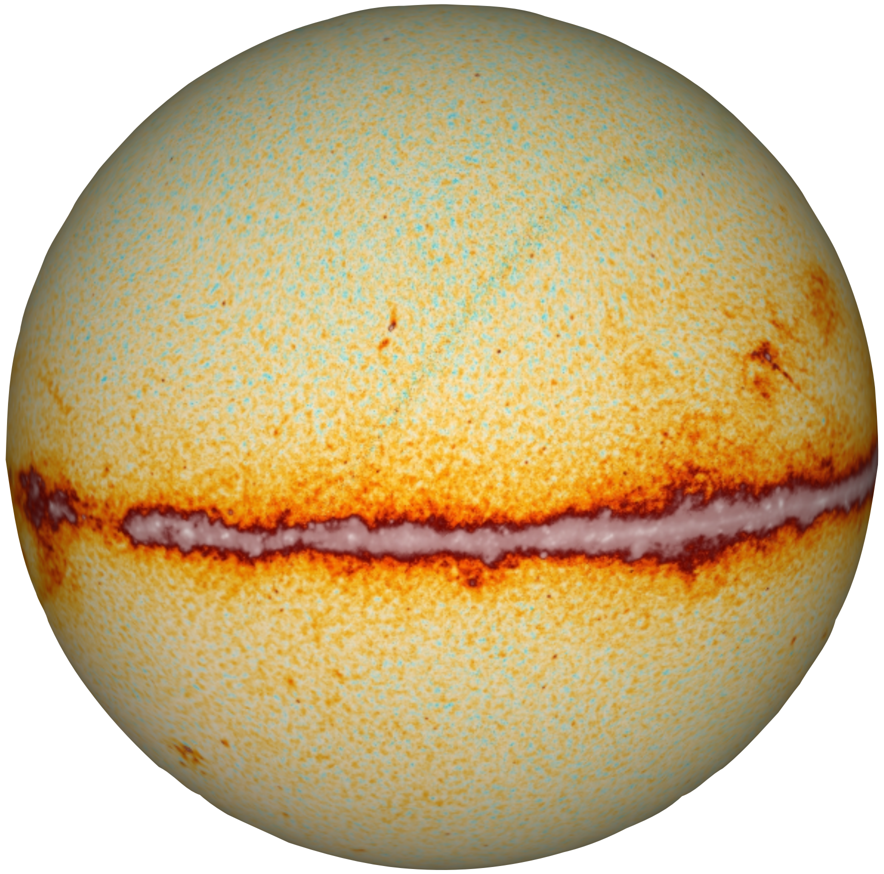
Towards an Analytic Theory of Large Scale Structure



Courtesy of thecmb.org

Daniel Green
Stanford

1304.4946 & 1310.0464 :
with Carrasco, Foreman and Senatore

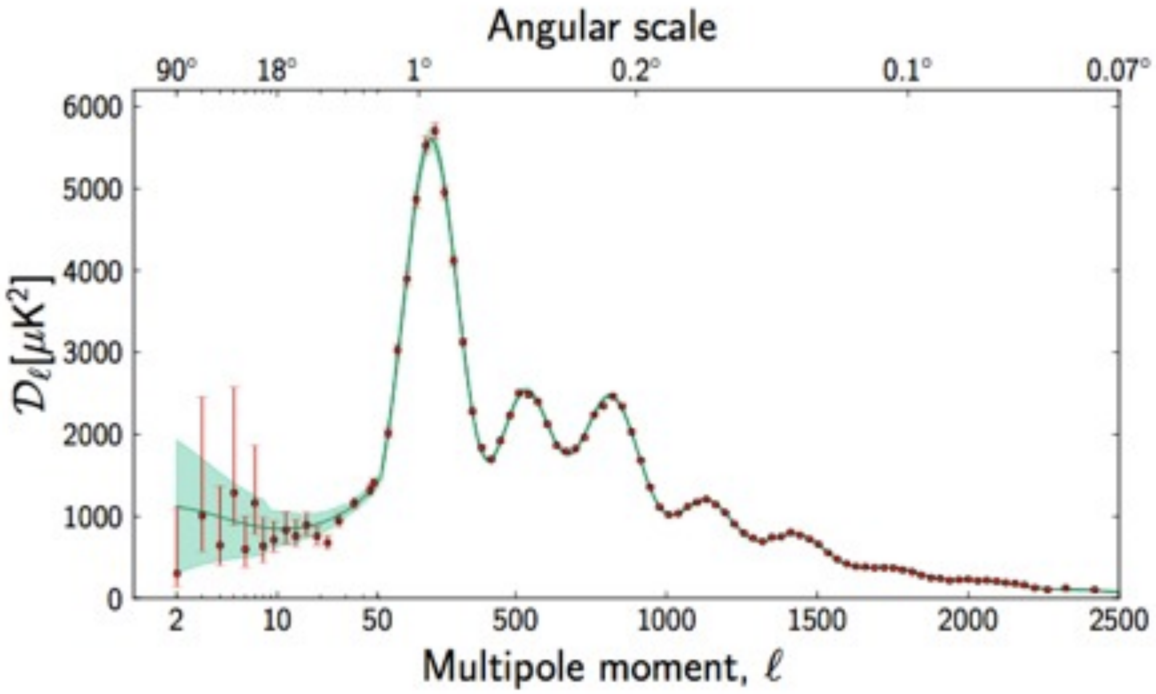
Motivation

Life after Planck

For may quantities of interest

$$\left(\frac{S}{N}\right) \sim \frac{1}{\sqrt{N_{\rm modes}}}$$

Planck has nearly saturated the modes in the CMB



$$\ell_{\rm max} \sim 1500 \rightarrow 2 \times 10^6 \; {\rm modes}$$

Life after Planck

For may quantities of interest

$$\left(\frac{S}{N}\right) \sim \frac{1}{\sqrt{N_{\rm modes}}}$$

For significant improvements we need LSS:

$$N^{\rm LSS}_{\rm linear} \sim \left(\frac{k_{\rm max}}{k_{\rm min}}\right)^3 \sim \left(\frac{.1\, h\, {\rm Mpc}^{-1}}{10^{-4}\, h\, {\rm Mpc}^{-1}}\right)^3 \sim 10^9$$

LSS contains a lot more information*

*if we measure the entire volume at low z

Life after Planck

In practice, near term surveys:

$$N^{\rm Euclid}_{\rm linear\ modes} \sim \left(\frac{k_{\rm max}}{k_{\rm min}}\right)^3 \sim \left(\frac{0.1\,h\,{\rm Mpc}^{-1}}{10^{-3}\,h\,{\rm Mpc}^{-1}}\right)^3 \sim 10^6$$

Just counting linear modes is comparable to CMB.

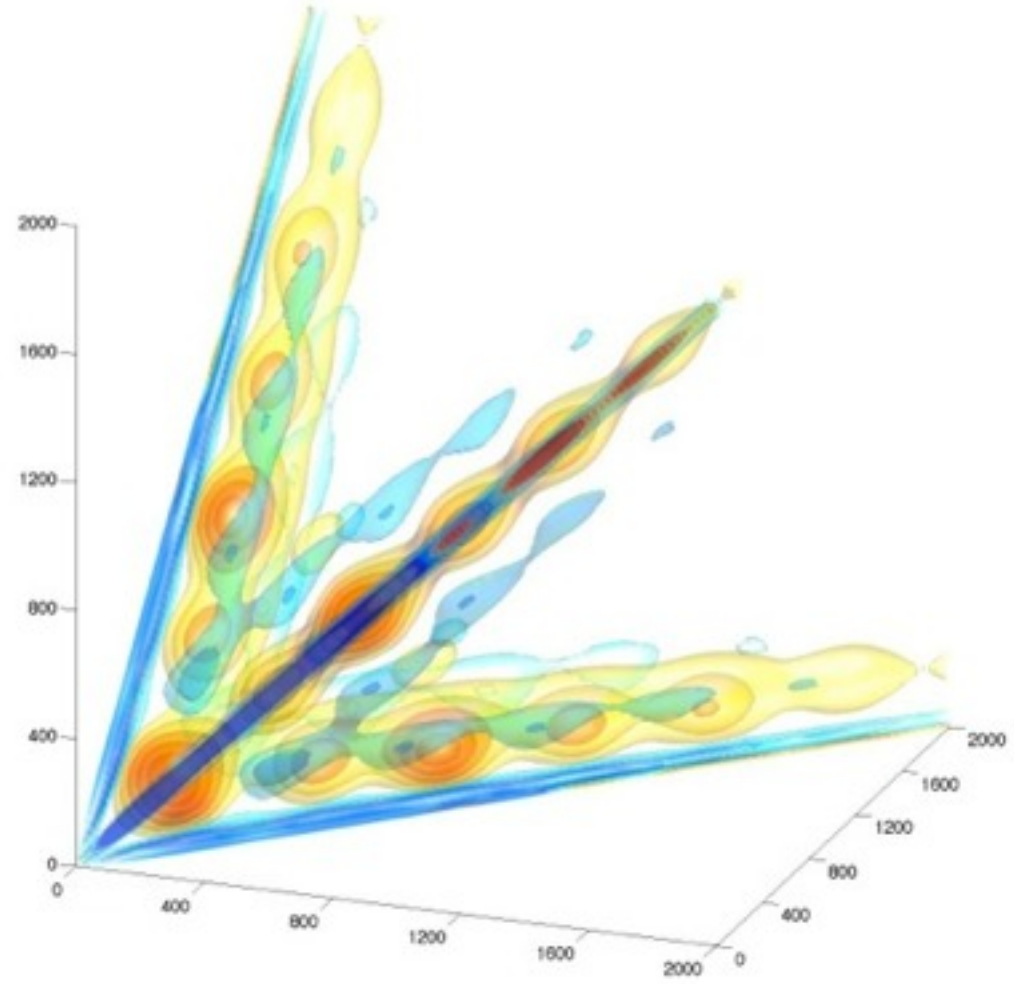
Can we do better than this? What is our goal?

I will focus on non-gaussanity

(similar results apply to Dark Energy, neutrino masses, etc.)

Life after Planck

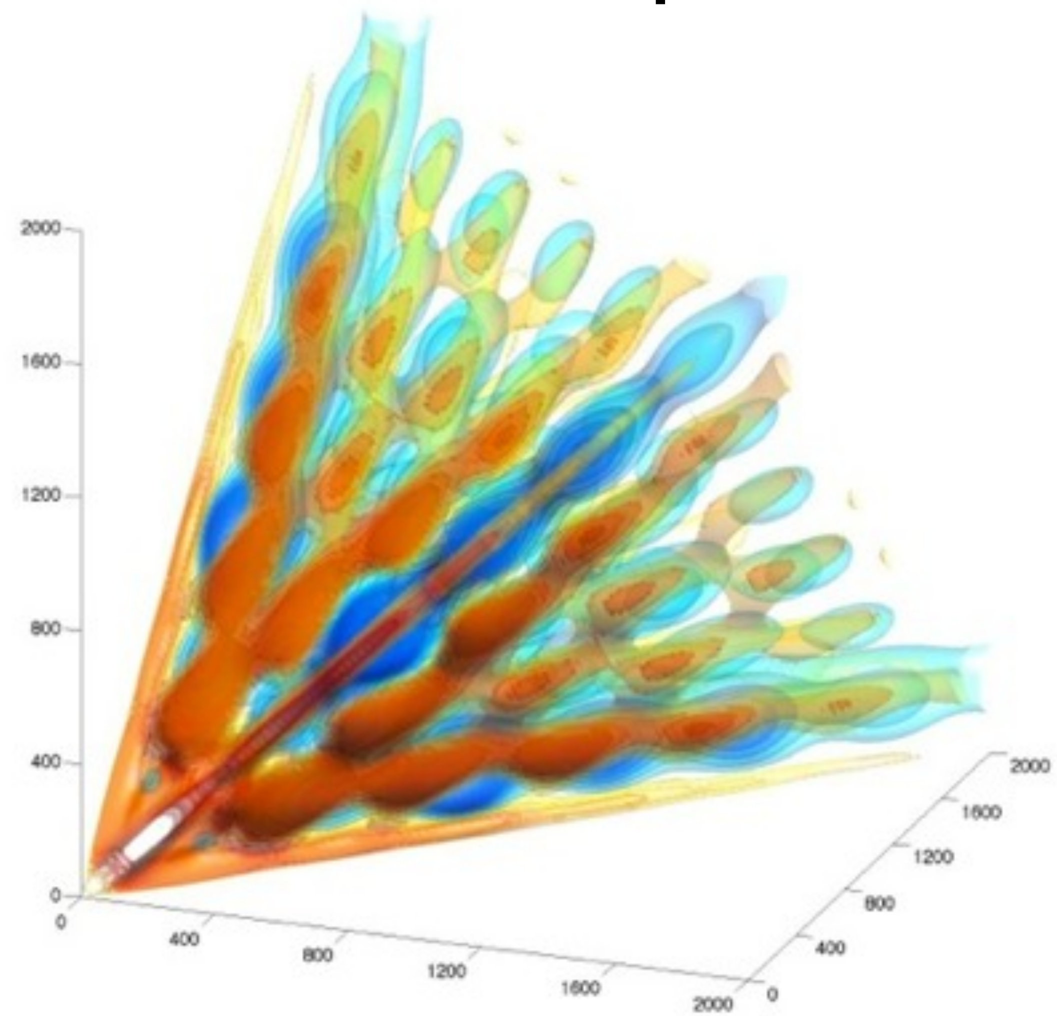
Planck reports limits on 3 templates:



$$f_{\rm NL}^{\rm local} = 2.7 \pm 5.8 \quad \text{(68\% C.I.)}$$

Life after Planck

Planck reports limits on 3 templates:

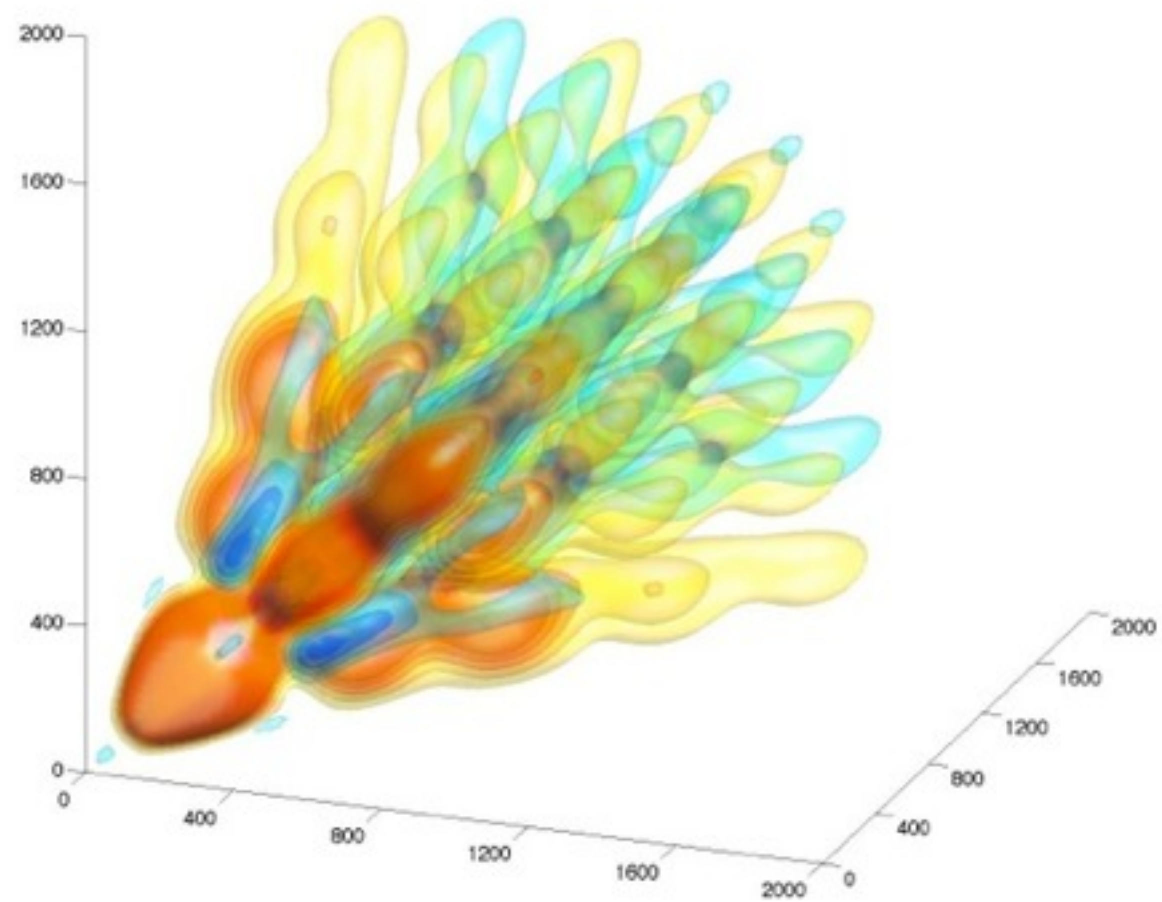


$$f_{\mathrm{NL}}^{\mathrm{ortho}} = -25 \pm 39 \text{ (68\% C.I.)}$$

Courtesy of Fergusson & Shellard

Life after Planck

Planck reports limits on 3 templates:



$$f_{\rm NL}^{\rm equil} = -42 \pm 75 \quad \text{(68\% C.I.)}$$

Courtesy of Fergusson & Shellard

Life after Planck

The bounds on equilateral/orthogonal are weak

Consider slow roll inflation + deformations Creminelli

$$\mathcal{L} = \mathcal{L}_{\text{slow roll}} + \frac{(\partial\phi)^4}{\Lambda^4}$$

For deformation to be under control $\Lambda^2 \gg \dot{\phi}$

$$f_{\text{NL}}^{\text{equilateral}} \sim \frac{\dot{\phi}^2}{\Lambda^4} \ll 1$$

In fact, single-field slow-roll would be ruled out by

$$f_{\text{NL}}^{\text{equilateral}} > 0.93 \quad (\,\Delta f_{\text{NL(Planck)}}^{\text{equilateral}} = 75\,)$$

This level of precision is needed to determine the mechanism

Life after Planck

LSS constraints on equilateral require bispectra:

$$\langle \delta_{m,g}(\mathbf{k}_1)\delta_{m,g}(\mathbf{k}_2)\delta_{m,g}(\mathbf{k}_3)\rangle$$

Non-linearity will also generate a bispectrum.

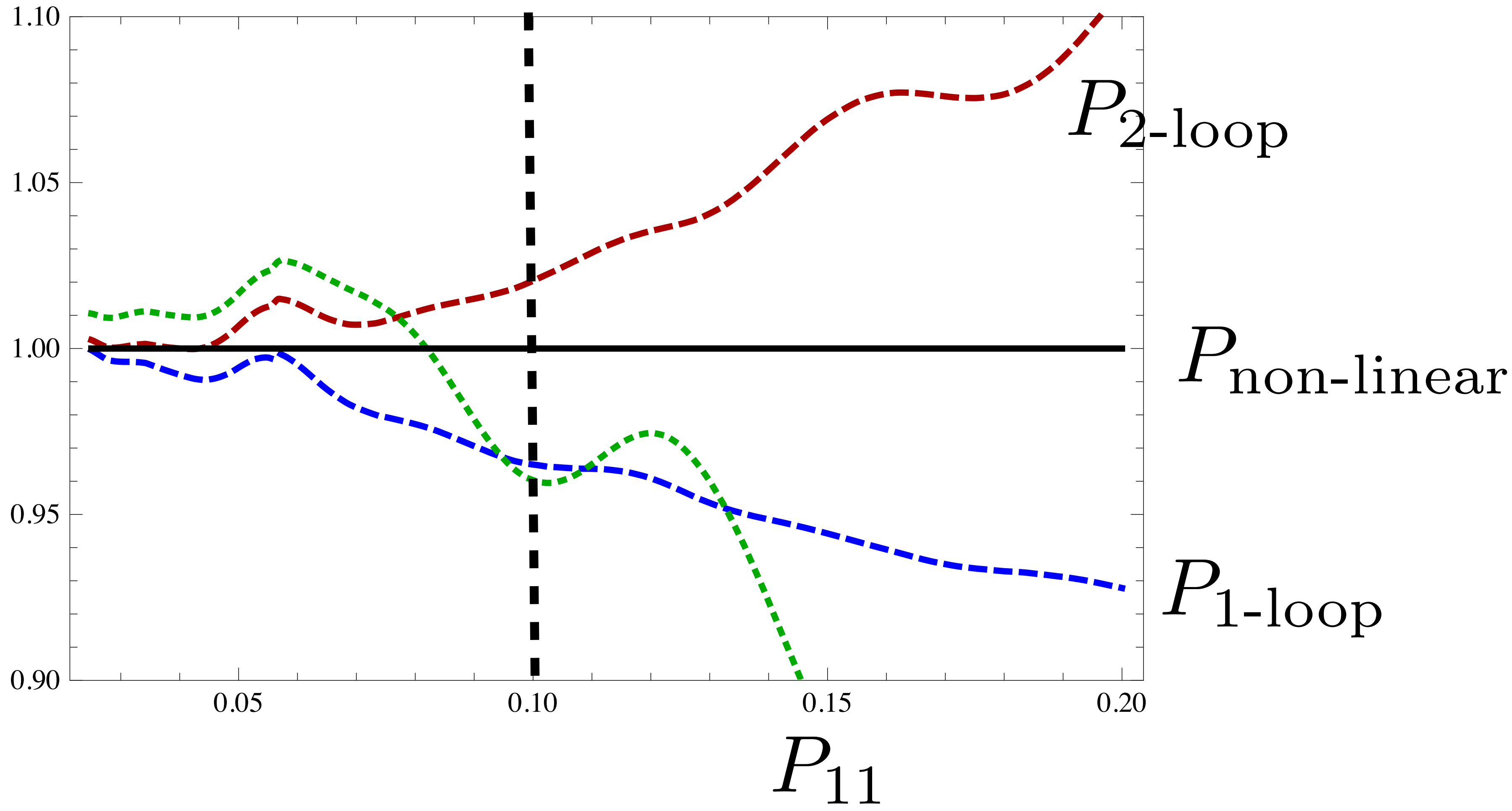
Need understand this well enough for $\Delta f_{\rm NL}^{\rm equi.} = \mathcal{O}(1)$

Often we use $N_{\rm modes}^{\rm max} \sim \frac{k_{\rm NL}^3}{k_{\rm min}^3} \sim \frac{(.1\, h\, {\rm Mpc}^{-1})^3}{k_{\rm min}^3}$

Is this really where non-linear effects come in?

Life after Planck

A common estimate is $P_{2\text{-loop}} \gtrsim P_{1\text{-loop}} \gtrsim P_{11}$



This would seem to give $k_{\text{NL}} \sim .1\, h\,\text{Mpc}^{-1}$

Life after Planck

Is this really correct?

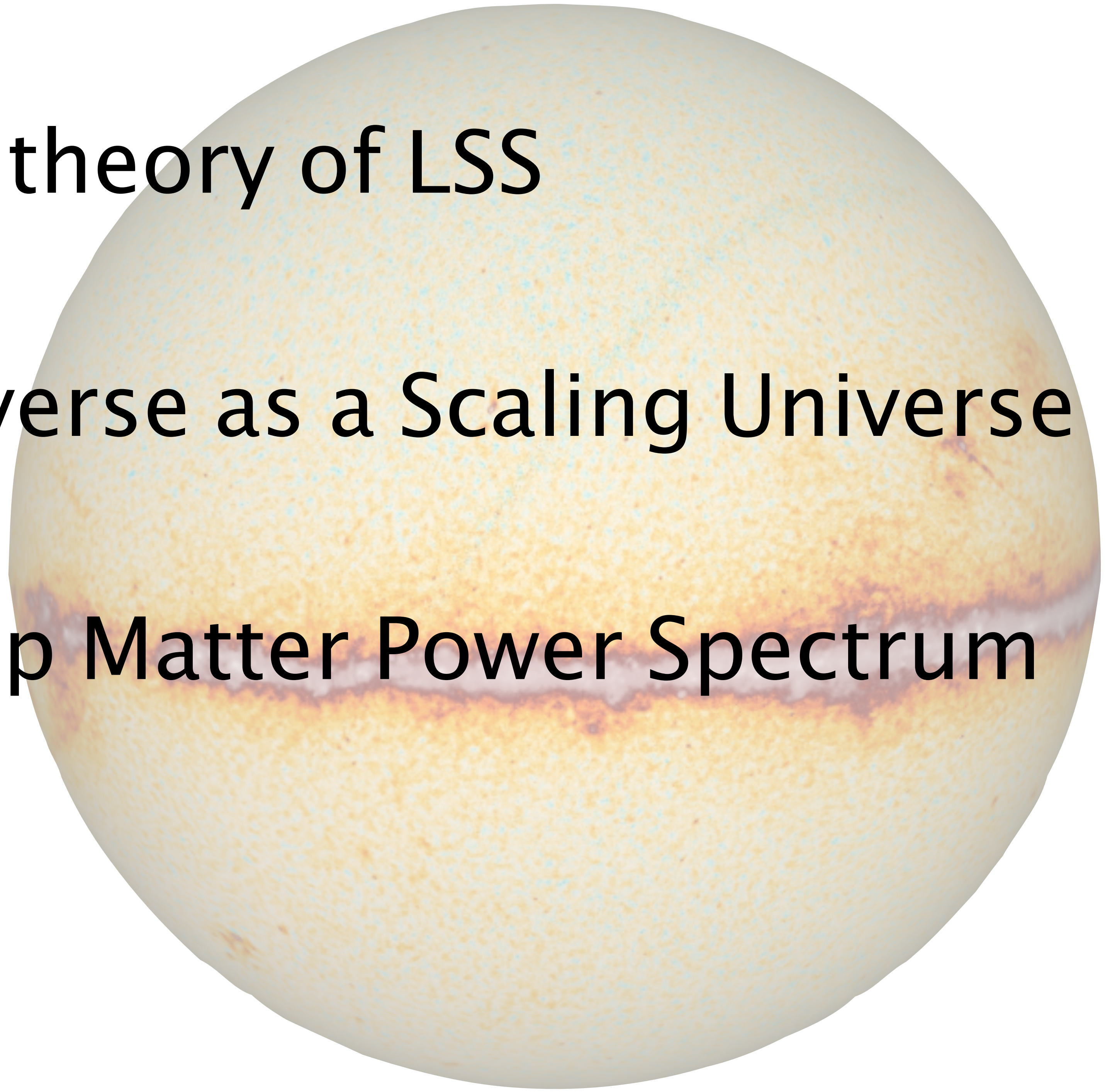
In many contexts: $P^{\text{STP}}_{\text{1-loop}} = \infty$ $P^{\text{STP}}_{\text{2-loop}} = \infty$

Our perturbation theory is missing something:
Dark matter is not a perfect fluid:

$$\dot{v}^i + H v_l^i + \frac{1}{a} v^j \partial_j v_l^i + \frac{1}{a} \partial^i \phi = -\tfrac{1}{a\rho} \partial_j \tau^{ij}$$

Many things will change when we include $\tau^{ij} \neq 0$

Outline



Effective theory of LSS

Real Universe as a Scaling Universe

Two-Loop Matter Power Spectrum

Outlook

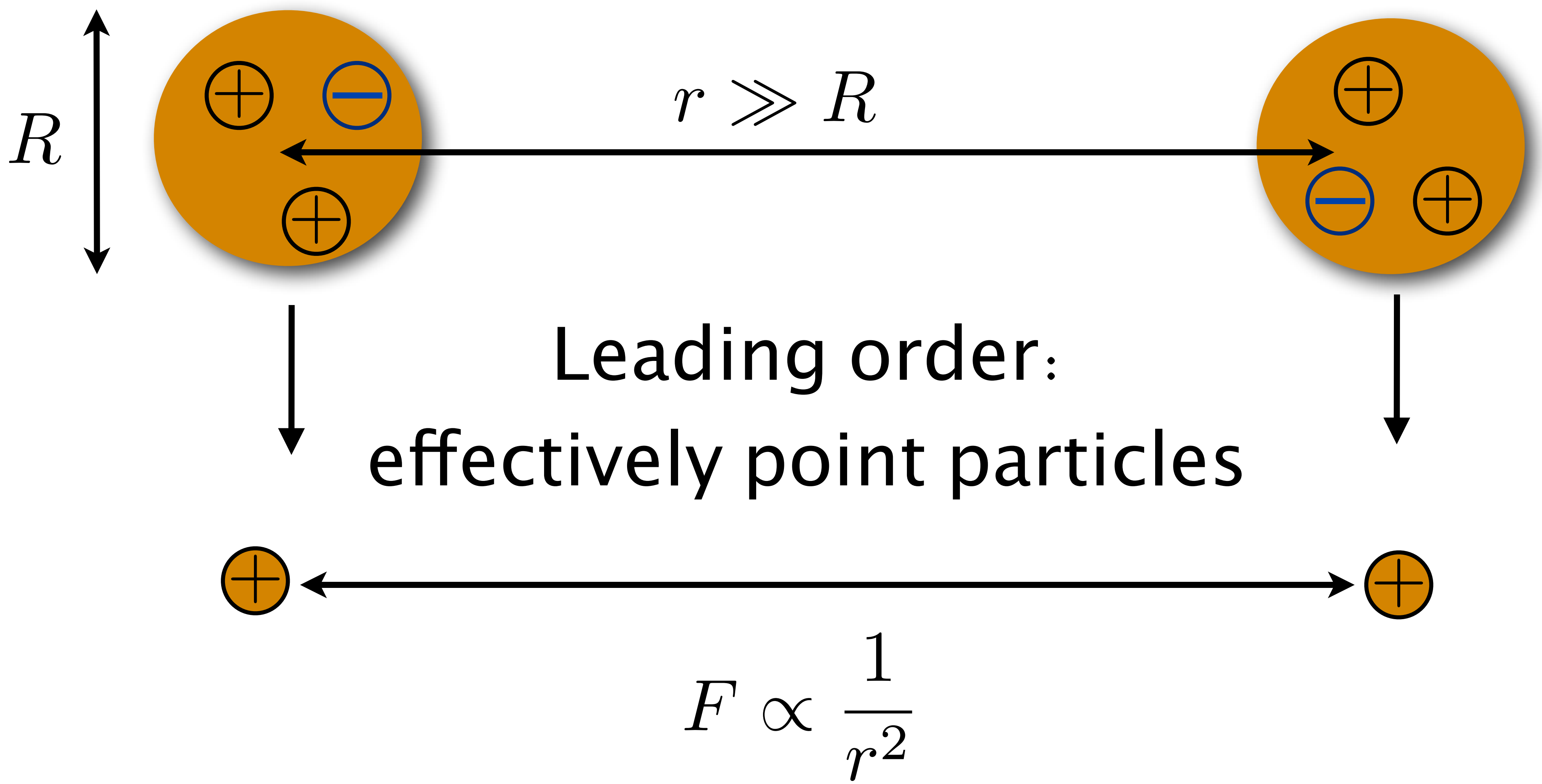
Courtesy of thecmb.org

Effective Theory of Large Scale Structure

Effective Field Theory

Often, EFT is a fancy term for normal physics

E.g. Forces between collections of charges



Effective Field Theory

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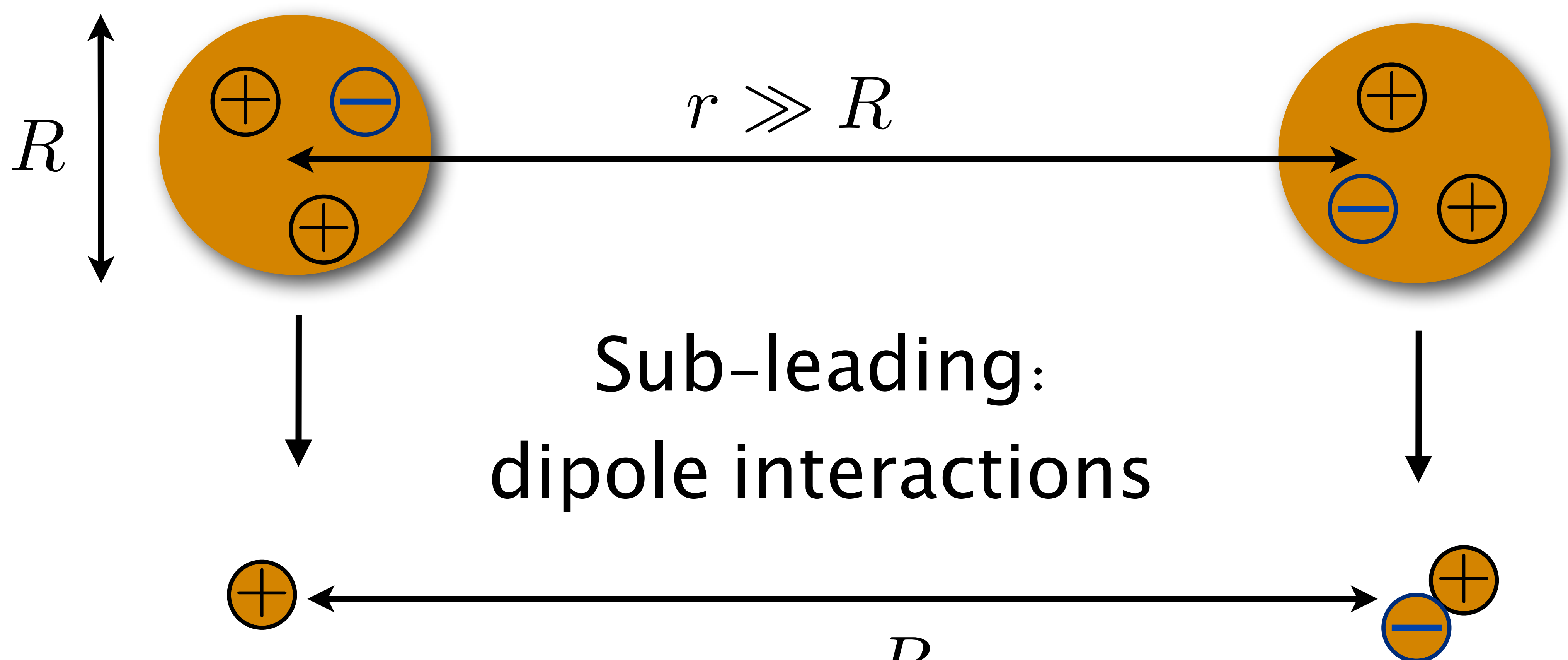


$$F_{\text{dipole}} \propto \frac{R}{r^3} \ll F$$

Effective Field Theory

Often, EFT is a fancy term for normal physics

E.g. Fluids

Start from the Boltzmann equation $\frac{df[\mathbf{x},\mathbf{p},t]}{dt} = C[f]$

Take moments - $\int d^3\mathbf{p}\,\mathbf{p}^n f[\mathbf{x},\mathbf{p},t]$

For perfect fluids, keep only $n = 0, 1$

To describe viscosity, etc. need to keep $n = 2, 3, \ldots$

$$\dot{\mathbf{v}} + \mathbf{v}\cdot\nabla\mathbf{v} = -\nabla p + c_b\nabla(\nabla\cdot\mathbf{v}) + c_v\nabla^2\mathbf{v}$$

Effective Field Theory

Small scale physics parameterized by a few numbers

However, in EFT, these "numbers" are not constant

Depends on: cutoff (regulator) Λ

renormalization scale μ

Then take μ to match the scale of measurements

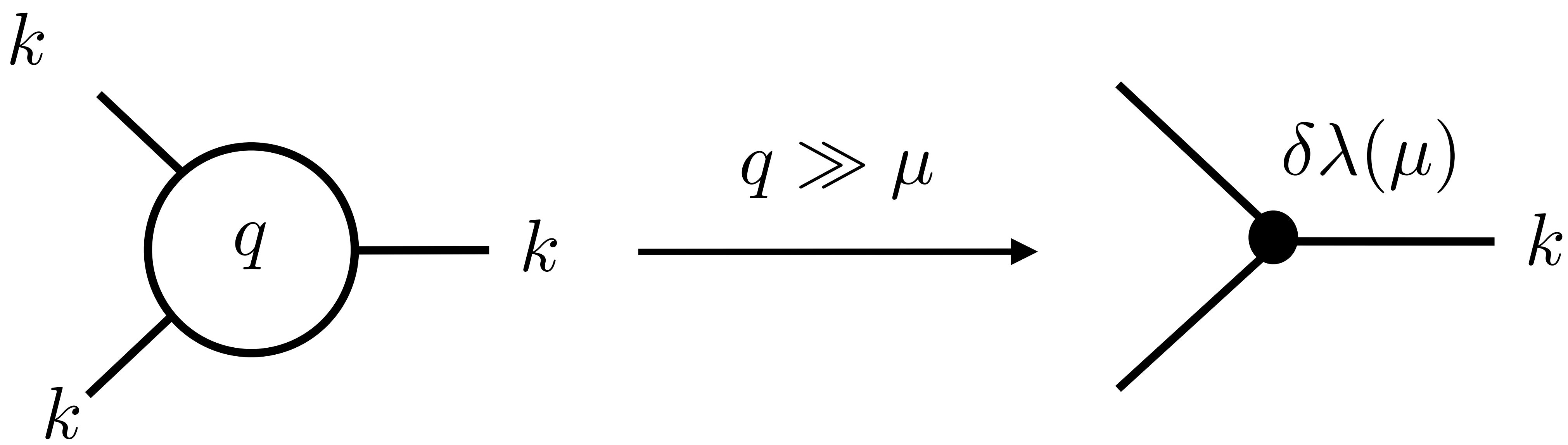
E.g. QED with massless electrons $\alpha \propto \dfrac{1}{\log(\Lambda/\mu)}$

Potential from massive charge $V(r) \propto \dfrac{1}{r\log(r\Lambda)}$

Effective Field Theory

Same is true in classical field theory

Simply capturing the mixing between scales

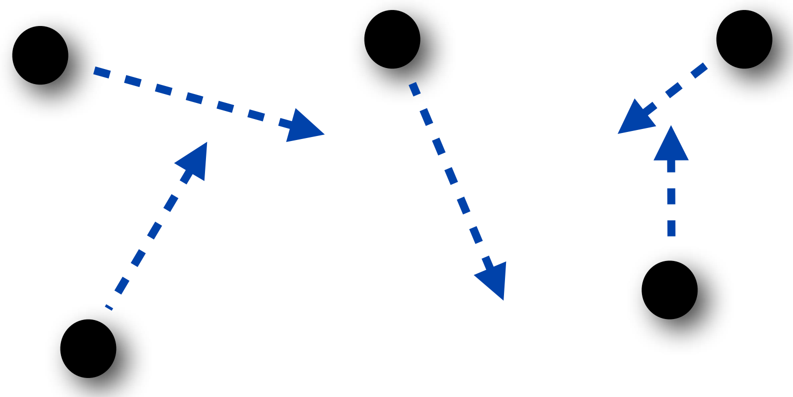


Coupling changes by including $\mu + \delta\mu > q > \mu$

EFT of LSS

Dark matter is NOT a pressureless fluid

It is just a bunch of collision-less particles



On large scales it looks like a fluid (DM moves slow):

$$\int d^3p \left(\frac{\mathbf{p}}{m}\right)^n f(\mathbf{k},\mathbf{p},t) \sim (x_{\rm MFP}k) \int d^3p \left(\frac{\mathbf{p}}{m}\right)^{n-1} f(\mathbf{k},\mathbf{p},t)$$

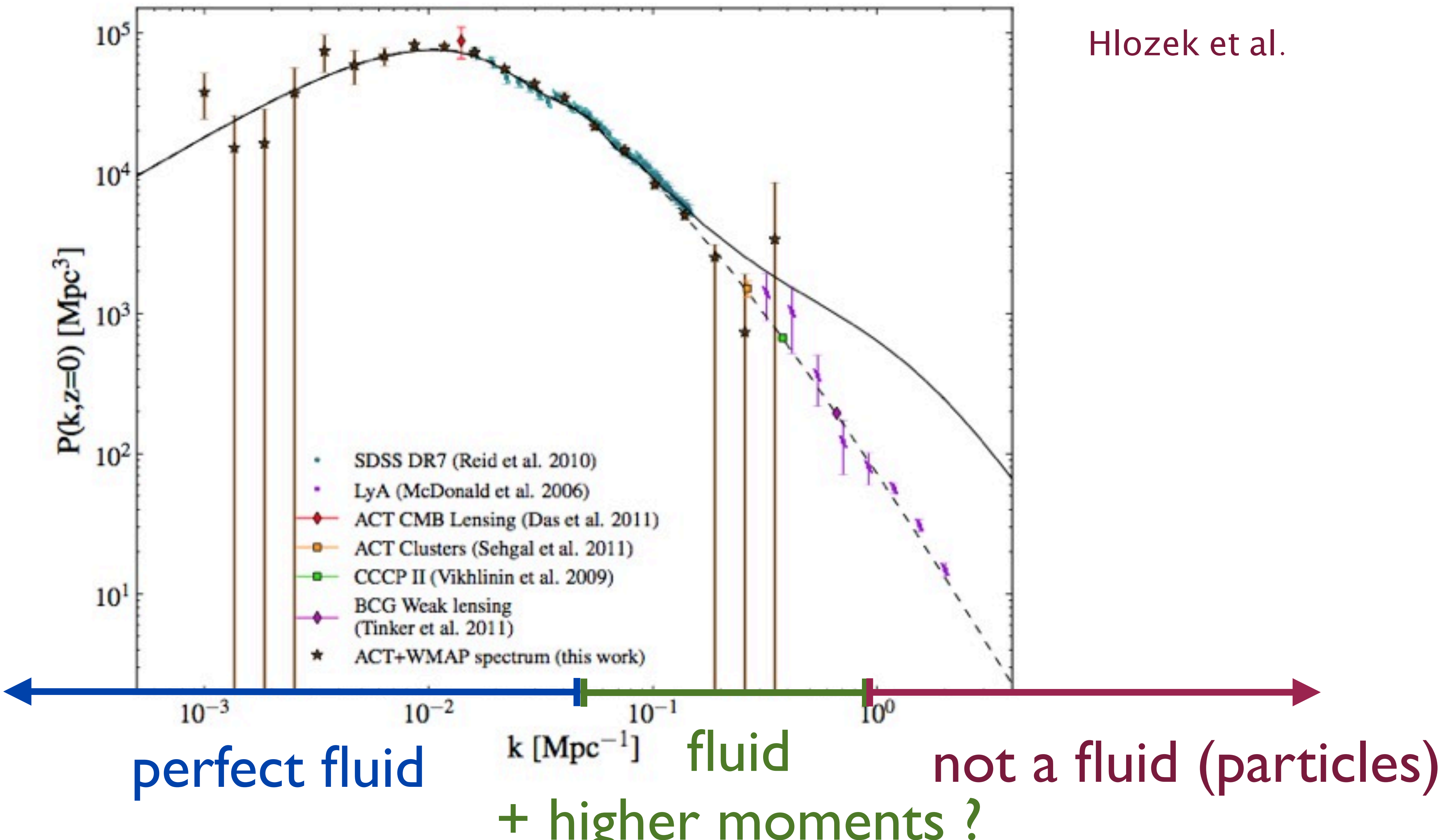
Like a perfect fluid when $k \ll x_{\rm MFP}^{-1}$

Baumann et al.
Carrasco, Hertzberg & Senatore

EFT of LSS

Dark matter is NOT a pressureless fluid

Hlozek et al.



EFT of LSS

Standard perturbation theory (SPT):

$$\nabla^2\phi = \frac{3}{2} H_0^2\,\Omega_m \frac{a_0^3}{a}\delta$$

$$\dot\delta = -\frac{1}{a}\partial_i\left([1+\delta]v^i\right)$$

$$\dot v^i + H v_l^i + \frac{1}{a} v^j \partial_j v_l^i + \frac{1}{a}\partial^i\phi = 0$$

EFT of LSS:

$$\dot v^i + H v_l^i + \frac{1}{a} v^j \partial_j v_l^i + \frac{1}{a}\partial^i\phi = -\tfrac{1}{a\rho}\partial_j \tau^{ij}$$

$$\tau^{ij} = \rho(c_s^2 \delta\,\delta^{ij} + \ldots)$$

EFT of LSS

Standard perturbation theory（SPT）：

Treat non-linear terms as perturbations （ $\theta \equiv \partial_i v^i$ ）

$$a\mathcal{H}\delta' + \theta = -\int \frac{d^3q}{(2\pi)^3}\alpha(p,k-p)\delta(k-p)\theta(p) \ ,$$

$$a\mathcal{H}\theta' + \mathcal{H}\theta + \frac{3}{2}\mathcal{H}_0^2\,\Omega_m\frac{a_0^3}{a}\delta = -\int \frac{d^3q}{(2\pi)^3}\beta(p,k-p)\theta(k-p)\theta(p)$$

EFT of LSS：

Also treat $\partial_i\partial_j\tau^{ij}$ as a perturbation

$$a\mathcal{H}\theta' + \mathcal{H}\theta + \frac{3}{2}\mathcal{H}_0^2\,\Omega_m\frac{a_0^3}{a}\delta = -\frac{1}{\rho}\partial^2\tau^2 - \int \frac{d^3q}{(2\pi)^3}\beta\theta^2$$

The Real Universe as a Scaling Universe

What is the small number?

SPT is an expansion in $\delta < 1$

Expect (hope?) loops are suppressed by $\delta^L \ll \delta$

The EFT of LSS wants us to add: $k^2\delta, k^2\delta^2, k^4\delta, \ldots$

Problem: How do I compare δ^L and $k^{2p}\delta^q$?

We need a better understanding of $\delta^L(k)$

SPT in the Scaling Universe

The basic building block of perturbation theory is

$$\langle \delta^{(1)}(k)\delta^{(1)}(k')\rangle = P_{11}(k)(2\pi)^3\delta^3(k+k')$$

We then solve for $\delta = \sum_n \delta^{(n)} = \sum_n F_n(\{q_i\})(\delta^{(1)})^n$

Simplest case to study is

$$P_{11}(k) = \frac{(2\pi)^3}{k_{\rm NL}^3}\left(\frac{k}{k_{\rm NL}}\right)^m$$

Only scale is $k_{\rm NL}$: dim. analysis works e.g. Jain & Bertschinger, Pajer & Zaldarriaga

SPT in the Scaling Universe

Finite parts (Λ- independent) are easy to estimate

$$P^{\text{finite}}_{\text{L-loop}} \sim (k^3 P_{11}(k))^L P_{11}(k) \sim \left(\frac{k}{k_{\rm NL}}\right)^{(3+m)L} P_{11}(k)$$

E.g. : $m = -\frac{3}{2}$ at one-loop:

$$P_{\text{1-loop}} = P_{31} + P_{22} \sim (\frac{k}{k_{\rm NL}})^{3/2} P_{11}(k)$$

There are also Λ-dependent contributions:

E.g. : $m = -\frac{3}{2}$ at two-loops

$$P_{\text{2-loop}} \sim [\frac{\Lambda}{k_{\rm NL}} \frac{k^2}{k_{\rm NL}^2} + \frac{k^3}{k_{\rm NL}^3}] P_{11}(k) + \mathcal{O}(\frac{k}{\Lambda})$$

SPT in the Scaling Universe

All Λ - dependent terms must be removeable

$$\partial_i \partial_j \tau^{ij} \sim (-\Lambda + c_0^2) \frac{k^2}{k_{\rm NL}^2} \delta \to P_{c_s^2} = (-\Lambda + c_0^2) \frac{k^2}{k_{\rm NL}^2} P_{11}(k)$$

These counter-terms also leave finite contributions:

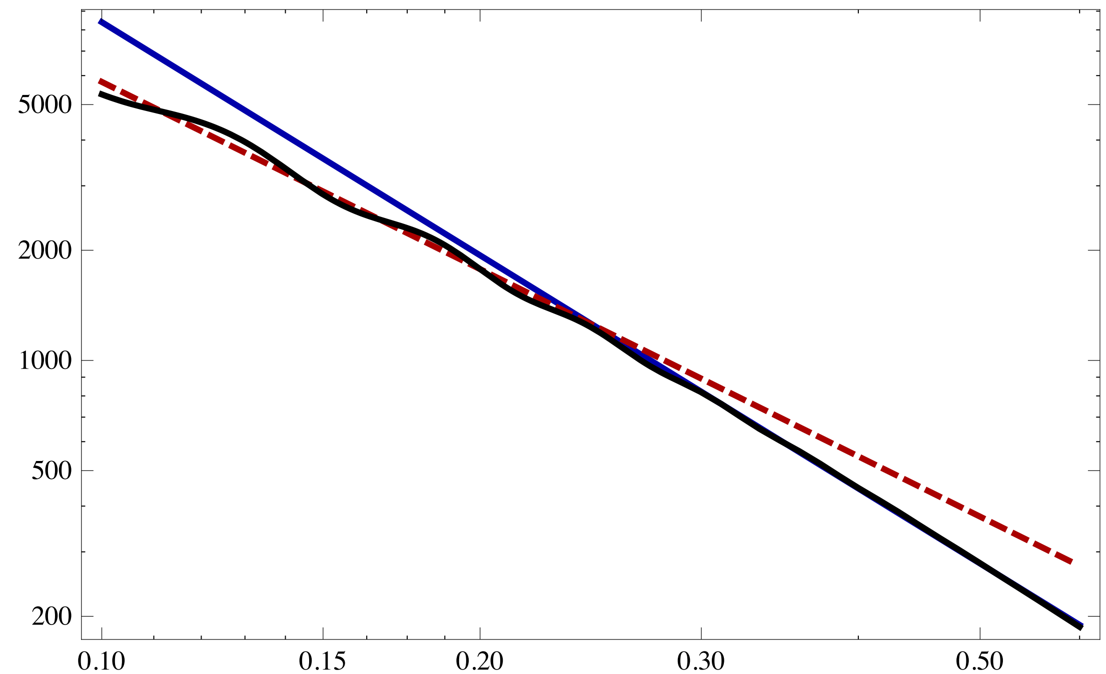
$$P_{\text{2-loop}} + P_{c_s^2} \sim \left(c_0^2 \frac{k^2}{k_{\rm NL}^2} + \frac{k^3}{k_{\rm NL}^3} \right) P_{11}(k)$$

The finite part (c_0^2) must be matched to simulations (not predicted by perturbation theory)

Scaling Behavior in the Real Universe

What does this have to do with the real universe?

$P_{11}^{\rm real}$



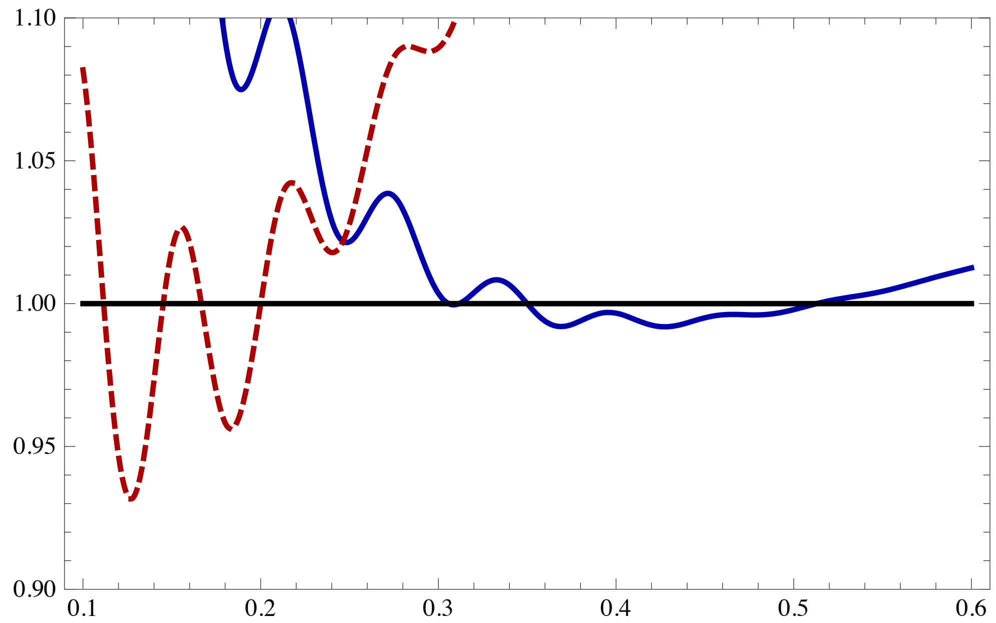
$$P_{11}^{\rm real}(k) \sim (2\pi)^3 \begin{cases} \frac{1}{k_{\rm NL}^3}\left(\frac{k}{k_{\rm NL}}\right)^{-2.1} & \text{for } k > k_{\rm tr} \\ \frac{1}{\tilde{k}_{\rm NL}^3}\left(\frac{k}{\tilde{k}_{\rm nl}}\right)^{-1.7} & \text{for } k < k_{\rm tr} \end{cases}$$

$$k_{\rm NL} \sim 4.6\, h\, {\rm Mpc}^{-1} \qquad k_{\rm tr} \sim .25\, h\, {\rm Mpc}^{-1}$$

Scaling Behavior in the Real Universe

What does this have to do with the real universe?

$P_{\text{fit}}/P_{11}^{\text{real}}$



$$P_{11}^{\text{real}}(k) \sim (2\pi)^3 \begin{cases} \dfrac{1}{k_{\text{NL}}^3}\left(\dfrac{k}{k_{\text{NL}}}\right)^{-2.1} & \text{for } k > k_{\text{tr}} \\ \dfrac{1}{\tilde{k}_{\text{NL}}^3}\left(\dfrac{k}{\tilde{k}_{\text{nl}}}\right)^{-1.7} & \text{for } k < k_{\text{tr}} \end{cases}$$

$$k_{\text{NL}} \sim 4.6\, h\, \text{Mpc}^{-1} \qquad k_{\text{tr}} \sim .25\, h\, \text{Mpc}^{-1}$$

Scaling Behavior in the Real Universe

What does this have to do with the real universe?

Above $k \sim .25\, h\, \mathrm{Mpc}^{-1}$, we can use $m = -2$

Estimate of error from 3-loop SPT

$$\frac{P_{\text{3-loop}}}{P_{\text{non-linear}}}(k = .5\, h\, \mathrm{Mpc}^{-1}) \sim 0.02 - 0.04$$

Estimate of required "counter-terms". Only need:

$$\partial_i \partial_j \tau^{ij} \sim [c_0^2 + c_{\text{2-loop}}(\Lambda)]\partial^2 \delta$$

All other counter-terms smaller than 3-loop SPT

Two-Loop Matter Power Spectrum

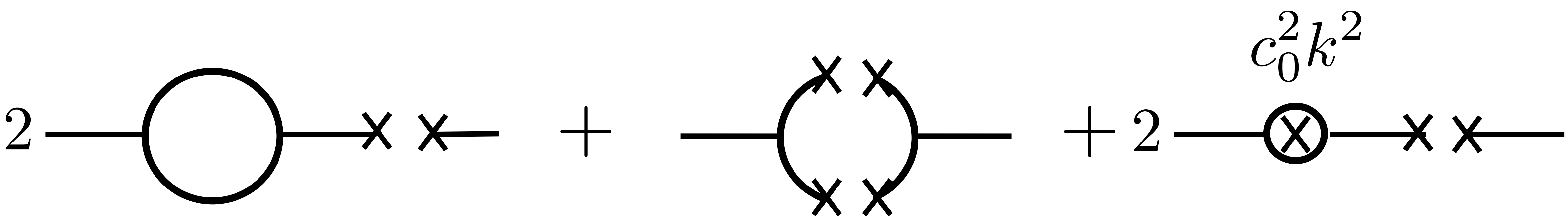
“Measuring” parameters

From scaling universe, at 1-loop we have

$$\partial_i \partial_j \tau^{ij} = c_0^2 \frac{\partial^2}{k_{\rm NL}^2} \delta$$

We can determine this using

$$P_{\text{1-loop}}^{\rm EFT} = P_{\text{1-loop}}^{\rm STP} + c_0^2 \frac{k^2}{k_{\rm NL}} P_{11}$$



$P_{31} \equiv \langle \delta^{(3)} \delta^{(1)} \rangle$

$P_{22} \equiv \langle \delta^{(2)} \delta^{(2)} \rangle$

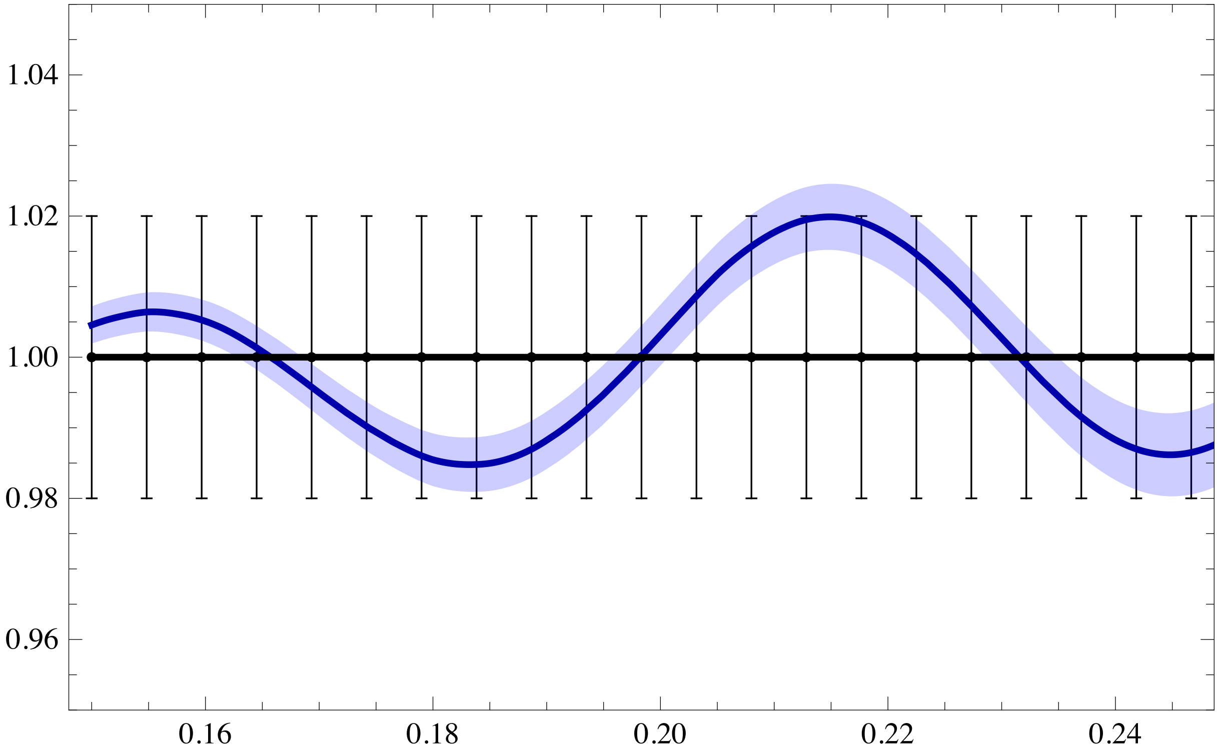
$c_0^2 k^2 P_{11}$

“Measuring” parameters

From scaling universe, at 1-loop we have

$$\partial_i \partial_j \tau^{ij} = c_0^2 \frac{\partial^2}{k_{\rm NL}^2} \delta$$

Fit to non-linear data (Coyote):



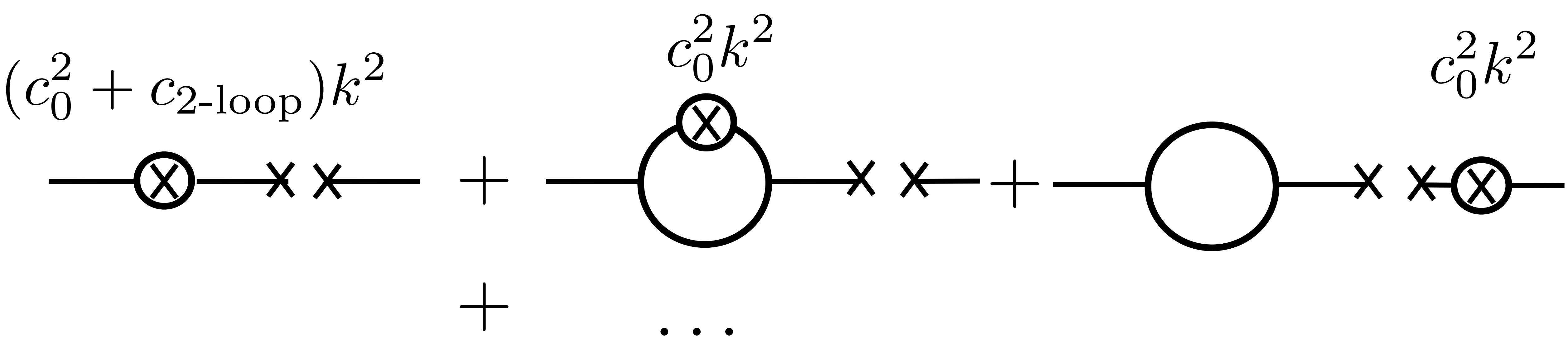
$$c_0^2 = (1.62 \pm 0.03) \left(\frac{k_{\rm NL}^2}{2\pi h^2 \, {\rm Mpc}^{-2}} \right)$$

“Measuring” parameters

From scaling universe, at 2-loops we have

$$\partial_i \partial_j \tau^{ij} = (c_0^2 + c_{2\text{-loop}}) \frac{\partial^2}{k_{\rm NL}^2} \delta$$

The two terms are evaluate at different orders:



c_0^2 counts as 1-loop and $c_{2\text{-loop}}$ counts as 2-loops

“Measuring” parameters

How do we determine $c_{\text{2-loop}}$?

In the m=-2 scaling universe:

$$P_{\text{2-loop}} = c^{\Lambda} \log(\Lambda/k) + \ldots$$

The two loop “counter-term” should be

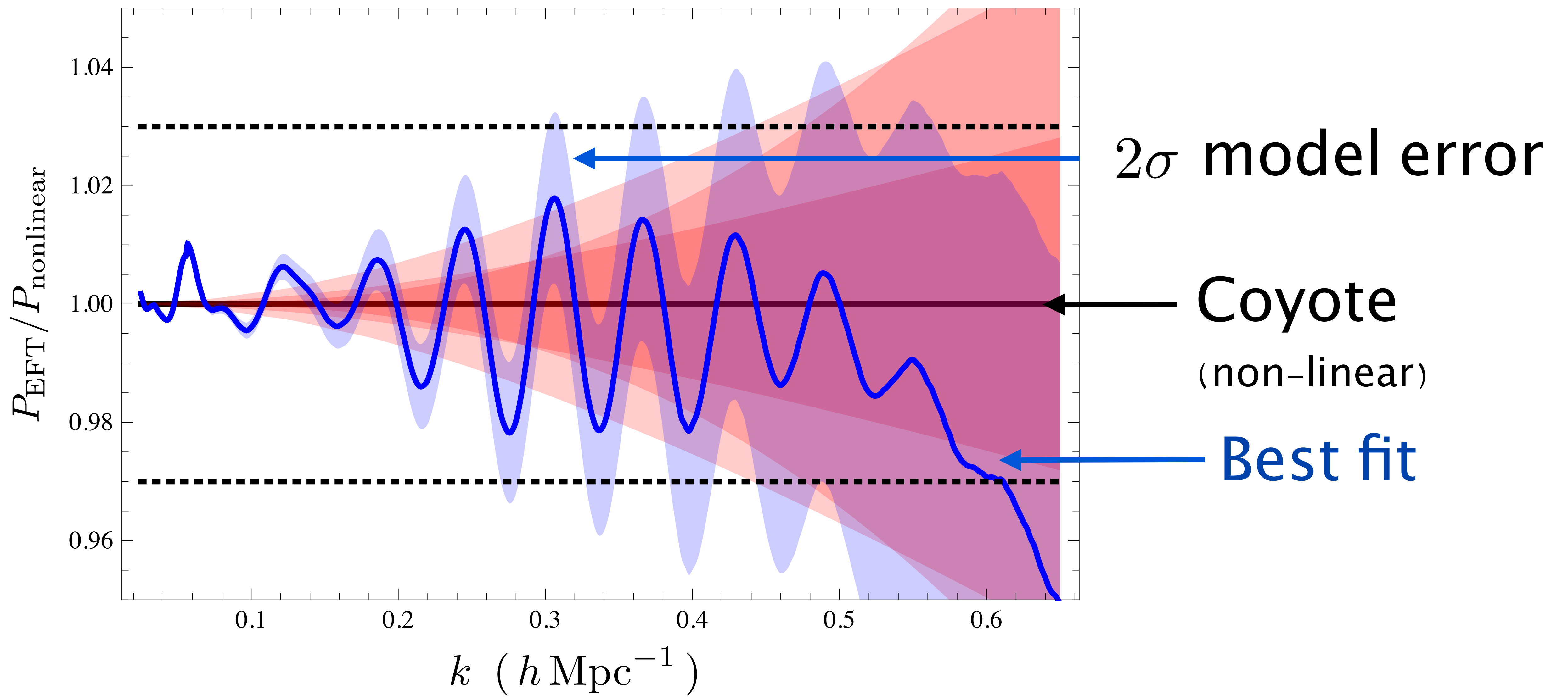
$$c_{\text{2-loop}} = -c^{\Lambda} \log(\Lambda/\mu)$$

This can be determined without non-linear data

Same idea works in real universe (but is more complicated)

Results

The 2-loop matter power spectrum:



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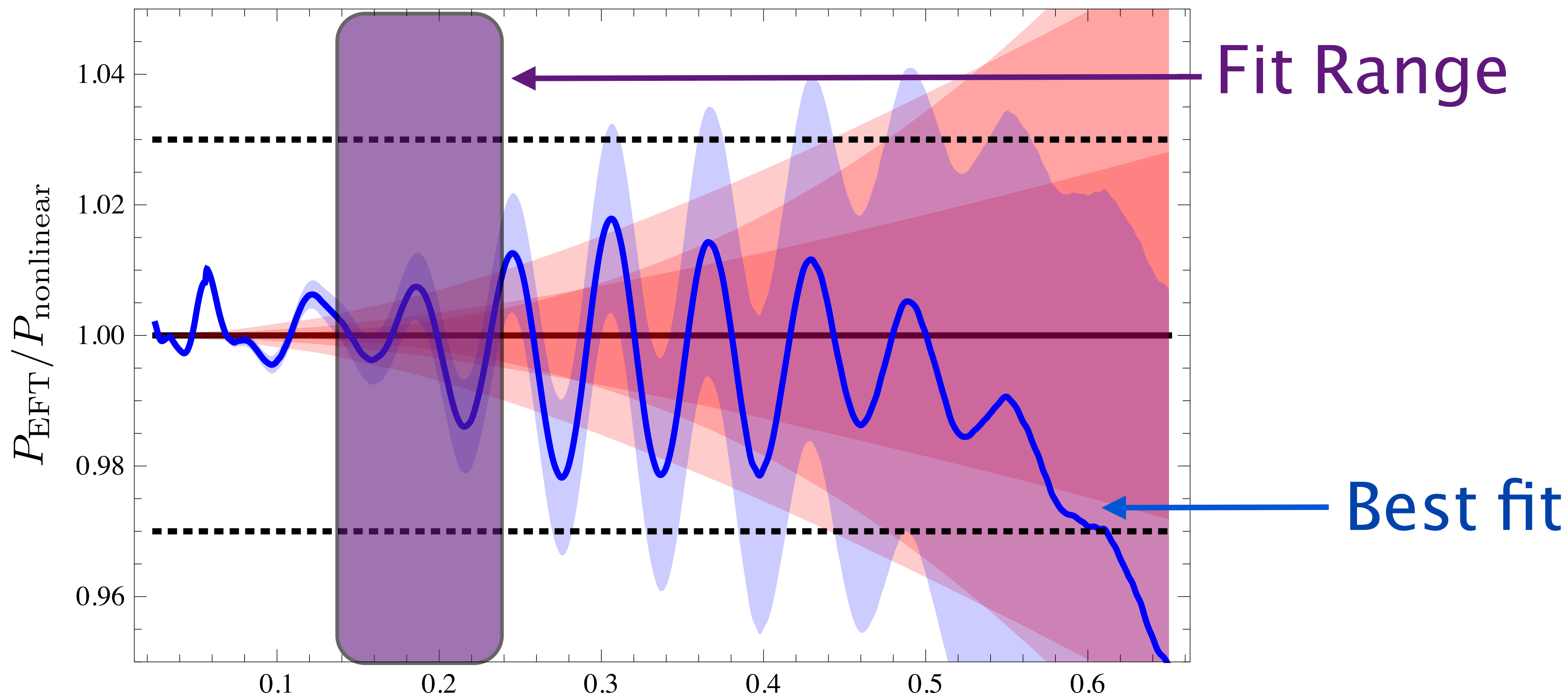

$$.15\, h\, \mathrm{Mpc}^{-1} < k_{\rm fit} < .25\, h\, \mathrm{Mpc}^{-1}$$

Results

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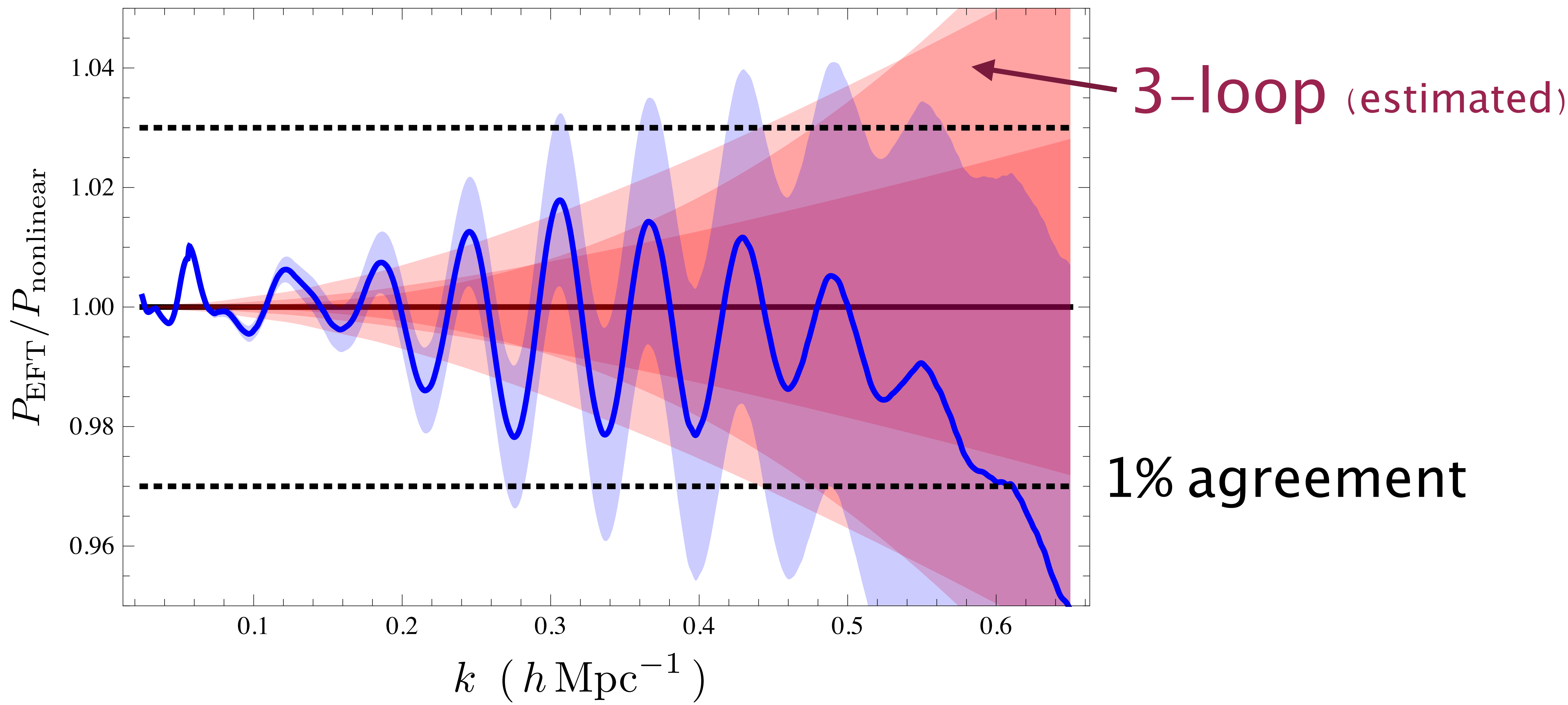

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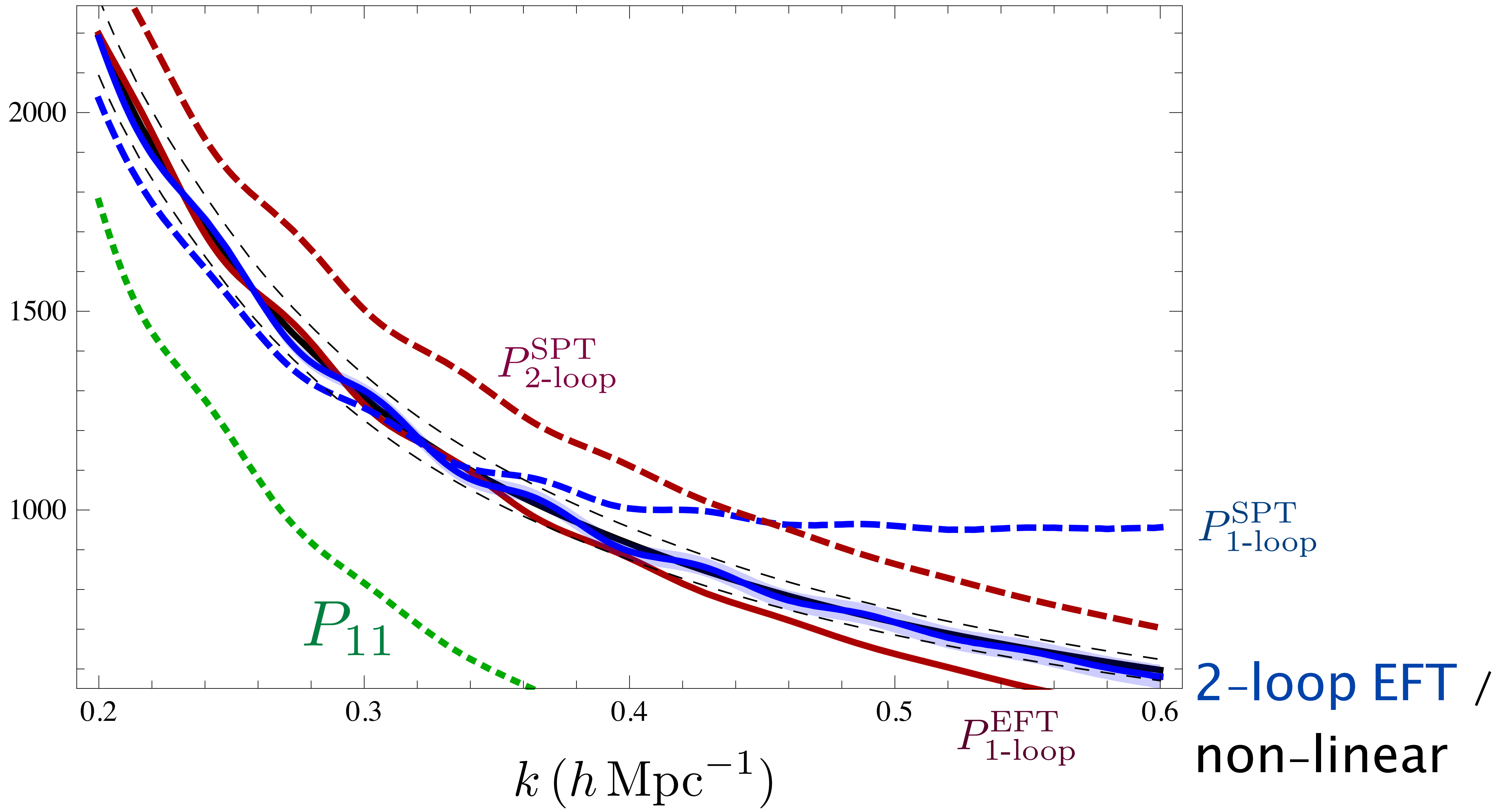

Results

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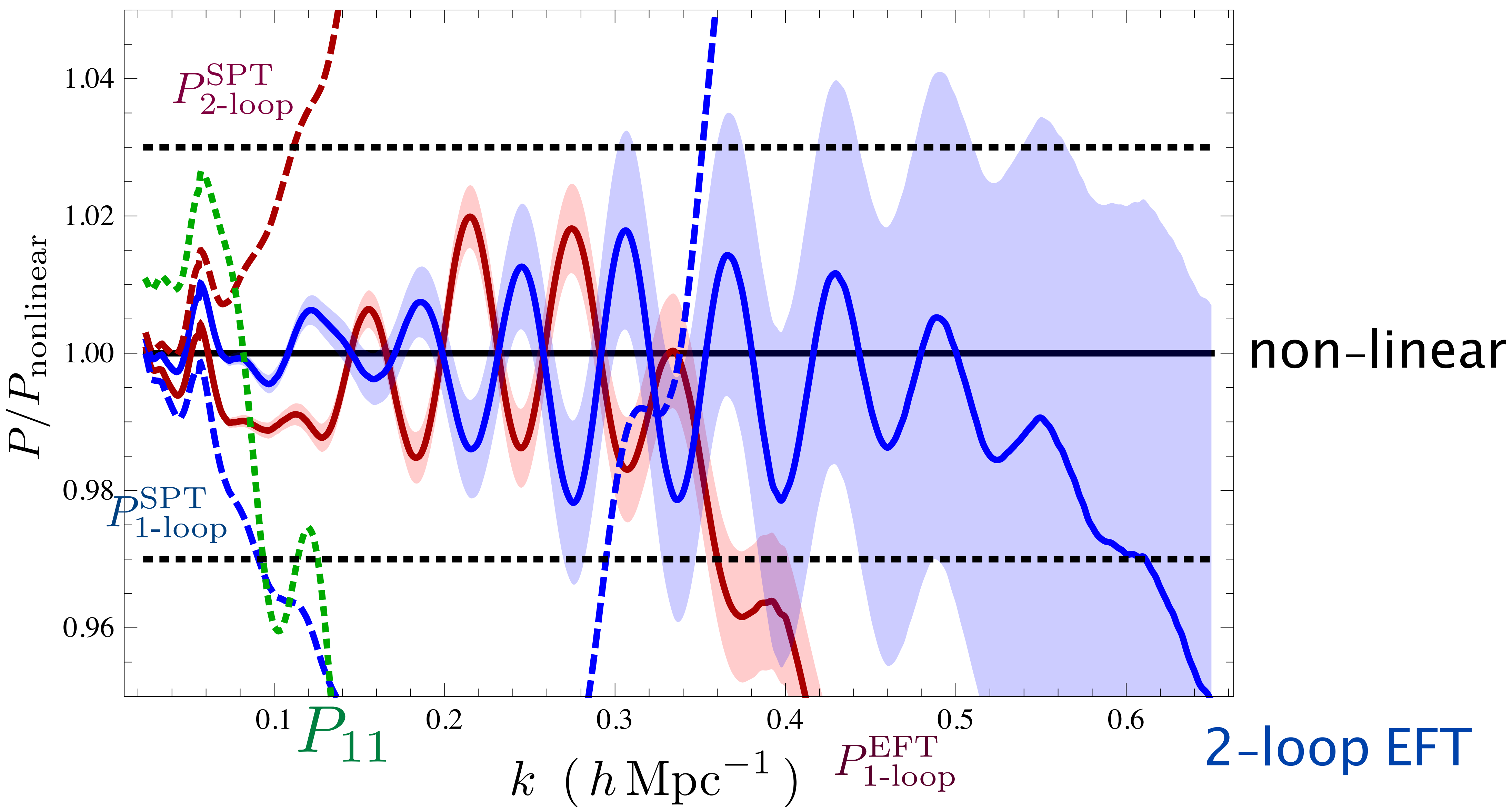

Implications for non-Gaussianity

Projections for future surveys give:

$$\Delta f_{\rm NL}^{\rm equilateral} \sim 10 \qquad \text{for} \qquad k_{\rm max} = 0.1\, h\, {\rm Mpc}^{-1}$$

Sefusatti et al.

If we used the 2-loop EFT range of validity

$$\Delta f_{\rm NL}^{\rm equilateral} \sim 1/2 \qquad \text{for} \qquad k_{\rm max} = 0.6\, h\, {\rm Mpc}^{-1}$$

Equivalent to a survey >150x larger than Euclid

Outlook

What we have shown

Estimating the non-linear scale is non-trivial:

Previous estimates used $P_{\text{2-loop}} \gtrsim P_{\text{1-loop}} \gtrsim P_{11}$

From the EFTofLSS we see this is not correct

Two loop EFT seems well behaved up to $k \gtrsim 0.6\, h\, \mathrm{Mpc}^{-1}$

Unfortunately, there is no rigorous definition:
(there is no equivalent of perturbative unitarity)

What is there to do?

The real universe contains more than dark matter:

We don't observed DM: halo & galaxy biasing

Or observe in real space: redshift space distortions

Even if we measure DM directly (weak lensing):

Can we ignore or include baryons well enough?
(is this an unmanageable mess?)