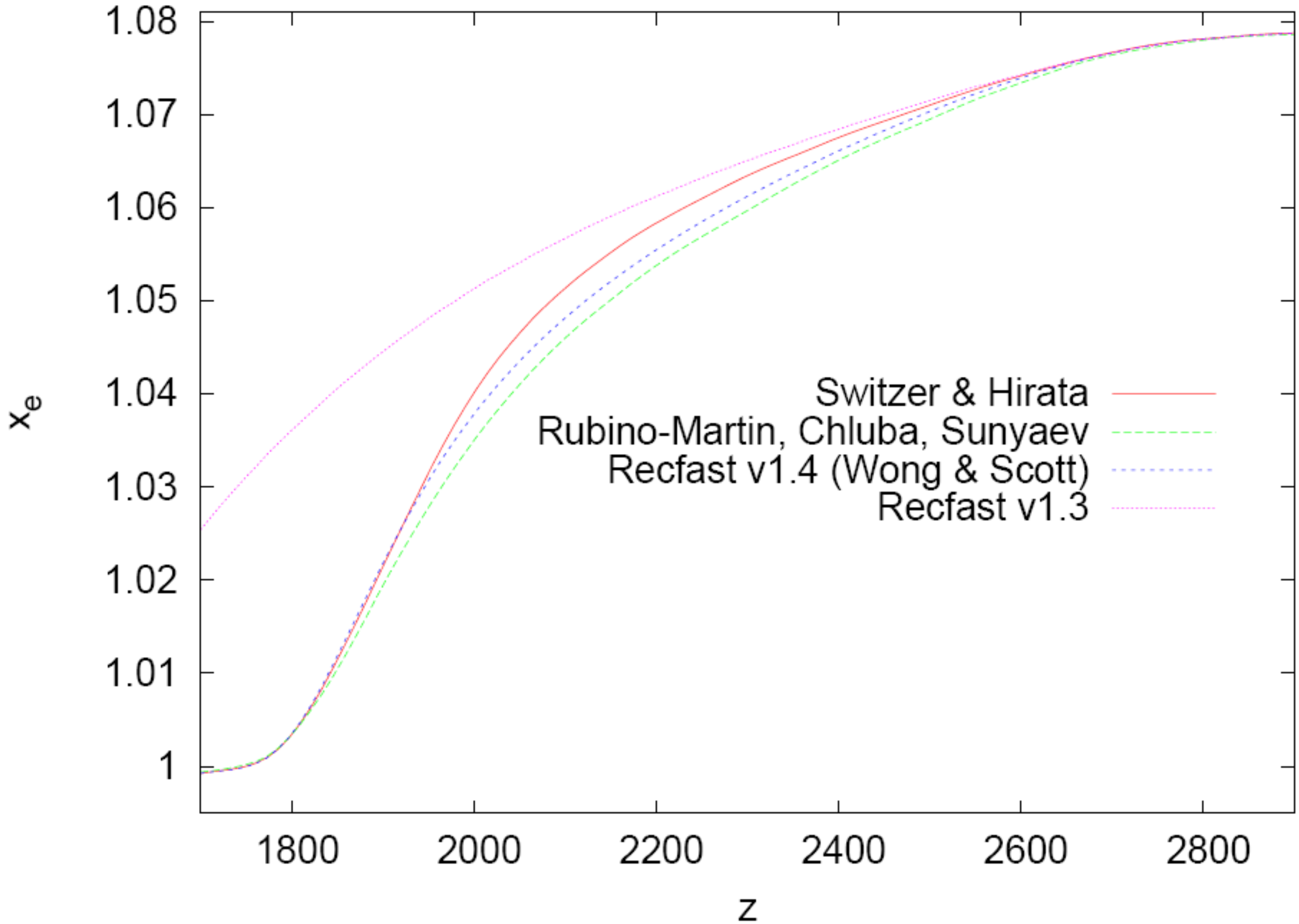
Issues in He recombination

- Mostly similar to H recombination except:
- Two line escape processes
  $He(2^1P_1) \rightarrow He(1^1S_0) + \gamma_{584Å}$
  $He(2^3P_1) \rightarrow He(1^1S_0) + \gamma_{591Å}$
- These are of comparable importance (Dubrovich & Grachev 2005).
- Feedback: redshifted radiation from blue line absorbed in redder line.
- Enhancement of escape probability by H opacity:
  $He(2^1P_1) \rightarrow He(1^1S_0) + \gamma_{584Å}$
  $H(1s) + \gamma_{584Å} \rightarrow H^+ + e^-$
  (Hu et al 1995)

The answer!


Accel. from
H opacity
Accel. from
2^3P_1 decay
Seager et al
2000
Switzer & Hirata
2007
thermal equilibrium
1.08
1.07
1.06
1.05
1.04
1.03
1.02
1.01
1.00
x_e
1600
1800
2000
2200
2400
2600
2800
3000
z

Current He recombination histories



Two-photon decays

- H(2s) → H(1s) + γ + γ (8.2 s^{-1}) included in all codes.
- But what about 2γ decays from other states?
- Selection rules: ns,nd only.
- Negligible under ordinary circumstances:
  H(3s,3d) → H(2p) + $\gamma_{6563Å}$, depopulates n≥3 levels.
- In cosmology:

$$\frac{n_{3d}}{n_{2s}} \approx 5e^{-E_{2,3}/kT_{CMB}} \approx 0.01$$

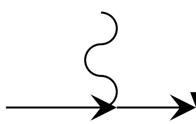
  so 3s,3d 2γ decays might compete with 2s (Dubrovich & Grachev 2005).
- Obvious solution: compute 2γ decay coefficients $\Lambda_{3s,3d}$, add to multilevel atom code.

Calculation p. 1

- Easy! This is tree-level QED.
- Feynman rules (in atomic basis set):

electron propagator $\dfrac{1}{E - E_{nlm} + i\varepsilon}$

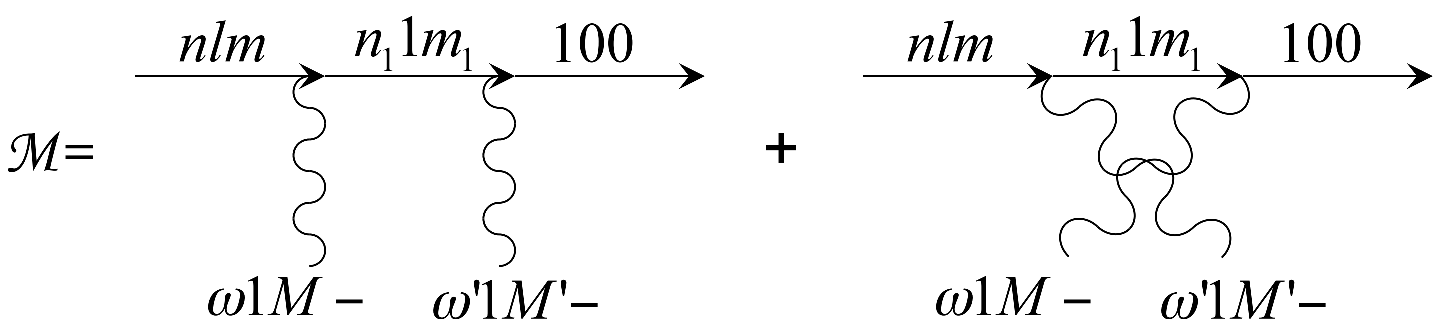
photon propagator $\dfrac{1}{E - k + i\varepsilon}$


vertex
(electric dipole) $(-1)^{m'} e k^{3/2} \langle n' l' \| r \| n l \rangle \begin{pmatrix} l' & 1 & l \\ -m' & M & m \end{pmatrix} \delta_{J\pi,1-}$

- Ignore positrons and electron spin – okay in nonrelativistic limit.

Calculation p. 2

- Two diagrams for 2γ decay:



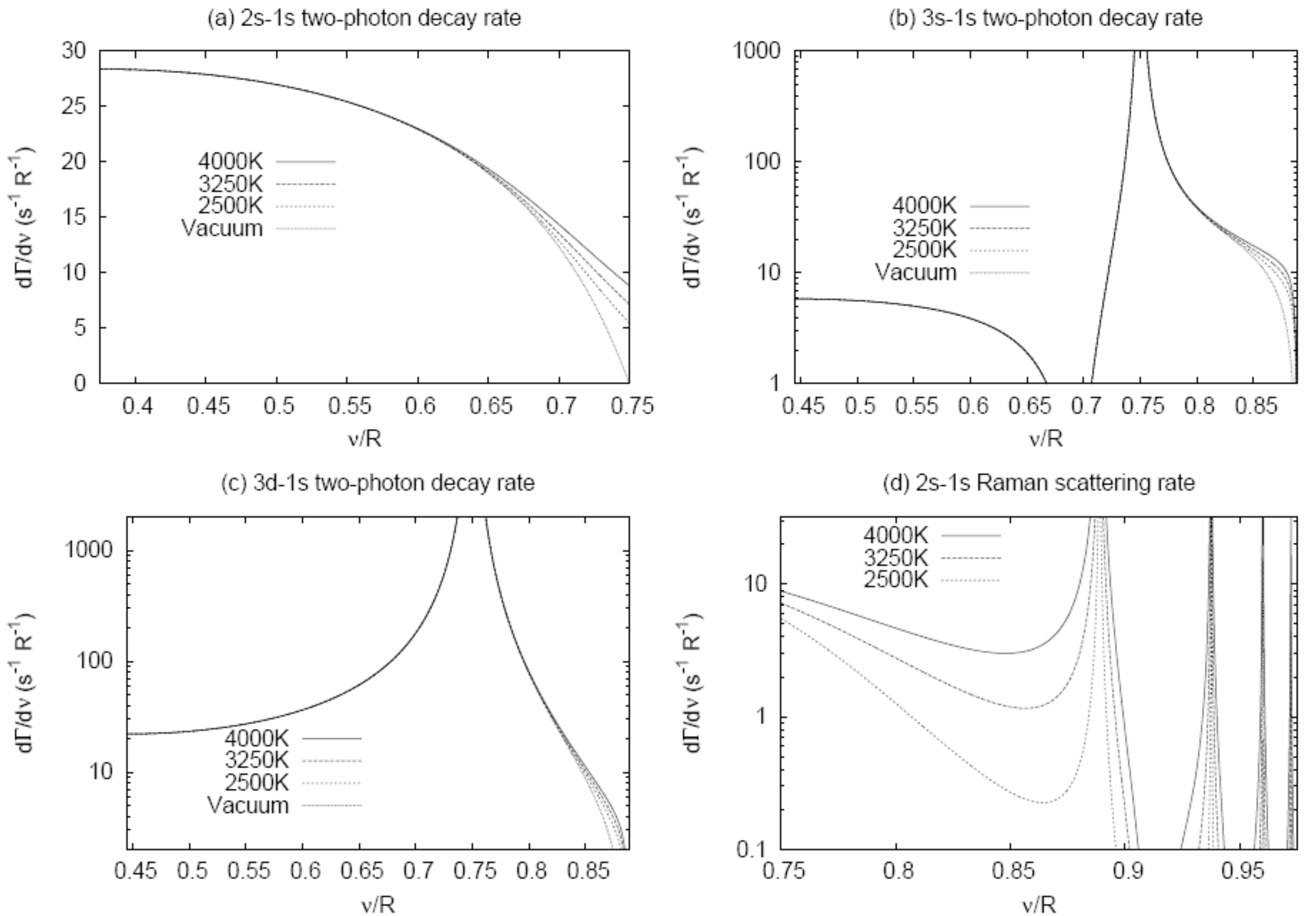
- Total decay rate: (ω=k for on-shell photon)

$$\Lambda_{nl} = \tfrac{1}{2}\int_0^{E(nl-1s)} |\mathcal{M}|^2 \, d\omega$$

$$\omega + \omega' = E(nl - 1s)$$

- Problem: infinite because $\mathcal{M}$ contains a pole if n_1=2 … n-1 (n_1p intermediate state is on-shell).

4. Two-photon decays in H



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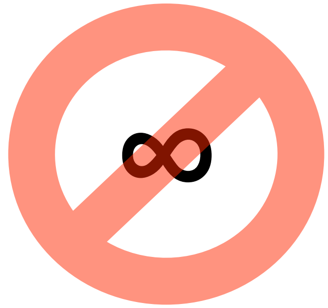


- The resolution to this problem in the cosmological context has provided some controversy. (See Dubrovich & Grachev 2005; Wong & Scott 2007; Hirata & Switzer 2007; Chluba & Sunyaev 2007; Hirata 2008).

- Pole displacement: rate still large, e.g. $\Lambda_{3d} = 6.5\times10^{7}\ s^{-1}$. Pole includes sequential 1γ decays, 3d→2p→1s.

- Re-absorption of 2γ radiation.

$$\mathrm{H(3d)} \leftrightarrow \mathrm{H(1s)} + \gamma_{>\mathrm{Ly}\alpha} + \gamma_{<\mathrm{H}\alpha}$$

$$\mathrm{H(2p)} \leftrightarrow \mathrm{H(1s)} + \gamma_{\mathrm{Ly}\alpha}$$

$$\mathrm{H(3d)} \leftrightarrow \mathrm{H(1s)} + \gamma_{<\mathrm{Ly}\alpha} + \gamma_{>\mathrm{H}\alpha}$$

No large rates or double-counting in optically thick limit.

Radiative transfer calculation

- A radiative transfer calculation is the only way to solve the problem.
- Must consistently include:
  - Stimulated 2γ emission (Chluba & Sunyaev 2006)
  - Absorption of spectral distortion (Kholupenko & Ivanchik 2006)
  - Decays from $n \geq 3$ levels.
  - Raman scattering – similar physics to 2γ decay, except one photon in initial state:
    $H(2s) + \gamma \rightarrow H(1s) + \gamma$
  - Two-photon recombination/photoionization.
- 2008 code did not have Lyman-α diffusion (now included – thanks to J. Forbes).

Radiative transfer calculation

- The Boltzmann equation:

$$\dot{f}_\nu = H\nu \frac{\partial f_\nu}{\partial \nu} + \frac{n_\mathrm{H} c^3}{8\pi\nu^2}\left(\sum_{nl} \Delta_{nl}^{2\gamma} + \sum_{nl} \Delta_{nl}^{\mathrm{Raman}} + \Delta^{2\gamma-\mathrm{rec}} \right)$$

- f_ν = photon phase space density
- Δ = number of decays / H nucleus / Hz / second

$$\Delta_{nl}^{2\gamma} = \frac{d\Lambda_{nl}}{d\nu}\left[(1+f_\nu)(1+f_{\nu'})x_{nl} - (2l+1) f_\nu f_{\nu'} x_{1s}\right]$$

- Ill-conditioned at Lyman lines: coefficients → ∞ (or large).
  - ➢ But solution is convergent:

$$\frac{d\Lambda_{nl}}{d\nu} \to \frac{A_{nl,2p}A_{2p,1s}}{4\pi^2(\nu-\nu_{\mathrm{Ly}\alpha})^2}; \qquad f_{\nu(\mathrm{Ly}\alpha)} \to \frac{\sum_{nl,n\geq 3} A_{nl,2p}(1+f_{\nu(nl,2p)})x_{nl}}{\sum_{nl,n\geq 3}(2l+1)A_{nl,2p} f_{\nu(nl,2p)} x_{1s}} \approx \frac{x_{2p}}{3x_{1s}}$$

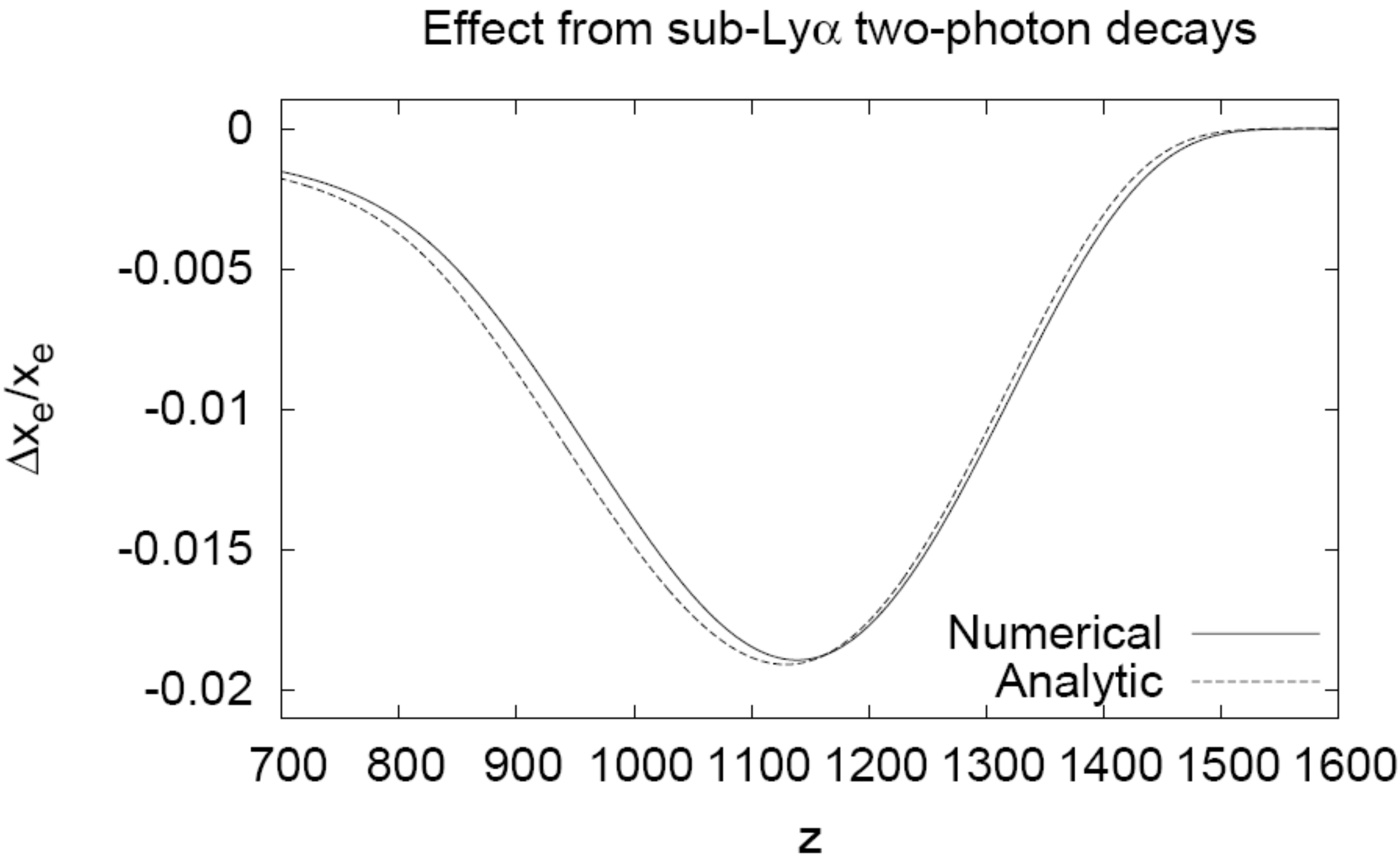
Physical effects 1

- Definitions:
  - a 2γ decay is "sub-Lyα" if both photons have E<E(Lyα).
  - a 2γ decay is "super-Lyα" if one photon has E>E(Lyα).
- Sub-Lyα decays:
  - Accelerate recombination by providing additional path to the ground state.
  - Delay recombination by absorbing thermal + redshifted Lyα photons.
  - The acceleration always wins, i.e. reaction:

$$H(nl) \leftrightarrow H(1s) + \gamma + \gamma$$

proceeds forward.

4. Two-photon decays in H



Physical effects 2

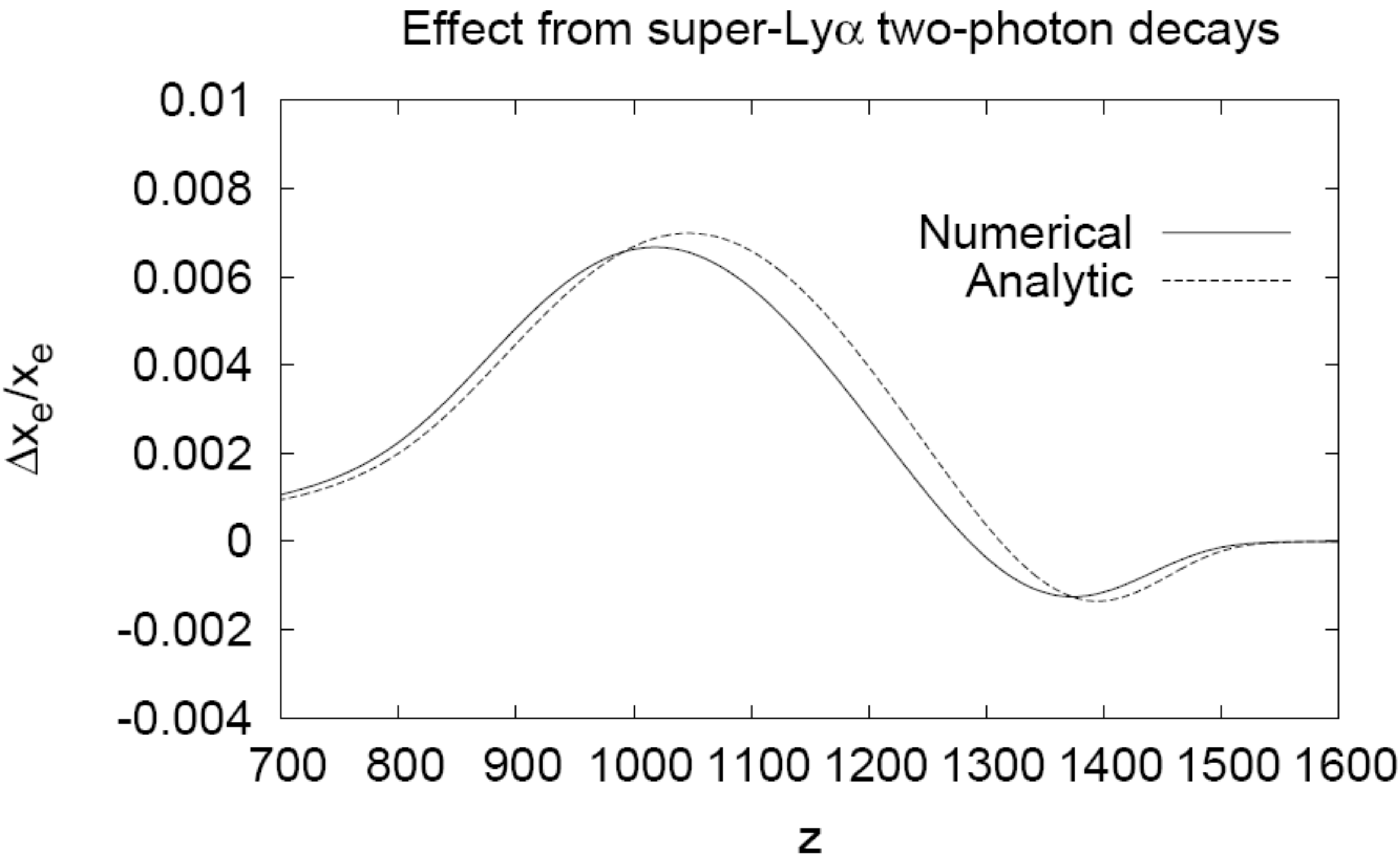
- Super-Lyα decays are trickier!
  - Also provide additional path to ground state.
  - But for every super-Lyα decay there will later be a Lyα excitation, e.g.:

$$H(3d) \rightarrow H(1s) + \gamma_{<1216Å} + \gamma_{>6563Å}$$

$$H(1s) + \gamma_{1216Å} \rightarrow H(2p)$$

- The net number of decays to the ground state is zero.
- But there is an effect:
  - Early, z>1260: accelerated recombination.
  - Later, z<1260: delayed recombination.
- Same situation for Raman scattering.

4. Two-photon decays in H



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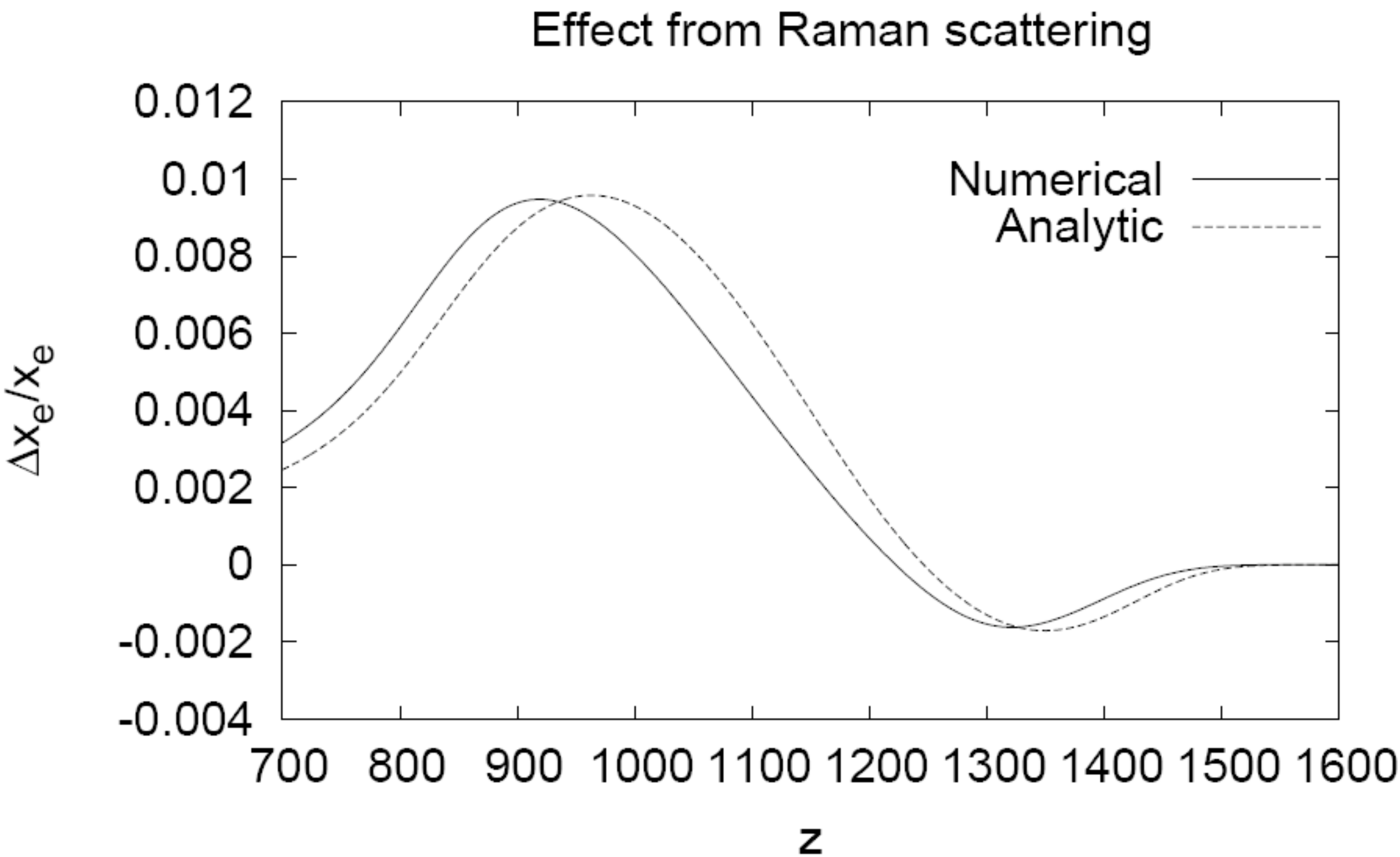
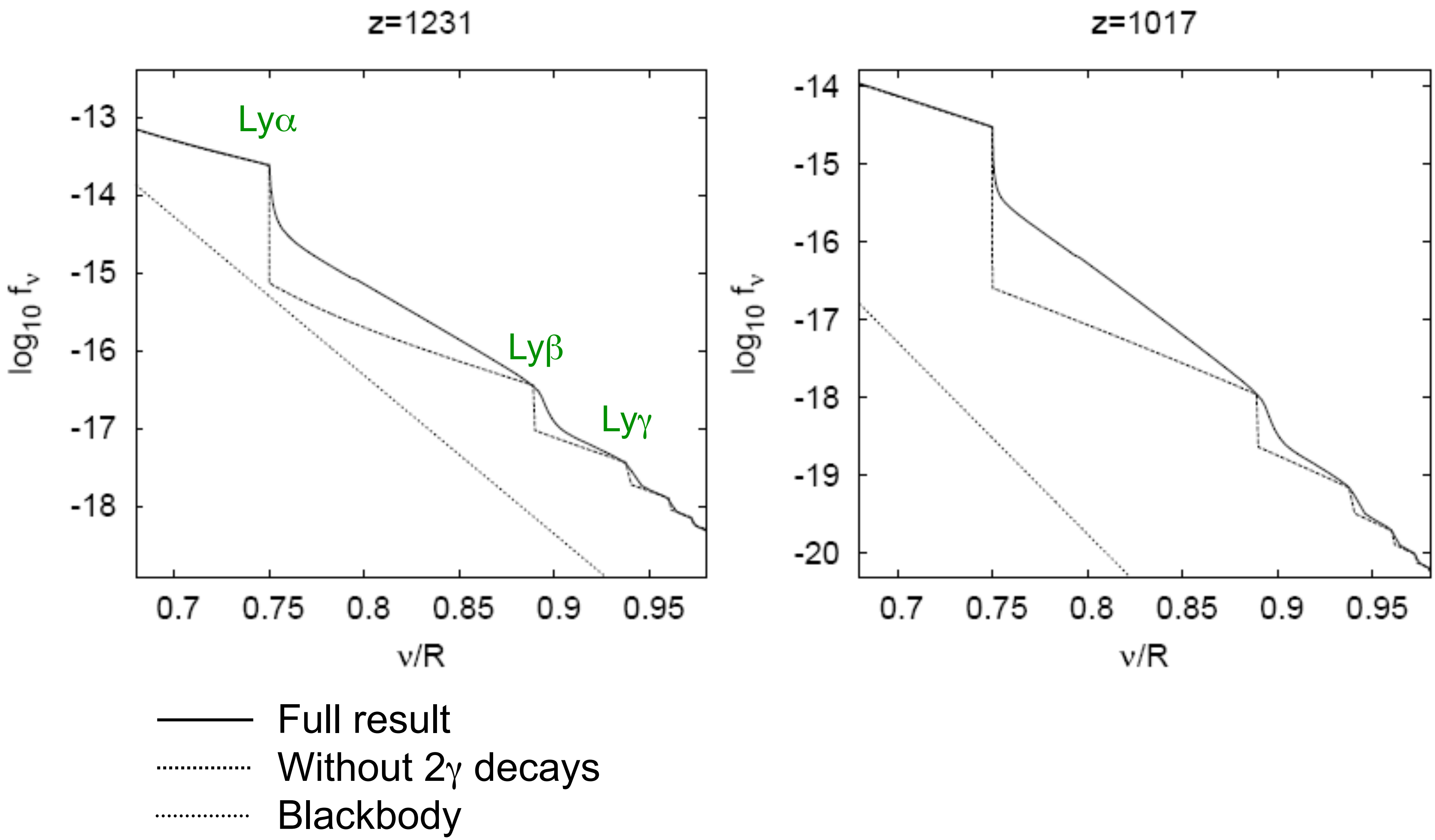


Phase space density



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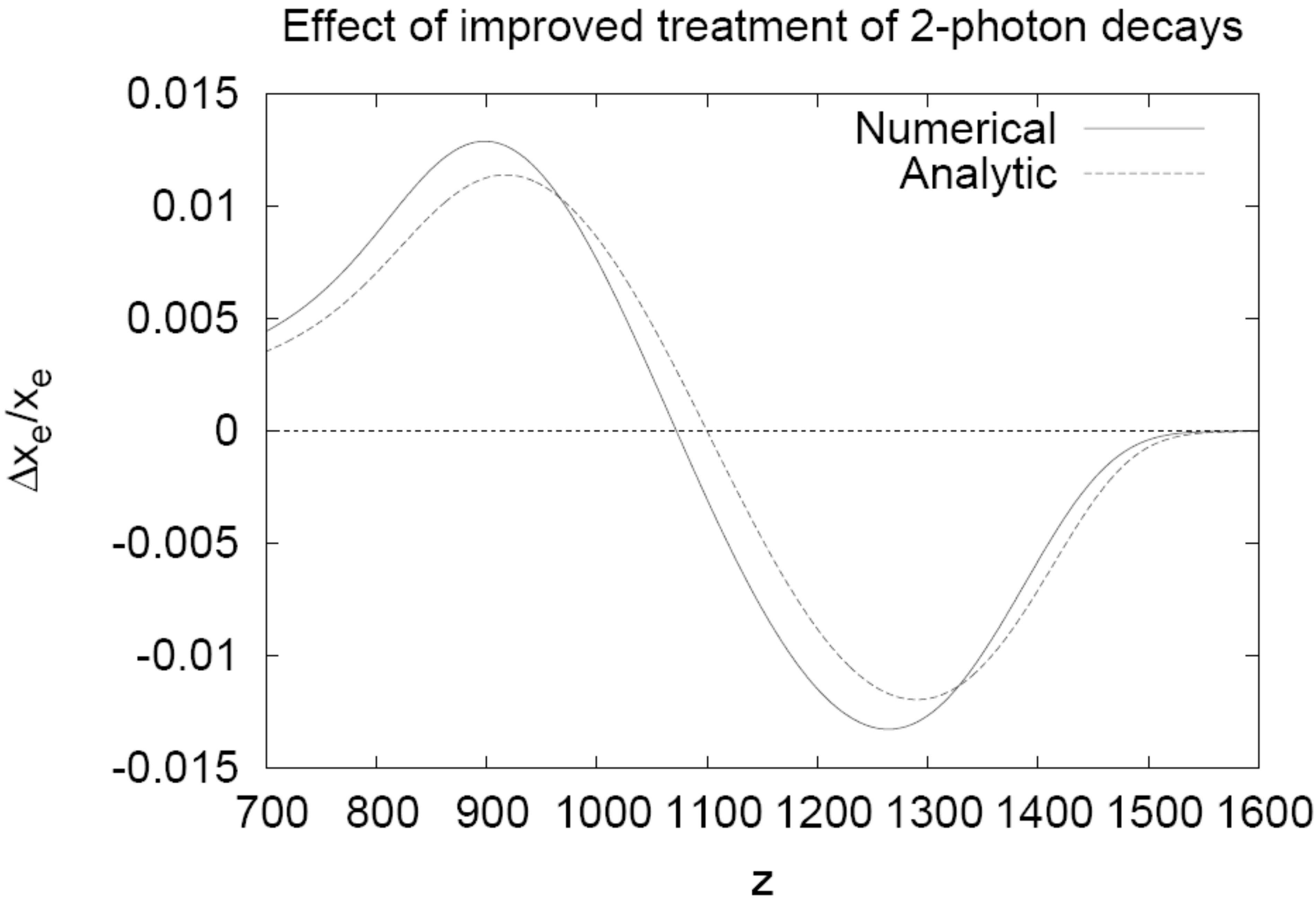

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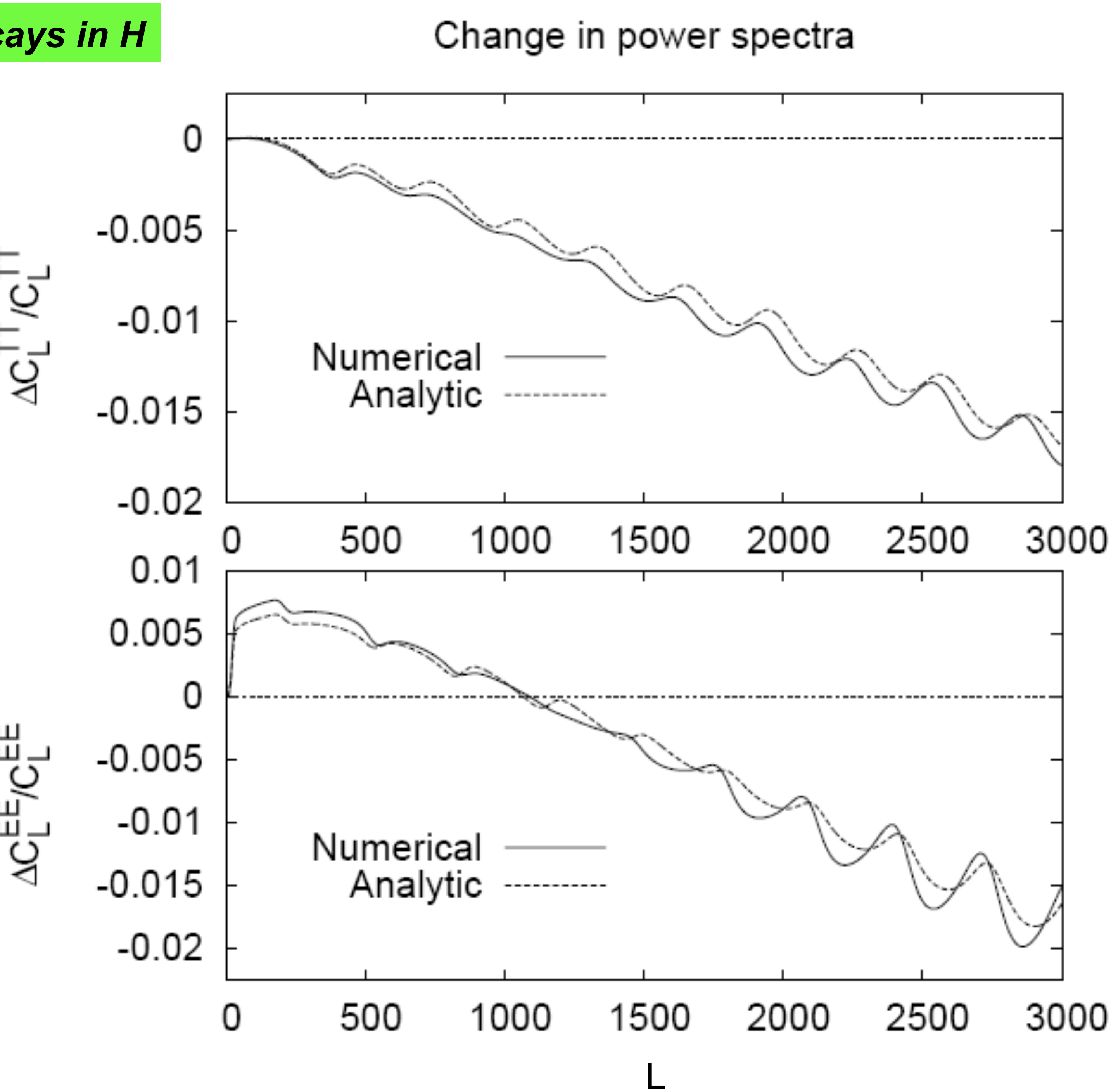


<u>Correction:</u>

0.19σ	ACBAR
0.27σ	WMAP5
7σ	Planck

Doppler shifts & Lyman-α diffusion

- So far we've neglected thermal motions of atoms.
- Main effect is in Lyman-α where resonant scattering

$$\mathrm{H(1s)} + \gamma_{\mathrm{Ly}\alpha} \to \mathrm{H(2p)}_{\mathrm{virtual}} \to \mathrm{H(1s)} + \gamma_{\mathrm{Ly}\alpha}$$

leads to diffusion in frequency space due to Doppler shift of H atoms. Diffusion coefficient is

$$D(\nu) = H\nu_{\mathrm{Ly}\alpha} \cdot \tau_{\mathrm{Ly}\alpha} f_{\mathrm{scat}} \phi(\nu) \cdot \frac{T\nu_{\mathrm{Ly}\alpha}^2}{m_{\mathrm{H}}}$$

- Rapid diffusion near line center, very slow in wings.

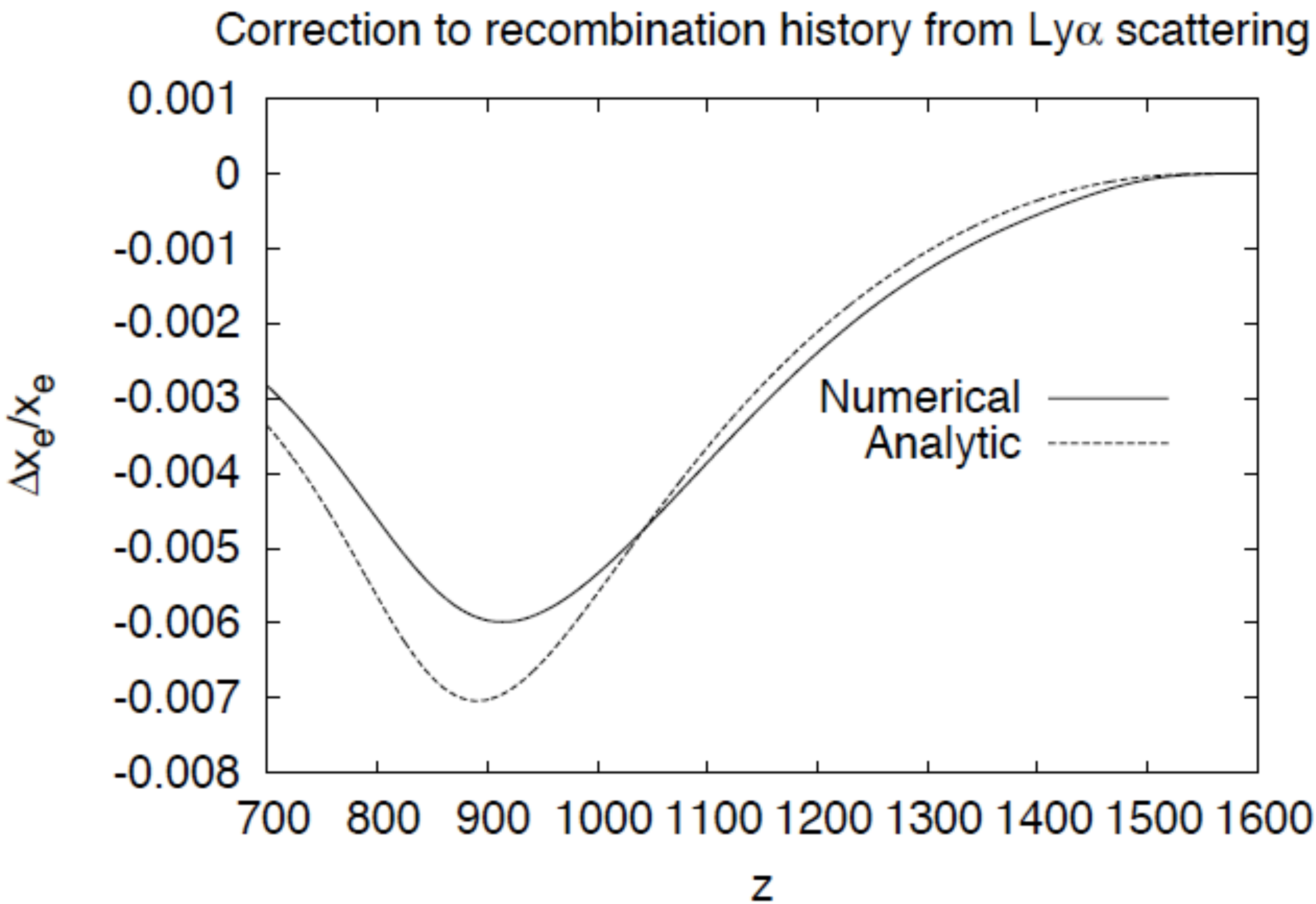
Doppler shifts & Lyman-α diffusion

- Can construct Fokker-Planck equation from two physical conditions (e.g. Rybicki 2006):
  - Exactly conserve photons in scattering
  - Respect the second law of thermodynamics: must preserve blackbody with μ-distortion, $f_\nu \sim e^{-h\nu/T}$.

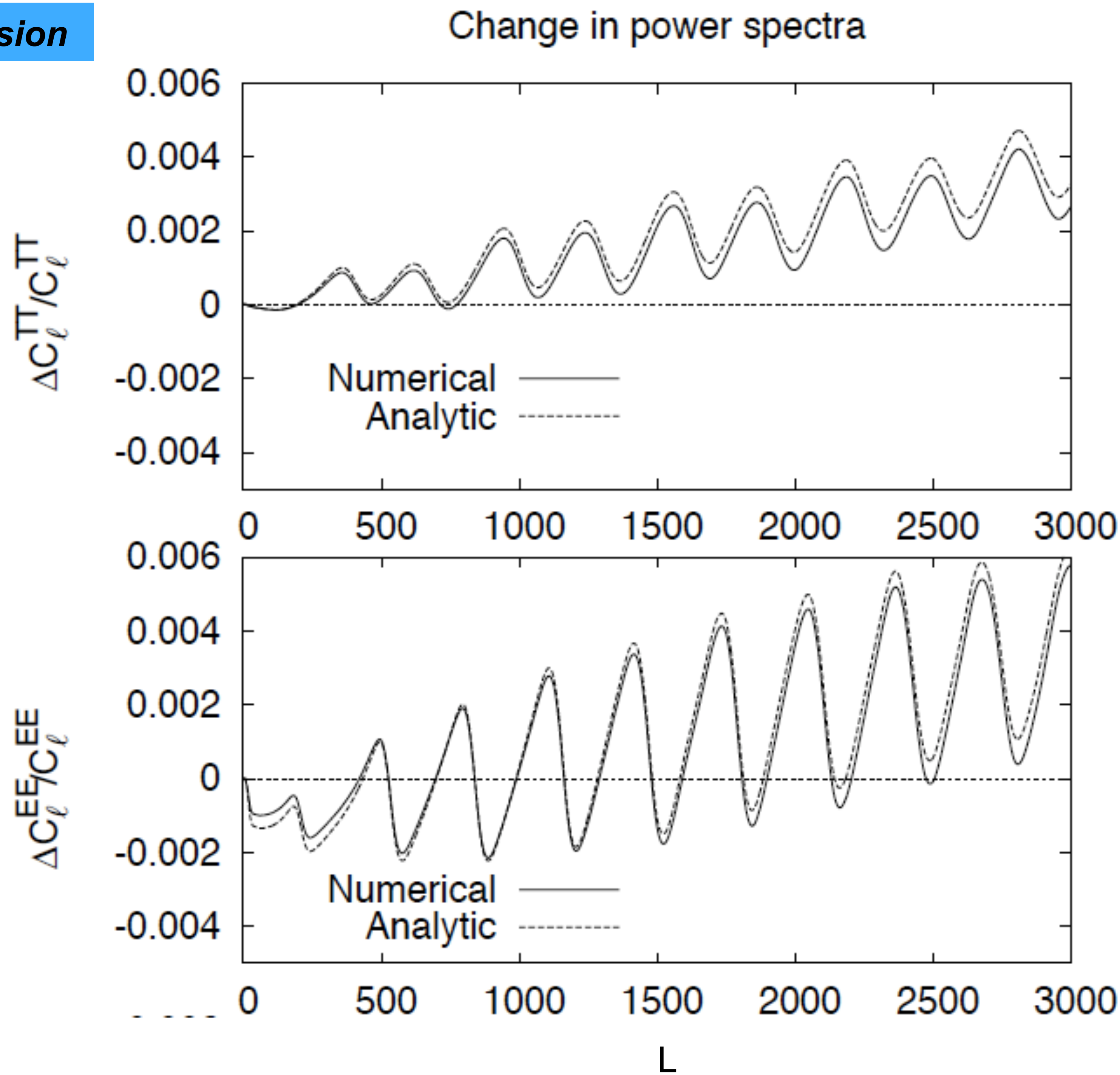
$$\dot{f}_\nu\Big|_{\rm diff} = \frac{1}{\nu^2}\frac{\partial}{\partial \nu}\left[\nu^2 D(\nu)\left(\frac{\partial f_\nu}{\partial \nu} + \frac{h f_\nu}{T}\right)\right]$$

- hf_ν/T term can be physically interpreted as due to recoil (e.g. Krolik 1990; Grachev & Dubrovich 2008). Effect is to push photons to red side of Lyman-α, speeding up recombination.
- With J. Forbes, diffusion now patched on to 2γ radiative transfer code.

5. Lyman-α diffusion



5. Lyman-α diffusion



Recombination theory

- Many known corrections to H recombination.
  - All current codes are missing some important effects.
  - Some processes (e.g. 2γ decays, Lyα diffusion) interact. Δx_e not additive.
- Lots of physics not yet investigated (forbidden lines, molecular opacity, fine structure) – most probably unimportant but not excluded.
- Existing codes far too slow for use in Markov chain parameter estimation (~1 day).
- Needs:
  - Comprehensive calculation of H by multiple groups.
  - Fast approximate code.

Ongoing efforts

- Feedback of He recombination photons on H (E. Switzer).
- Incorporation of high-n levels (D. Grin) – sparse matrix techniques should allow us to follow up to n~1000 with full l resolution. (Current: n_{max}=200.)
- The following are being worked but the preliminary result is that they appear to be insignificant:
  - Molecular opacity
  - Fine structure, e.g. 11 GHz line, $2p_{3/2} \rightarrow 2p_{1/2}$. (Population inverted but optically thin.)
  - Quadrupole lines
  - Thomson scattering
  - Modification of Sobolev depth in Doppler-broadened high-n Lyman lines (Y. Ali-Haïmoud).

Implications for experiments

- Recfast v1.4 (Wong, Moss, Scott 2008; in CAMB) good enough for WMAP/ACBAR – but don't push it any farther.
- No show-stoppers yet in the theory
  - Given the resources, predictions to cosmic variance accuracy are achievable (but not achieved).
  - But this will take time (~1 year?)
- Testing the results:
  - We need to get the polarization. EE:TT ratio is a signature of modified H recombination history. Cannot be mimicked by modifying n_s,w,etc.
  - Spectral distortion – in principle most direct and informative test.
  - More laboratory tests of atomic data.