From Einstein through Vainstein to dRGT design

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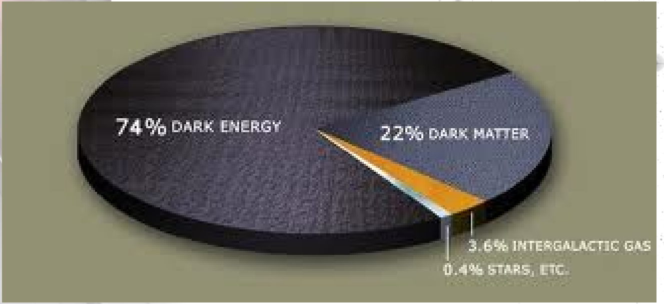
outline

Universe is accelerating

An accelerating Universe

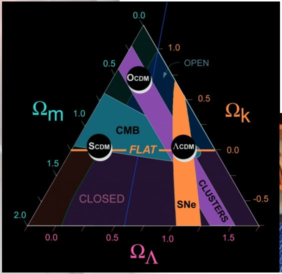
- **Maybe this is more illustrative**

Camembert of the Universe, DE$\approx 70\%$



"The Universe never did make sense; I suspect it was built on government contract". (Robert A. Heinlein)

Observations indicate an accelerating Universe

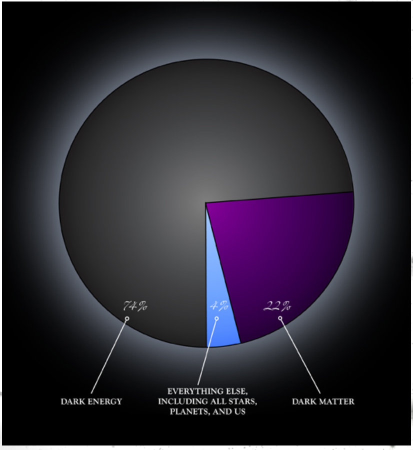


- homogeneous and isotropic universe $ds^2 = dt^2 - a(t)^2 d\vec{x}^2$
- equations of general relativity $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$
- for an accelerating universe:
  $\ddot{a} \gg 0 \rightarrow \qquad p \ll -\frac{1}{3}\rho$

What is Dark Energy?

3 options?

- **Cosmological Constant**
  (Why is it so small?)
  $\rightarrow$ cosmological constant problem?
- **Dark Energy**
  (Why don't we see them? Similar fine-tuning problem?)
- **Dark Gravity**
  (Is there any viable model?)
  $\rightarrow$ massive gravity?



Infra-red Modification of GR

Motivations for IR Modification of GR

- a very nice alternative to the CC or dark energy for explaining the recent acceleration of the Hubble expansion
- a way of attacking the Cosmological Constant problem (fine-tuning problem)
  $\Lambda_{\text{phys}} = \Lambda_{\text{bare}} + \Delta\Lambda \sim (10^{-3}\text{eV})^4$ with $\Delta\Lambda \sim \text{TeV}^4$
- learn more about GR by modifying it!
- fun!

Dark Gravity

Lets concentrate on the third option: **Modifying gravity**

Maybe not modifying that much! only close to the horizon scale (~ 1Gpc$/h$) , corresponding to modifying gravity today.

Modifying Gravity in the Infrared (IR)

typically requires new degrees of freedom

New degrees of freedom (dof) in the IR

Modifying gravity in the IR typically requires new dof
usually: scalar field

$$\mathcal{L} = -\frac{1}{2}\mathcal{Z}_\phi(\partial\delta\phi)^2 - \frac{1}{2}{m_\phi}^2(\delta\phi)^2 - g_\phi\delta\phi T$$

where these quantities $\mathcal{Z}_\phi, m_\phi, g_\phi$ depend on the field.

Density dependent mass

- Chameleon
  m_ϕ depends on the environment
  (Khoury, Weltman 2004)
  $\rightarrow f(R)$ theories

Density dependent coupling

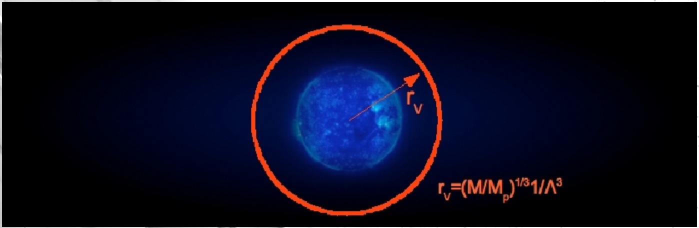
- **Vainshtein**(1971)
  $\mathcal{Z}_\phi$ depends on the environment
  $\rightarrow$ massive gravity, Galileons
- Symmetron
  g_ϕ depends on the environment
  (Hinterbichler, Khoury 2010)

Vainshtein mechanism (massive gravity, Galileons)

$$\mathcal{L} = -\frac{1}{2}\mathcal{Z}_\phi(\partial\delta\phi)^2 - \frac{1}{2}{m_\phi}^2(\delta\phi)^2 - g_\phi\delta\phi T$$

Screening with

- effective coupling to matter depends on the self-interactions of these new dof
  $\Box\delta\phi \sim \frac{1}{M_p}\frac{\delta\phi}{\sqrt{\mathcal{Z}}}T$
  $\rightarrow$ coupling small for properly canonically normalized field!
  ($\mathcal{Z} \gg 1 \rightarrow$ coupling small)
- non-linearities dominate within Vainshtein radius



Chameleon mechanism ($f(R)$ theories)

important ingredients: a conformal coupling between the scalar and the matter fields $\tilde{g}_{\mu\nu} = g_{\mu\nu}A^2(\phi)$, and a potential for the scalar field $V(\phi)$ which includes relevant self-interaction terms.

$S = \int d^4x\sqrt{-g}\left(\frac{M_p^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right) + S_{matter}[g_{\mu\nu}A^2(\phi)]$

The equation of motion for ϕ:

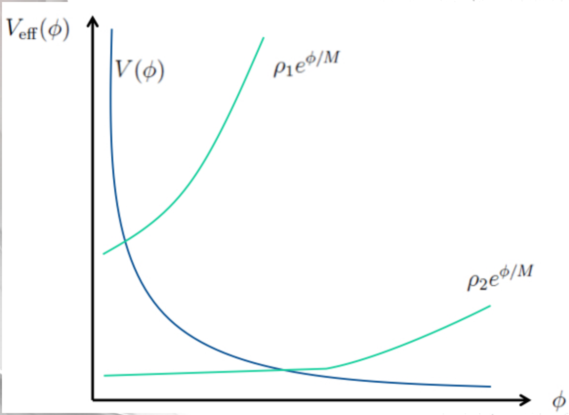
$$\nabla^2\phi = V_{,\phi} - A^3(\phi)A_{,\phi}\tilde{T} = V_{,\phi} + \rho A_{,\phi}$$

where $\tilde{T} \sim \rho/A^3(\phi)$

giving rise

to an effective potential
$V_{\text{eff}}(\phi) = V(\phi) + \rho A(\phi)$

Chameleon mechanism ($f(R)$ theories)



- mass of the new dof depends on the local density (m_ϕ large in regions of high density)

Symmetron mechanism

important ingredients:

$$S = \int d^4x\sqrt{-g}\left(\frac{M_p^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right) + S_{matter}[g_{\mu\nu}A^2(\phi)]$$

with a symmetry-breaking potential

$$V(\phi) = \frac{-1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$$

and a conformal coupling to matter of the form

$$A(\phi) = 1 + \frac{\phi^2}{2M^2} + \mathcal{O}(\phi^4/M^4)$$

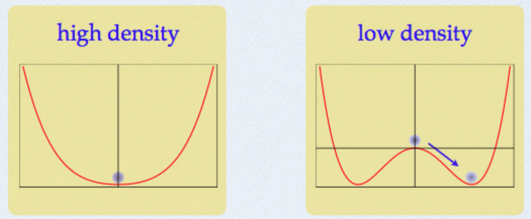
giving rise

to an effective potential

$$V_{\text{eff}} = \frac{1}{2}\left(\frac{\rho}{M^2} - \mu^2\right)\phi^2 + \frac{1}{4}\lambda\phi^4$$

$\rho > \mu^2 M^2 \rightarrow$ the field sits in a minimum at the origin

Symmetron mechanism



- perturbations couple as $\frac{\bar{\phi}}{M^2}\delta\phi\rho$
- In high density symmetry-restoring environments, the scalar field vev $\sim 0 \to$ fluctuations of the field do not couple to matter
- As the local density drops the symmetry of the field is spontaneously broken and the field falls into one of the two new minima with a non-zero vev.
- $\to$ coupling to matter depends on the enviroment (g small in regions of high density)

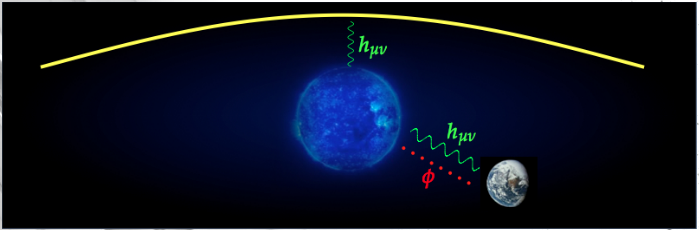
Massive Gravity

A general linear mass term for the graviton is

$$\mathcal{L}_{mass} = -\frac{1}{2} M_p^2 (m_1^2 h^{\mu\nu} h_{\mu\nu} + m_2^2 h^2)$$

The only **ghost-free**: $m_1^2 = -m_2^2$ Fierz-Pauli tuning

$\rightarrow$ **vDVZ discontinuity**



Massive Gravity

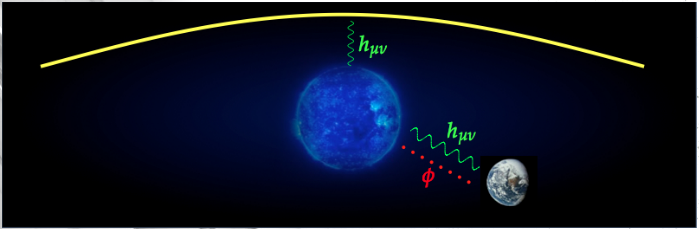
Introduce the Stueckelberg fields A_μ and ϕ to restore back the gauge symmetry

$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu$ and $A_\mu \to A_\mu + \partial_\mu \phi$

After canonically normalizing the fields, diagonalizing the interactions, taking the decoupling limit $m \to 0$ one obtains

$$-h_{\mu\nu}\mathcal{E}^{\mu\nu}_{\alpha\beta}h^{\alpha\beta} - \frac{1}{2}F_{\mu\nu}F^{\mu\nu} + 3\phi\Box\phi + \frac{1}{M_p}h_{\mu\nu}T^{\mu\nu} + \frac{1}{M_p}\phi T$$

$\to$ **vDVZ discontinuity**



Massive Gravity

Artifact:

The vDVZ discontinuity is just an artifact of the linear approximation
$\rightarrow$ need: non-linear extension

Issue:

The ghost we have cured by Fierz-Pauli tuning seems to come back at non-linear level (the sixth degree of freedom is associated to higher derivative terms)

Ghost

challenging task: non-linear extension of FP **without ghost**

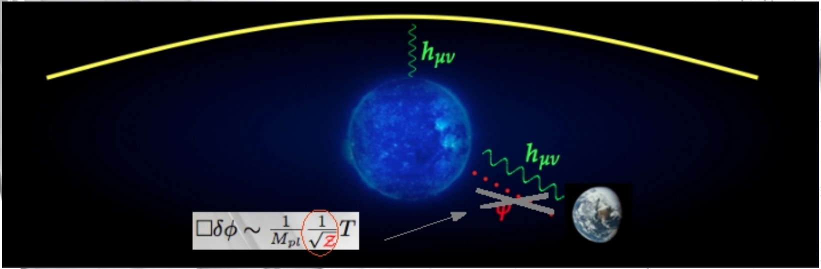
Massive Gravity

dRGT

Vainshtein mechanism at work in dRGT:
The effective coupling to matter depends on the self-interactions of the helicity-0 mode ϕ

$$\Box\delta\phi \sim \frac{1}{M_p}\frac{\delta\phi}{\sqrt{Z}}T$$

$\rightarrow$ The non-linearities cure the vDVZ discontinuity



Ghost-free extension of FP = dRGT

a 4D covariant theory of a massive spin-2 field

$$\mathcal{L}=\frac{M_p^2}{2}\sqrt{-g}\left(R-\frac{m^2}{4}\mathcal{U}(g,H)\right)$$

the most generic potential that bears no ghosts is
$\mathcal{U}(g,H)=-4\left(\mathcal{U}_2+\alpha_3\,\mathcal{U}_3+\alpha_4\,\mathcal{U}_4\right)$ where the covariant tensor $H_{\mu\nu}=h_{\mu\nu}+2\Phi_{\mu\nu}-\eta^{\alpha\beta}\Phi_{\mu\alpha}\Phi_{\beta\nu}$ and the potentials:

$$\begin{aligned}
\mathcal{U}_2 &= [\mathcal{K}]^2-[\mathcal{K}^2]\\
\mathcal{U}_3 &= [\mathcal{K}]^3-3[\mathcal{K}][\mathcal{K}^2]+2[\mathcal{K}^3]\\
\mathcal{U}_4 &= [\mathcal{K}]^4-6[\mathcal{K}^2][\mathcal{K}]^2+8[\mathcal{K}^3][\mathcal{K}]+3[\mathcal{K}^2]^2-6[\mathcal{K}^4]
\end{aligned}$$

where $\mathcal{K}^\mu_\nu(g,H)=\delta^\mu_\nu-\sqrt{\delta^\mu_\nu-H^\mu_\nu}$, $\Phi_{\mu\nu}=\partial_\mu\partial_\nu\phi$ and $[..]$ =trace.
(de Rham, Gabadadze, Tolley (Phys.Rev.Lett.106,231101))

Decoupling limit (DL)

Decoupling limit
($M_p \to \infty$, $m \to 0$ with $\Lambda_3^3 = m^2 M_p \to$ const)
and decomposition of $H_{\mu\nu}$ in terms of the canonically normalized helicity-2 and helicity-0 fields

$$H_{\mu\nu} = \frac{h_{\mu\nu}}{M_p} + \frac{2\partial_\mu\partial_\nu\phi}{\Lambda_3^3} - \frac{\partial_\mu\partial^\alpha\phi\partial_\nu\partial_\alpha\phi}{\Lambda_3^6}$$

gives the following scalar-tensor interactions

$$\mathcal{L} = -\frac{1}{2}h^{\mu\nu}\mathcal{E}_{\mu\nu}{}^{\alpha\beta}h_{\alpha\beta} + h^{\mu\nu}\sum_{n=1}^{3}\frac{a_n}{\Lambda_3^{3(n-1)}}X^{(n)}_{\mu\nu}[\Phi]$$

where $a_1 = -\frac{1}{2}$ and $a_{2,3}$ are two arbitrary constants and $X^{(1,2,3)}_{\mu\nu}$ denote the interactions of the helicity-0 mode

$$X^{(1)}_{\mu\nu} = \Box\phi\eta_{\mu\nu} - \Phi_{\mu\nu}$$
$$X^{(2)}_{\mu\nu} = \Phi^2_{\mu\nu} - \Box\phi\Phi_{\mu\nu} - \frac{1}{2}([\Phi^2] - [\Phi]^2)\eta_{\mu\nu}$$
$$X^{(3)}_{\mu\nu} = 6\Phi^3_{\mu\nu} - 6[\Phi]\Phi^2_{\mu\nu} + 3([\Phi]^2 - [\Phi^2])\Phi_{\mu\nu} - \eta_{\mu\nu}([\Phi]^3 - 3[\Phi^2][\Phi] + 2[\Phi^3])$$

Diagonalized interactions

The transition to Einsteins frame is performed by the change of variable

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - 2a_1\phi\eta_{\mu\nu} + \frac{2a_2}{\Lambda_3^3}\partial_\mu\phi\partial_\nu\phi$$

one recovers Galileon interactions for the helicity-0 mode of the graviton

$$\mathcal{L} = -\frac{1}{2}\bar{h}(\mathcal{E}\bar{h})_{\mu\nu} + 6a_1^2\phi\Box\phi - \frac{6a_2a_1}{\Lambda_3^3}(\partial\phi)^2[\Phi] + \frac{2a_2^2}{\Lambda_3^6}(\partial\phi)^2([\Phi^2] - [\Phi]^2) + \frac{a_3}{\Lambda_3^6}h^{\mu\nu}X^{(3)}_{\mu\nu}$$

with the coupling

$$\frac{1}{M_p}\left(\bar{h}_{\mu\nu} - 2a_1\phi\eta_{\mu\nu} + \frac{2a_2}{\Lambda_3^3}\partial_\mu\phi\partial_\nu\phi\right)T^{\mu\nu}$$

Differences to Galileon interactions

Common

- IR modification of gravity as due to a light scalar field with non-linear derivative interactions
- respects the symmetry $\phi(x) \to \phi(x) + c + b_\mu x^\mu$
- Second order equations of motion, containing at most two time derivatives

Different

- undiagonazable interaction $+\frac{a_3}{\Lambda_3^6} h^{\mu\nu} X^{(3)}_{\mu\nu}$ $\to$ important for the self-accelerating solution
- extra coupling $\partial_\mu\phi\partial_\nu\phi T^{\mu\nu}$ $\to$ important for the degravitating solution
- only 2 free-parameters
- **observational difference** due to $\frac{a_3}{\Lambda_3^6} h^{\mu\nu} X^{(3)}_{\mu\nu}$ and $\partial_\mu\phi\partial_\nu\phi T^{\mu\nu}$

Two branches

The Lagrangian in the decoupling limit

$$\mathcal{L} = -\frac{1}{2}h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} + h^{\mu\nu}\sum_{n=1}^{3}\frac{a_n}{\Lambda_3^{3(n-1)}}X^{(n)}_{\mu\nu}[\Phi] + \frac{1}{M_p}h^{\mu\nu}T_{\mu\nu}$$

Self-accelerating solution

- $T_{\mu\nu} = 0$
- $H \neq 0$

Degravitating solution

- $T_{\mu\nu} \neq 0$
- $H = 0$

Equation of motions

The equation of motions for the helicity-2 mode

$$-\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} + \sum_{n=1}^{3}\frac{a_n}{\Lambda_3^{3(n-1)}}X^{(n)}_{\mu\nu}[\Phi] = -\frac{1}{M_p}T_{\mu\nu}$$

and for helicity-0 mode

$$\partial_\alpha\partial_\beta h^{\mu\nu}\left(a_1\epsilon_\mu{}^{\alpha\rho\sigma}\epsilon_\nu{}^\beta{}_{\rho\sigma} + 2\frac{a_2}{\Lambda_3^3}\epsilon_\mu{}^{\alpha\rho\sigma}\epsilon_\nu{}^{\beta\gamma}{}_\sigma\Phi_{\rho\gamma} + 3\frac{a_3}{\Lambda_3^6}\epsilon_\mu{}^{\alpha\rho\sigma}\epsilon_\nu{}^{\beta\gamma\delta}\Phi_{\rho\gamma}\Phi_{\sigma\delta}\right) = 0$$

de Rham, Gabadadze, Heisenberg, Pirtskhalava (Phys.Rev.D 83,103516)

- Gravitons form a condensate whose energy density sources self-acceleration
- Gravitons form a condensate whose energy density compensates the cosmological constant

Self-accelerating solution

de Sitter as a small perturbation over Minkowski space-time

$$ds^2 = [1 - \frac{1}{2}H^2 x^\alpha x_\alpha]\eta_{\mu\nu}dx^\mu dx^\nu$$

For the helicity-0 field we look for the solution of the following isotropic form

$$\phi = \frac{1}{2}q\Lambda_3^3 x^a x_a + b\Lambda_3^2 t + c\Lambda_3$$

The equations of motion for the helicity-0 and helicity-2 fields are then given by

$$H^2\left(-\frac{1}{2} + 2a_2 q + 3a_3 q^2\right) = 0 \qquad H^2 \neq 0$$

$$M_p H^2 = 2q\Lambda_3^3\left[-\frac{1}{2} + a_2 q + a_3 q^2\right]$$

Self-accelerating solution

$H^2 = m^2\left(2a_2q^2 + 2a_3q^3 - q\right)$ and $q = -\frac{a_2}{3a_3} + \frac{(2a_2^2+3a_3)^{1/2}}{3\sqrt{2}a_3}$

Consider perturbations

$$h_{\mu\nu} = h^b_{\mu\nu} + \chi_{\mu\nu} \quad \text{and} \quad \phi = \phi^b + \pi$$

the Lagrangian for the perturbations

$$\mathcal{L} = -\frac{\chi^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}\chi_{\alpha\beta}}{2} + 6(a_2 + 3a_3q)\frac{H^2M_p}{\Lambda_3^3}\pi\Box\pi - 3a_3\frac{H^2M_p}{\Lambda_3^6}(\partial_\mu\pi)^2\Box\pi$$
$$+\frac{a_2+3a_3q}{\Lambda_3^3}\chi^{\mu\nu}X^{(2)}_{\mu\nu}[\Pi] + \frac{a_3}{\Lambda_3^6}\chi^{\mu\nu}X^{(3)}_{\mu\nu}[\Pi] + \frac{\chi^{\mu\nu}T_{\mu\nu}}{M_p}$$

- abscence of ghost implies $a_2 + 3a_3q > 0$
- the perturbation of the helicity-0 mode keeps a kinetic term in this decoupling limit $\rightarrow$ no strong coupling issues

Self-accelerating solution

$$H^2 = m^2\left(2a_2q^2 + 2a_3q^3 - q\right) \text{ and } q = -\frac{a_2}{3a_3} + \frac{(2a_2^2+3a_3)^{1/2}}{3\sqrt{2}a_3}$$

stability

- $H^2 > 0$ and $a_2 + 3a_3 q > 0$
- stable self-accelerating solution: $a_2 < 0$ and $\frac{-2a_2^2}{3} < a_3 < \frac{-a_2^2}{2}$
- **interaction $h^{\mu\nu}X^{(3)}_{\mu\nu}$ plays a crucial role for the stability since $a_3 = 0 \to$ ghost**
- there is no quadratic mixing term between χ and π
- cosmological evolution very similar to ΛCDM

Degravitating solution

We now focus on a pure cosmological constant source $T_{\mu\nu} = -\lambda\eta_{\mu\nu}$ and make the following ansatz

$$h_{\mu\nu} = -\frac{1}{2}H^2x^2M_p\eta_{\mu\nu}$$
$$\phi = \frac{1}{2}qx^2\Lambda_3^3$$

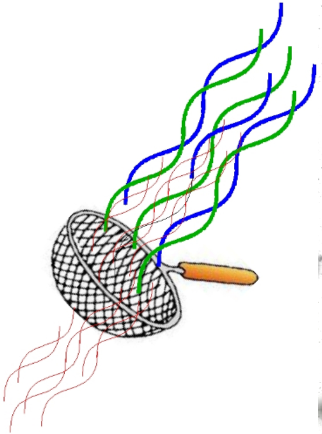
The equations of motion then simplify to

$$\left(-\frac{1}{2}M_pH^2 + \sum_{n=1}^{3} a_nq^n\Lambda_3^3\right)\eta_{\mu\nu} = -\frac{\lambda}{6M_p}\eta_{\mu\nu}$$
$$H^2\left(a_1 + 2a_2q + 3a_3q^2\right) = 0$$

$$\rightarrow a_1q + a_2q^2 + a^3q^3 = \frac{-\lambda}{\Lambda_3^3M_p}$$

Degravitating solution

- degravitating solution: high pass filter modifying the effect of long wavelength sources such as a CC $\rightarrow$ vacuum energy gravitates very weakly
- $H = 0 \quad \rightarrow \quad g_{\mu\nu} = \eta_{\mu\nu}$
- $a_1 q + a_2 q^2 + a_3 q^3 = \frac{-\lambda}{\Lambda_3^3 M_p}$
  as long as the parameter a_3 is present, this equation has always at least one real root
- this static solution is stable for any region of the parameter space for which $2(a_1 + 2a_2 q + 3a_3 q^2) \neq 0$ and real



Proxy theory

We had the following Lagrangian in the decoupling limit

$$\mathcal{L} = -\frac{1}{2}h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta}+h^{\mu\nu}X^{(1)}_{\mu\nu}+\frac{a_2}{\Lambda^3}h^{\mu\nu}X^{(2)}_{\mu\nu}+\frac{a_3}{\Lambda^6}h^{\mu\nu}X^{(3)}_{\mu\nu}+\frac{1}{2M_p}h^{\mu\nu}T_{\mu\nu}$$

lets integrate by part the first interaction $h^{\mu\nu}X^{(1)}_{\mu\nu}$:

$$\begin{aligned} h^{\mu\nu}X^{(1)}_{\mu\nu} = h^{\mu\nu}(\Box\phi\eta_{\mu\nu} - \partial_\mu\partial_\nu\phi) &= h^{\mu\nu}(\partial_\alpha\partial^\alpha\phi\eta_{\mu\nu} - \partial_\mu\partial_\nu\phi) \\ &= (\Box h - \partial_\mu\partial_\nu h^{\mu\nu})\phi \\ &= -R\phi \end{aligned}$$

so covariantization of the first interaction: $h^{\mu\nu}X^{(1)}_{\mu\nu} \longleftrightarrow -R\phi$

Proxy theory

Similarly, we can covariantize the other interaction terms. One finds the following correspondences:

$$
\begin{aligned}
h^{\mu\nu} X^{(1)}_{\mu\nu} &\longleftrightarrow -\phi R \\
h^{\mu\nu} X^{(2)}_{\mu\nu} &\longleftrightarrow -\partial_\mu\phi\partial_\nu\phi G^{\mu\nu} \\
h^{\mu\nu} X^{(3)}_{\mu\nu} &\longleftrightarrow -\partial_\mu\phi\partial_\nu\phi\Phi_{\alpha\beta} L^{\mu\alpha\nu\beta}
\end{aligned}
$$

such that the Lagrangian becomes

$$
\mathcal{L}^\phi = M_p \left(-\phi R - \frac{a_2}{\Lambda^3}\partial_\mu\phi\partial_\nu\phi G^{\mu\nu} - \frac{a_3}{\Lambda^6}\partial_\mu\phi\partial_\nu\phi\Phi_{\alpha\beta} L^{\mu\alpha\nu\beta} \right).
$$

with the dual Riemann tensor

$$
\begin{aligned}
L^{\mu\alpha\nu\beta} = 2R^{\mu\alpha\nu\beta} &+ 2(R^{\mu\beta}g^{\nu\alpha} + R^{\nu\alpha}g^{\mu\beta} - R^{\mu\nu}g^{\alpha\beta} - R^{\alpha\beta}g^{\mu\nu}) \\
&+ R(g^{\mu\nu}g^{\alpha\beta} - g^{\mu\beta}g^{\nu\alpha})
\end{aligned}
$$

Proxy theory

Instead of focusing on the entire complicated model, study a proxy theory:

$$\mathcal{L} = \sqrt{-g}M_p(M_pR + -\phi R - \frac{a_2}{\Lambda^3}\partial_\mu\phi\partial_\nu\phi G^{\mu\nu} - \frac{a_3}{\Lambda^6}\partial_\mu\phi\partial_\nu\phi\Phi_{\alpha\beta}L^{\mu\alpha\nu\beta})$$

- in 4D $G_{\mu\nu}$ and $L^{\mu\alpha\nu\beta}$ are the only divergenceless tensors $\rightarrow \nabla_\mu G^{\mu\nu} = 0$ and $\nabla_\mu L^{\mu\alpha\nu\beta} = 0$
- All eom are 2^{nd} order $\rightarrow$ No instabilities
- Reproduces the decoupling limit $\rightarrow$ Exhibits the Vainsthein mechanism

Chkareuli, Pirtskhalava (Phys.Lett. B713 (2012) 99-103)
de Rham, Heisenberg (PRD84 (2011) 043503)

Self-accelerating solution

- self-acceleration solution: H =const and $\dot{H}=0$.
- make the ansatz $\dot{\phi}=q\frac{\Lambda^3}{H}$.
- assume that we are in a regime where $H\phi \ll \dot{\phi}$

The Friedmann and field equations can be recast in

$$H^2=\frac{m^2}{3}(6q-9a_2q^2-30a_3q^3)$$
$$H^2(18a_2q+54a_3q^2-12)=0$$

Assuming $H\neq 0$, the field equation then imposes,

$$q=\frac{-a_2\pm\sqrt{a_2^2+8a_3}}{6a_3}$$

$\rightarrow$ similar to DL our proxy theory admits a self-accelerated solution, with the Hubble parameter set by the graviton mass.

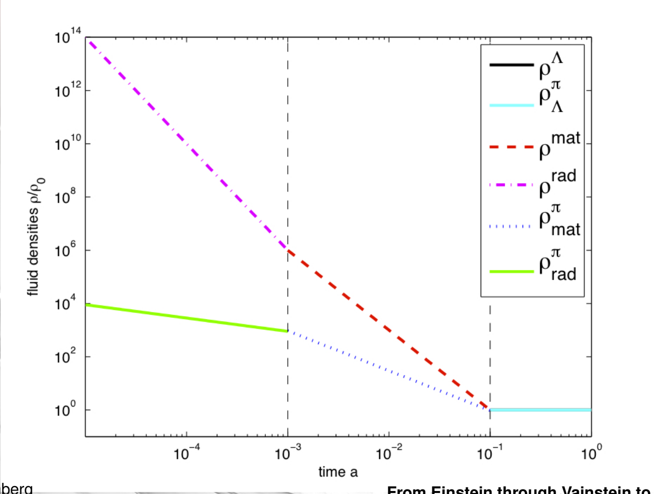
Proxy theory

$$\mathcal{L}^\phi = M_p\left(-\phi R - \frac{a_2}{\Lambda^3}\partial_\mu\phi\partial_\nu\phi G^{\mu\nu} - \frac{a_3}{\Lambda^6}\partial_\mu\phi\partial_\nu\phi\Phi_{\alpha\beta}L^{\mu\alpha\nu\beta}\right)$$

- We recover some decoupling limit results:
  - stable self-accelerating solutions within the space parameter space
- During the radiation domination the energy density for ϕ goes as $\rho^\phi_{\rm rad} \sim a^{-1/2}$ and during matter dominations as $\rho^\phi_{\rm mat} \sim a^{-3/2}$ and is constant for later times $\rho^\phi_\Lambda = \text{const}$
- At early time, interactions for scalar mode are important $\rightarrow$ cosmological screening effect
- Below a critical energy density, screening stop being efficient $\rightarrow$ scalar contribute significantly to the cosmological evolution
- But still the cosmological evolution different than in ΛCDM

Densities

$$\mathcal{L}^{\phi} = M_p \left(-\phi R - \frac{a_2}{\Lambda^3}\partial_\mu\phi\partial_\nu\phi G^{\mu\nu} - \frac{a_3}{\Lambda^6}\partial_\mu\phi\partial_\nu\phi\Phi_{\alpha\beta}L^{\mu\alpha\nu\beta}\right)$$



Degravitation solution

The effective energy density of the field ϕ is

$$\rho^{\phi} = M_p\left(6H\dot{\phi} + 6H^2\phi - \frac{9a_2}{\Lambda^3}H^2\dot{\phi}^2 - \frac{30a_3}{\Lambda^6}H^3\dot{\phi}^3\right)$$

- If one takes $\phi = \phi(t)$ and $H = 0 \rightarrow \rho^{\phi} = 0$
  $\rightarrow$ so the field has absolutely no effect and cannot help the background to degravitate.
- Fab Four has similar interactions, they find degravitation solution! (arXiv:1208.3373)
  BUT they rely strongly on spatial curvature
- in the absence of spatial curvature $\kappa = 0$, the contribution from the scalar field vanishes if $H = 0$.
- $\rightarrow$ BUT relying on spatial curvature brings concerns over instabilities

Conclusion

- decoupling limit of dRGT
  - stable self-accelerating solution similar to ΛCDM
  - degravitating solution
- Proxy theory
  - stable self accelerating solution
  - no degravitating solution
  - the scalar mode does not decouple around the self-accelerating background
  - leads to an extra force during the history of the Universe
  - would influence the time sequence of gravitational clustering and the evolution of peculiar velocities, as well as the number density of collapsed objects.

Ongoing and Future work

Quantum corrections

- the mass needs to be tuned $m \lesssim H_0$
  same tuning as Cosmological Constant
  $\frac{\Lambda}{M_p^4} \sim \frac{H_0^2}{M_p^2} \sim \frac{m^2}{M_p^2} \sim 10^{-120}$
- But the graviton mass is expected to remain stable against quantum corrections
- Check quantum corrections beyond the decoupling limit
  $\delta m^2 \sim m^2 \rightarrow$ the theory would be tuned but technically natural

constraining dRGT through observations

- put observational constraint on the free parameters of dRGT and test it against ΛCDM.

observations are always a challenging task!

challenging task: observation!

we would like to study

- the distance-redshift relation of supernovae
- the angular diameter distance as a function of redshift
  - CMB
  - BAO
- Weak Lensing
- integrated Sachs-Wolfe Effect
- Gravitational Clustering and Number density of collapsed objects

for massive gravity!

cosmological observations

two categories:

geometrical probes

measurement of the Hubble function

- distance-redshift relation of supernovae
- measurements of the angular diameter distance as a function of redshift (CMB+BAO)

structure formation probes

measurement of the Growth function

- homogeneous growth of the cosmic structure
  $\rightarrow$ integrated Sachs-Wolfe effects
- non-linear growth
  $\rightarrow$ gravitational lensing
  $\rightarrow$ formation of galaxies
  $\rightarrow$ clusters of galaxies by gravitational collapse

(e.g Kimura et al.)

Attention: possible degeneracy

Naturally, the changes coming from massive gravity are degenerate with a different choice of cosmological parameters and with introducing non-Gaussian initial conditions.