Percolation and the Large Scale Structure of the Universe

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- Understanding the formation of structure in the Universe
- Precision cosmology vs. qualitative relationships
- New tools for theory and observations
- Connections to statistical mechanics?

Dark matter simulation using MC^2

“Precision” Cosmology

Requirements for precision science:

- **Accurate observations with good statistics**
- **Accurate theory that connects directly to observations**
  OR
- **Controllable phenomenology that connects to observations**

Touchstone: CMB

Nonlinear regime of structure formation now plays a key role in all of the above. Accuracy figure of merit is ~1%!

Large-scale, quantitative, and accurate simulations, based on solid theory, are essential for future progress.

“The Universe is far too complicated a structure to be studied deductively, starting from initial conditions and solving the equations of motion.”

-- Robert Dicke (Jayne Lectures, 1969)

Times have changed!

Evolution under Gravity

The matter fluid evolves via a collisionless Vlasov-Poisson equation in an expanding background geometry. Initial conditions are prescribed by the spectrum of perturbative (Gaussian) linear fluctuations at an early epoch.

$$\frac{\partial f_i}{\partial t} + \dot{\mathbf{x}}\frac{\partial f_i}{\partial \mathbf{x}} - \nabla\phi\frac{\partial f_i}{\partial \mathbf{p}} = 0, \qquad \mathbf{p} = a^2\dot{\mathbf{x}},$$

$$\nabla^2\phi = 4\pi G a^2(\rho(\mathbf{x},t) - \langle\rho_{\rm dm}(t)\rangle) = 4\pi G a^2\Omega_{\rm dm}\delta_{\rm dm}\rho_{\rm cr},$$

$$\delta_{\rm dm}(\mathbf{x},t) = (\rho_{\rm dm} - \langle\rho_{\rm dm}\rangle)/\langle\rho_{\rm dm}\rangle),$$

$$\rho_{\rm dm}(\mathbf{x},t) = a^{-3}\sum_i m_i \int d^3\mathbf{p} f_i(\mathbf{x},\dot{\mathbf{x}},t).$$

Because of high dimensionality and strong nonlinearity effects, direct solution as a PDE is essentially impossible (unlike the case in plasma physics).

For small perturbations, the gravitational Jeans instability in an expanding Universe predicts:

$$P(k,z) = b(z,z_{in})^2 P(k,z_{in})$$

As evolution proceeds, linear theory fails for $k > k_{NL}$ where k_{NL} is determined very roughly by the dimensionless power spectrum being of order unity.

Accurate results in the nonlinear regime require N-body simulations.

Large Scale Structure Theory/Simulations

I. Beginnings (60's/70's)

Theory: Eulerian perturbations.
Simulations: direct N^2 methods; N~100, no theory of initial conditions.

II. Medieval period (80's)

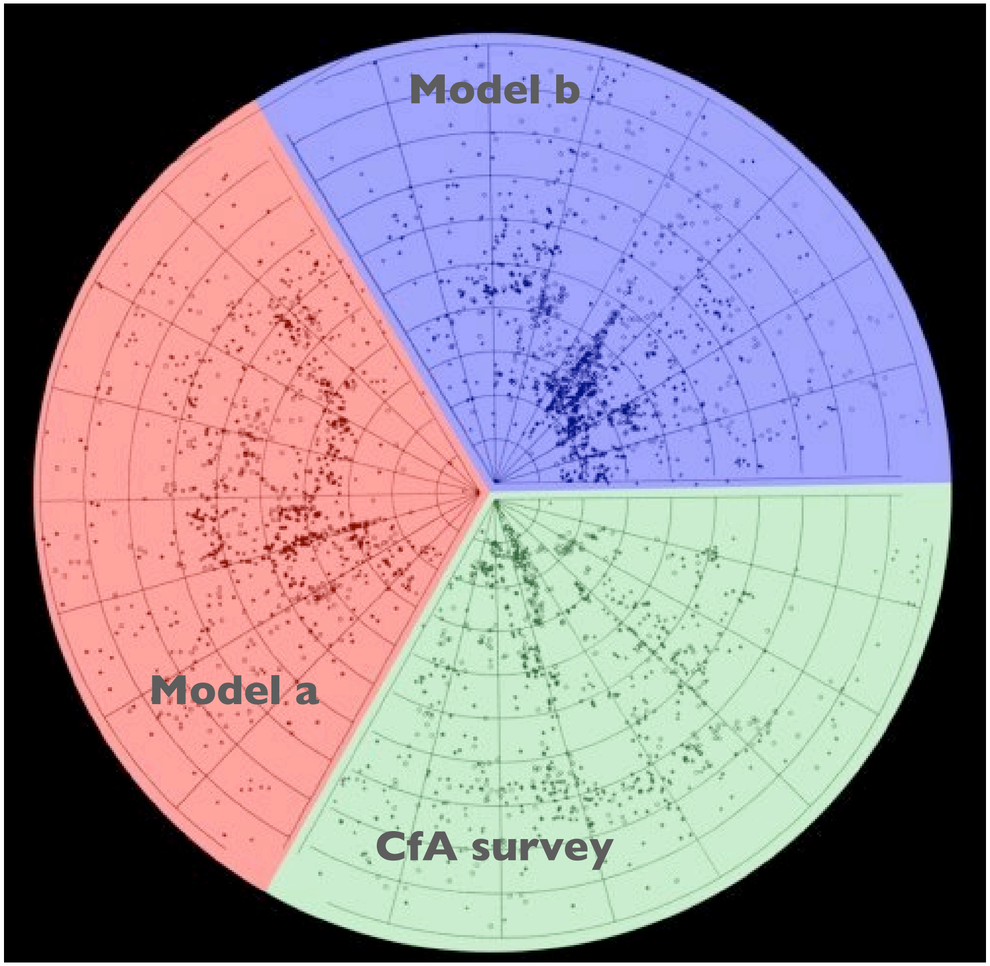
Theory: Lagrangian methods appear.
Simulations: Zeldovich approximation allows systematic approach to initial conditions. O(N) methods -- and their adaptive extensions -- implemented.

III. Modern period (90's)

Theory: Attempts to control perturbation theory.
Simulations: Multi-resolution parallel codes; hydrodynamics simulations approach maturity.

IV. Current era

Transition from qualitative analysis to quantitative prediction underway driven by observational advances, but nonlinear sector resists "theory" --



Davis et al 1985

Mock catalogs from 20 years ago for "eyeball" comparisons with the CfA galaxy survey

Large Scale Structure of the Universe

I. Two-point Statistics

Relatively robust, "easy" to compute and compare to observations. Clean theoretical interpretation.

II. Shape Indicators

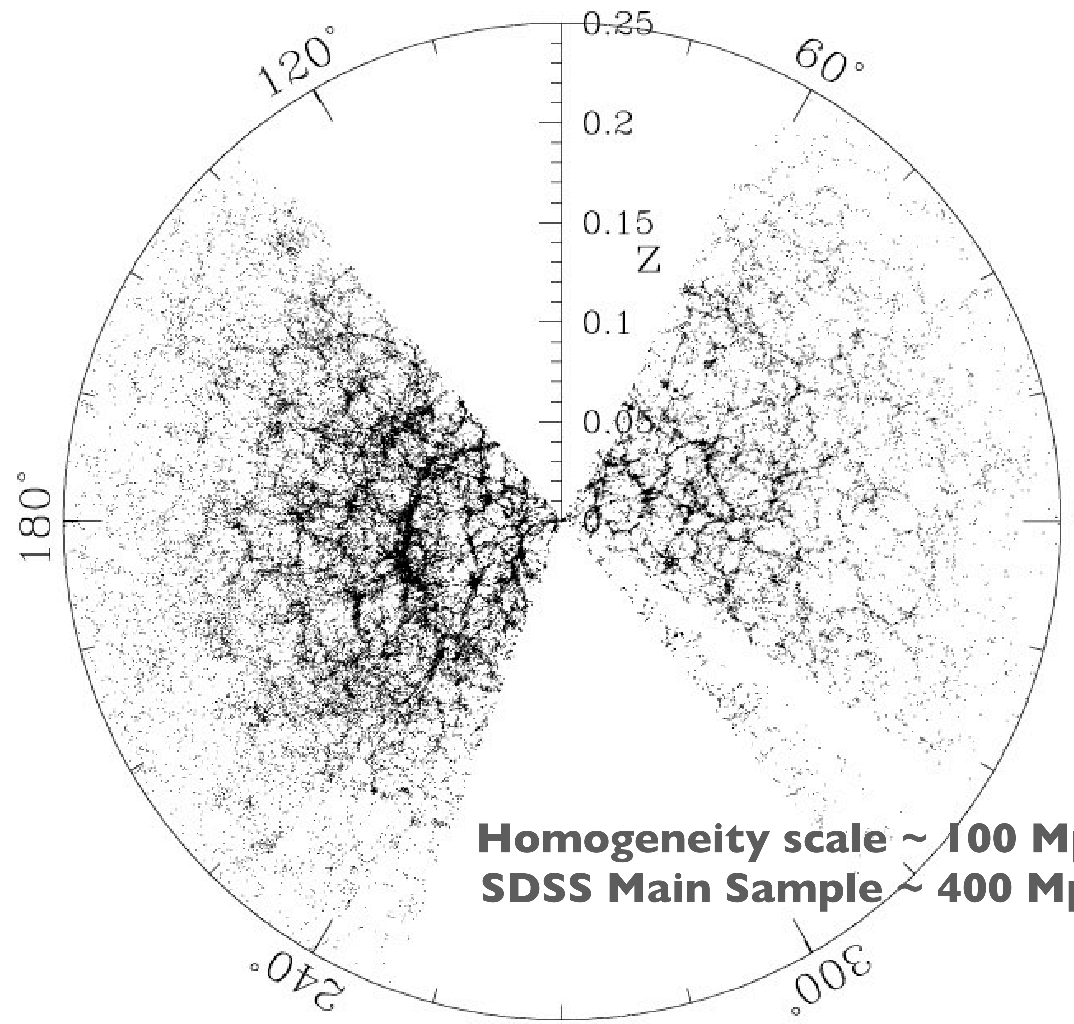
Useful as characterization tools, but connection to underlying gravitational physics unclear.

III. Higher-point statistics

Tedious to compute, theoretical interpretations not so straightforward.

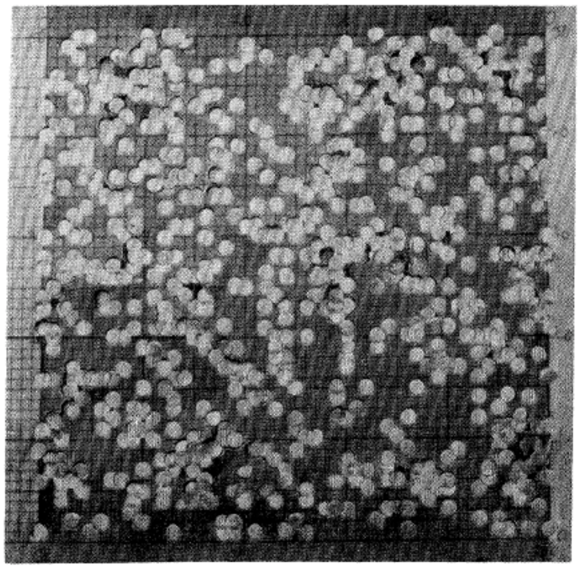
IV. Phenomenology

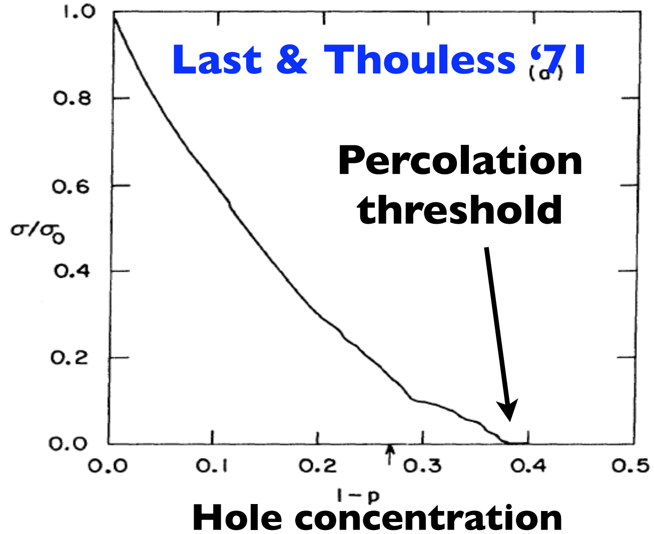
Halo models useful as statistical descriptors and to provide basic intuition, but connection to underlying theory is very indirect.



Homogeneity scale ~ 100 Mpc
SDSS Main Sample ~ 400 Mpc

Percolation





I. Continuous Structural Transition

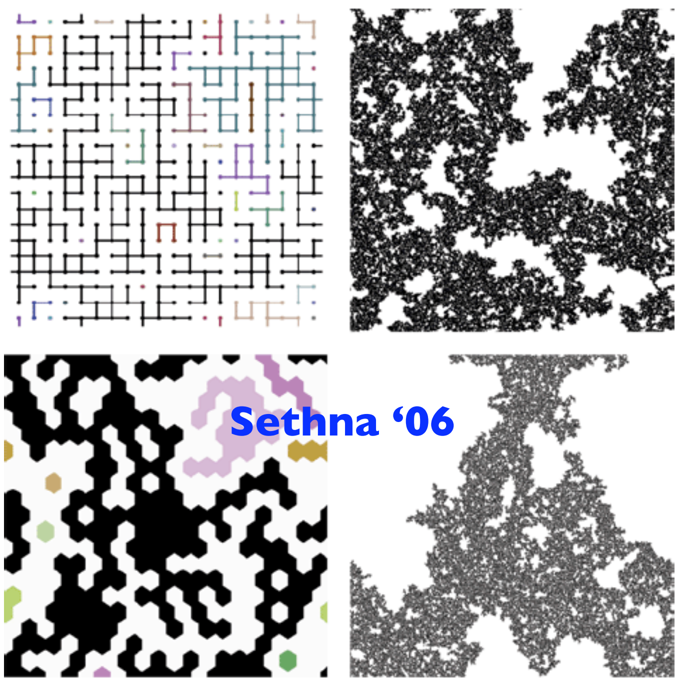
As a function of some control parameter, a physical property changes continuously near a singular point.

II. Percolation I

Percolation = probabilistic models with continuous (percolation) transition

III. Percolation II

No concepts from equilibrium statistical mechanics or the existence of Hamiltonians required to study percolation.

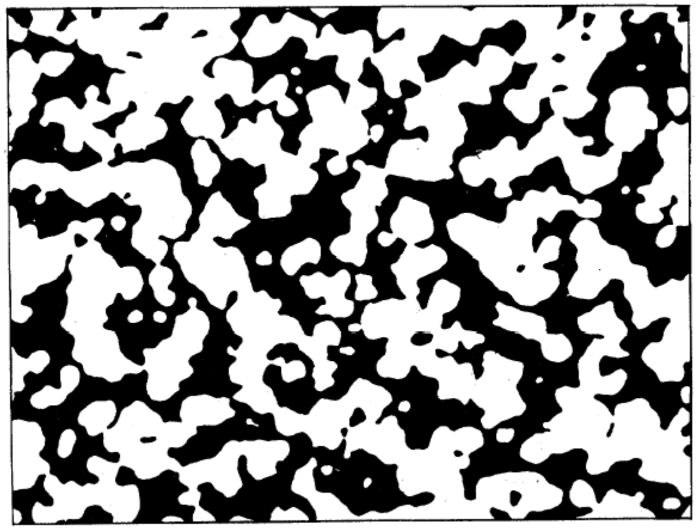


IV. Universality/ Scaling

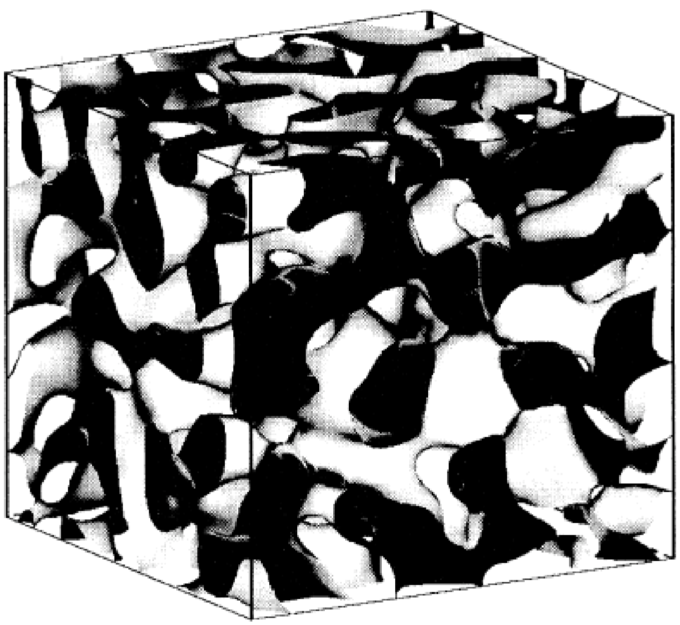
Near the transition point, percolation properties should split up into a small number of universality classes (e.g., morphology of percolating cluster).

Simple scaling laws expected near the transition --

Continuum Percolation



Smith & Lobb '79



Roberts & Teubner '95

I. Continuum Models

Instead of lattice-based models, consider continuous random fields, control parameter being amplitude or density, etc. Dual models map to random networks.

II. Gaussian Random Fields

A popular, very simple, class of continuum model; no exact results available for percolation properties!

III. Scaling Ansatz (single-variable)

$$n_s(p) = s^{-\tau} f[(p - p_c)s^{\sigma}], \;\; (p \to p_c, \; s \to \infty)$$

Normalized cluster number (per lattice site) of a given size, as a function of on-site occupation probability, near the percolation transition, and for large sizes.

Simple (continuum) versions of this ansatz provide very good fits to numerical results.

Basis of application to cosmology --

Cosmic Voids

I. Voids

Underdense regions in the Universe largely devoid of bright galaxies (suppressed mass function)

II. Observation

As surveys cover large contiguous volumes (SDSS), analysis of voids becomes possible (void scale ~ 10 Mpc)

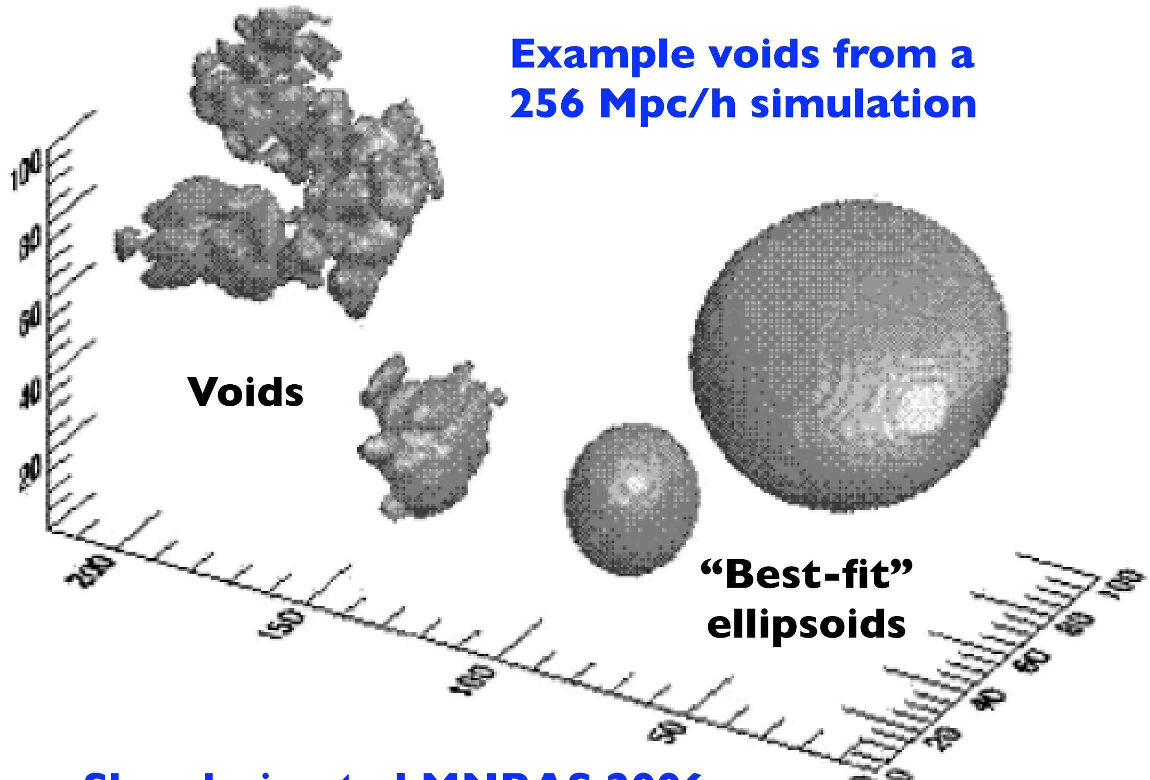
III. What is a Void?

Various ambiguities in operational definitions. We use a simple underdensity threshold definition.

IV. Voids are Complex

Strategy: Use scaling ansatz to characterize void properties near the void percolation transition

Example voids from a 256 Mpc/h simulation

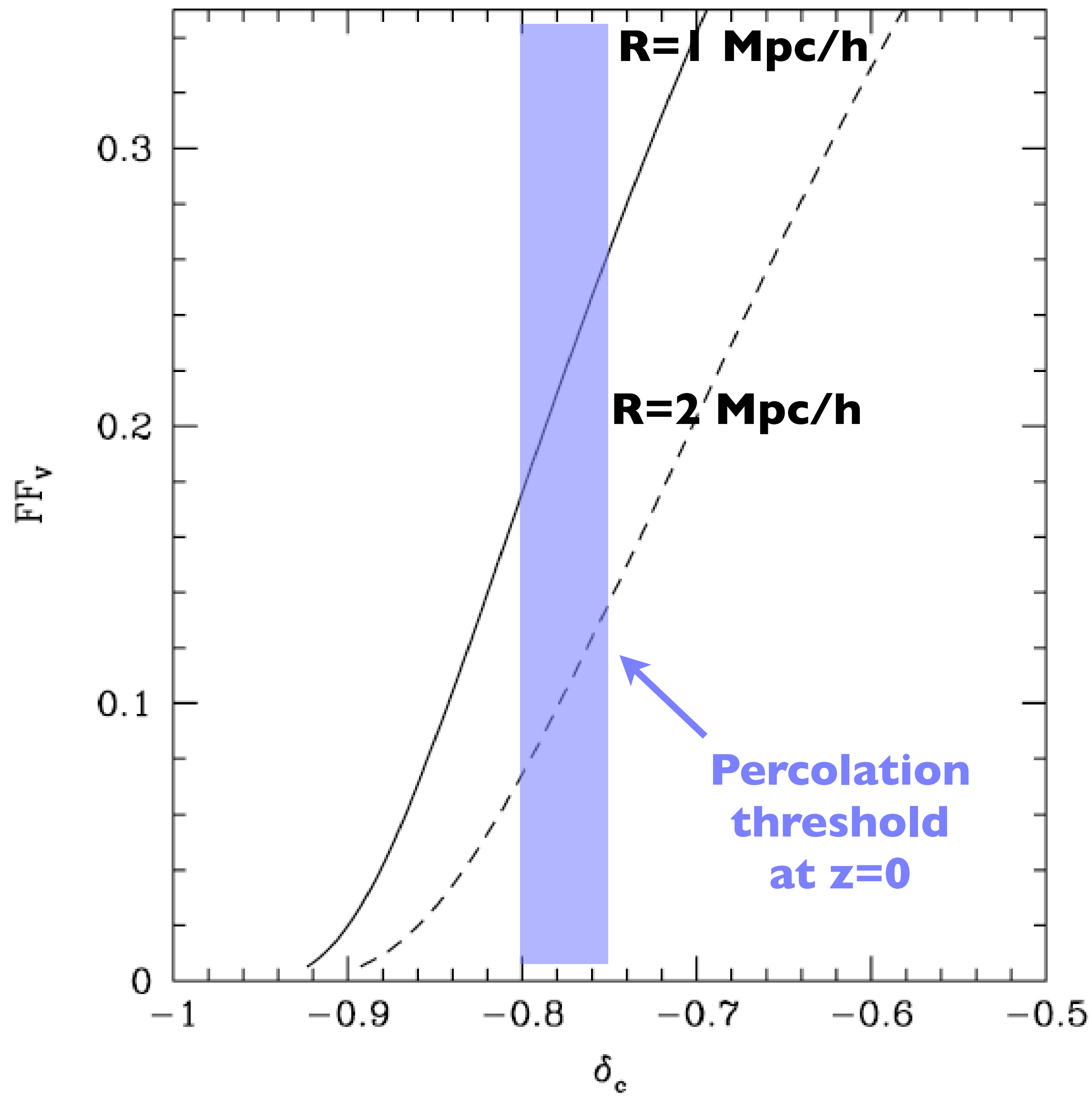


Shandarin et al MNRAS 2006

V. Void Percolation

At percolation, the largest void quickly dominates the excursion set (exclude this); near this point the individual voids reach their largest sizes and volumes

Voids and Percolation I



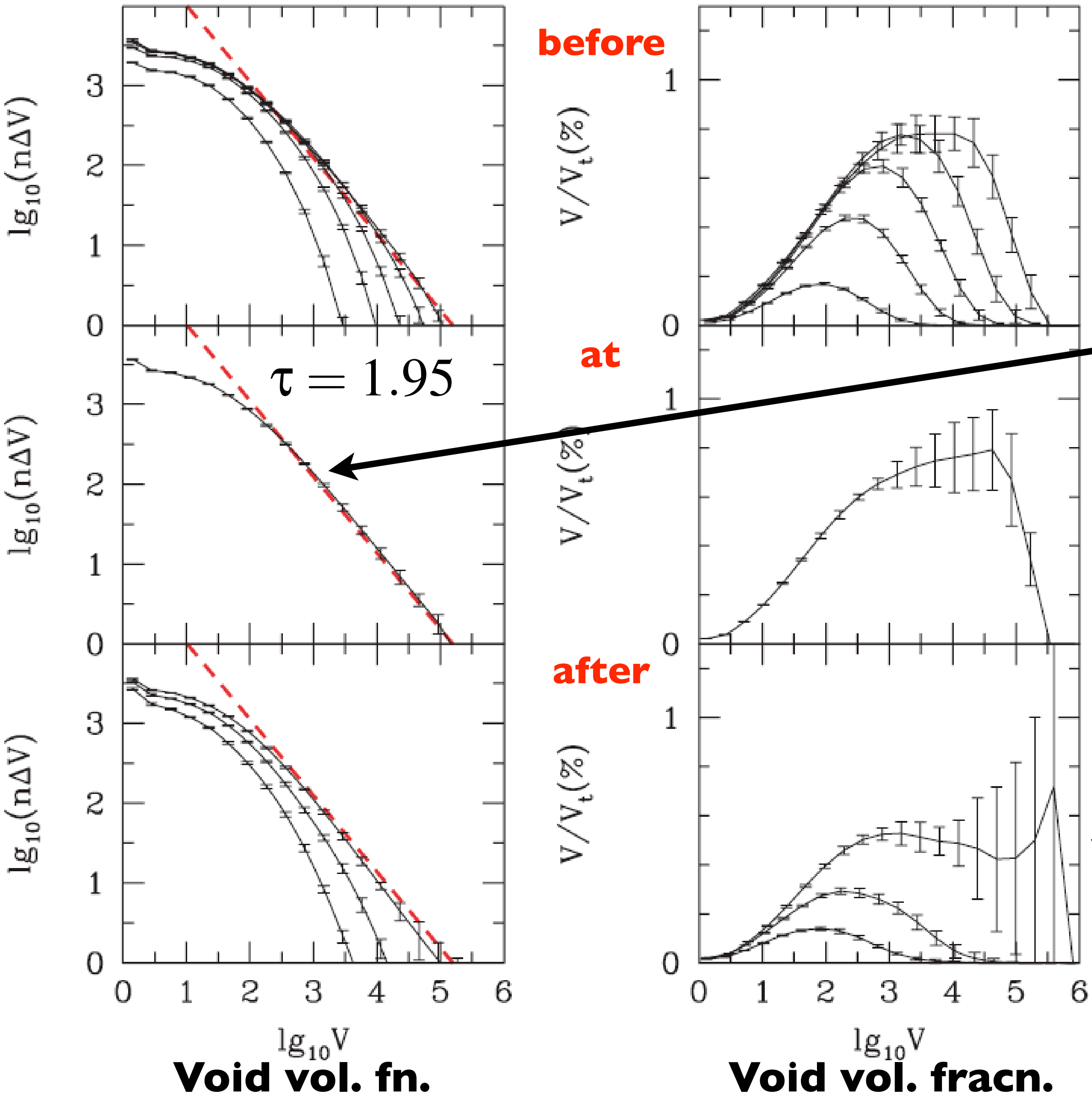
I. Void Filling Factor

The void filling factor is a monotonic function of the (under)density threshold over the range of interest.

II. Percolating Void

At percolation, about half of the volume of the underdense excursion set is already in one percolating void.

Voids and Percolation II: Gaussian Fields



LCDM Initial Conditions at z=150

Percolation Ansatz Works

$$n(V) \sim V^{-\tau}\exp(-cV)$$

$$c \sim |FF - FF_c|^{1/\sigma}$$

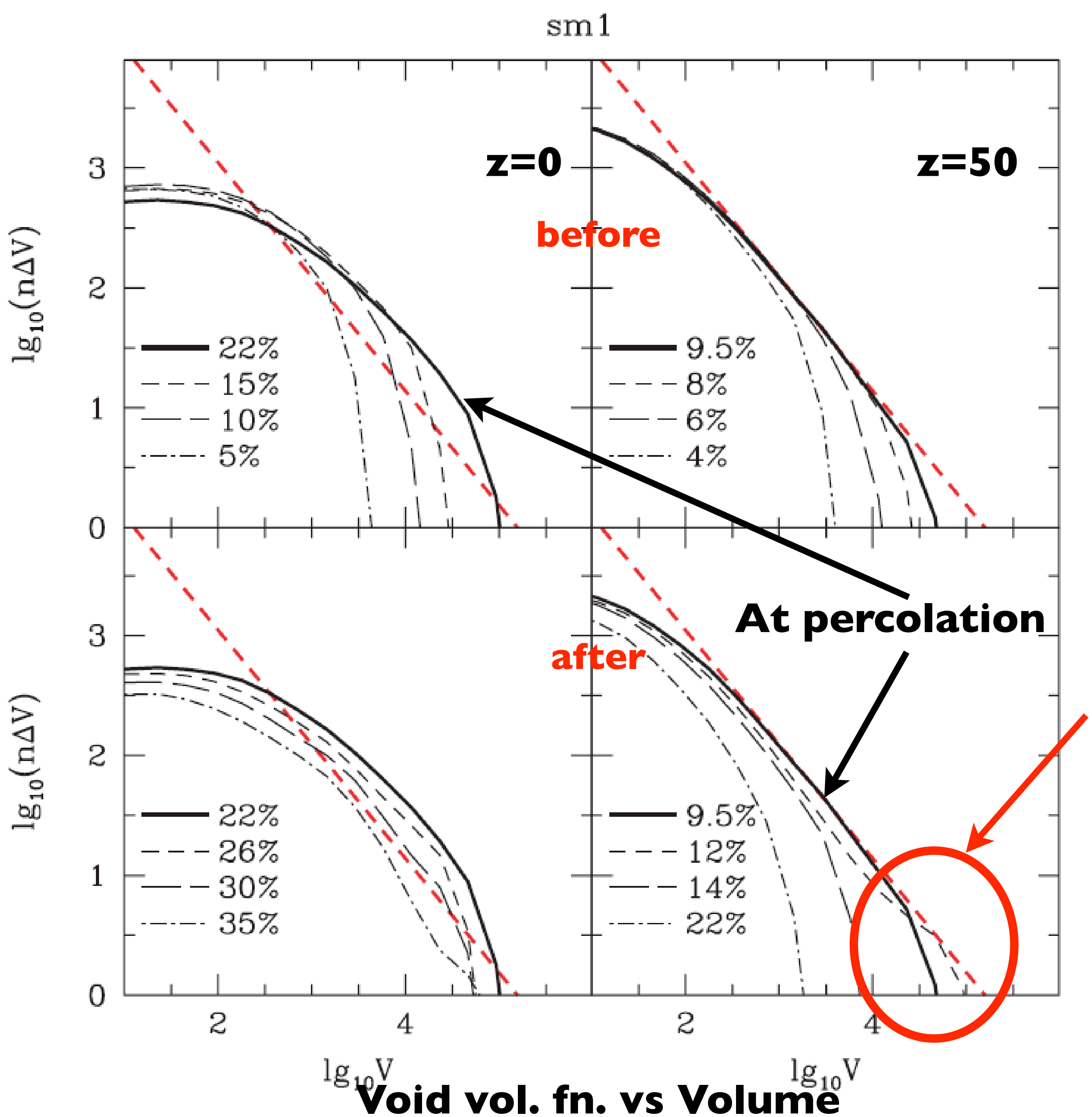
I. Void Volume Function

# of voids of volume V, analog of halo mass function

II. Void Volume Fraction

Fractional amount of total volume in voids of volume V.

Voids and Percolation III: Evolution



I. Void Evolution

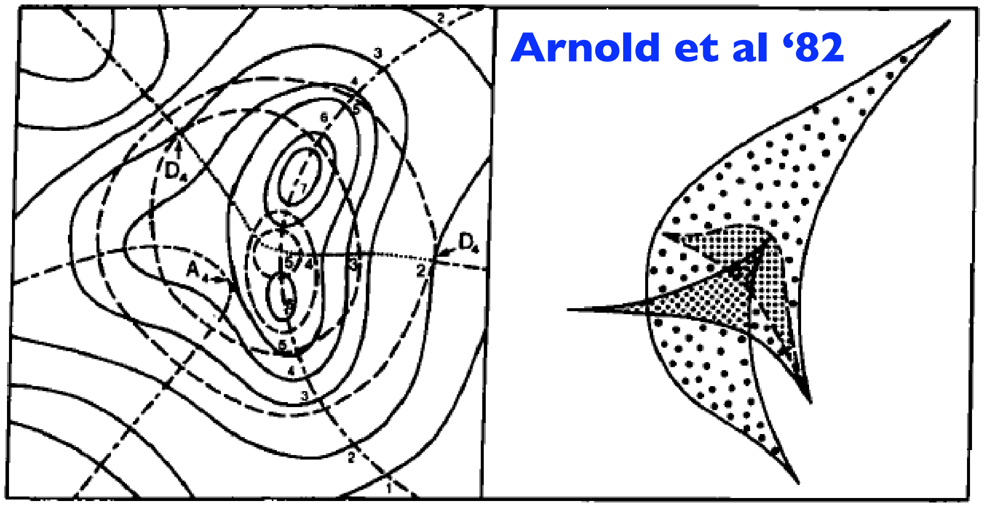
Number of small voids decreases, number of large voids increases.

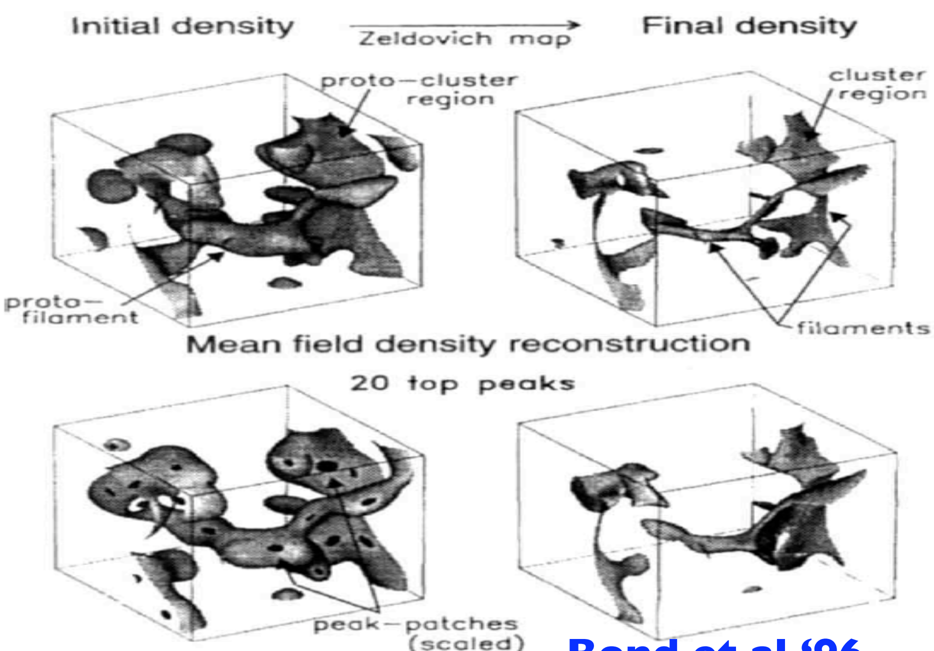
II. Scaling Violation?

Percolation ansatz does not hold at z=0.

Not clear why: finite volume, large-scale coherence, --

Local Descriptions of Structure Formation





Bond et al '96

I. Singularities in Lagrangian Space

Singularity structure of local map approximations:

$$\vec{x}(\vec{q},t) = \vec{q} + D(t)\vec{s}_R(\vec{q})$$

$$d_{ik} = \frac{\partial s_i}{\partial q_k}$$

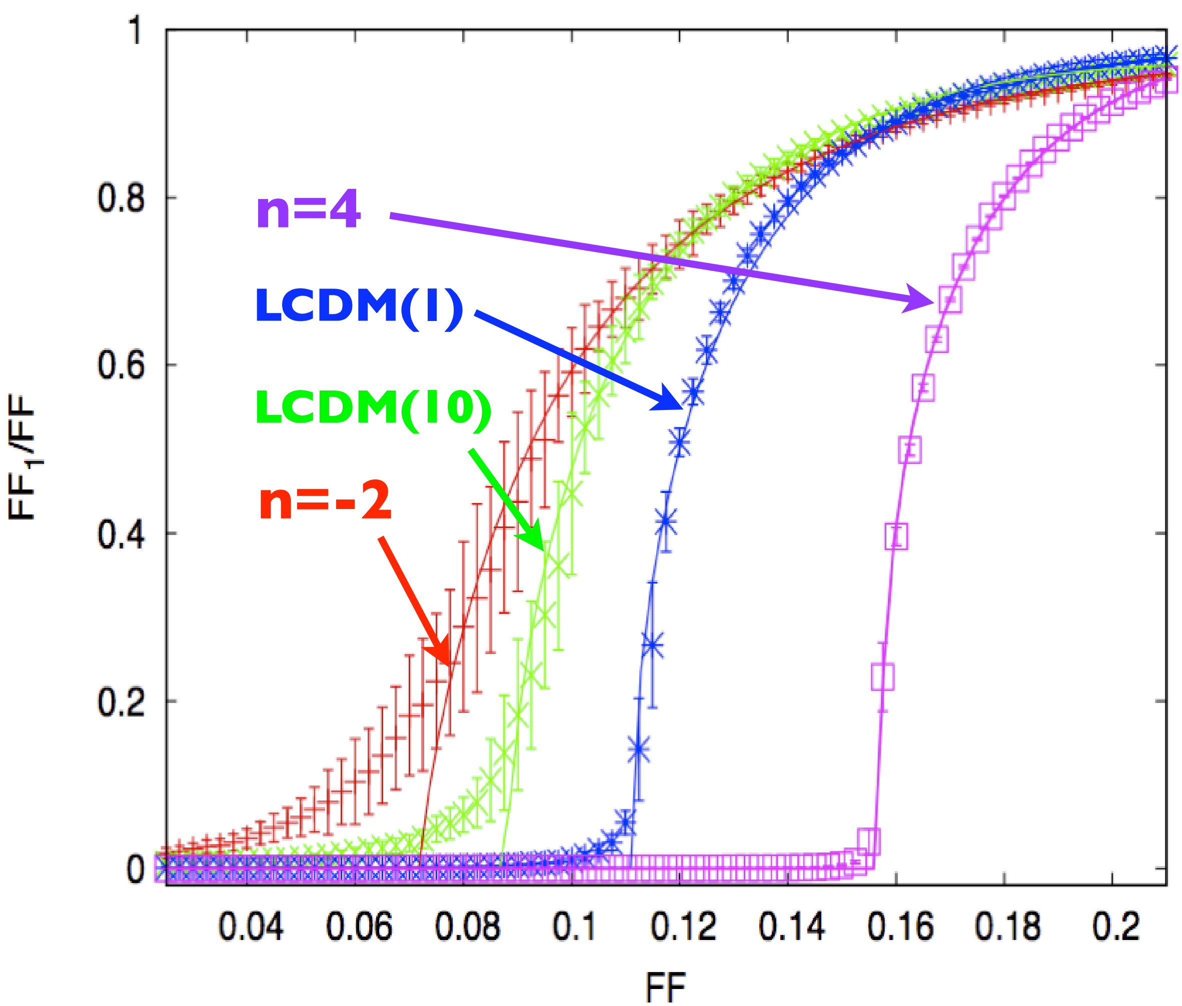
II. Cosmic Web

"Correlation bridges" from considering conditional multi-point correlation functions (e.g., of the primordial shear field)

II. Structural "Building Blocks"

Although the basic units of structure may be so indentified, we desire a global, quantitative measure of network structure.

Percolation in Gaussian Random Fields



Percolation threshold decreases with increase of large-scale power

I. Gaussian Fields

Uniquely specified by their two-point statistics (power spectra).

II. Symmetry

Exact symmetry between overdense and underdense excursion sets.

III. Percolation Ansatz

$$FF_1 = A(FF - FF_c)^{\nu}$$

FF_1 **is the filling factor of the percolating region.** FF_c **is the filling factor when percolation occurs. The ansatz applies when** $FF > FF_c$**.**

Percolation Coefficients

Model	FF_p	A	ν	δ/σ_δ
NL underdense	0.228	1.80	0.76	δ= -0.80
n=4	0.157	0.61	0.38	1.006
ΛCDM (10)	0.111	0.66	0.51	1.22
ΛCDM (1)	0.089	0.75	0.62	1.35
n=-2	0.072	0.89	0.76	1.46
NL overdense	0.035	0.73	0.75	δ=3.31

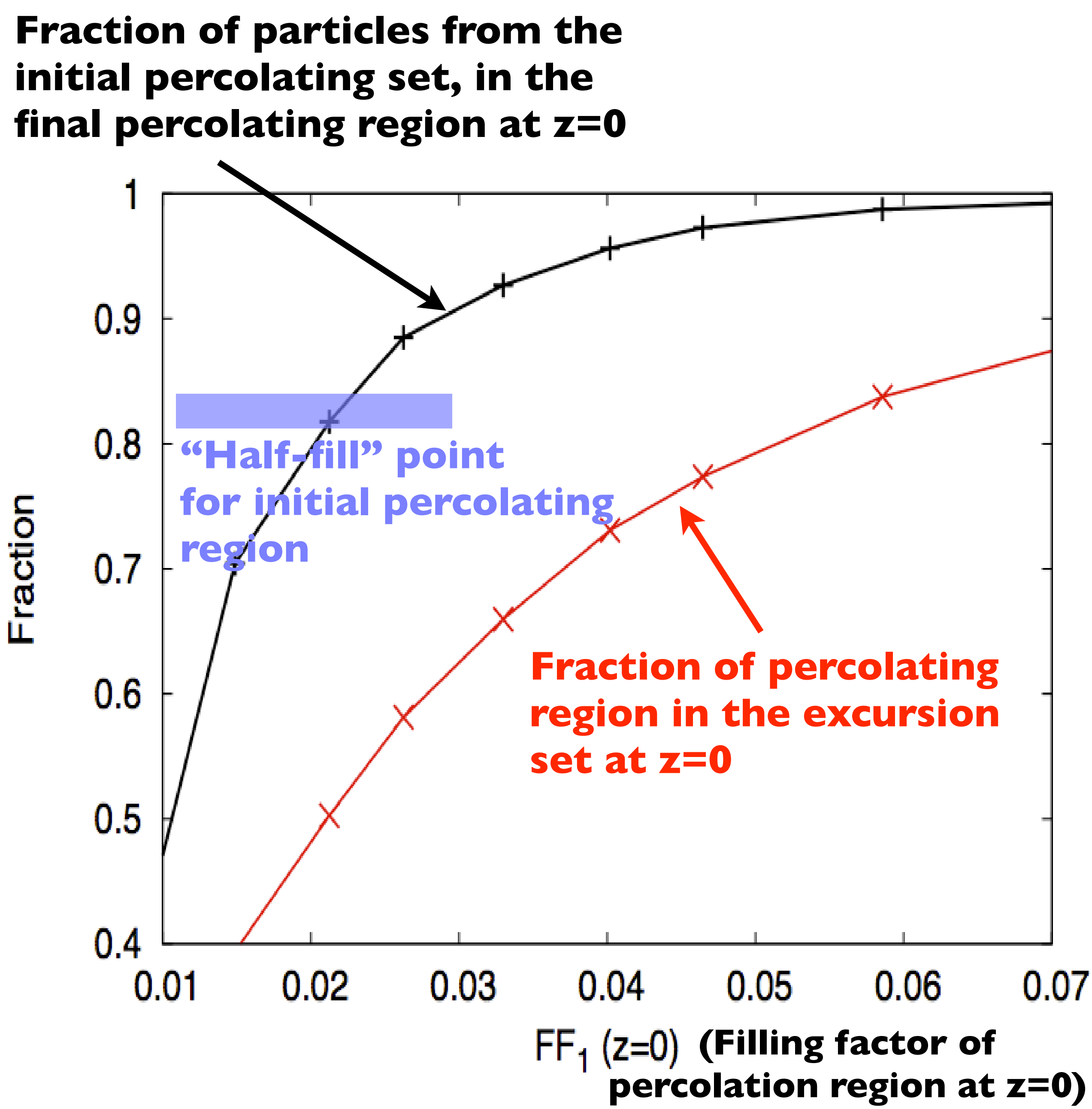
Multiple realizations capture percolation coefficients accurately.

LCDM(1) = "Concordance" model, 15 realizations of a 340 Mpc/h box, with smoothing scale R = 1 Mpc/h.

LCDM (10) = As above but with 10 realizations of a 3.4 Gpc/h box, with R = 10 Mpc/h

NL is the evolved LCDM(1) case at z = 0.

Nature or Nurture?



I. "Percolating" Particles

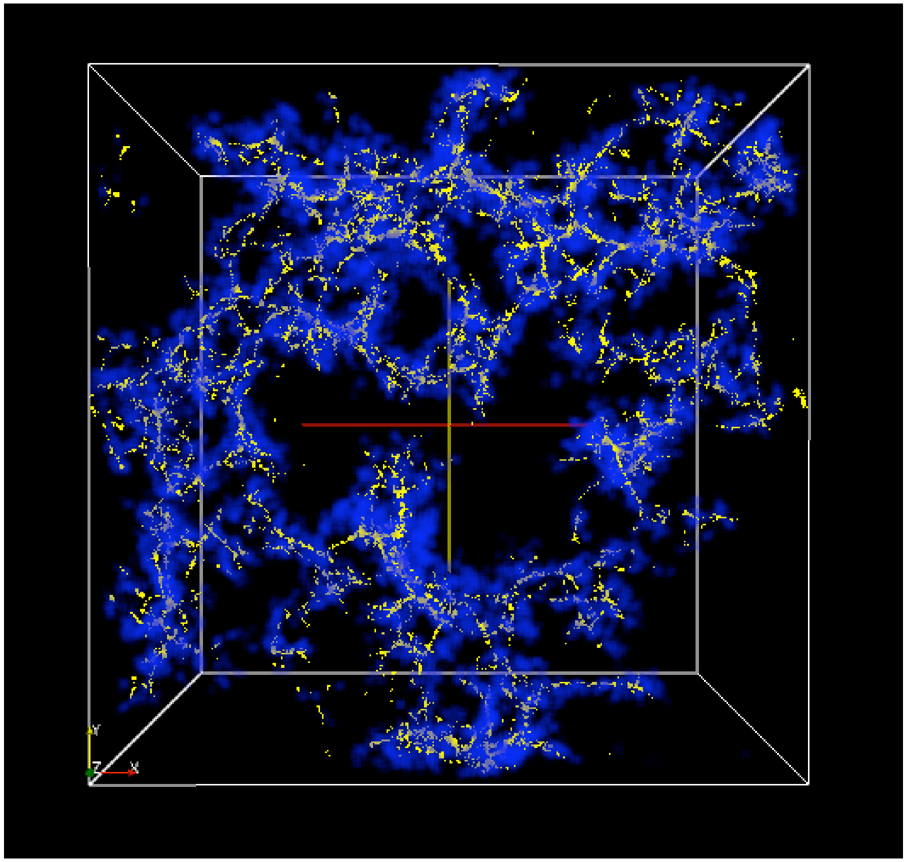
All particles in a percolating region (not equivalent to density cut)

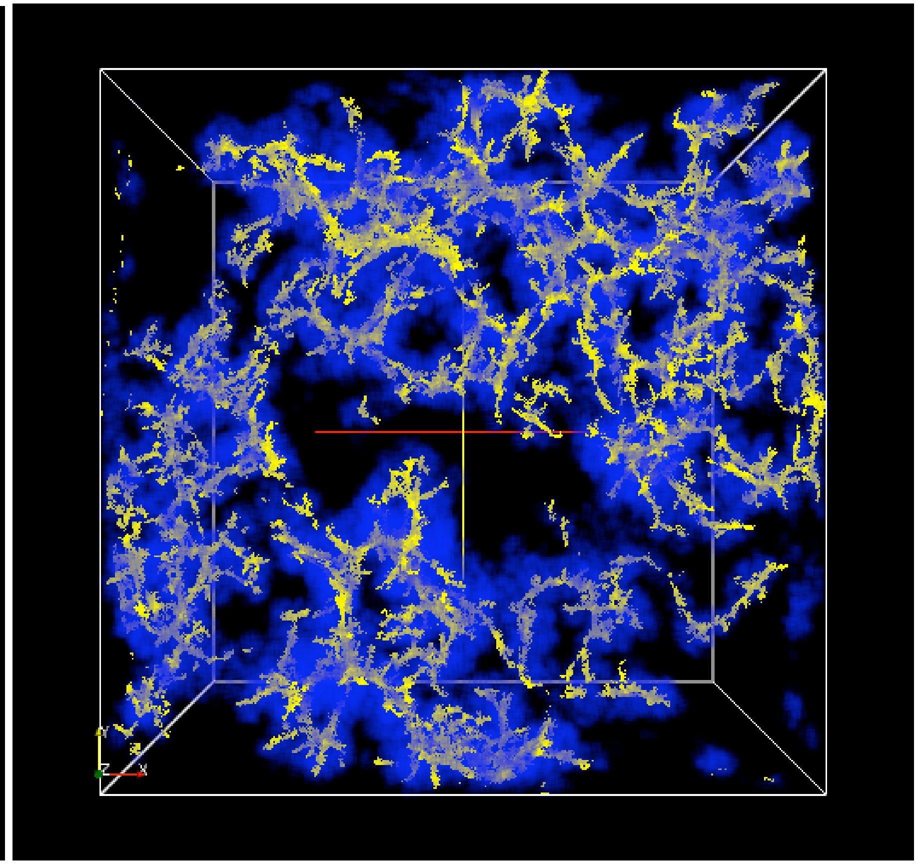
II. Forward/Inverse Maps

Particle from initial percolating region(s) are mapped to final percolating regions. But these particles do not themselves form a percolating cluster: they fragment into a very large number of isolated regions (overdense regions collapse), a compression factor of more than an order of magnitude.

Inverse particle map percolates --

Forward and Backward Maps

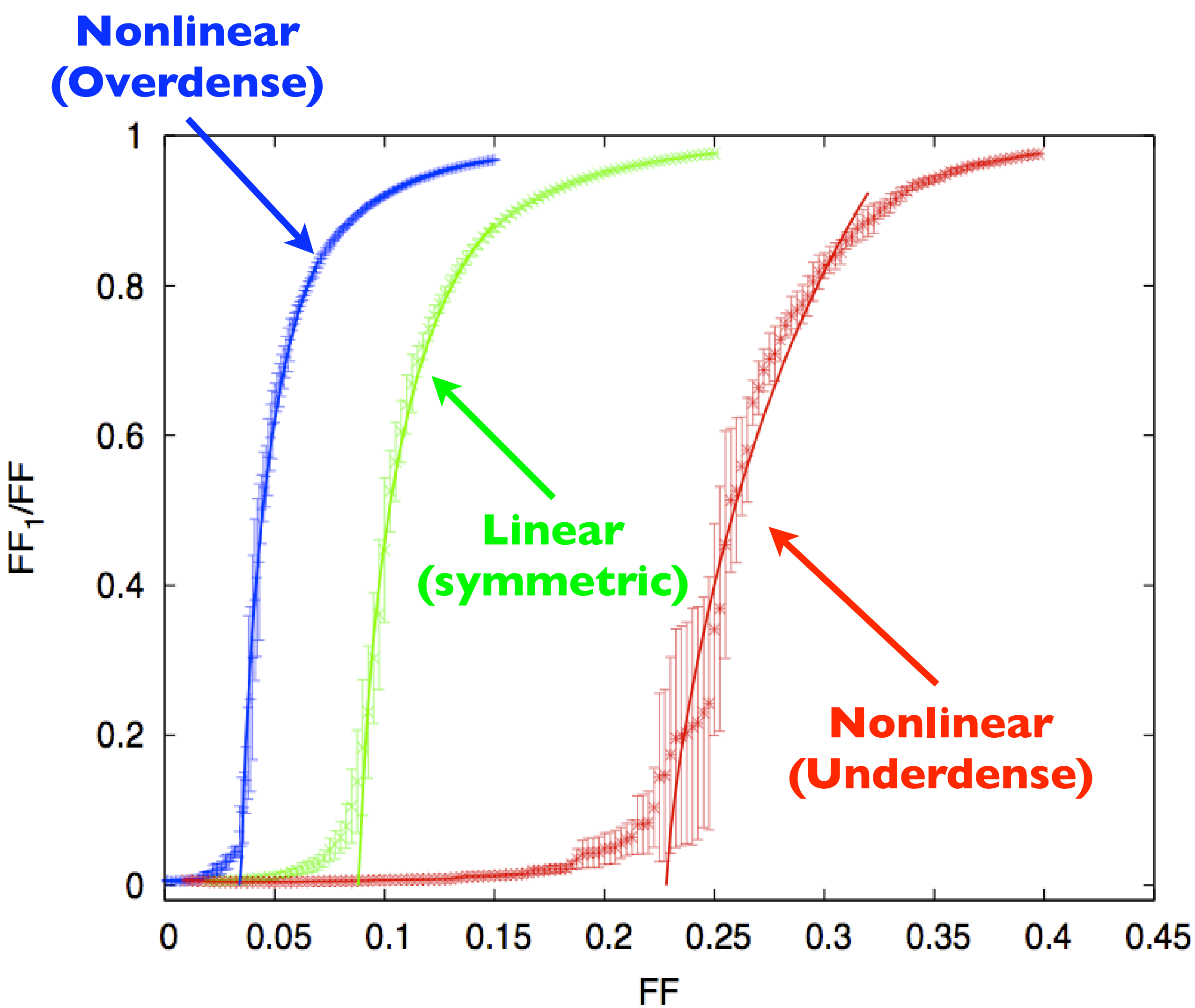




Forward Map
z=50, percolating region (blue)
z=0, percolating region (yellow)
Slab thickness=70 Mpc/h

Backward Map
z=0, percolating region (yellow)
z=50, percolating region (blue)
Slab thickness=70 Mpc/h

LCDM Percolation Transition at z=0



I. Broken Symmetry

Symmetry between underdense and overdense excursions is broken by gravitational evolution

II. Percolation

Ansatz still holds separately for the under and overdense sets. Overdense set percolates much more easily (more large-scale power), underdense percolation set goes the other way: Nonlinear evolution amplifies the network structure present in the cosmic web.

Summary

I. With current computational capabilities, percolation statistics can be calculated both robustly and accurately for cosmological density fields.

II. Percolation provides a useful global measure of the nature of cosmological structure, how much is controlled by the initial condition, and how much by gravitational evolution.

III. Percolation measures are easy to compute and should be applicable to large-volume galaxy surveys (both 2-D and 3-D). No harder than the power spectrum or the two-point function? Explore with mock catalogs.

IV. In statistical mechanics, percolation scaling laws have been predicted using RG methods. Can this -- or some other approach -- be an alternative to conventional perturbation theory to understand the gravitational instability?

V. Can particle percolation statistics be connected to phenomenological approaches to structure formation, such as the halo model?