

Density and transverse velocity in the local Universe as probes of cosmology

Alex Hall

University of Edinburgh

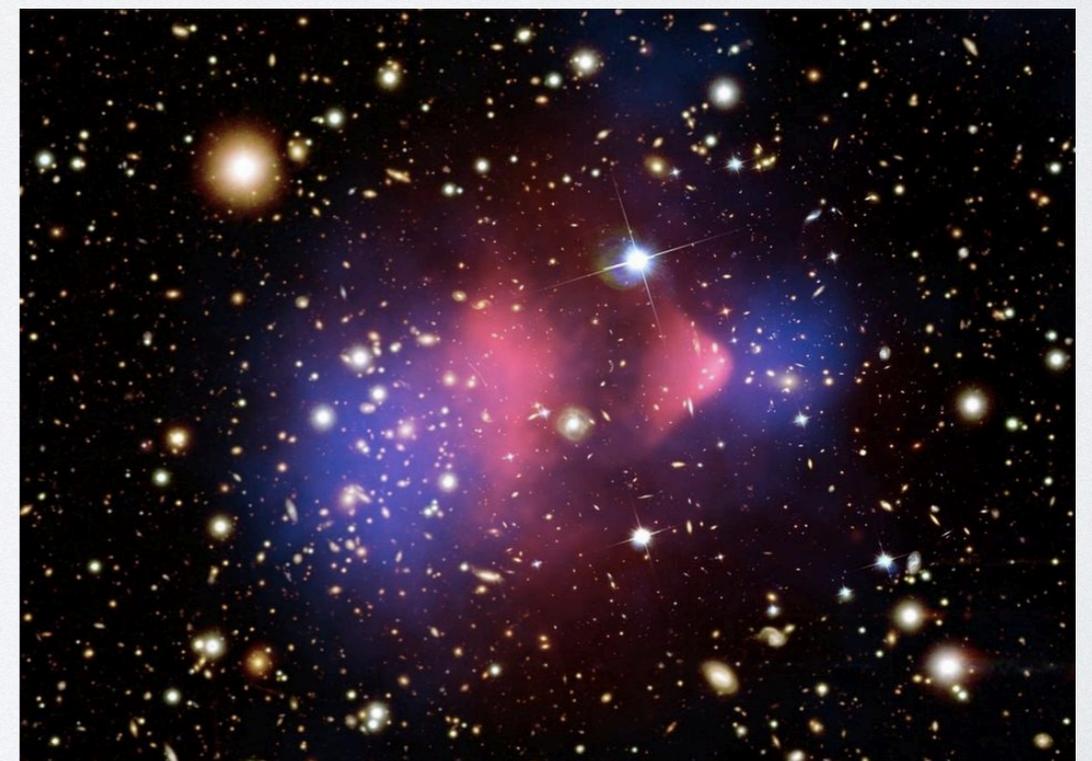
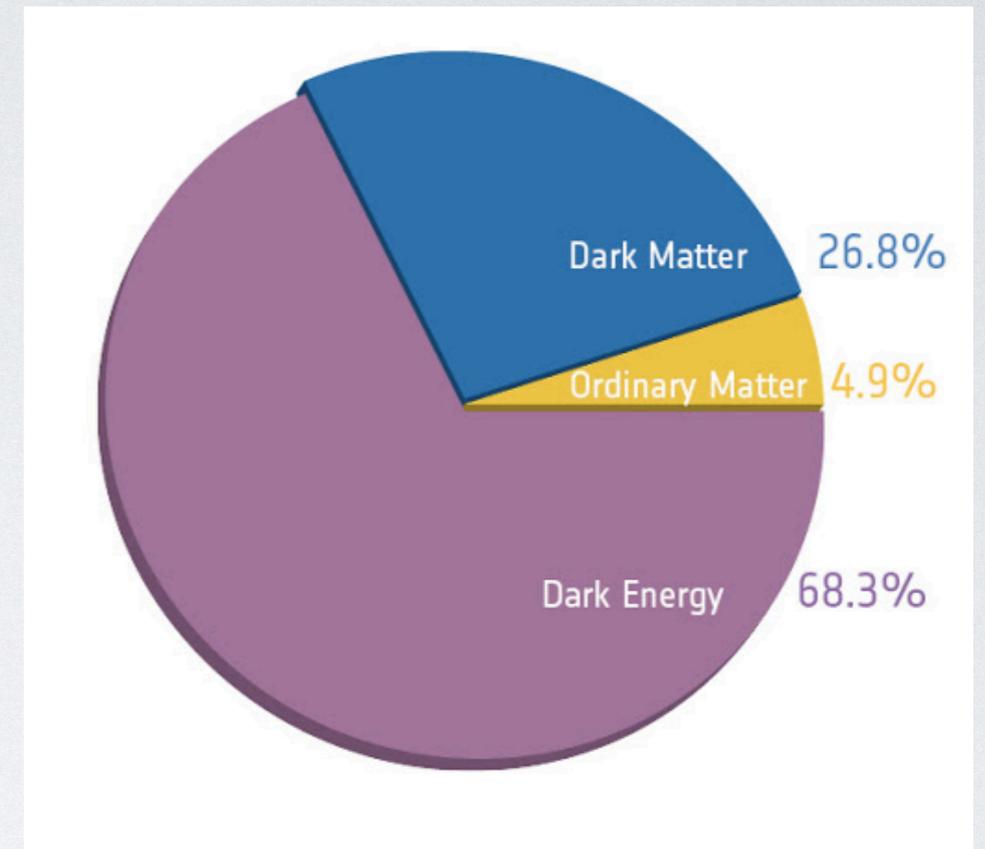
**Based on arXiv: 1811.05454 (Hall 2019)
and arXiv: 1911.07855 (Hall 2020)**



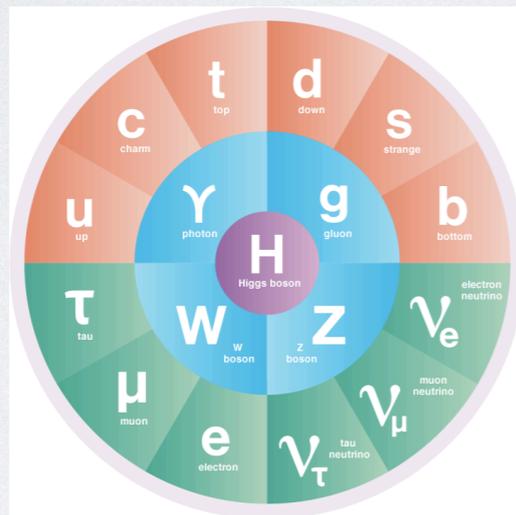
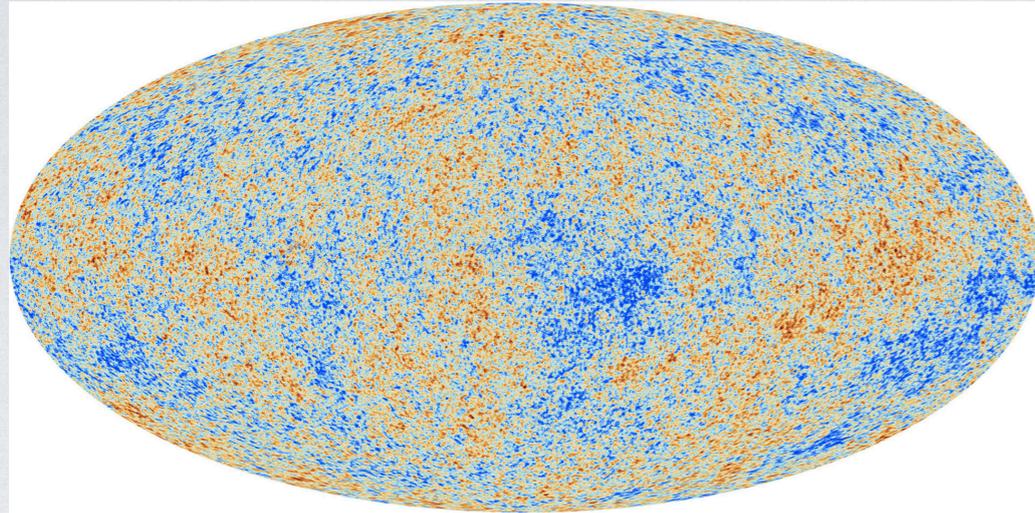
Outstanding questions in cosmology

'Late' times

- **What is causing the Universe to accelerate in its expansion?**
- **What is dark matter?**
- **What is the mass of the neutrino?**
- **How do galaxy formation processes interact with the cosmic web?**
- **Are the apparent 'tensions' due to new physics?**

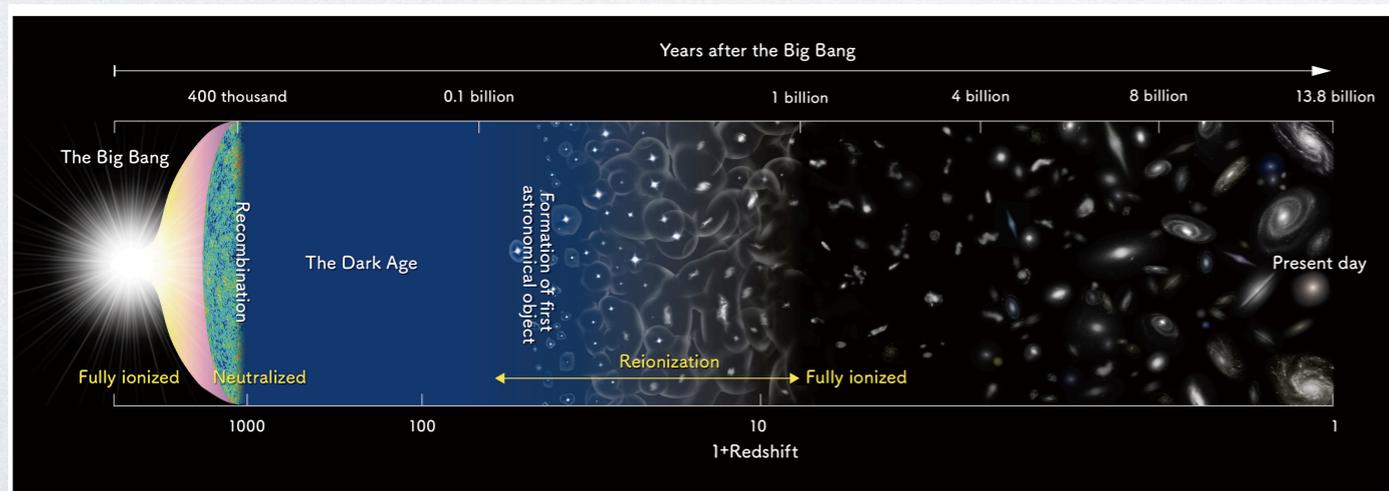


Outstanding questions in cosmology



'Early' times

- **What is the correct physical description of Inflation?**
- **What is the origin of the Standard Model? How does gravity fit in? What caused baryon asymmetry?**
- **How and when did reionization occur?**



Outstanding questions in cosmology

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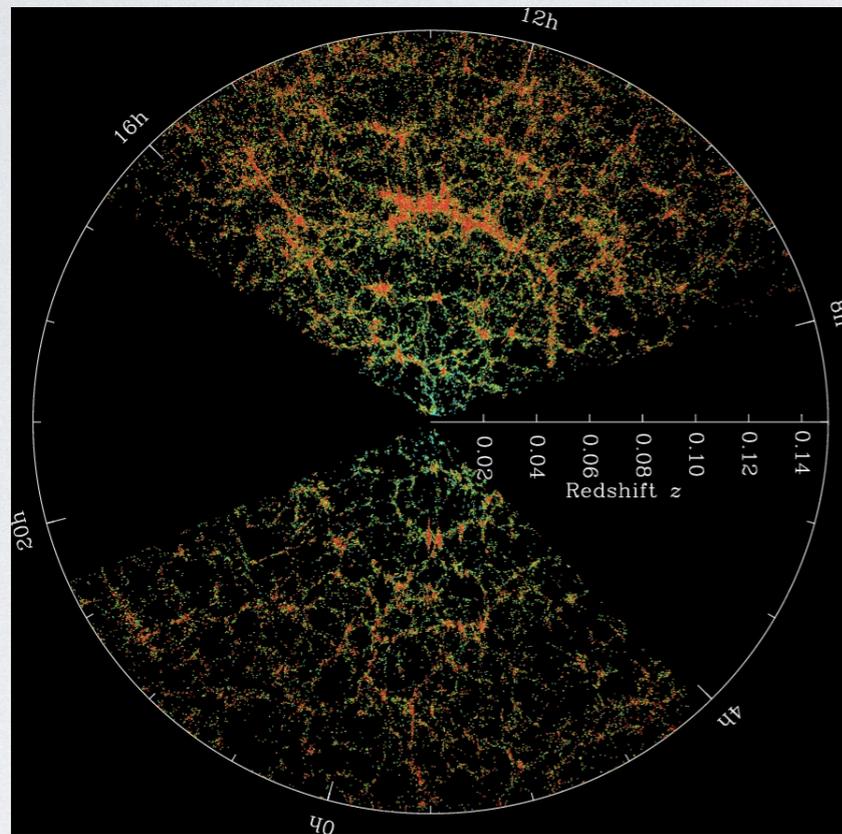
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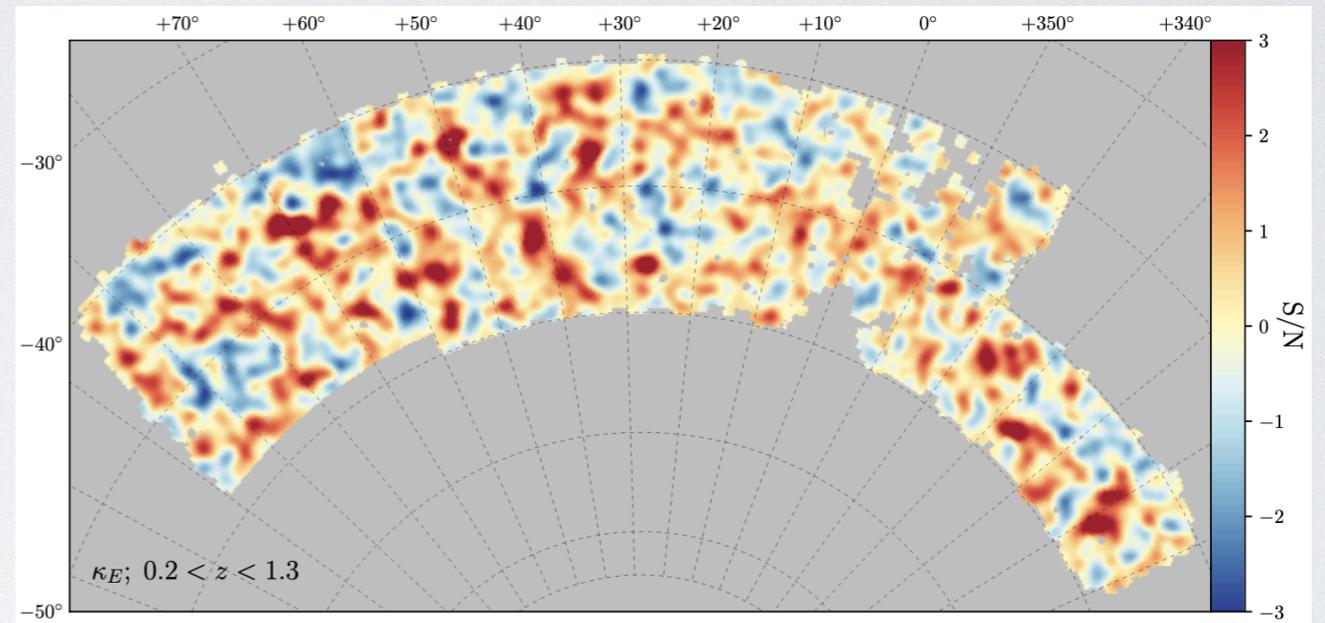
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Cosmological information in the late-time Universe



Galaxy redshift surveys

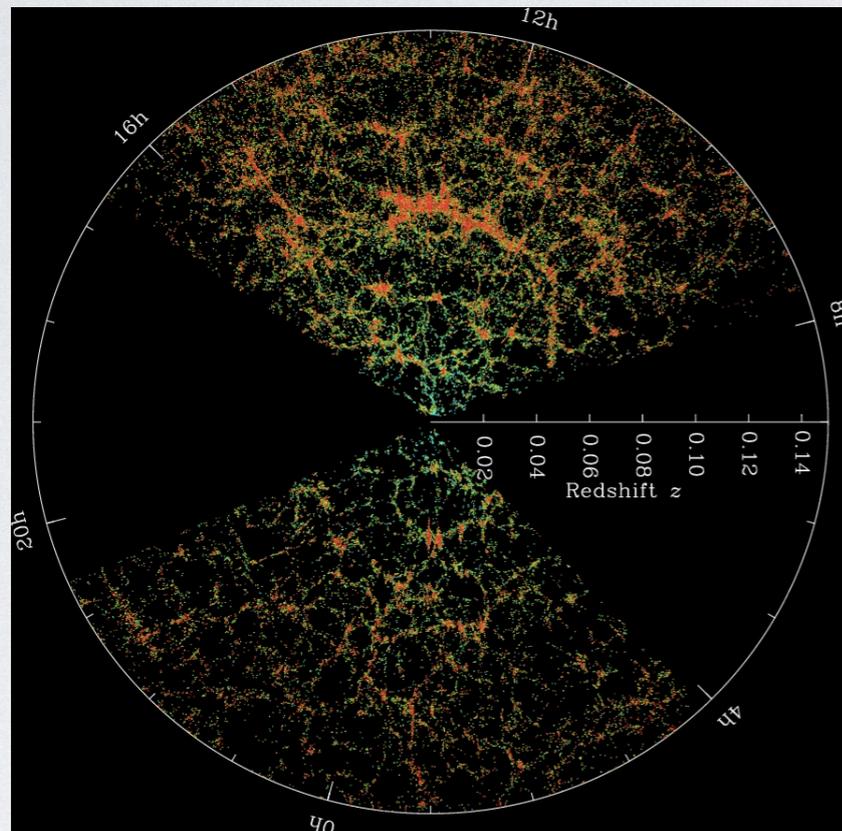


Weak gravitational lensing surveys

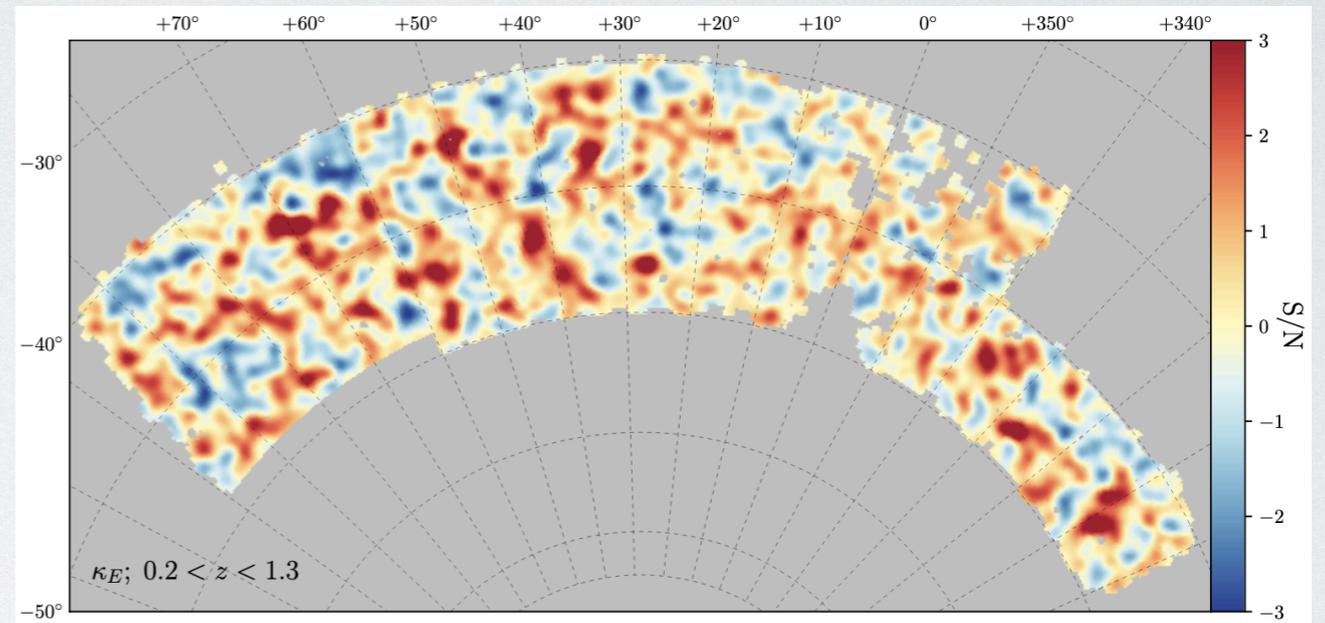
See also:

Geometric probes (Sn Ia, BAO, H_0), Lyman- α Forest, cluster number counts, SZ surveys, 21 cm, gravitational waves, ...

Cosmological information in the late-time Universe



Galaxy redshift surveys



Weak gravitational lensing surveys

Ultimate aim is to study the *statistics* of the matter distribution, via its gravitational influence on galaxies and light

Outline of this talk

Part I

Transverse velocities of galaxies as a probe of cosmic flows, large-scale structure, and cosmology

Part II

The influence of our local environment on the statistics of large-scale structure and implications for cosmology

Part I

Cosmology with extragalactic proper motions

**Based on arXiv:1811.05454
(AH 2019)**

Disruption as summer winds batter Scotland

🕒 7 August 2016

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Source: BBC News

Proper motion of luminous sources at cosmological distances

Questions

Can we ever measure the real-time motion of galaxies and quasars across the sky?

What would such a measurement tell us about our Universe?

Contributions to the proper motion of galaxies

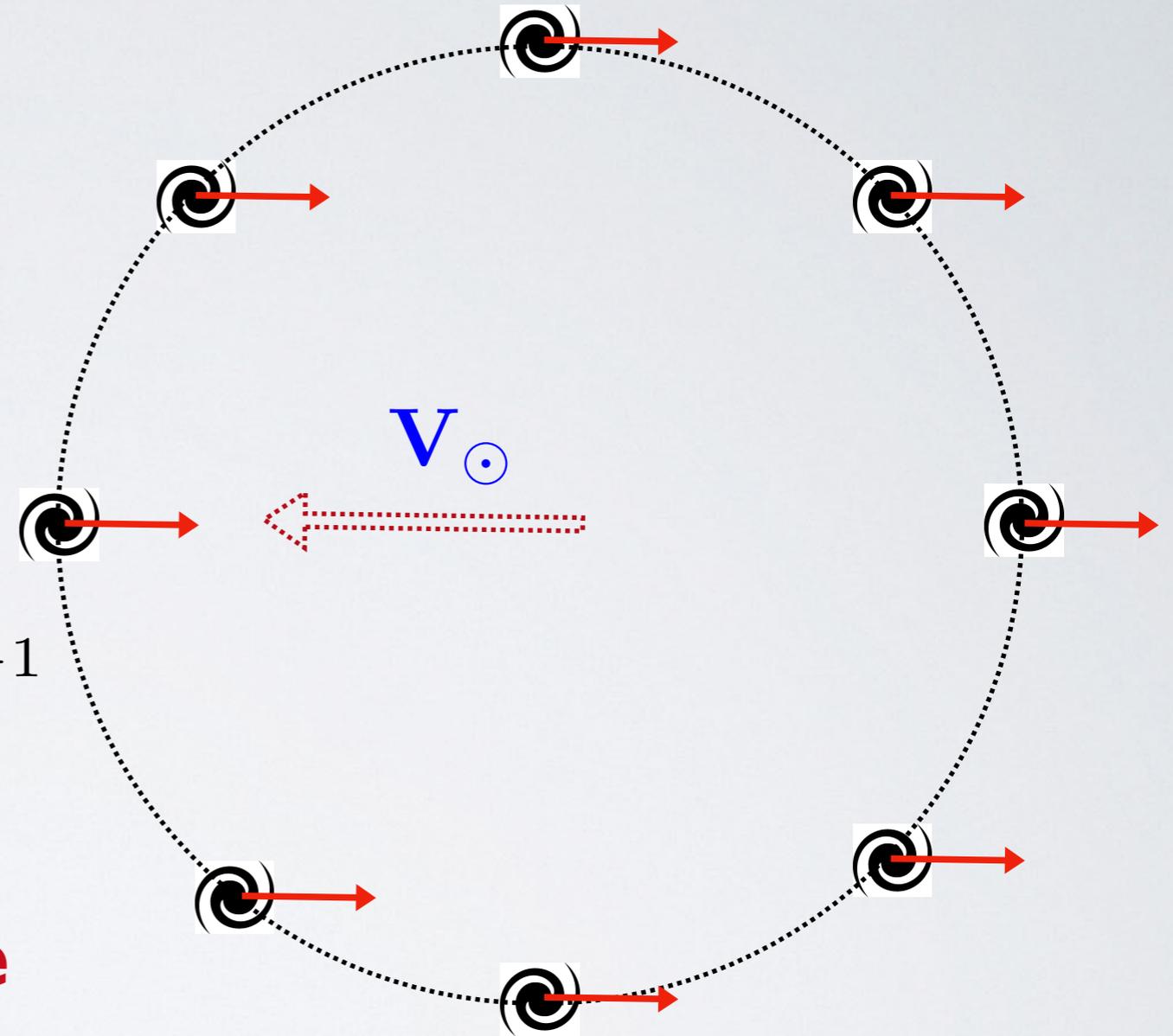
- **Motion of the Solar System: Secular parallax**
- **Secular aberration drift**
- **Transverse peculiar velocities**

Contributions to the proper motion of galaxies

1) Solar System moves relative to galaxies ('secular parallax')

$$\mu(r) \sim 80 \left(\frac{r}{1 \text{ Mpc}} \right)^{-1} \mu\text{as yr}^{-1}$$

Dipolar angular structure



McCrea (1935), Weinberg (1972), Kardashev, Pariiskii and Umarbaeva (1973), Novikov (1977), Kardashev (1986), Kasai (1988), Ding & Croft (2009), Darling & Truebenbach (2018), Paine, Darling, Graziani, Courtois (2020)

Contributions to the proper motion of galaxies

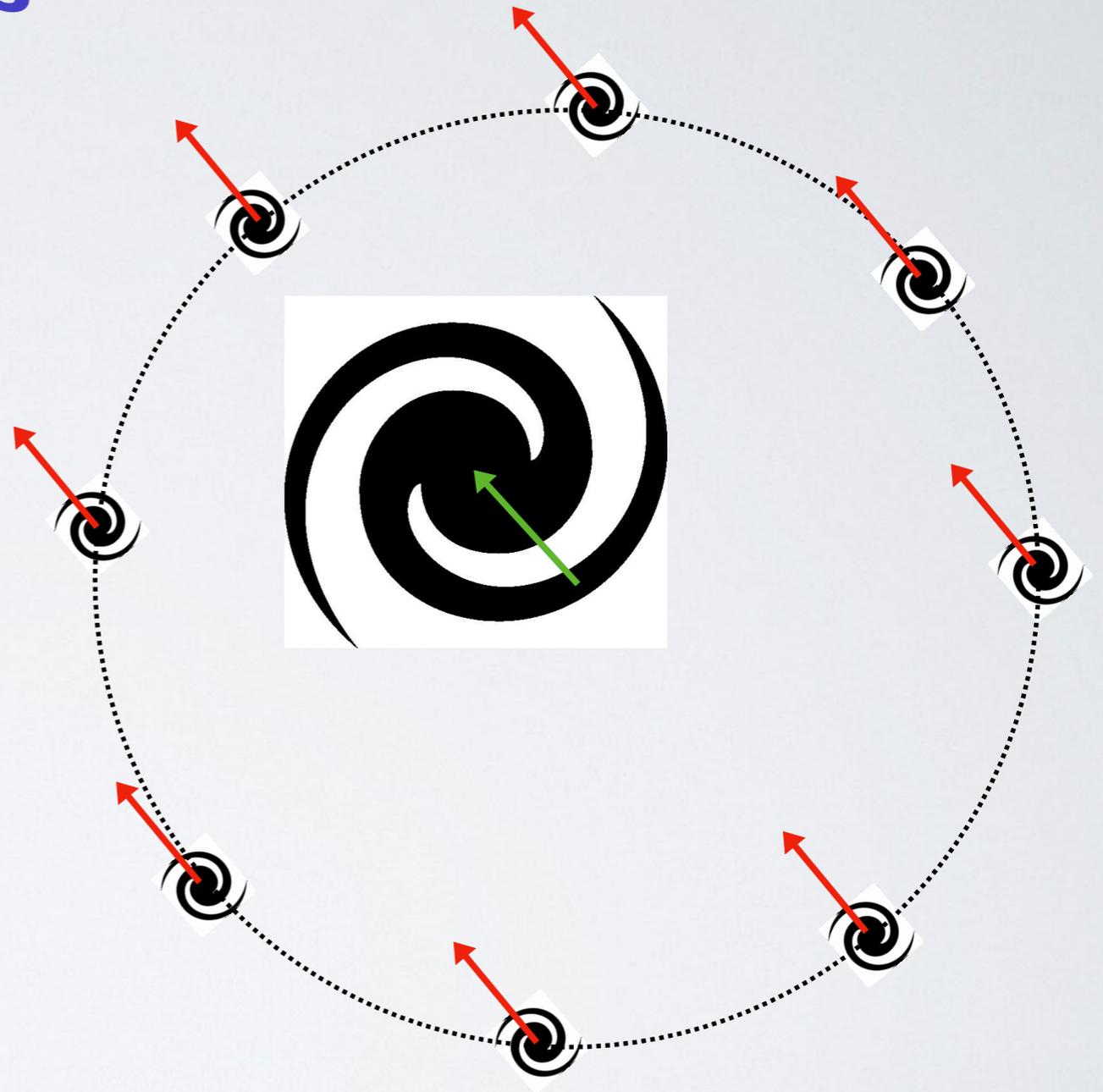
2) Solar System accelerates towards galactic centre ('secular aberration drift')

$$\mu(r) \sim 5 \mu\text{as yr}^{-1}$$

Titov et al (2011) : VLBI
 $6.4 \pm 1.5 \mu\text{as yr}^{-1}$

Klioner et al (Gaia EDR3) :
 $5.05 \pm 0.35 \mu\text{as yr}^{-1}$

**Dipolar angular structure
(independent of distance)**

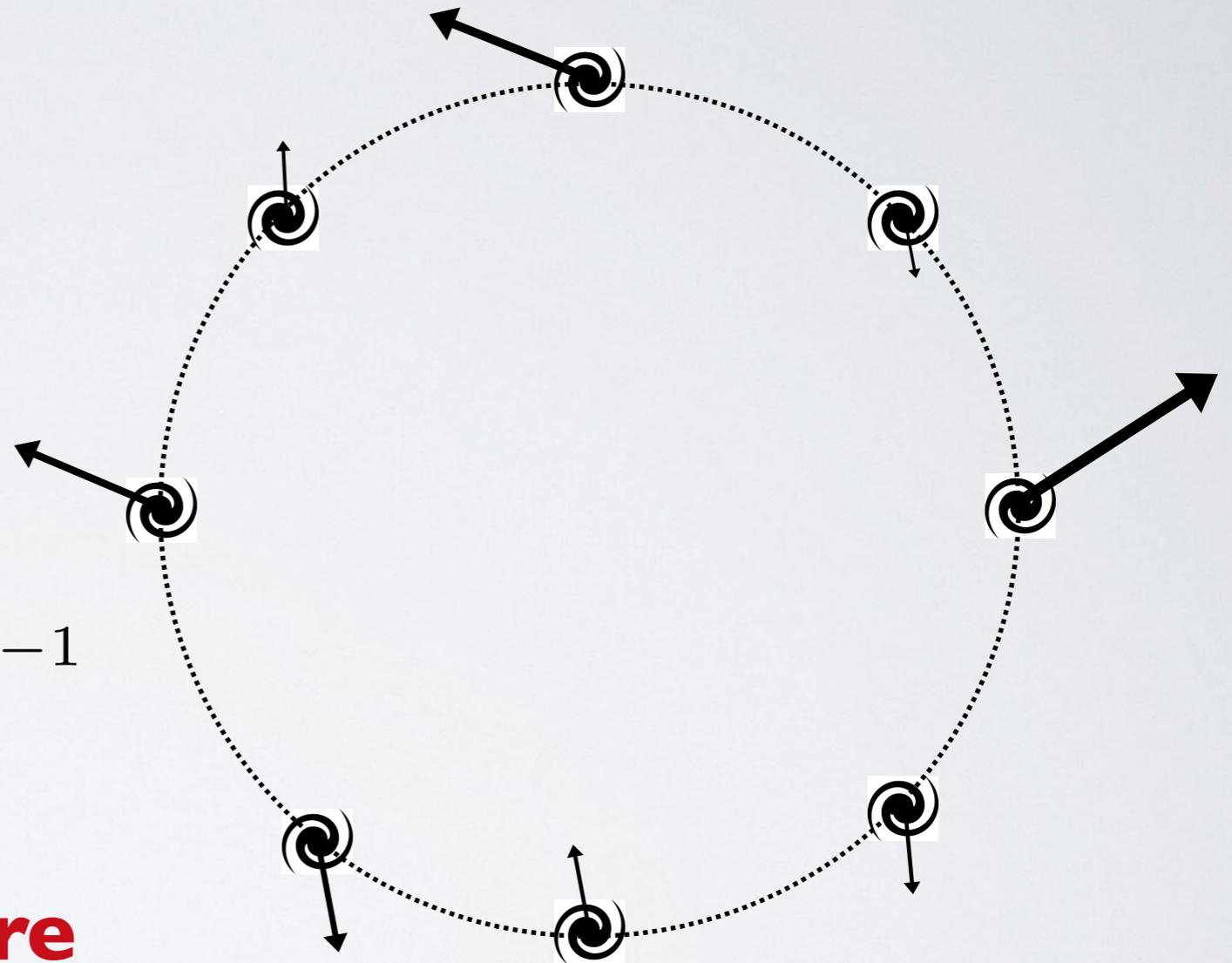


Contributions to the proper motion of galaxies

3) Galaxies move due to gravitational structure formation

$$\mu(r) \sim 90 \left(\frac{r}{1 \text{ Mpc}} \right)^{-1} \mu\text{as yr}^{-1}$$

Complex angular structure



Contributions to the proper motion of galaxies

- **Motion of the Solar System: Secular parallax**

$$\mu(r) \sim 80 \left(\frac{r}{1 \text{ Mpc}} \right)^{-1} \mu\text{as yr}^{-1}$$

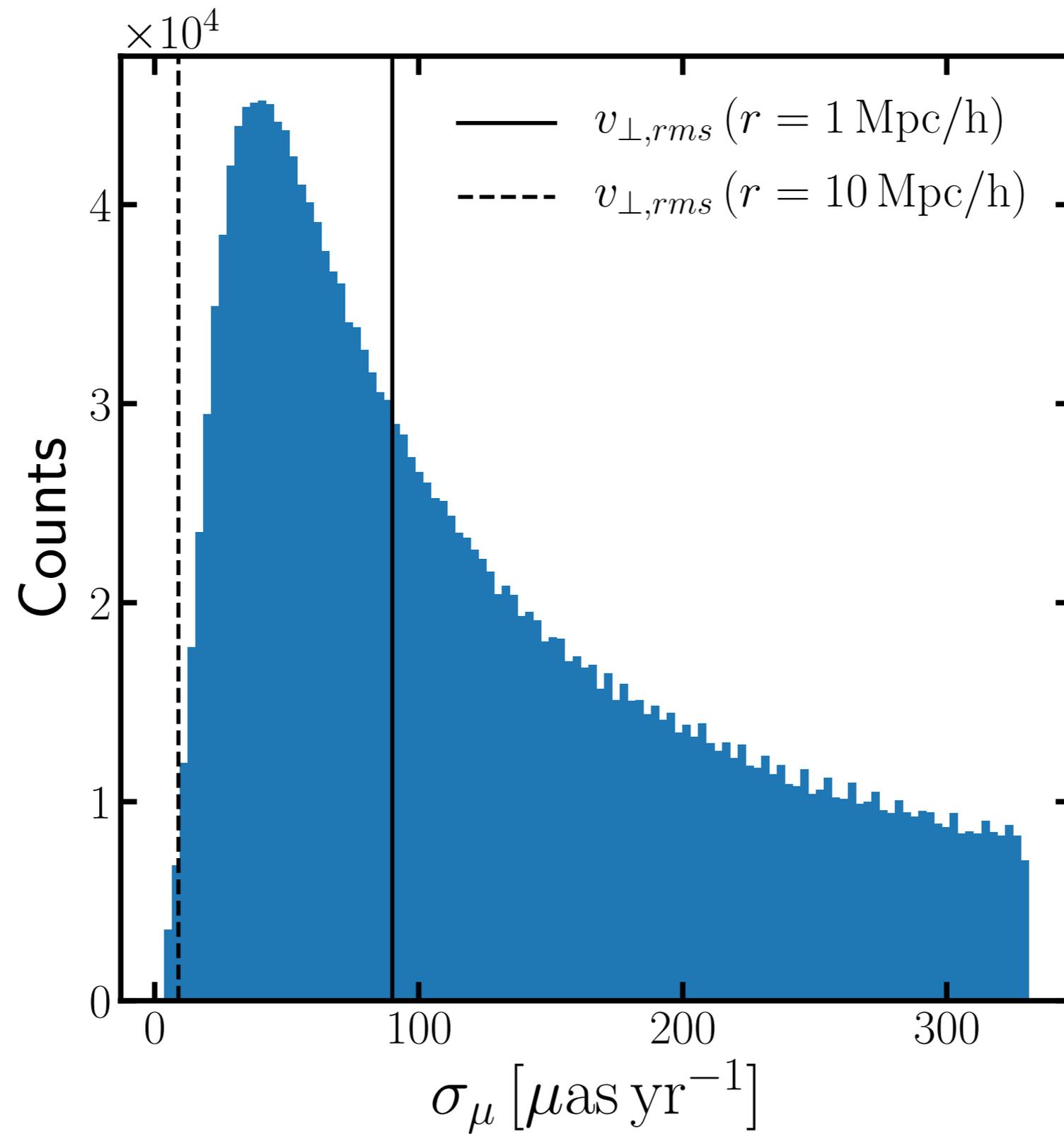
- **Secular aberration drift**

$$\mu(r) \sim 4 \mu\text{as yr}^{-1}$$

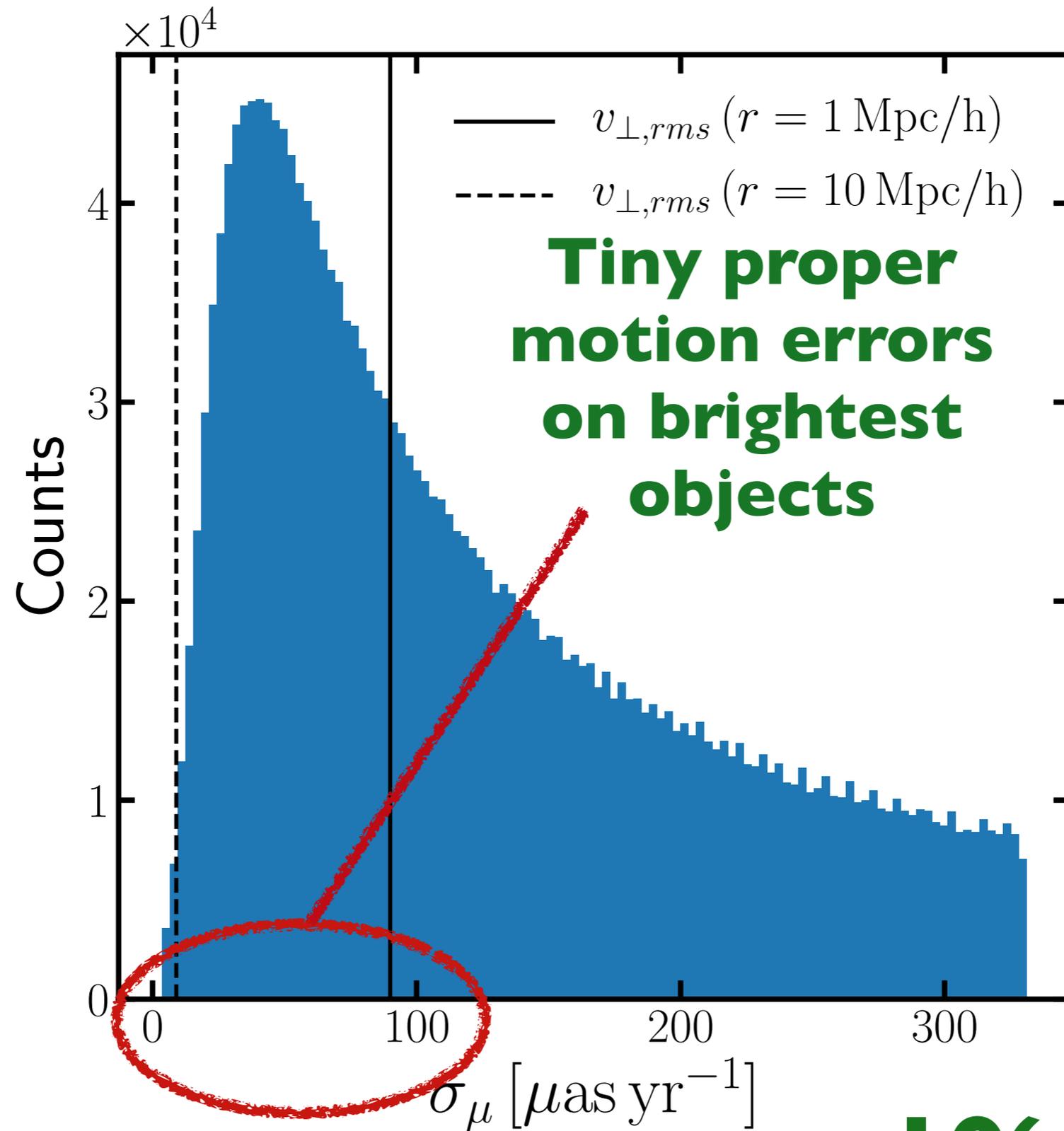
- **Transverse peculiar velocities**

$$\mu(r) \sim 90 \left(\frac{r}{1 \text{ Mpc}} \right)^{-1} \mu\text{as yr}^{-1}$$

End-of-mission Gaia proper motion uncertainties



End-of-mission Gaia proper motion uncertainties



Gaia end-of-mission simulated proper motions

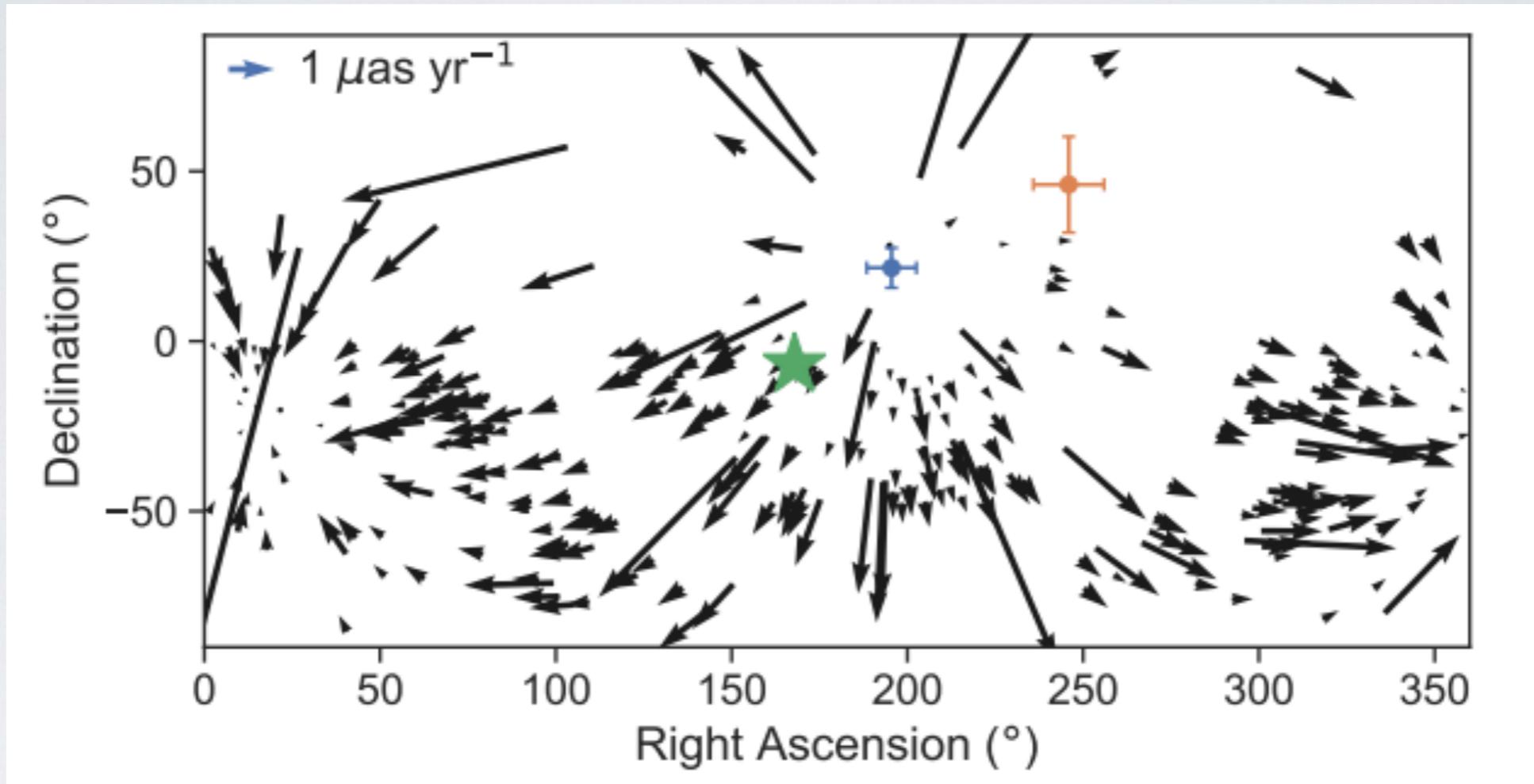
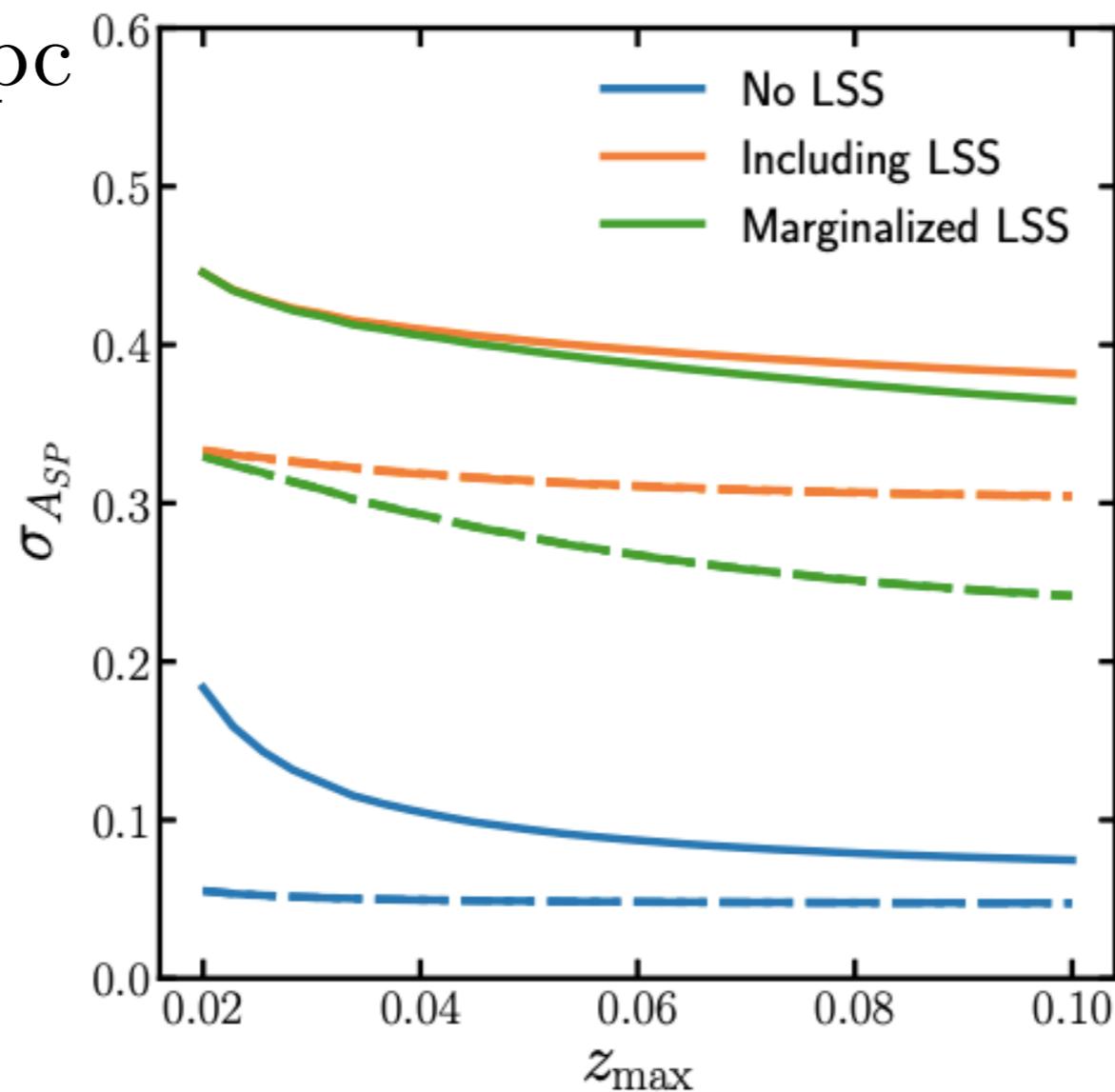


Figure from Paine et al (2020)

(First attempt at measuring secular parallax)

Gaia end-of-mission forecasts on secular parallax

$$r_{min} = 20 h^{-1} \text{Mpc}$$

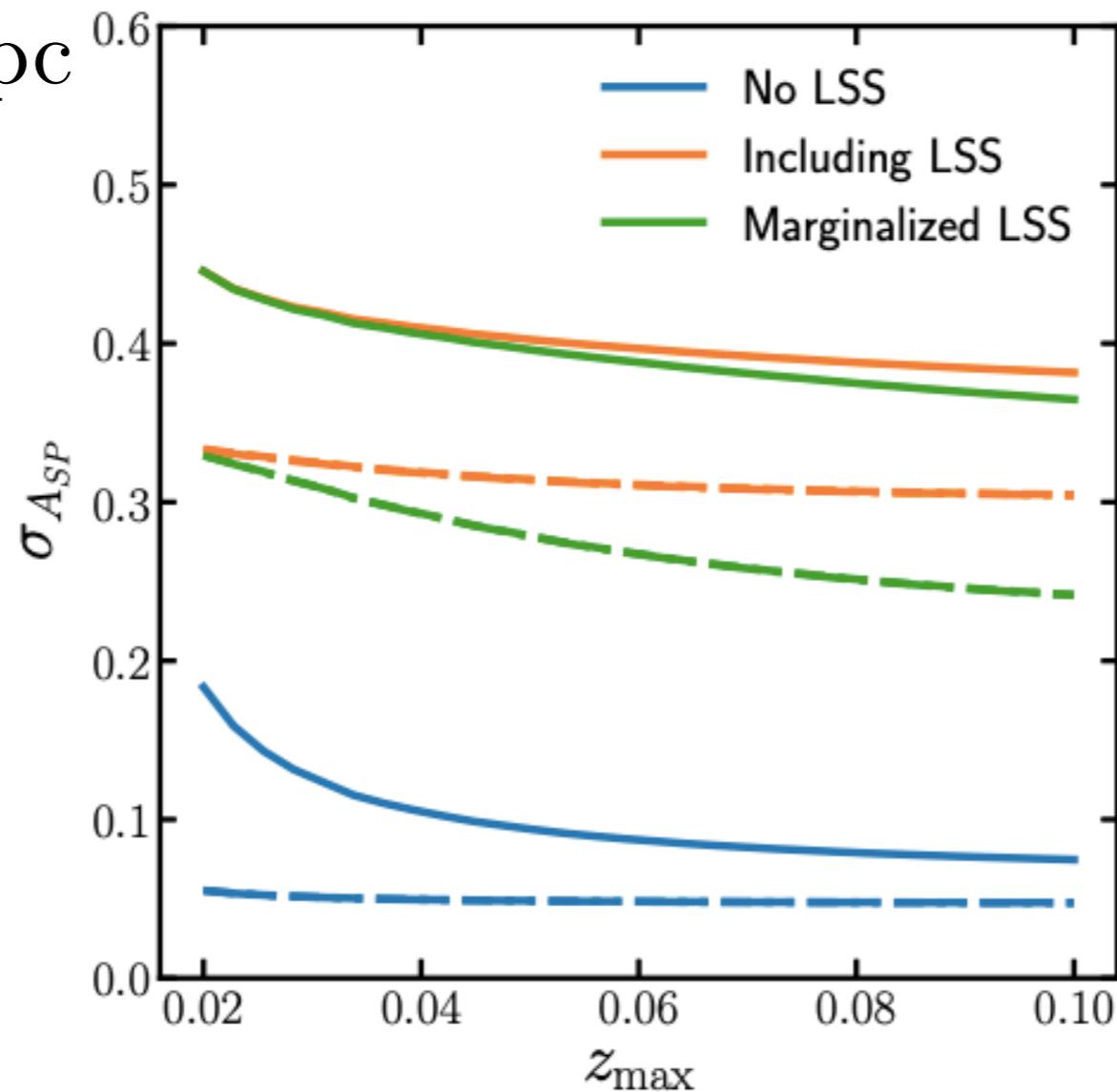


Solid:
Unknown secular aberration drift amplitude and direction

Dashed:
Fixed secular aberration drift direction, unknown amplitude

Gaia end-of-mission forecasts on secular parallax

$$r_{min} = 20 h^{-1} \text{Mpc}$$

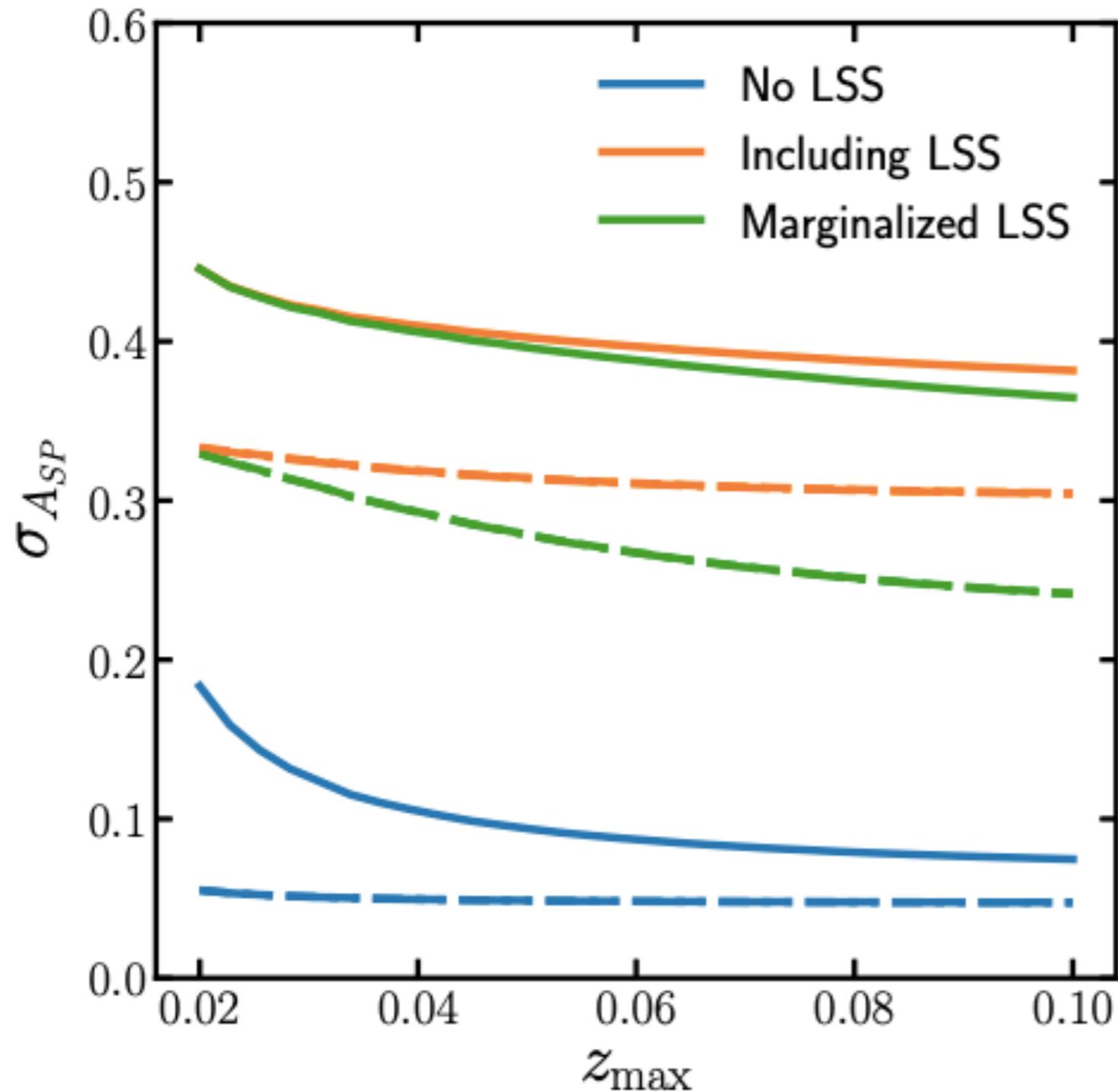


Solid:
Unknown secular aberration drift amplitude and direction

Dashed:
Fixed secular aberration drift direction, unknown amplitude

Could measure Hubble constant if we could identify proper motion dipole with CMB dipole; bulk flow complicates this!

Gaia end-of-mission forecasts on secular parallax



- Even with prior knowledge of velocity dipole from CMB, detection significance of Solar System-CMB motion is limited if peculiar dipole unknown.
- Potentially can make a significant detection of Solar System-Galaxy dipole: need to go to close, bright objects for this (Paine+20).
- Can get $\sim 3\%$ on H_0 with LSST, $\sim 1\%$ with NGRST with 3D velocity reconstruction + bias model (Croft 20)

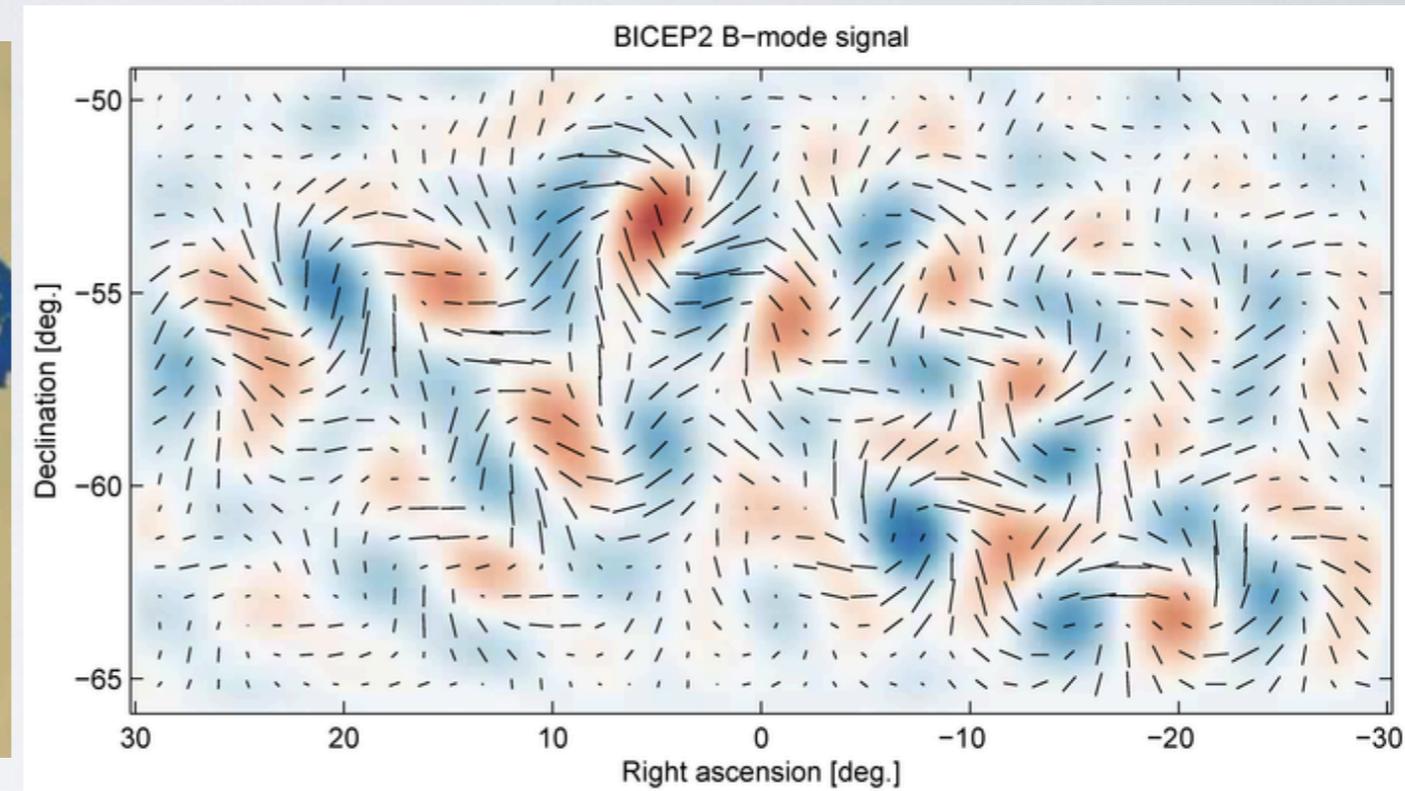
What could we measure with galaxy proper motions?

- Dipole due to acceleration (Secular Aberration Drift)
 - Independent of distance, so best off using QSOs (cf recent detections)
 - Partially degenerate with Secular Parallax (both E-mode dipoles) if range of distances limited.

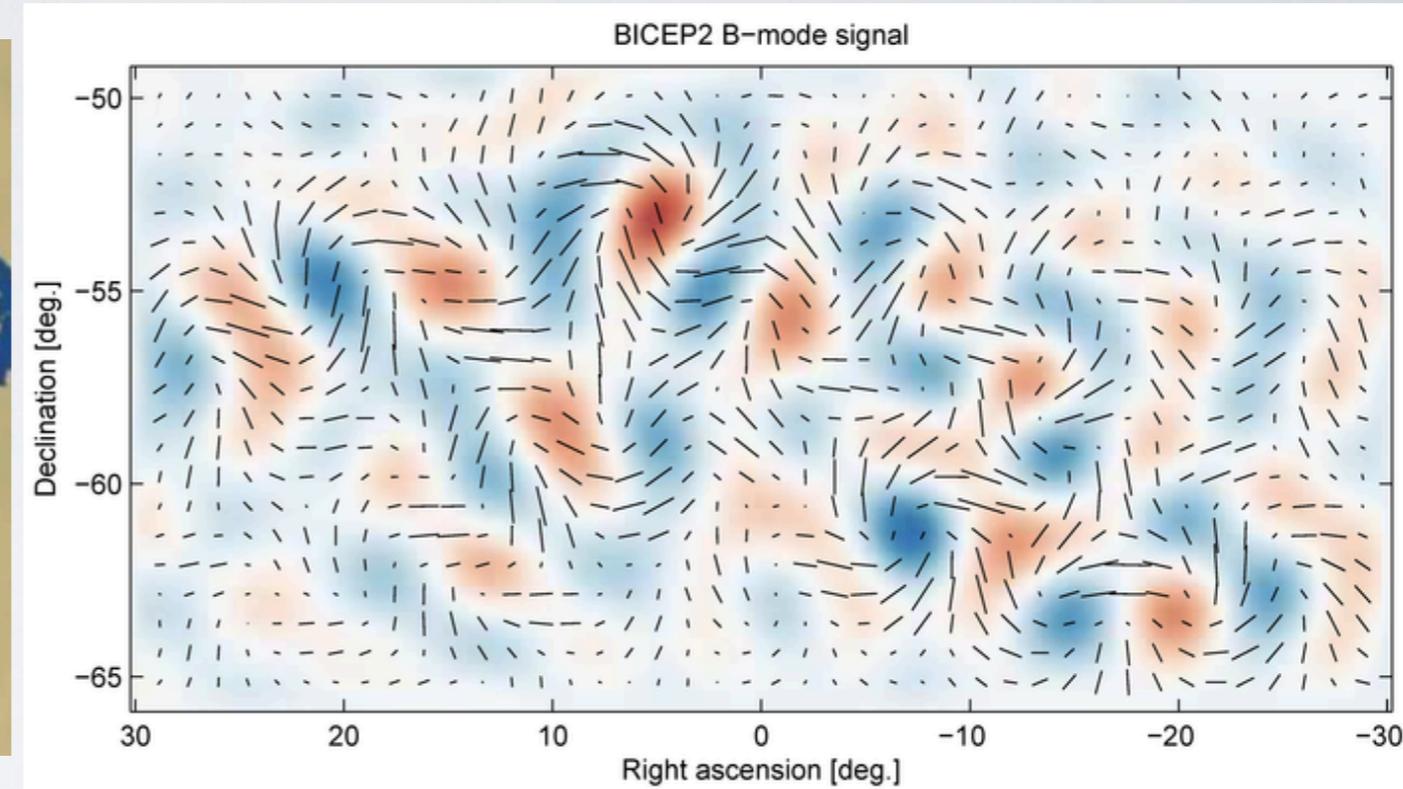
What could we measure with galaxy proper motions?

- Dipole due to acceleration (Secular Aberration Drift)
 - Independent of distance, so best off using QSOs (cf recent detections)
 - Partially degenerate with Secular Parallax (both E-mode dipoles) if range of distances limited.
- Peculiar velocities of galaxies
 - Have angular structure at all scales.
 - Still an E-mode.
 - If you want to compare with models you need at least:
 - 1) distances (TF/FP, or spectroscopic redshifts and incur extra noise from radial velocities).
 - 2) correlation functions, with predictions from large-scale structure theory (cosmology!)

The statistics of transverse velocities

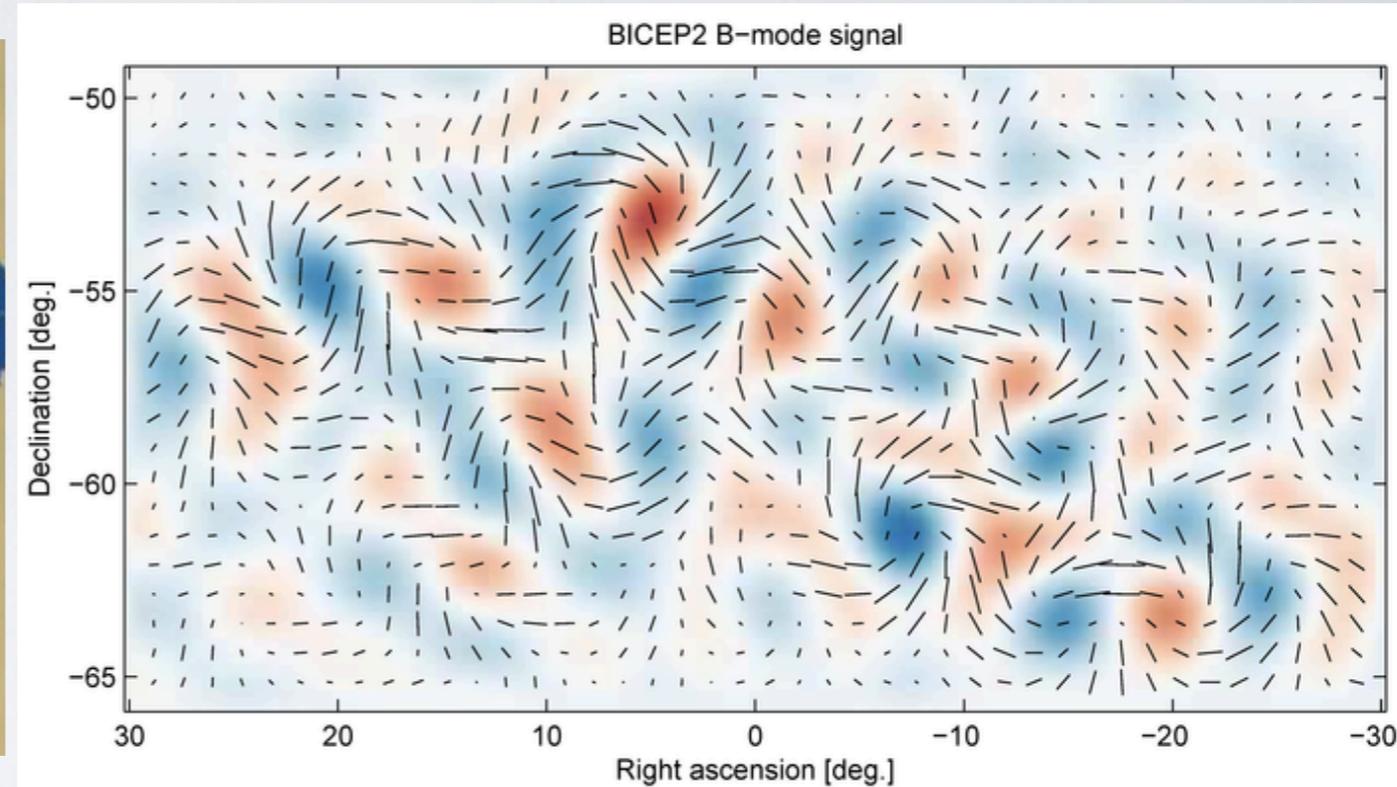


The statistics of transverse velocities



Vector field on the sphere

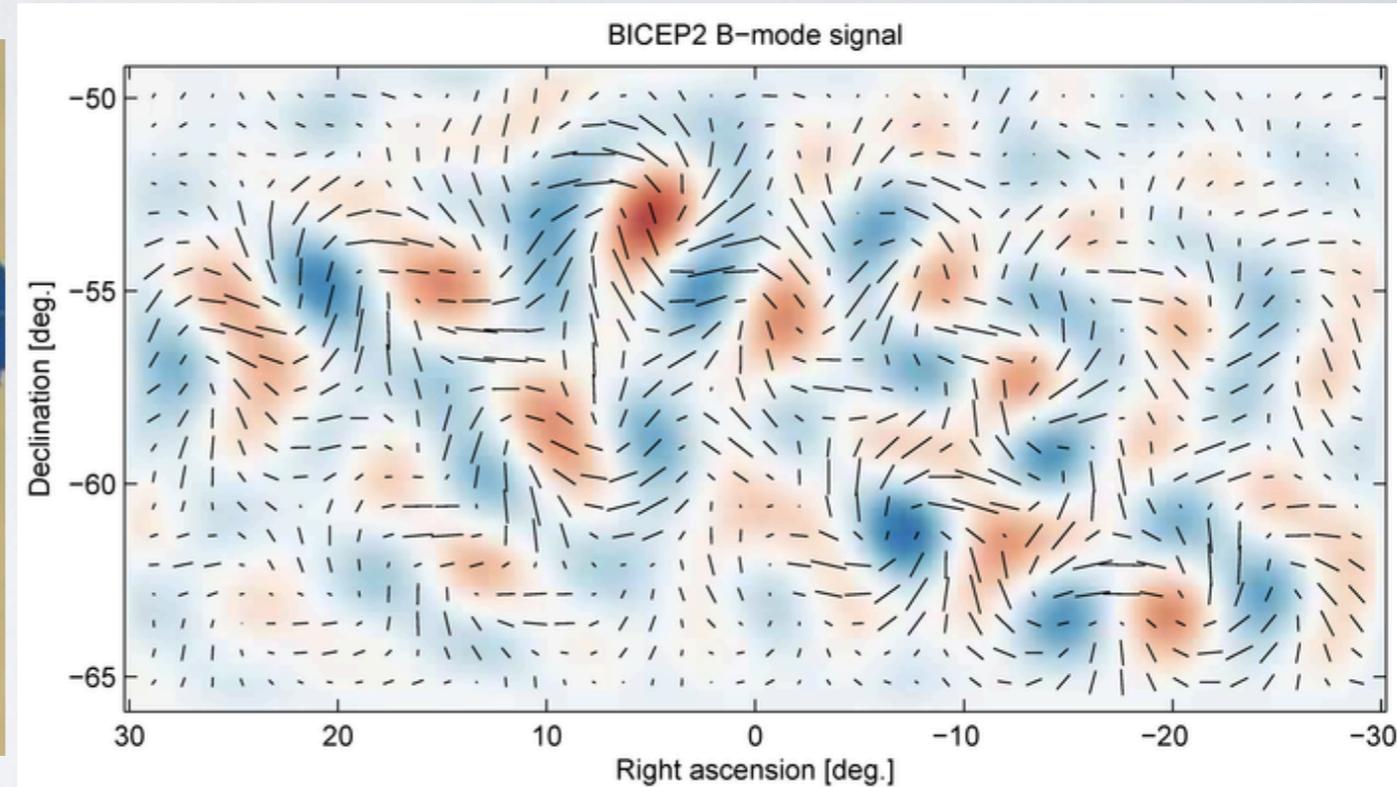
The statistics of transverse velocities



Vector field on the sphere

Gradient and curl part

The statistics of transverse velocities

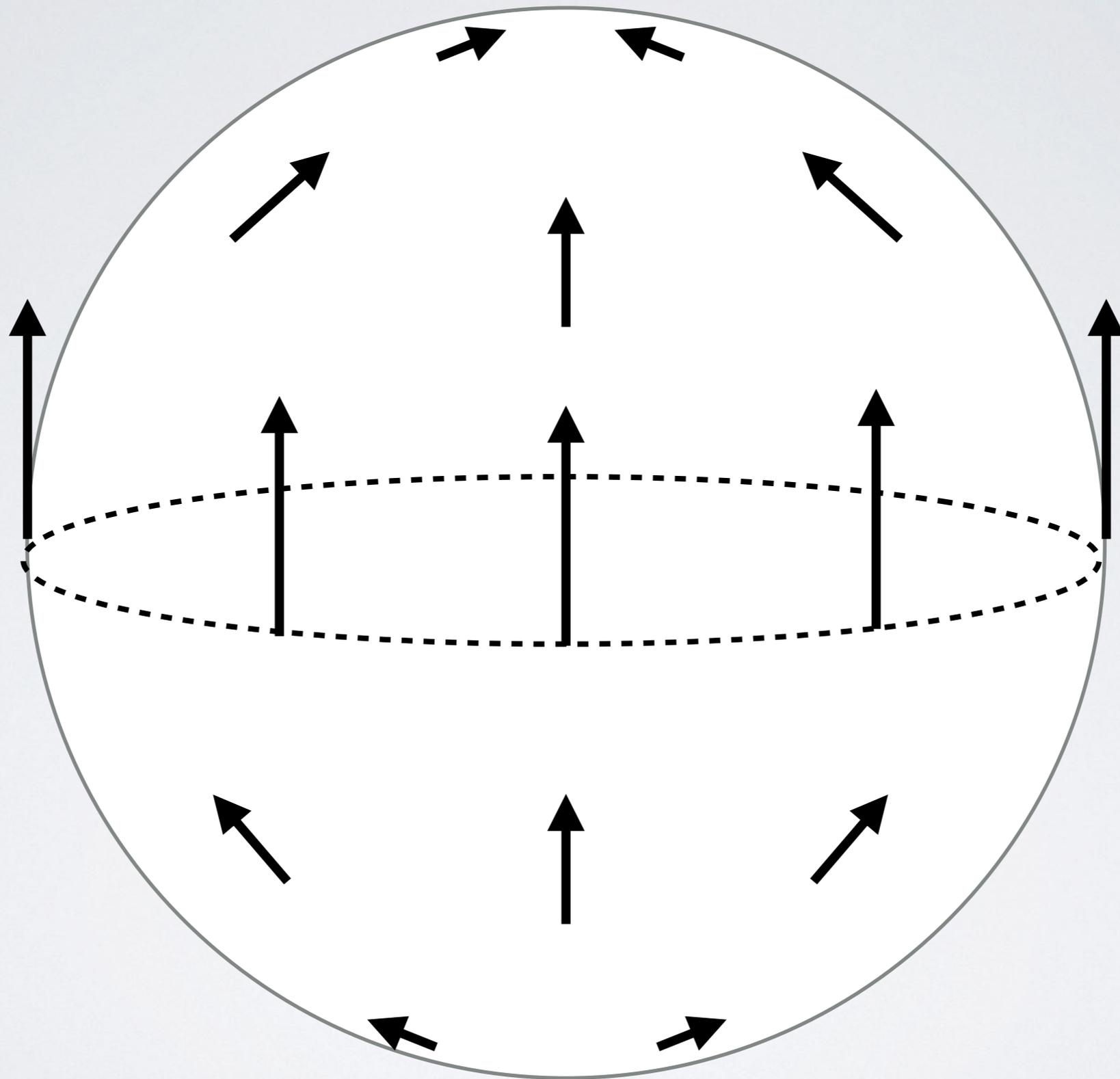


Vector field on the sphere

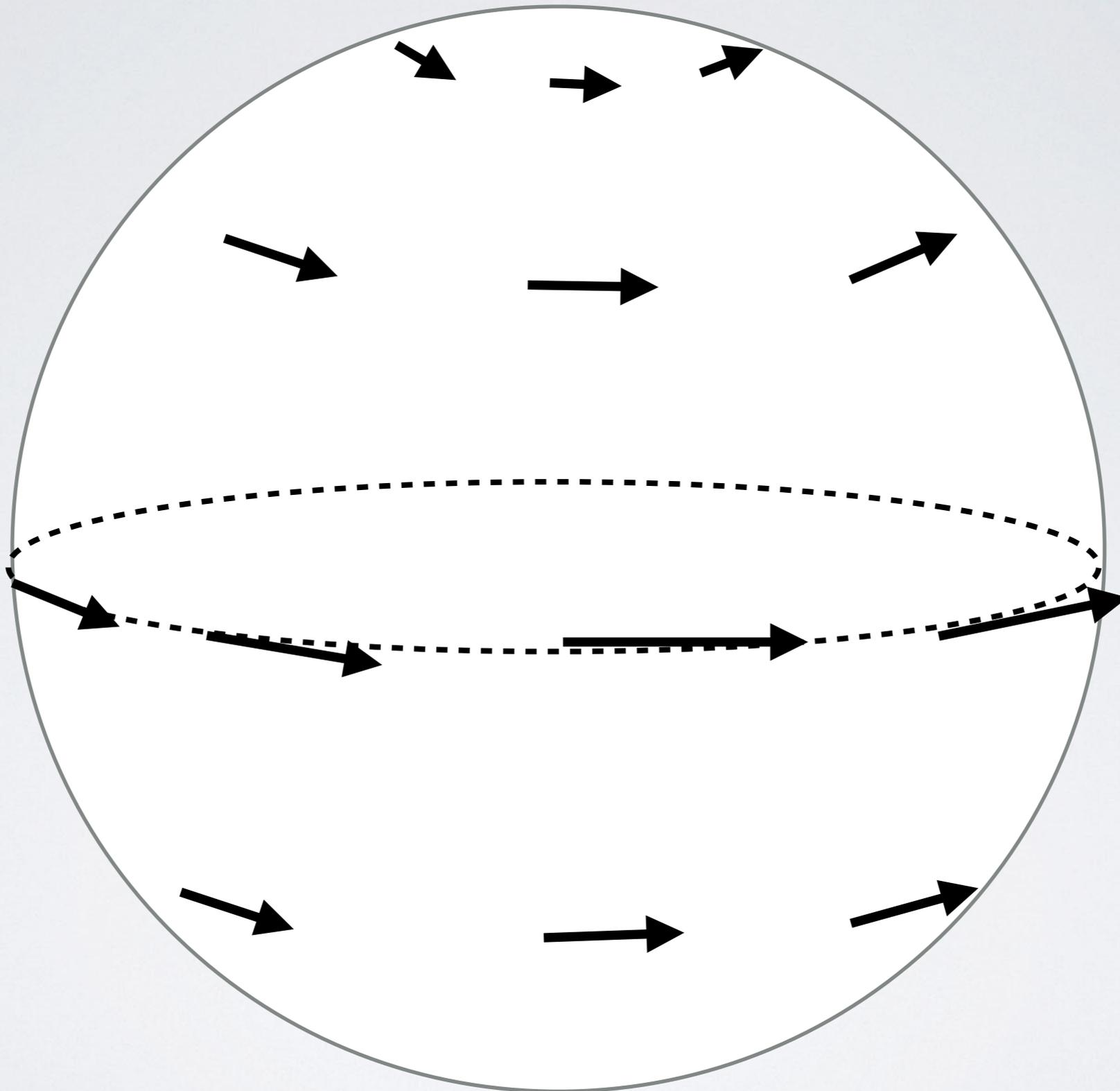
Gradient and curl part

E and B modes...

E-mode dipole



B-mode dipole



Spin-weighted spherical harmonics

$$V_{\pm} = V_{\theta} \pm iV_{\phi}$$

$$V_{\pm}(\hat{\mathbf{n}}) = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} (\mp\epsilon_{\ell m} + i\beta_{\ell m})_{\pm 1} Y_{\ell m}(\hat{\mathbf{n}})$$

$${}_{\pm 1}Y_{\ell m}(\hat{\mathbf{n}})$$

- Complete set for square-integrable vector fields on the sphere.
- Simple rotational properties.
- Orthonormal.
- Explicit form depends on local basis choice.
- Fast implementations available in LIBSHARP (packaged with HEALPIX).

Spin-weighted spherical harmonics

Constant relative velocity
(e.g. Secular Parallax):

$$\epsilon_{lm} = -\frac{8\pi}{3} \frac{|\mathbf{V}|}{\sqrt{2}} Y_{1m}^*(\hat{\mathbf{V}}) \delta_{l1},$$
$$\beta_{lm} = 0.$$

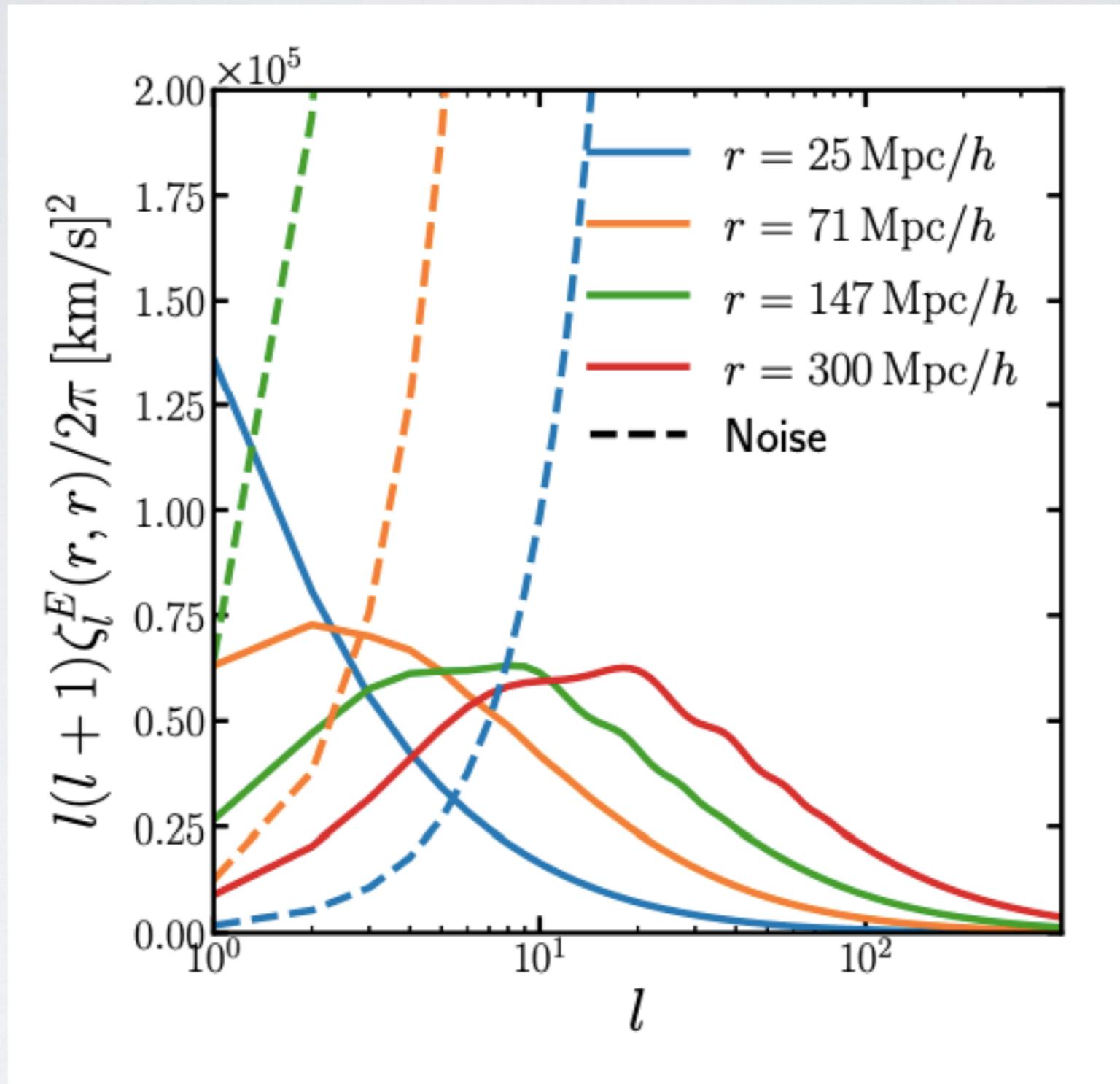
Large-scale structure:

$$\epsilon_{lm}(r) = -4\pi i^l \sqrt{l(l+1)} \int \frac{d^3\mathbf{k}}{(2\pi)^3} v(\mathbf{k}, r) \frac{j_l(kr)}{kr} Y_{lm}^*(\hat{\mathbf{k}}),$$
$$\beta_{lm}(r) = 0$$

Relation with VSH:

$$(\hat{\boldsymbol{\theta}} \pm i\hat{\boldsymbol{\phi}}) \cdot \mathbf{S}_{lm} = \mp_{\pm 1} Y_{lm},$$
$$(\hat{\boldsymbol{\theta}} \pm i\hat{\boldsymbol{\phi}}) \cdot \mathbf{T}_{lm} = i_{\pm 1} Y_{lm}.$$

Angular power spectrum of transverse velocities



Optimal power spectrum estimators

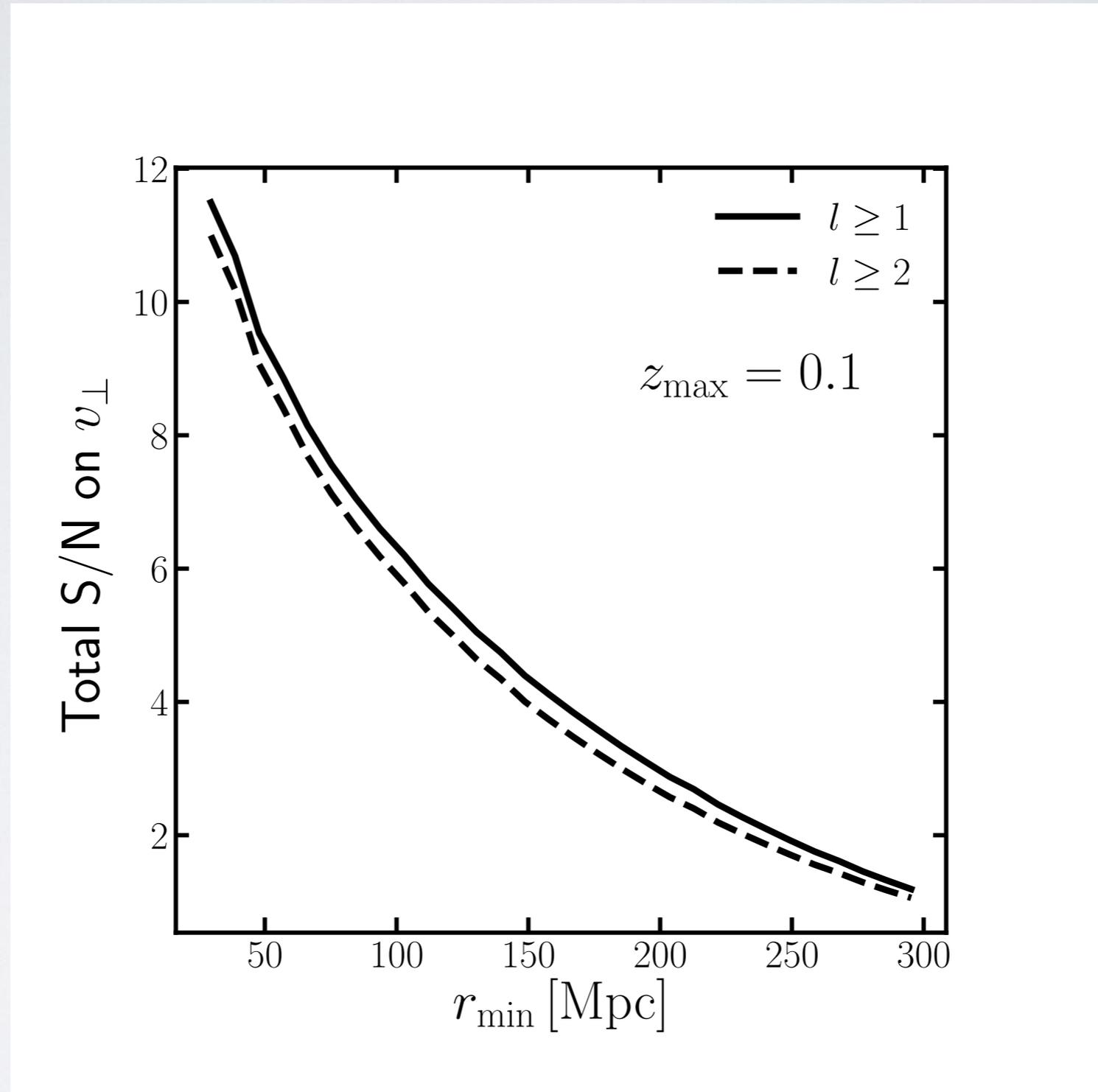
Use likelihood function of proper motions to derive optimal estimators

Project out the dipole mode

Inverse-variance weighted two-point correlation function of proper motions — rotate each vector to physical basis defined by geodesic connecting the two points

Optimal 'stack' of galaxy proper motions

Gaia end-of-mission forecast on transverse velocity power spectrum (optimistic)



Amplitude a proxy for $f \sigma_8 H_0$

AH 2019

Conclusions

arXiv:1811.05454

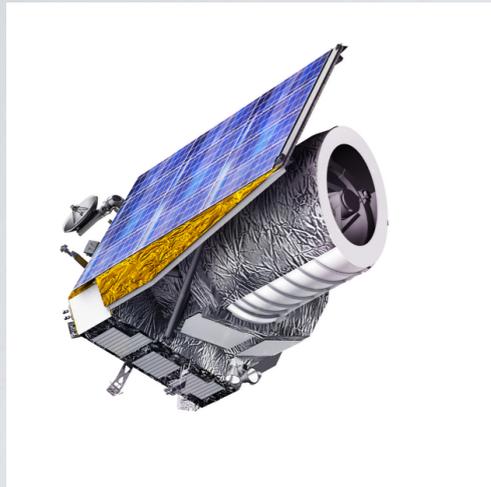
- **Gaia will measure proper motions for a million galaxies**
- **A probe of the Solar System's local motion with respect to the CMB and the Hubble constant**
- **A probe of transverse velocities due to gravitational collapse - a probe of dark energy**
- **Gaia: 2-sigma detection of local motion, 10-sigma detection of LSS transverse velocities**
- **Complicated by relativistic aberration, centroid errors, cosmic variance, bulk flow...**

Part II

The impact of our local environment on cosmological statistics

Based on arXiv:1911.07855

The 2020s - the decade of wide multi-purpose LSS surveys



Euclid

Launch: June 2022

Wide survey: 15,000 sq. deg.

Weak lensing and galaxy clustering



LSST

Science ops begin: 2023

Main survey: 18,000 sq. deg.

Weak lensing and galaxy clustering



DESI

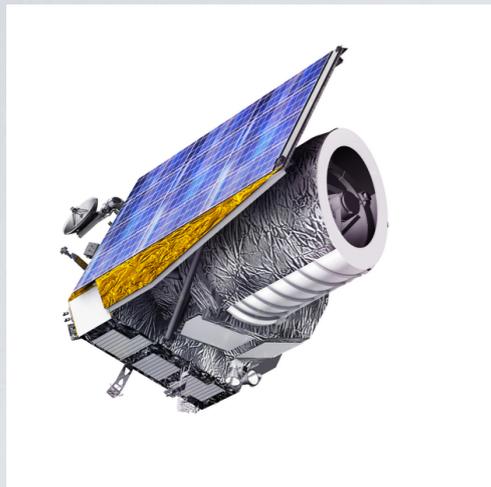
Science ops begin: 2020

Main survey: 14,000 sq. deg.

BAO, RSD

Also SKA, SO, 4MOST,
LiteBIRD, HIRAX,
WFIRST, CMBS4, MSE,
etc. etc.

The 2020s - the decade of wide multi-purpose LSS surveys



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Forecasts

$$\sigma(\Sigma m_\nu) \sim 0.03 \text{ eV}$$

$$\sigma(w_0) \sim 0.01 - 0.03$$

$$\sigma(w_a) \sim 0.1$$

Precision

constraints on dark energy, neutrino mass, primordial NG, modified gravity etc.

The 2020s - the decade of wide multi-purpose LSS surveys



Euclid

Forecasts

Tiny statistical error bars

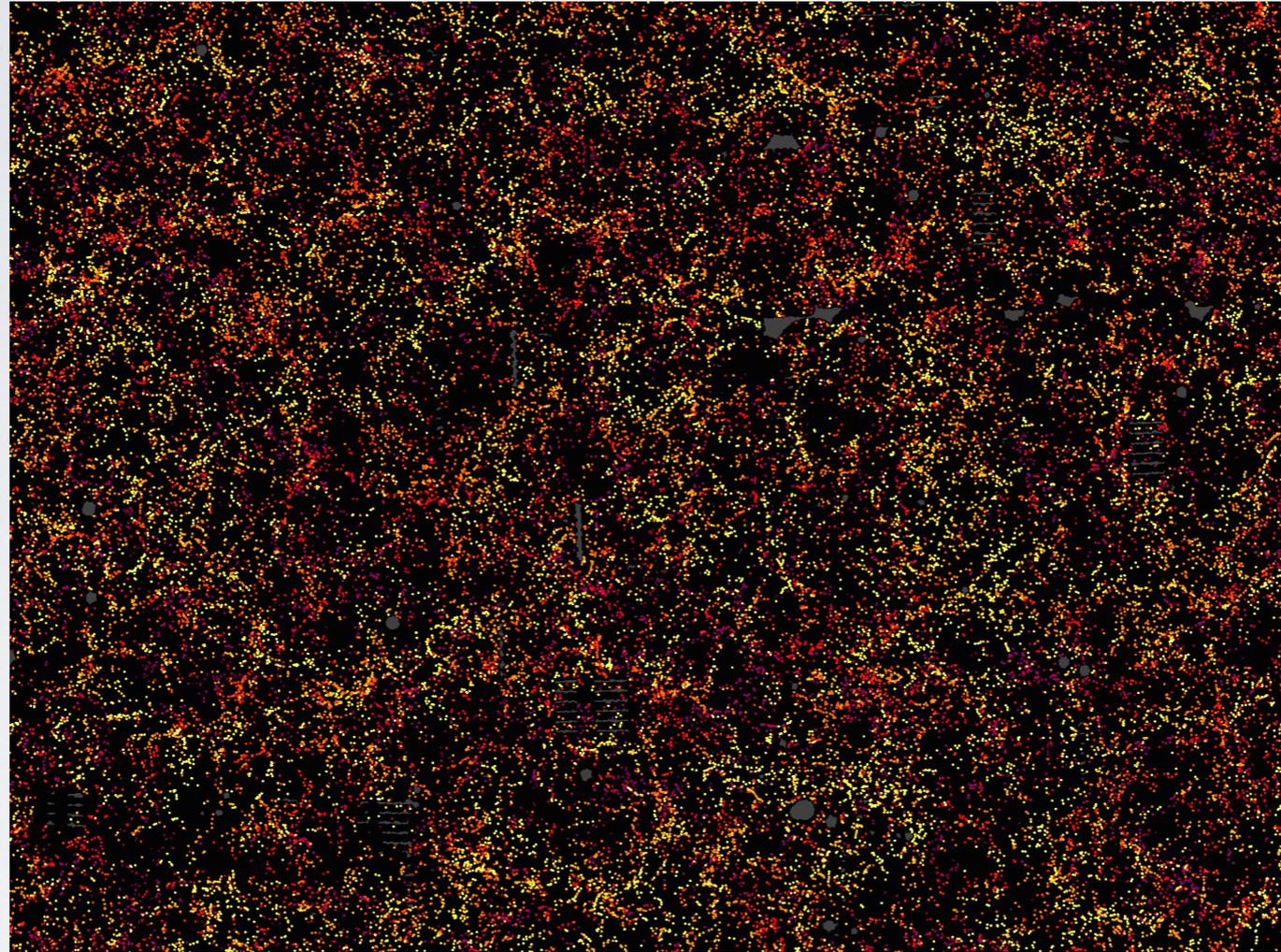
**Systematics need to be kept
under control**

Small effects can matter!

Main survey: 14,000 sq. deg.
BAO, RSD

modified gravity etc.

A schematic illustration of what LSS surveys measure



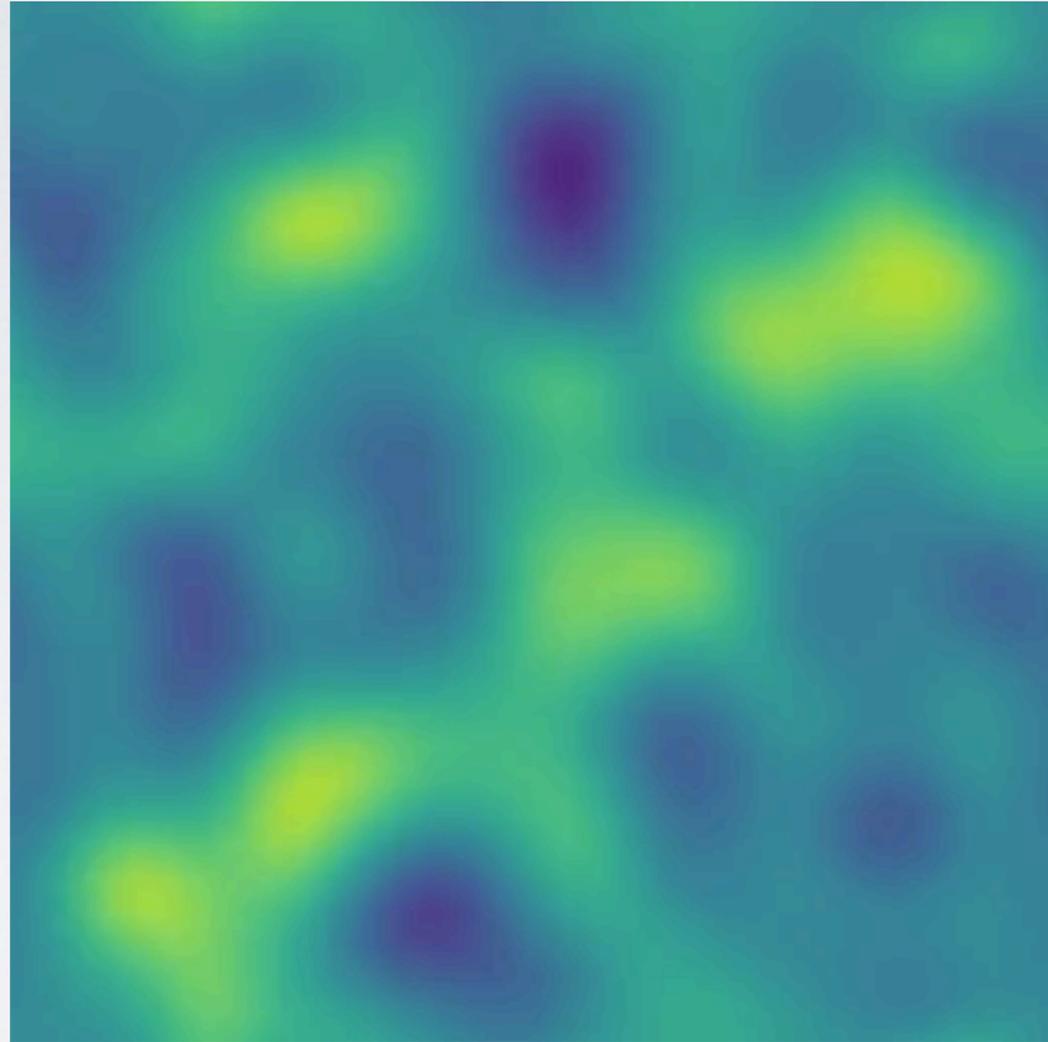
BOSS CMASS. Image credit: Daniel Eisenstein and the SDSS-III collaboration

Measure power spectrum or correlation function of map

Fit an ensemble-averaged power spectrum

Infer posterior probability of cosmological parameters

The ensemble - Gaussian initial conditions



- Initial curvature fluctuations laid down during inflation
- Initial power spectrum constrained by CMB
- Evolution imposes physical scales (equality scale, Silk damping scale, non-linear scale)

Our local environment

All astronomical measurements are made from within the Local Group, a **dense and **atypical** position in the Universe**



Virgo Cluster. Image: Wikimedia Commons/Kees Scherer

Our local environment

All astronomical measurements are made from within the Local Group, a **dense and **atypical** position in the Universe**



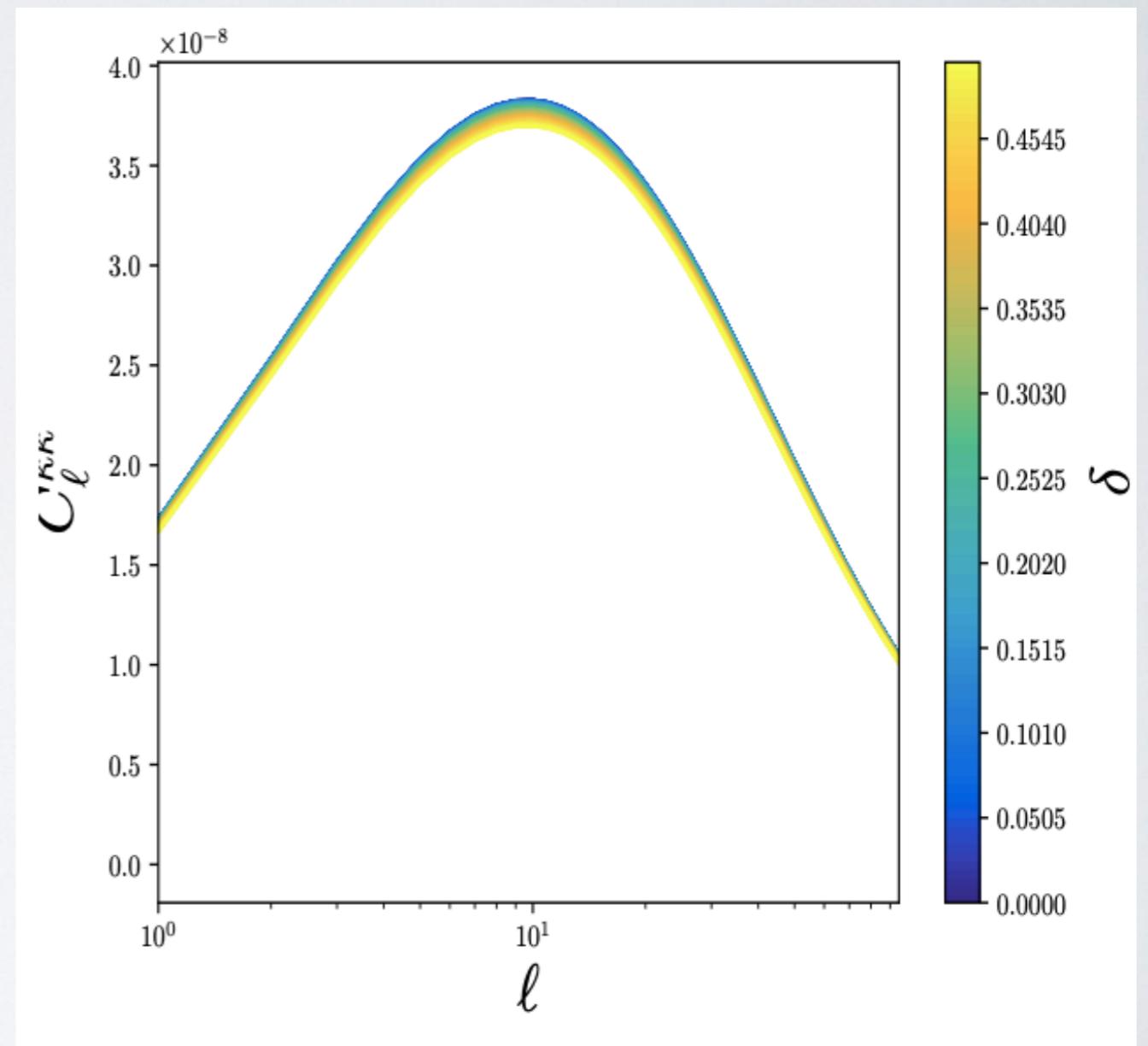
Virgo Cluster. Image: Wikimedia Commons/Kees Scherer

Q: Does our dense local environment influence large-scale structure through spatial correlations in the density field?

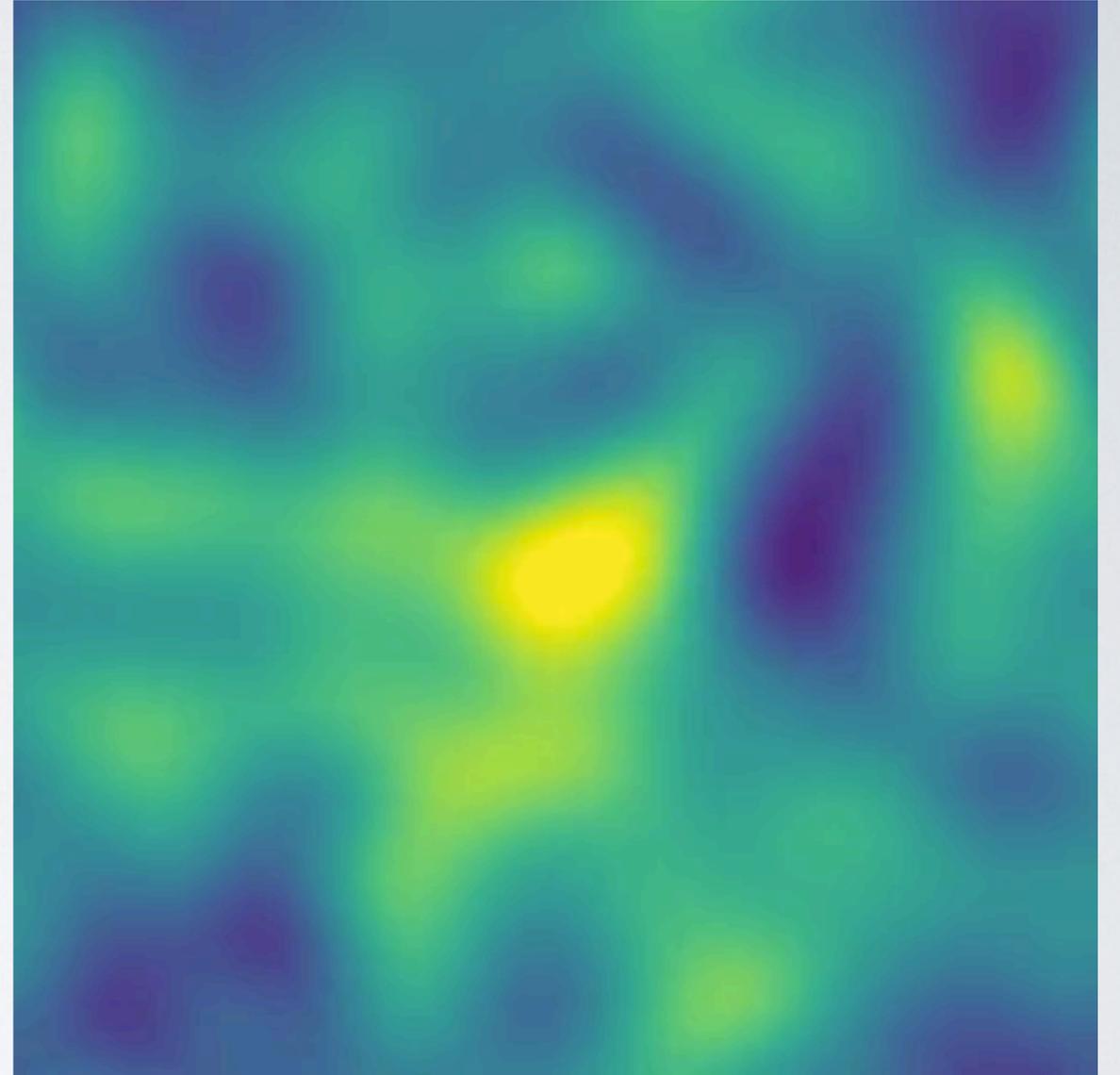
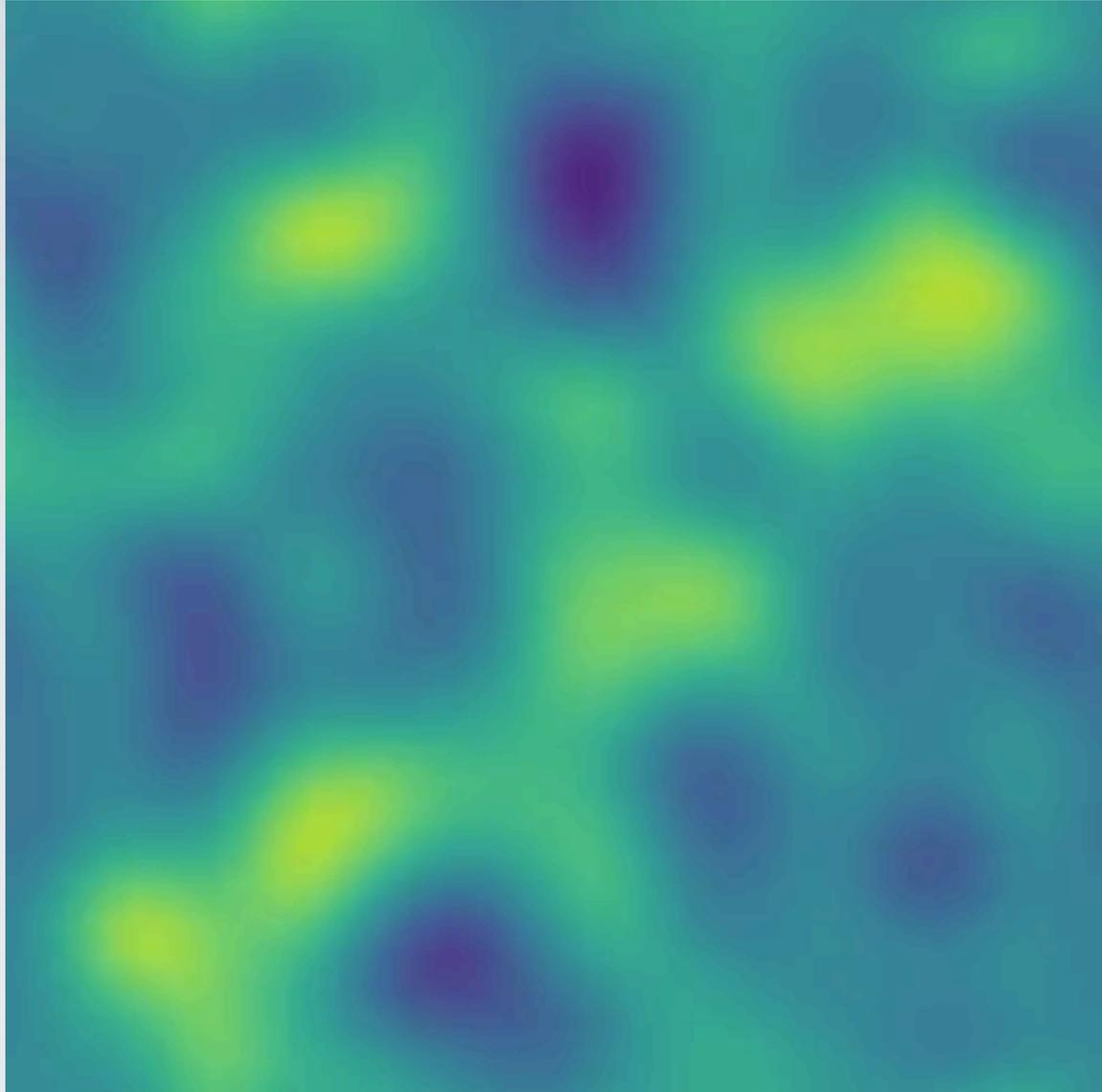
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**Reischke et al. 2019 :
percent-level effects in
weak lensing power
spectra across all
angular scales.**

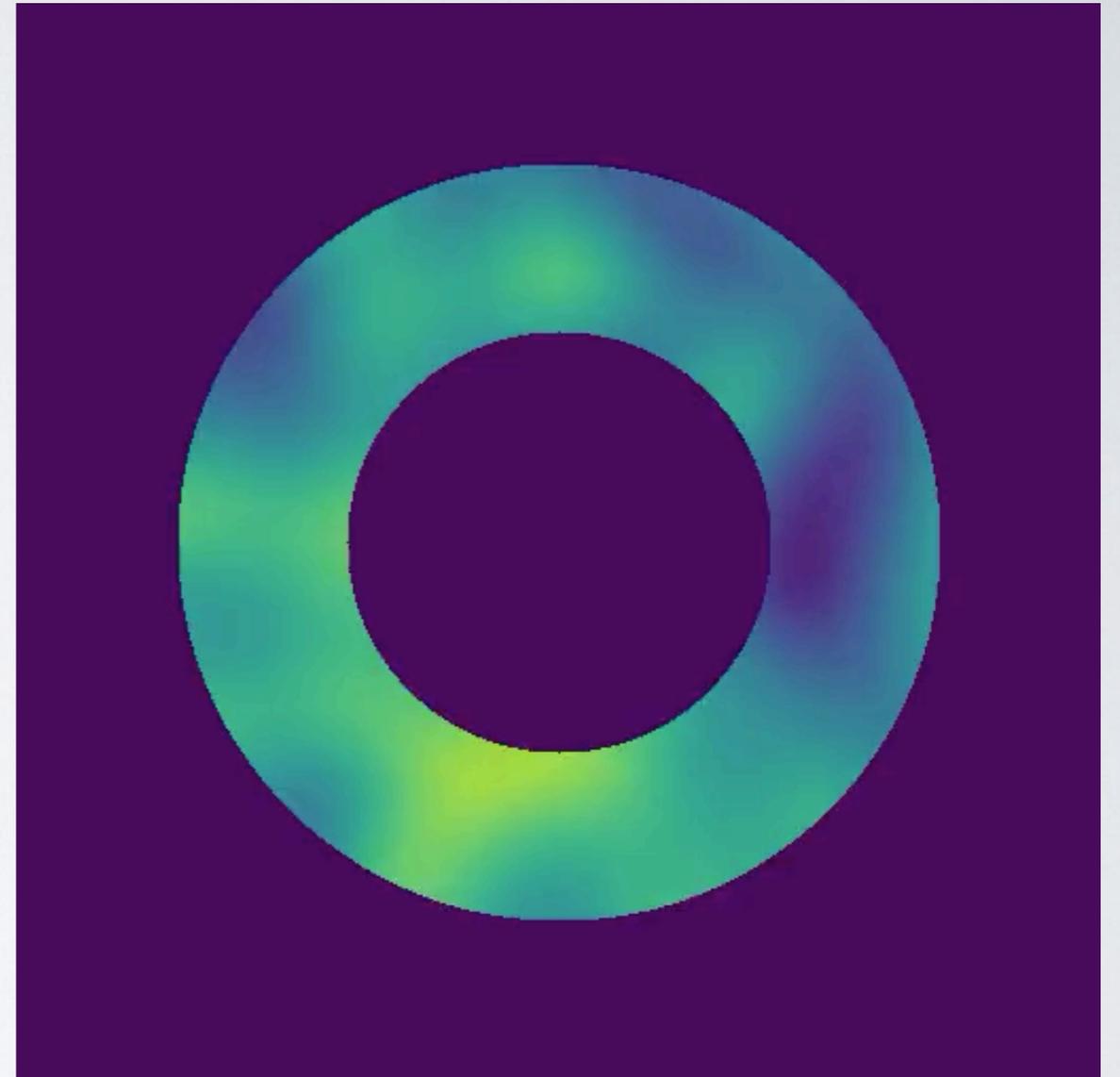
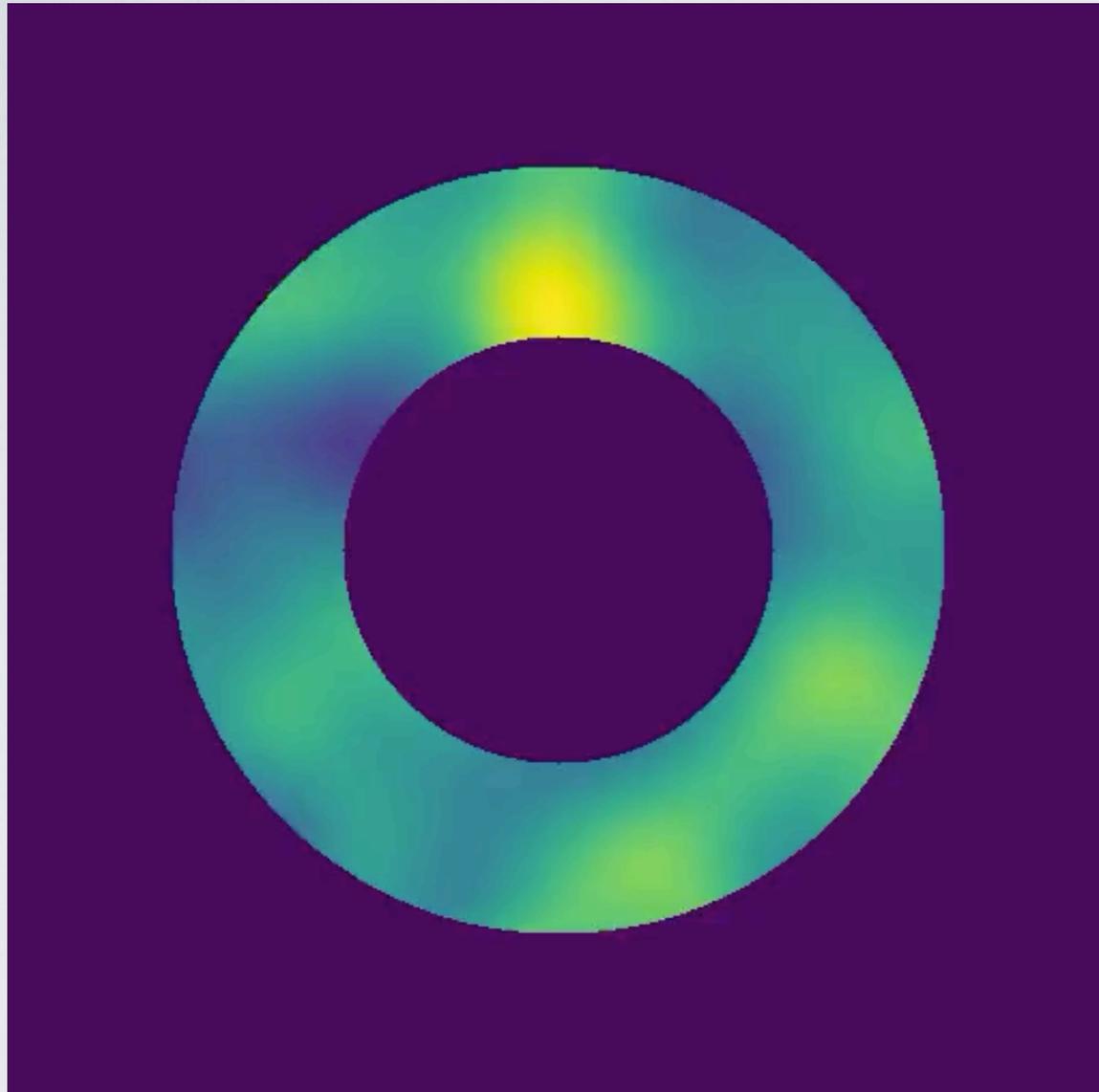
**Would be important
for Euclid!**



Constrained and unconstrained ensembles



Constrained and unconstrained ensembles



Q: Given we reside in an over density which cannot be 'averaged out', which ensemble should be used to compute model predictions?

Q: Is there a significant difference between constrained and unconstrained statistics for cosmological fields?

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AH 2020 (1911: 07855)

- For Gaussian fields, **no** effect for $l > 0$.
- Need to have **non-Gaussian** (i.e. non-linear) fields to see any effect at all.
- Can predict conditional power spectra analytically using second-order perturbation theory.
- Result is proportional to **bispectrum** times the **local smoothed density contrast**.

Conditional statistics

Conditional mean:

$$\langle \delta_i | \delta_0 \rangle = \langle \delta_i \delta_0 \rangle \frac{\delta_0}{\sigma^2} + \frac{1}{2} \left[\langle \delta_i \delta_0^2 \rangle - \frac{\langle \delta_i \delta_0 \rangle \langle \delta_0^3 \rangle}{\sigma^2} \right] \left(\frac{\delta_0^2}{\sigma^4} - \frac{1}{\sigma^2} \right)$$

Conditional covariance:

$$\begin{aligned} \text{cov}(\delta_i, \delta_j | \delta_0) = & \langle \delta_i \delta_j \rangle - \frac{\langle \delta_i \delta_0 \rangle \langle \delta_j \delta_0 \rangle}{\sigma^2} + \\ & \left[\langle \delta_i \delta_j \delta_0 \rangle - \frac{\langle \delta_i \delta_0 \rangle \langle \delta_j \delta_0^2 \rangle -}{\sigma^2} \right. \\ & \left. \frac{\langle \delta_j \delta_0 \rangle \langle \delta_i \delta_0^2 \rangle}{\sigma^2} + \frac{\langle \delta_i \delta_0 \rangle \langle \delta_j \delta_0 \rangle \langle \delta_0^3 \rangle}{\sigma^4} \right] \frac{\delta_0}{\sigma^2}. \end{aligned}$$

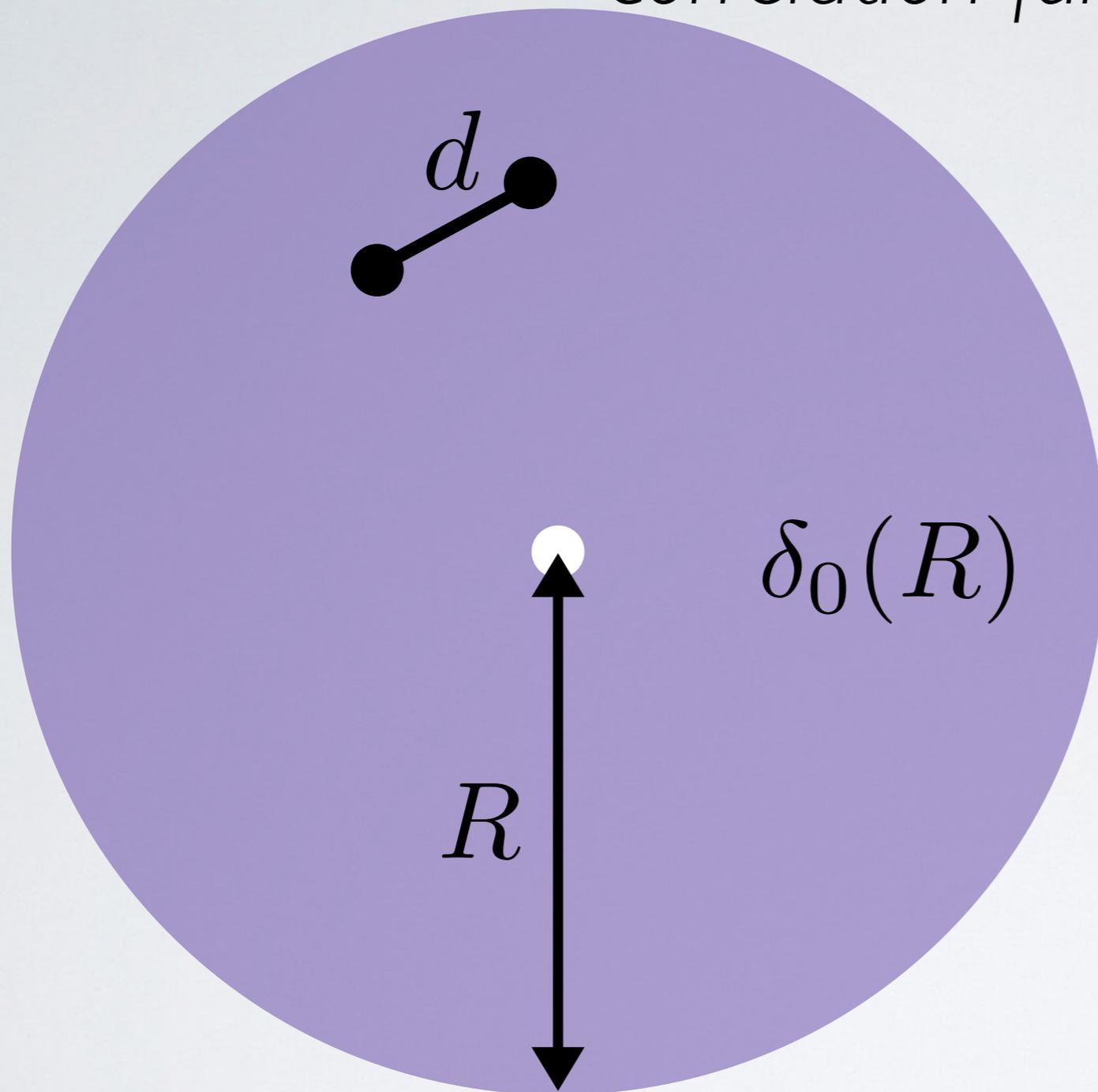
Conditional angular power spectrum

$$\begin{aligned}\tilde{C}_l(r_i, r_j) = & C_l(r_i, r_j) - \frac{4\pi\xi(r_i)\xi(r_j)}{\sigma^2}\delta_{l0}^K \\ & + [b_{ll0}(r_i, r_j, 0) - \xi(r_i)b_{000}(r_j, 0, 0)\delta_{l0}^K \\ & - \xi(r_j)b_{000}(r_i, 0, 0)\delta_{l0}^K + \xi(r_i)\xi(r_j)b_{000}(0, 0, 0)\delta_{l0}^K] \frac{\delta_0}{4\pi\sigma^2},\end{aligned}$$

Correction at $l > 0$ is

$$\tilde{C}_\ell(r_i, r_j) = C_\ell(r_i, r_j) + b_{\ell\ell 0}(r_i, r_j, 0) \frac{\delta_0}{4\pi\sigma^2}$$

Effect of a local overdensity on the matter correlation function

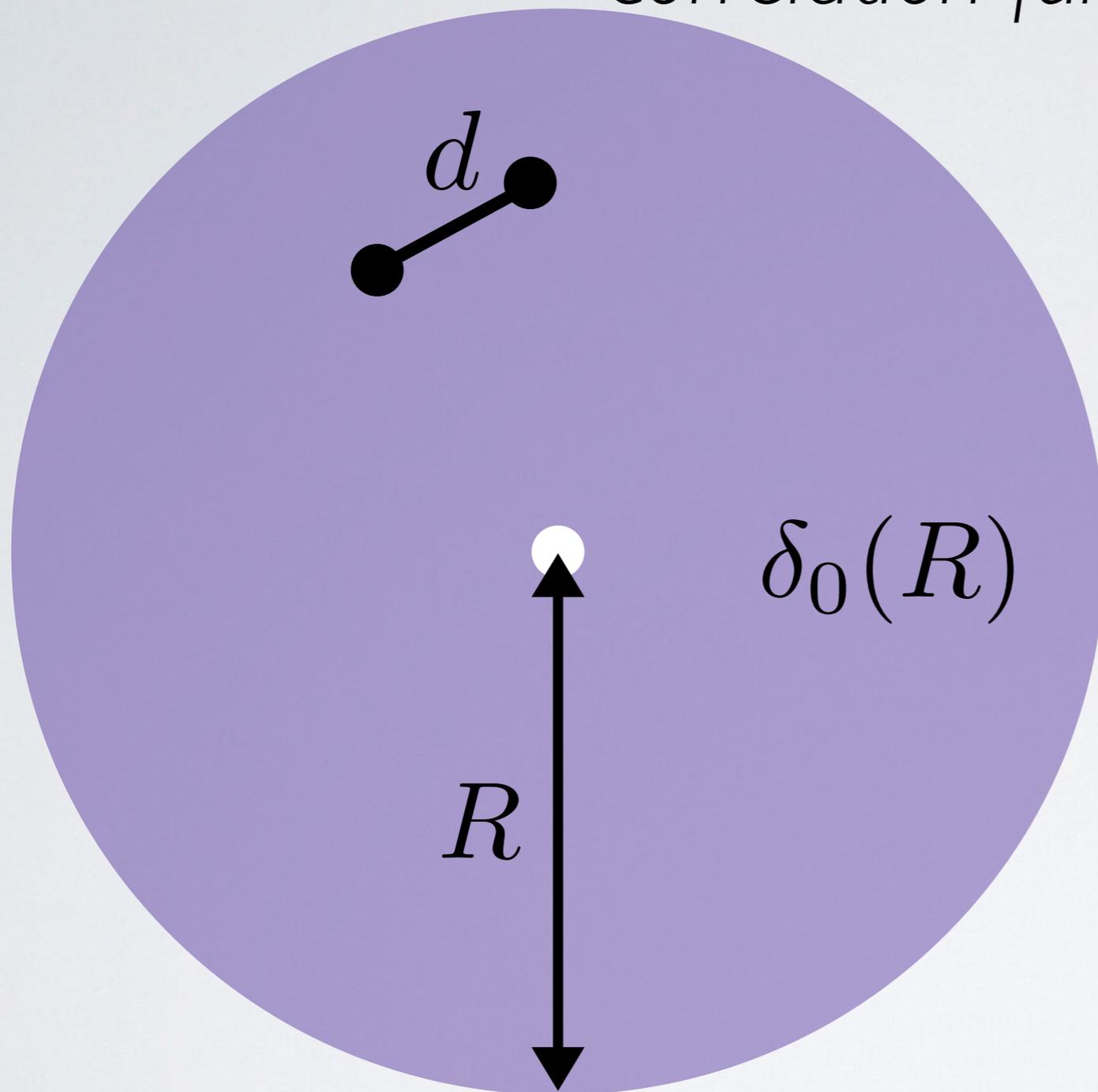


Matter fluctuations within a large-scale overdensity effectively live in an overdense 'closed' Universe

- Matter fluctuations grow faster in a closed Universe
- All distances dilated by effective background curvature
- Mean matter density renormalised

Correction to correlation function is $\approx \left[\frac{68}{21} \xi(d) + \frac{d}{3} \xi'(d) \right] \delta_0(R)$

Effect of a local overdensity on the matter correlation function

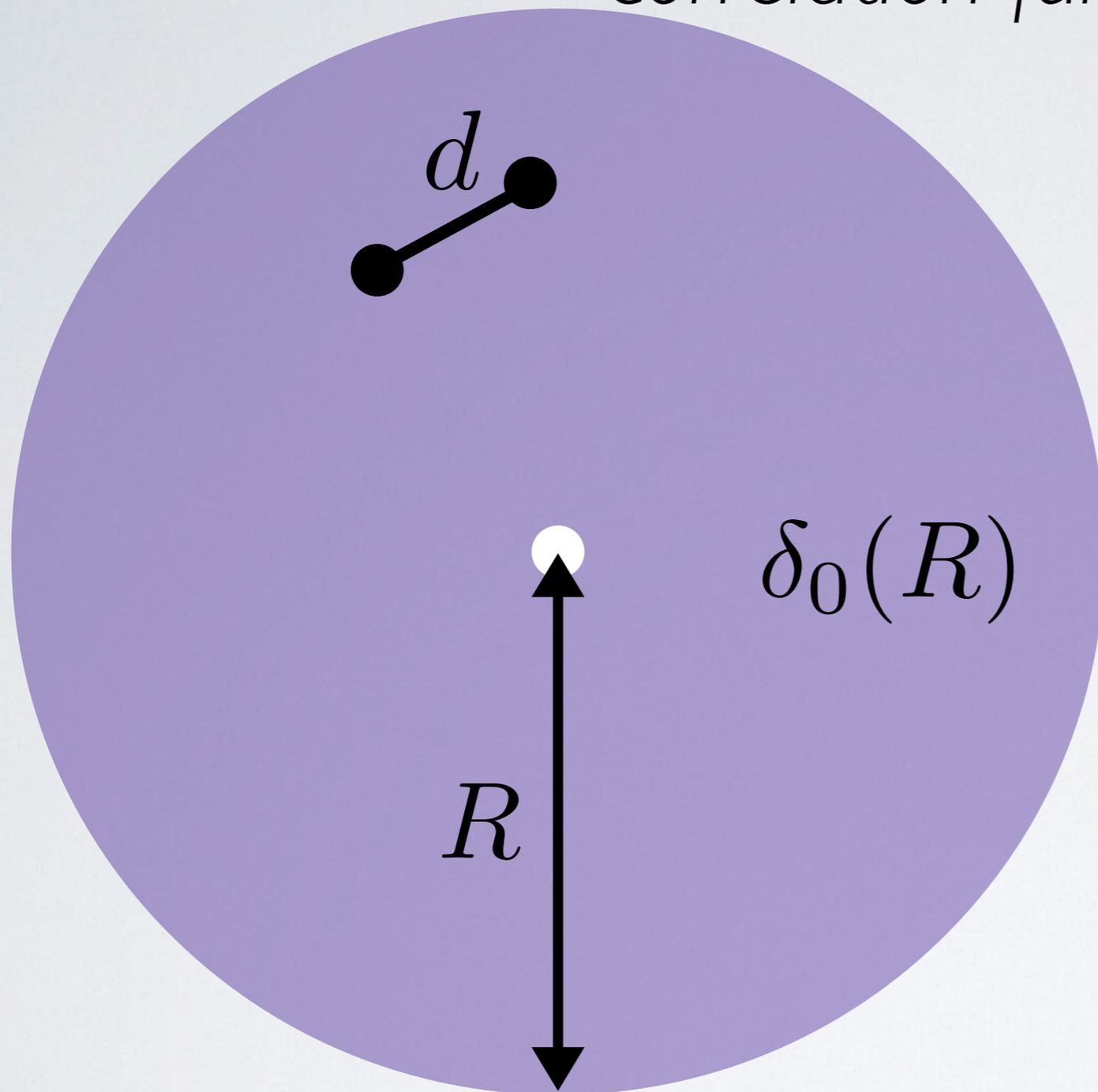


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With $\xi(d) \sim d^{-n}$ fractional correction is $\sim \left(\frac{68}{21} - \frac{n}{3} \right) \delta_0(R)$

Effect of a local overdensity on the matter correlation function

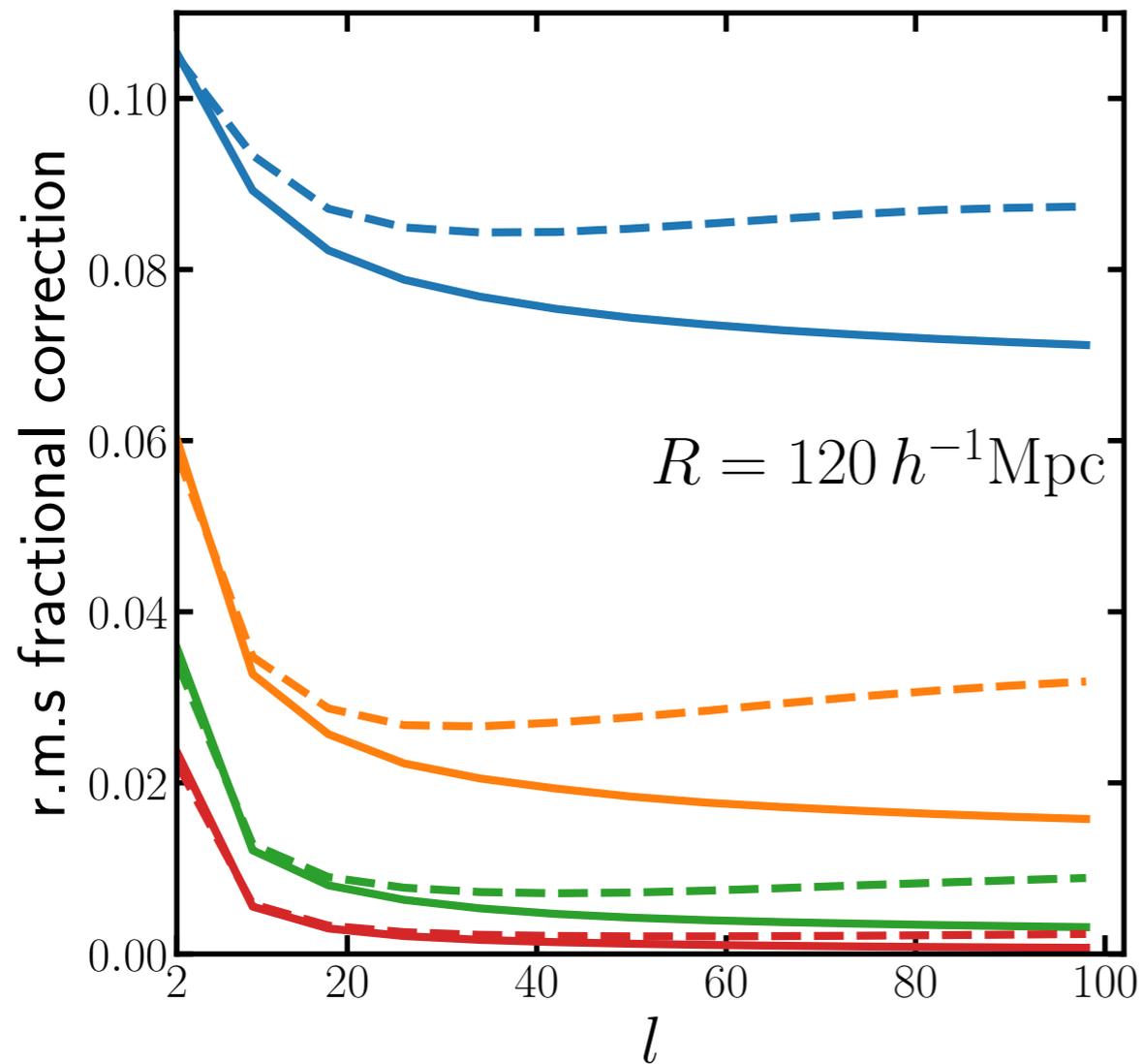


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Correlation of fluctuations **enhanced** within a local overdensity

Q: Is there a significant difference between constrained and unconstrained statistics for cosmological fields?



Effect on lensing angular power spectrum

Source redshifts

$z_s = 0.1$ (blue)

$z_s = 0.2$ (orange)

$z_s = 0.4$ (green)

$z_s = 0.8$ (red)

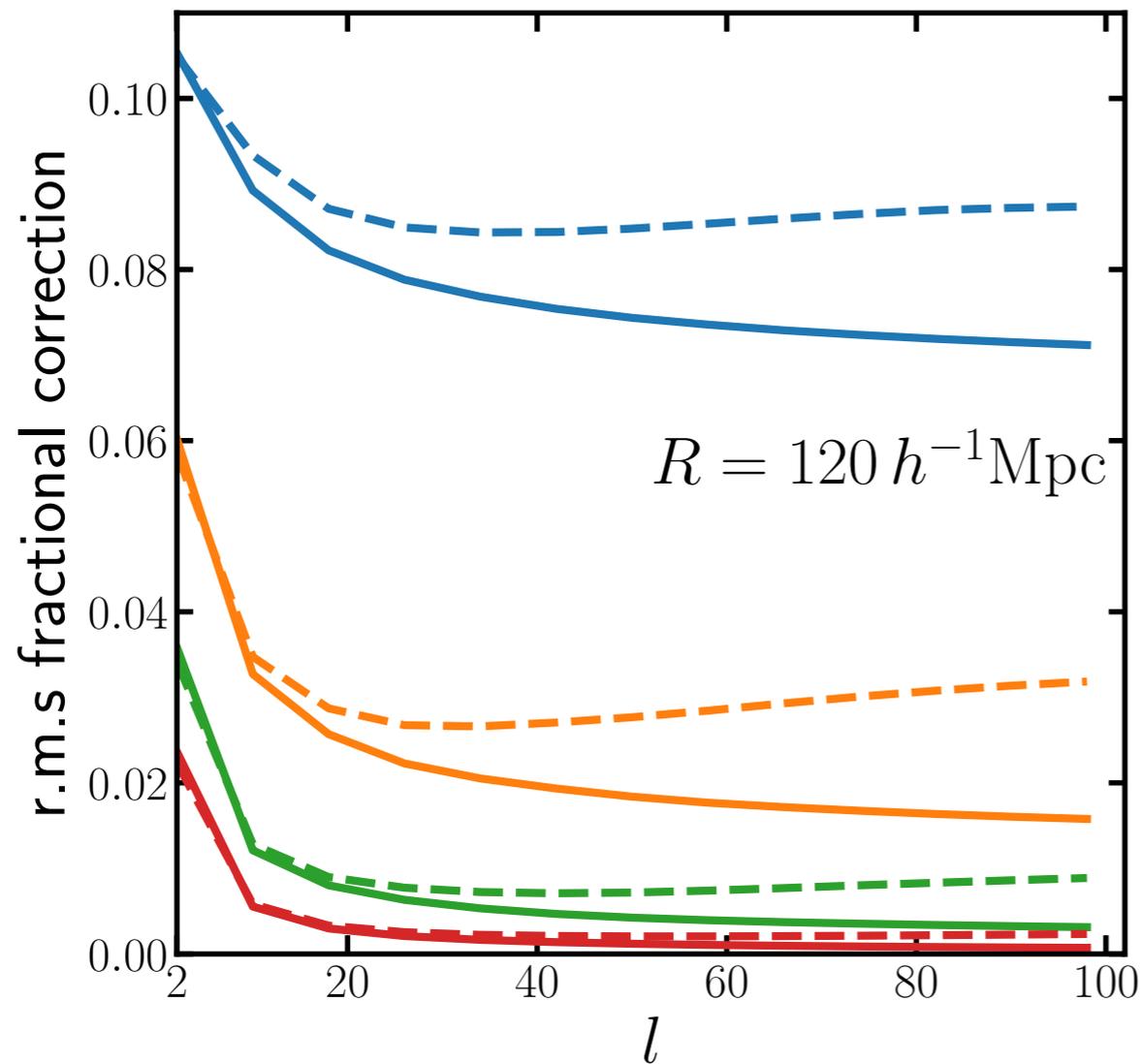
Solid:

Linear matter power

Dashed:

Non-linear matter power

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Effect on lensing angular power spectrum

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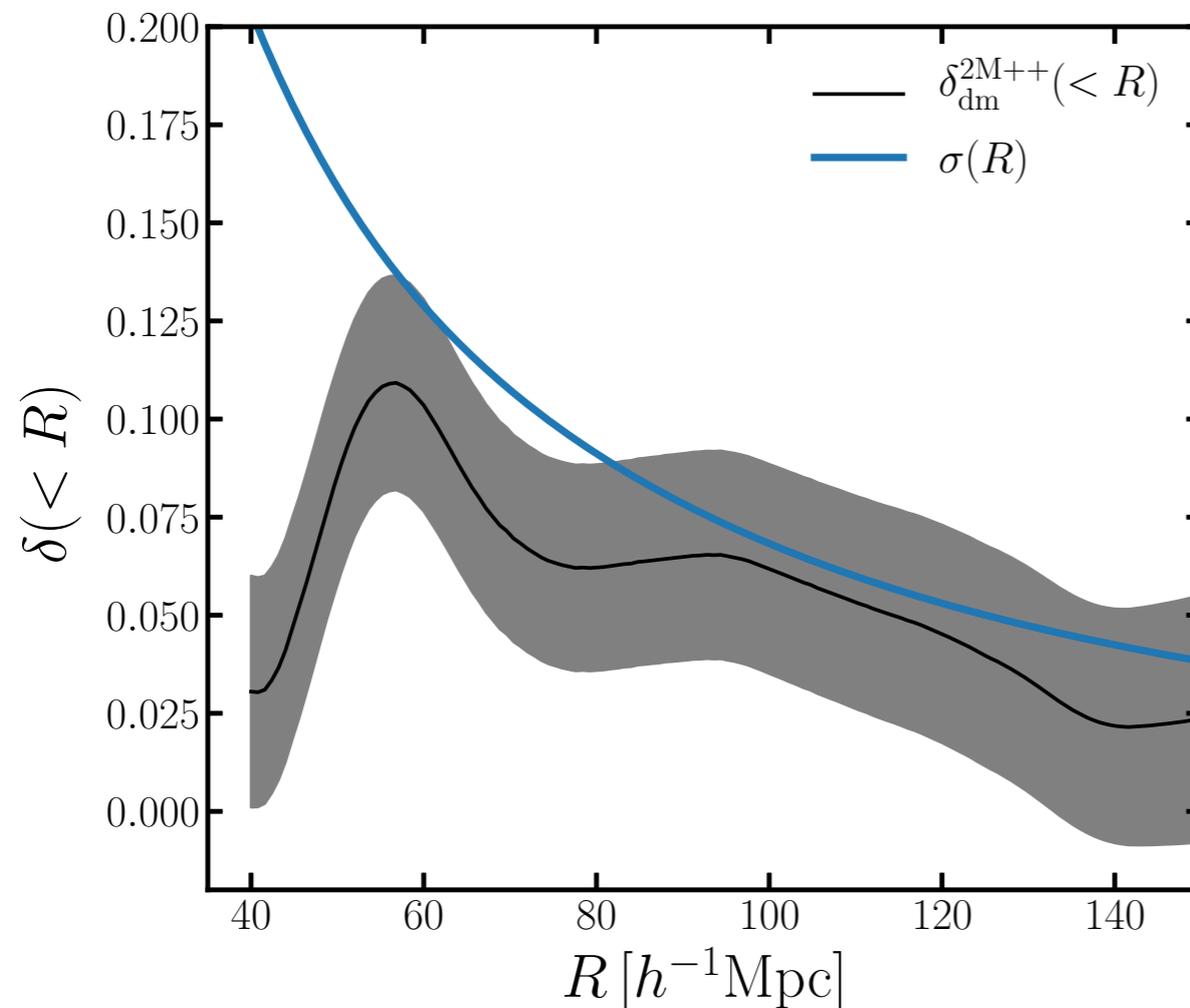
$z_s = 0.2$ (orange)

$z_s = 0.4$ (green)

$z_s = 0.8$ (red)

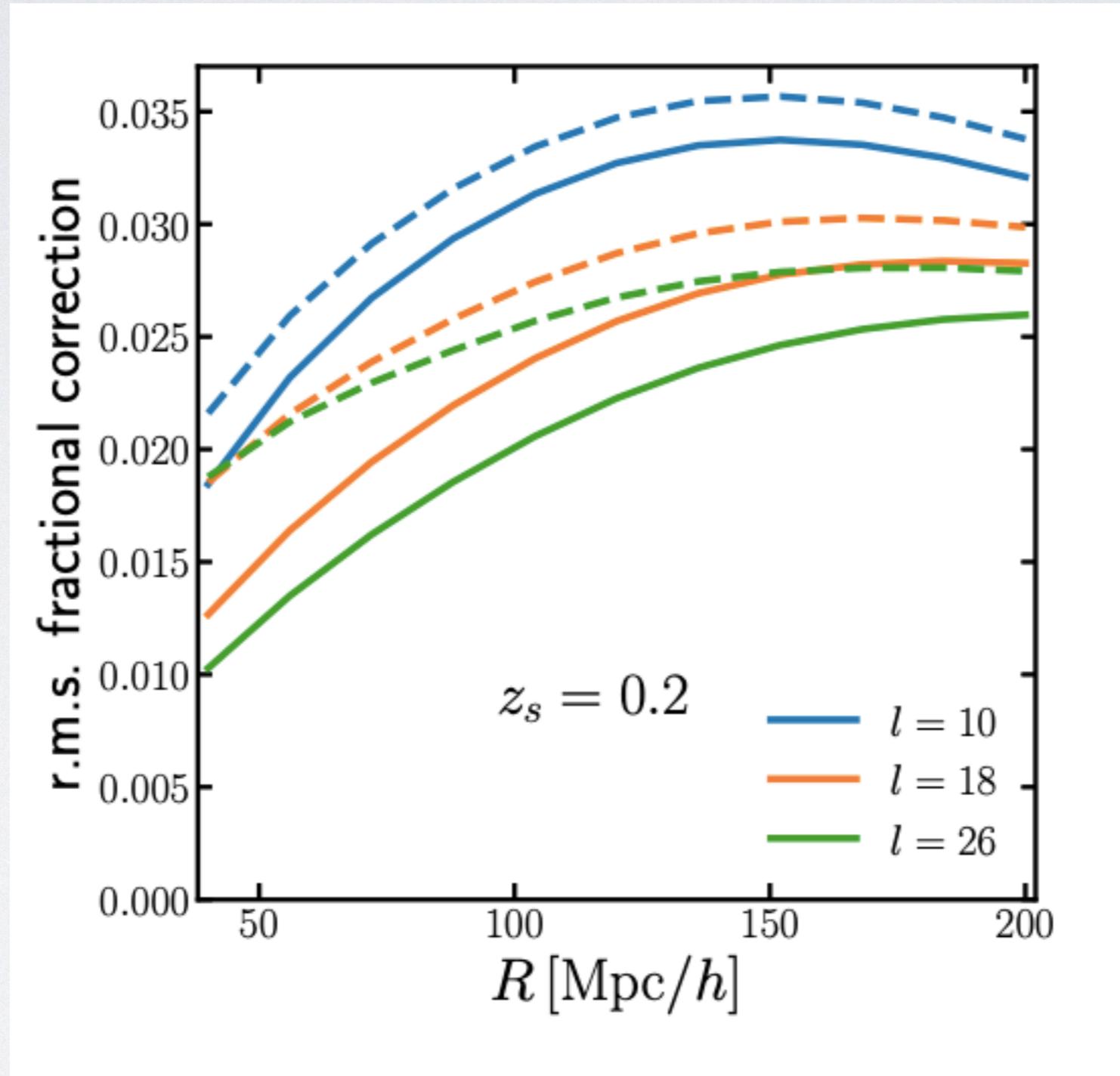
Percent-level corrections only for very nearby sources and very large angular scales

Q: Is there a significant difference between constrained and unconstrained statistics for cosmological fields?



Local dark matter density field inferred from local galaxy and peculiar velocity surveys
2M++ [Carrick et al. 2015]

Dependence on smoothing scale



Solid:
Linear matter
power

Dashed:
Non-linear
matter power

Conclusions

arXiv:1911.07855

- For Gaussian fields there is no effect on two-point statistics from conditioning on our local density fluctuation, in contrast to the claims of Reischke et al. 2019.
- Need to have **non-Gaussian** (i.e. non-linear) fields to see any effect at all. Can predict conditional power spectra analytically using second-order perturbation theory.
- Result is proportional to **bispectrum** times the **local smoothed density contrast**.
- Effects confined to large angular scales and well below cosmic variance, hence **negligible** for upcoming surveys.
- Potentially bigger effects hidden on very non-linear scales — would need constrained N-body simulations to probe this.
- Consistency test of cosmological models and extra information.

Final conclusions

- Upcoming surveys aiming to measure dark energy and neutrino mass will be systematics limited.
- We will need complementary approaches, even if those approaches are individually not competitive.
- Many small systematic errors which were previously negligible are now important for the statistical constraining power of upcoming survey to be realised.
- Real-time transverse velocities of galaxies are a realistic prospect — a new probe of cosmic flows, the growth rate of large-scale structure, and hence dark energy and modified gravity.
- A local density fluctuation can modify cosmological power spectra, in particular for weak lensing, but effect much smaller than previously thought — not a limiting systematic for upcoming surveys.

AH 2019, MNRAS, 1811.05454

AH 2020, Phys. Rev. D, 1911.07855