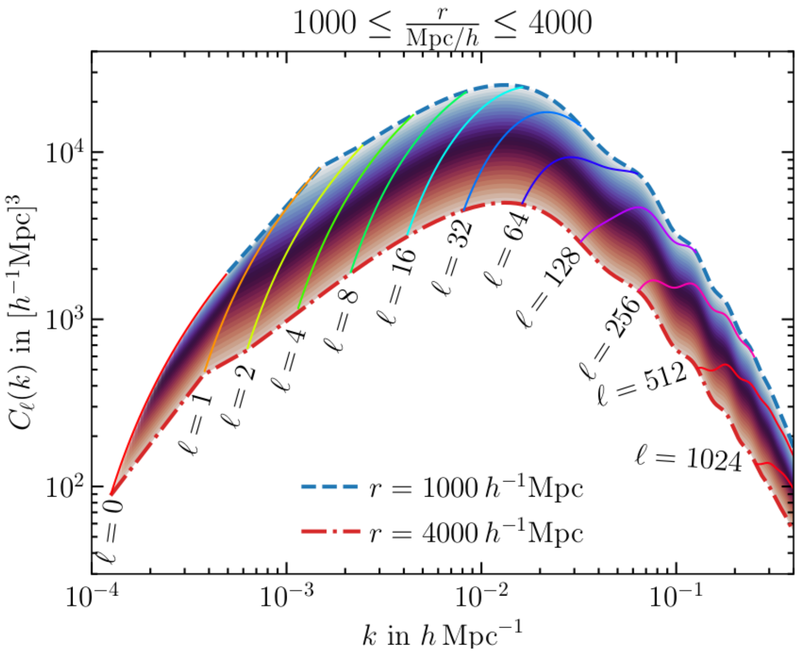
SuperFaB: a fabulous code for Spherical Fourier-Bessel decomposition



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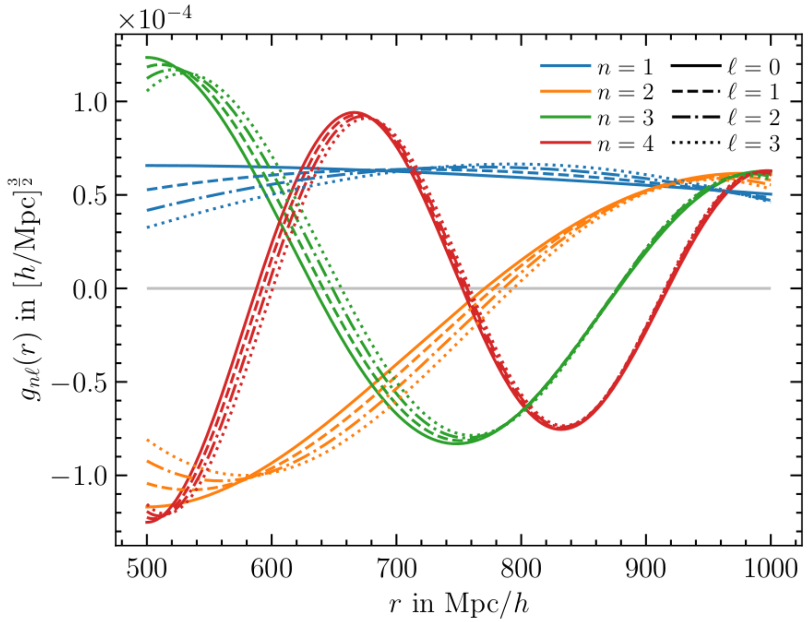


arXiv:2102.10079

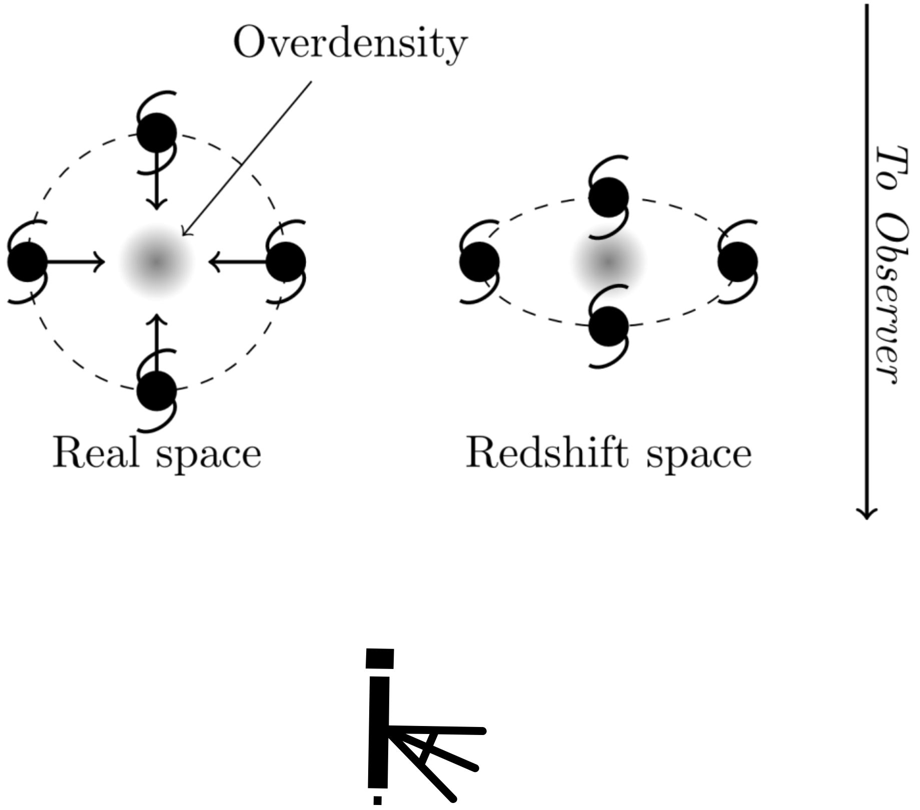
Outline

- Motivation: Cosmology with the Power Spectrum
  - RSD
  - Redshift-evolution
- Motivation: Spherical Fourier-Bessel (SFB) vs. Fourier
- Intuition for the SFB power spectrum
- Measuring SFB modes
- Results: *Roman*-, *SPHEREx*-, *Euclid*-like

Motivation: Roman, SPHEREx, Euclid, PFS, DESI, …

- Galaxy surveys bridge the gap between low-redshift supernova Ia and high-redshift CMB measurements. (e.g. BAO)
- Redshift-space distortions (RSD) probe the growth of structure over cosmic time. → Tests for modified gravity.
- Large scales especially important for non-Gaussianity.

Redshift-space distortions I: Kaiser effect



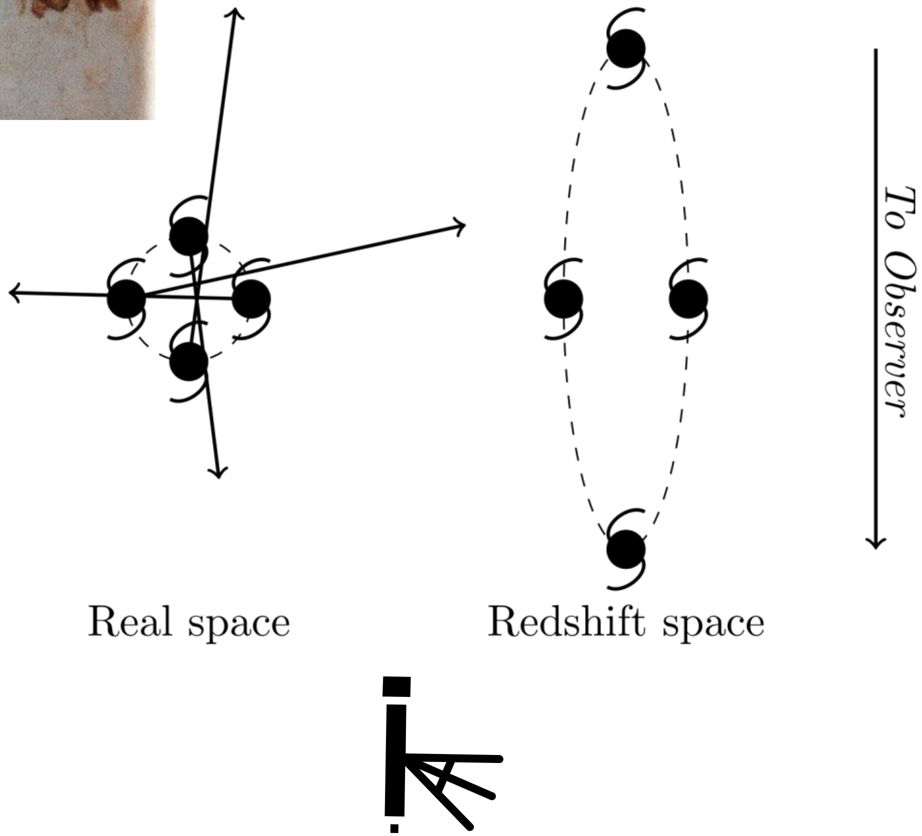
Coherent flow of galaxies towards overdensities leads to increased clustering along the line of sight direction.

→ Non-zero quadrupole!

$$s = r + \frac{v_{\parallel}}{aH}$$

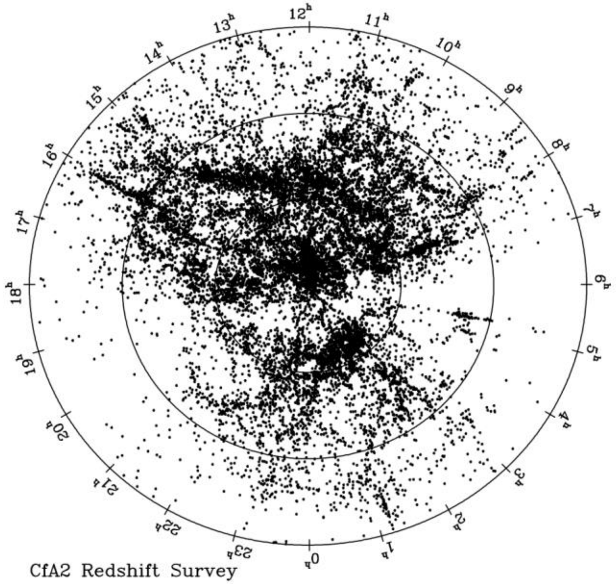
“Pancakes of God”

Redshift-space distortions II: fingers of God

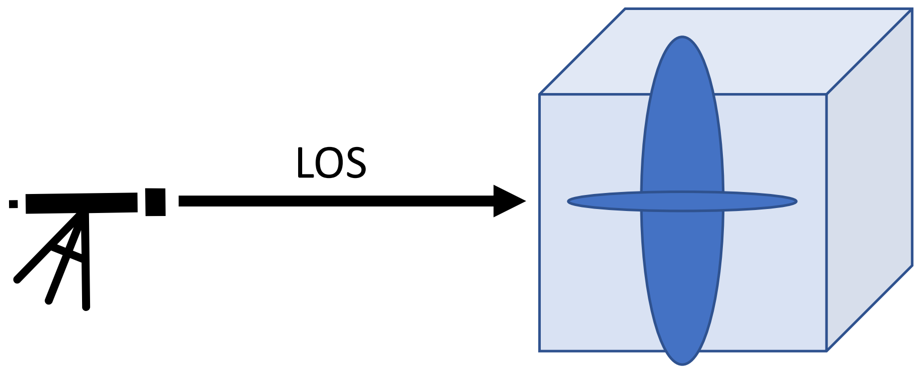


Stochastic motions of galaxies lead to a suppression of clustering along line-of-sight direction.

→ Small-scale suppression of power.



Homogeneity and ~~Isotropy~~ with Redshift-space Distortions (RSD)



$$\delta(\vec{r}) = \frac{n(\vec{r}) - \bar{n}}{\bar{n}}$$

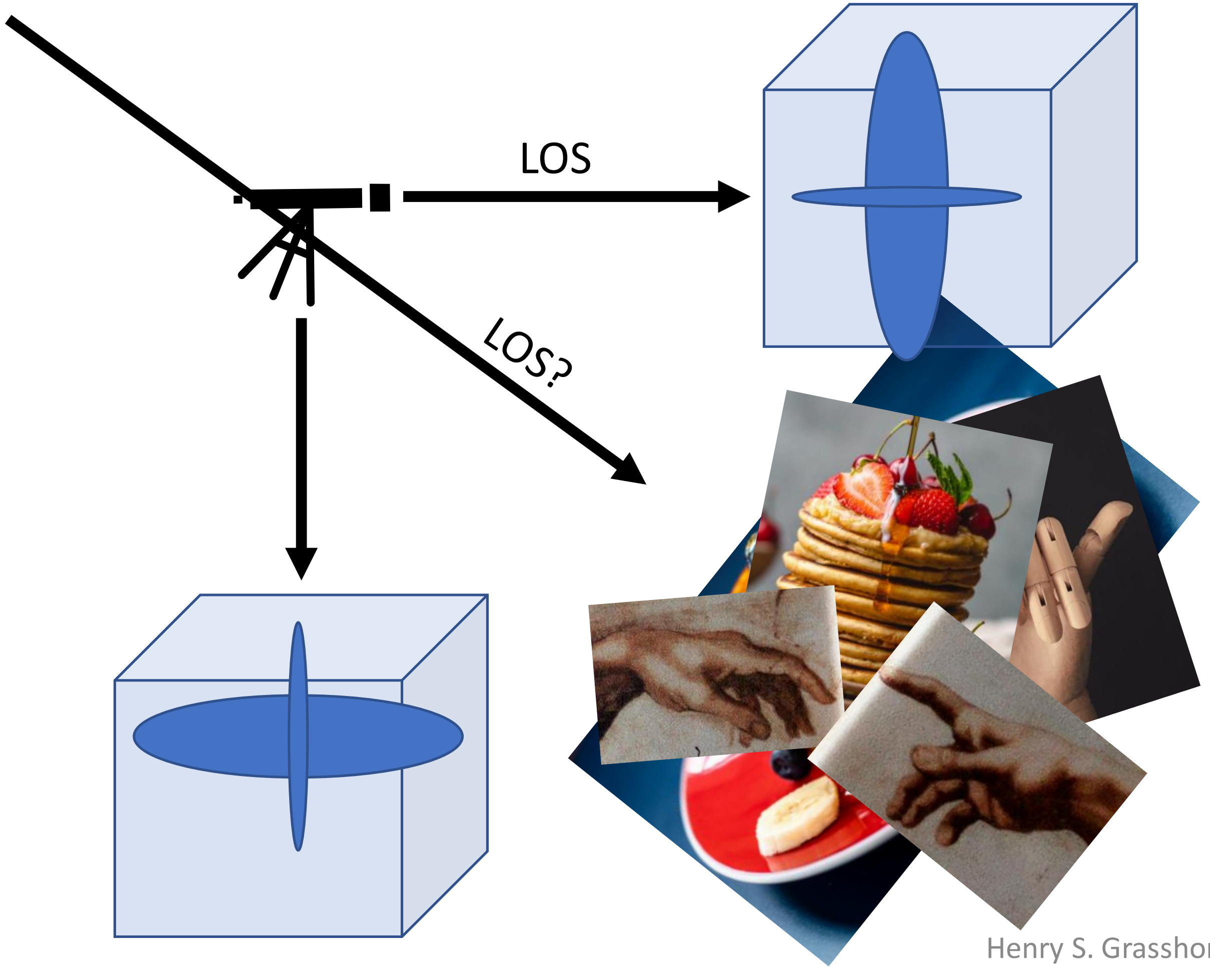
$$\delta(\vec{k}) = \int d^3r\, e^{-i\vec{k}\cdot\vec{r}}\, \delta(\vec{r})$$

$$\langle \delta(\vec{k})\delta^*(\vec{k}')\rangle = (2\pi)^3 \delta^D(\vec{k}' - \vec{k}) P(k)$$

Isotropy

Nope: $P(k_\perp, k_\parallel) = (b + f\mu^2)^2\, P(k)$

~~Homogeneity~~ and ~~Isotropy~~ with Redshift-space Distortions (RSD)



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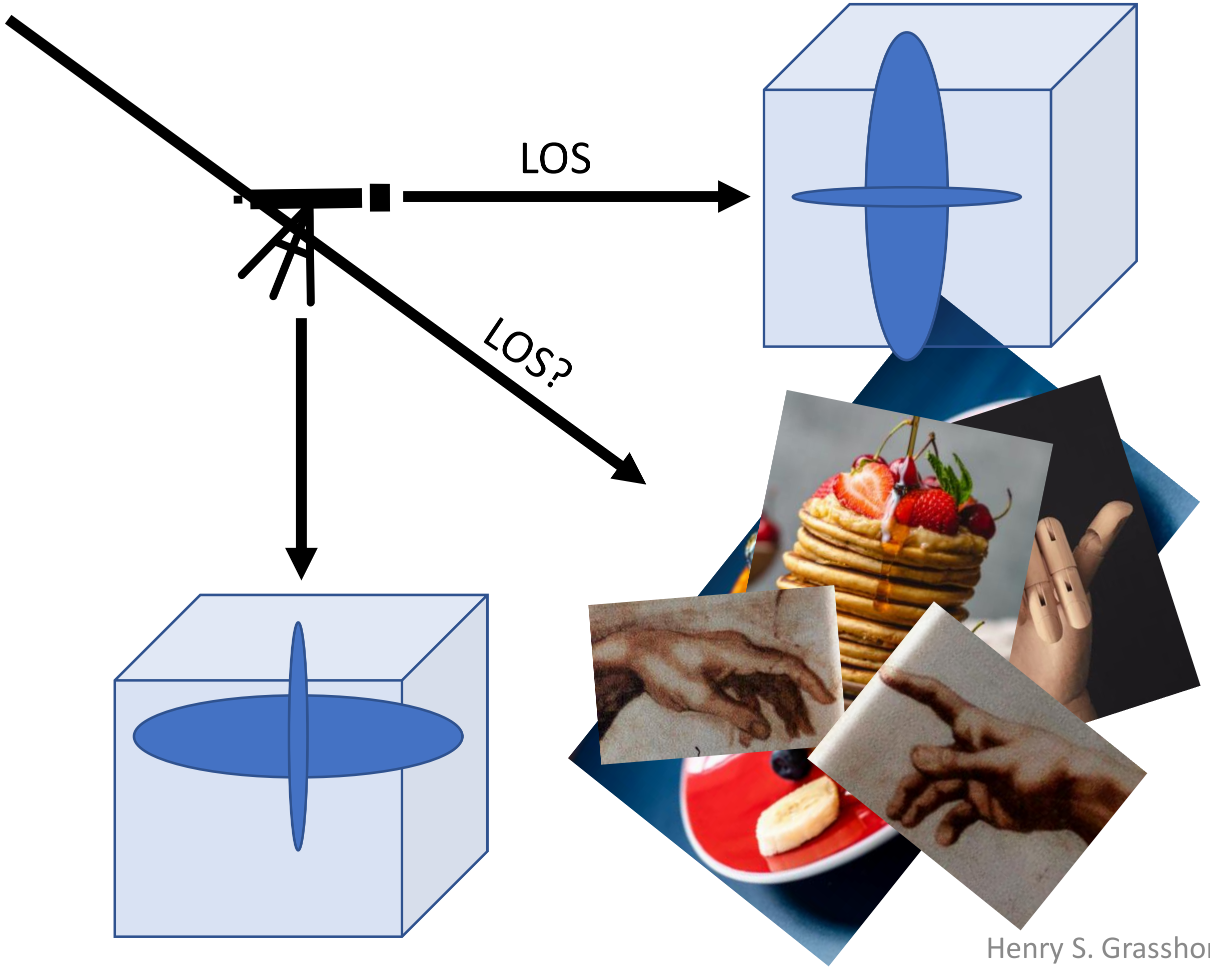
Homogeneity — Isotropy

Nope: $P(k_\perp, k_\parallel) = (b + f\mu^2)^2\, P(k)$

Nope: Wide angles

~~Homogeneity~~ and ~~Isotropy~~ with Redshift-space Distortions (RSD)



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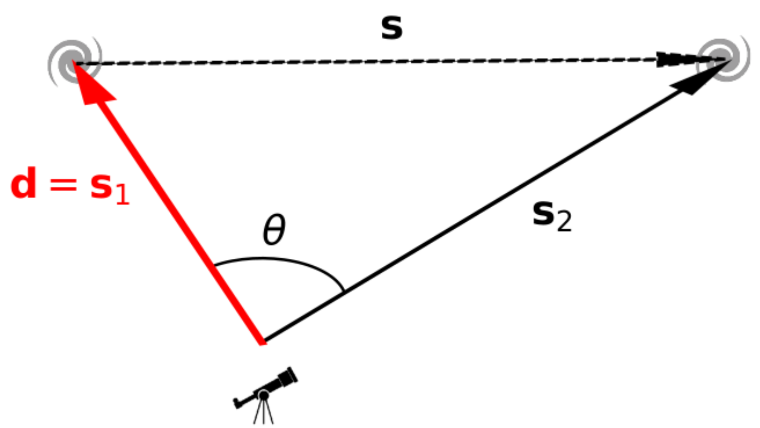
Homogeneity Isotropy

Nope: $P(k_\perp, k_\parallel) = (b + f\mu^2)^2\, P(k)$

Nope: Wide angles

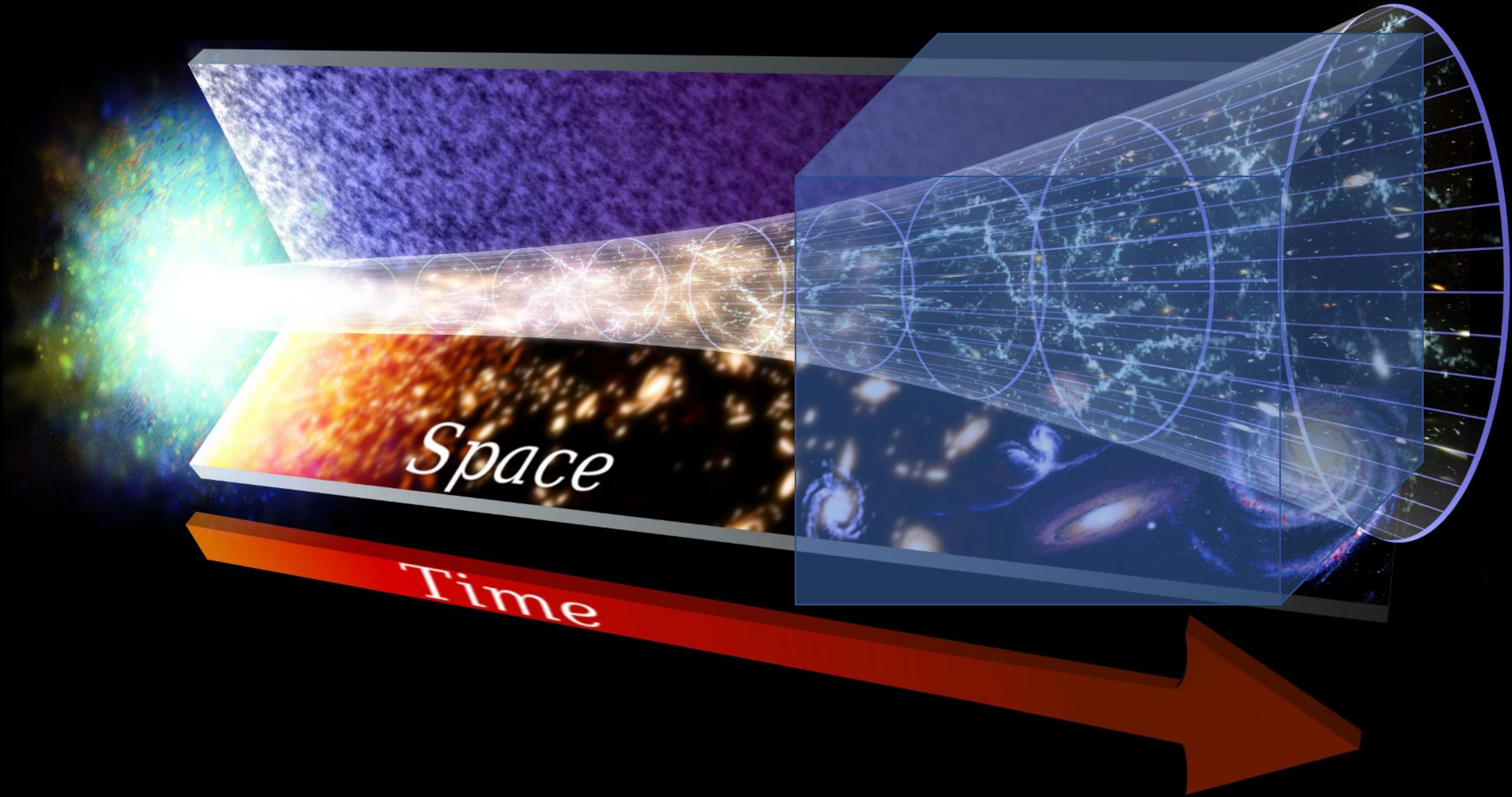
Yamamoto to the rescue: Great for small angles!



- Choose one LOS per galaxy pair.
- Perturbative expansion for modeling.
- Suboptimal for the largest angles, because RSD can cancel.

(Figure from Beutler, Castorina, Zhang 2019)

Redshift-evolution: Our past lightcone is not homogeneous.



What to do?

Back to Basics: Choose a coordinate system that is better suited for the fixed position of the observer than a Cartesian.

Laplacian eigenfunctions: $\nabla^2 f(\vec{r}) = -k^2 f(\vec{r})$

Cartesian Coordinates:

$$f(\vec{r}) = e^{-i\vec{k}\cdot\vec{r}}$$

Fourier transform:

$$\delta(\vec{k}) = \int d^3r\, e^{-i\vec{k}\cdot\vec{r}}\, \delta(\vec{r})$$

Spherical Coordinates:

$$f(\vec{r}) = j_\ell(kr)\, Y_{\ell m}(\hat{r})$$

Spherical Fourier-Bessel transform:

$$\delta_{\ell m}(k) = \int d^3r\, j_\ell(kr)\, Y^*_{\ell m}(\hat{r})\, \delta(\vec{r})$$

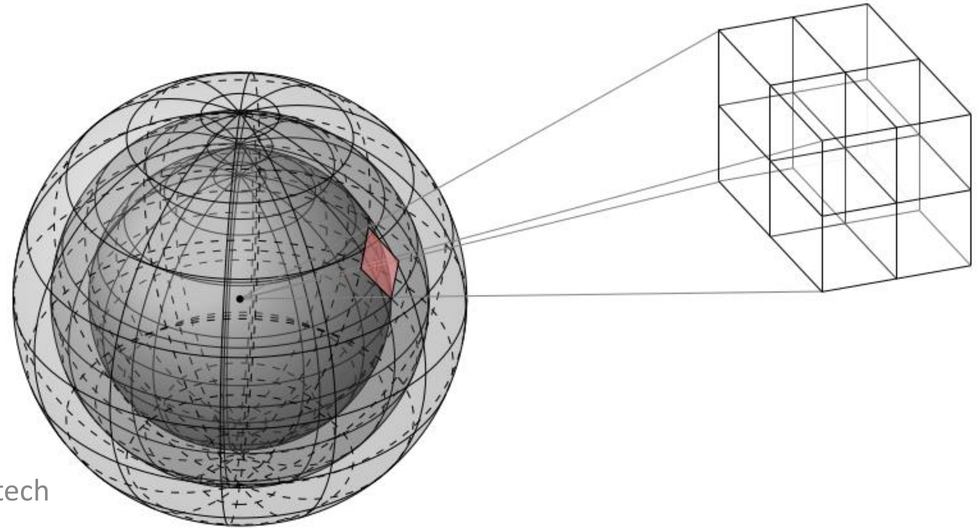
SFB power spectrum

$$\langle \delta_{\ell m}(k)\, \delta_{\ell' m'}(k') \rangle = \underbrace{\delta^K_{\ell\ell'} \delta^K_{mm'}}_{\text{Isotropy}}\, C_\ell(k, k')$$

$$C_\ell(k, k') = \underbrace{\delta^D(k' - k)}_{\text{Homogeneity}}\, P(k)$$

The SFB power spectrum is ideally suited to deep and wide galaxy surveys

- Natural separation between angular and radial coordinates
  - Individual line of sights for each galaxy
    - Maximal information from RSD
  - All wide-angle effects
  - Redshift-evolution (e.g., growth of the non-linear power spectrum)
- Nearly diagonal covariance matrix



Limber's Approximation

$$C_\ell(k,k') = P(k)\, e^{-\sigma_u^2 k^2} \delta^D(k-k') \times \phi^2\left(\frac{\ell+\frac{1}{2}}{k}\right) D^2\left(\frac{\ell+\frac{1}{2}}{k}\right) b^2\left(\frac{\ell+\frac{1}{2}}{k}, k\right) \times \left[1-\beta\left(f^\ell_{-2}+f^\ell_0+f^\ell_2\right)\right]^2 .$$

Power spectrum

Fingers of God

Redshift evolution

SFB power spectrum

Scale-dependent bias

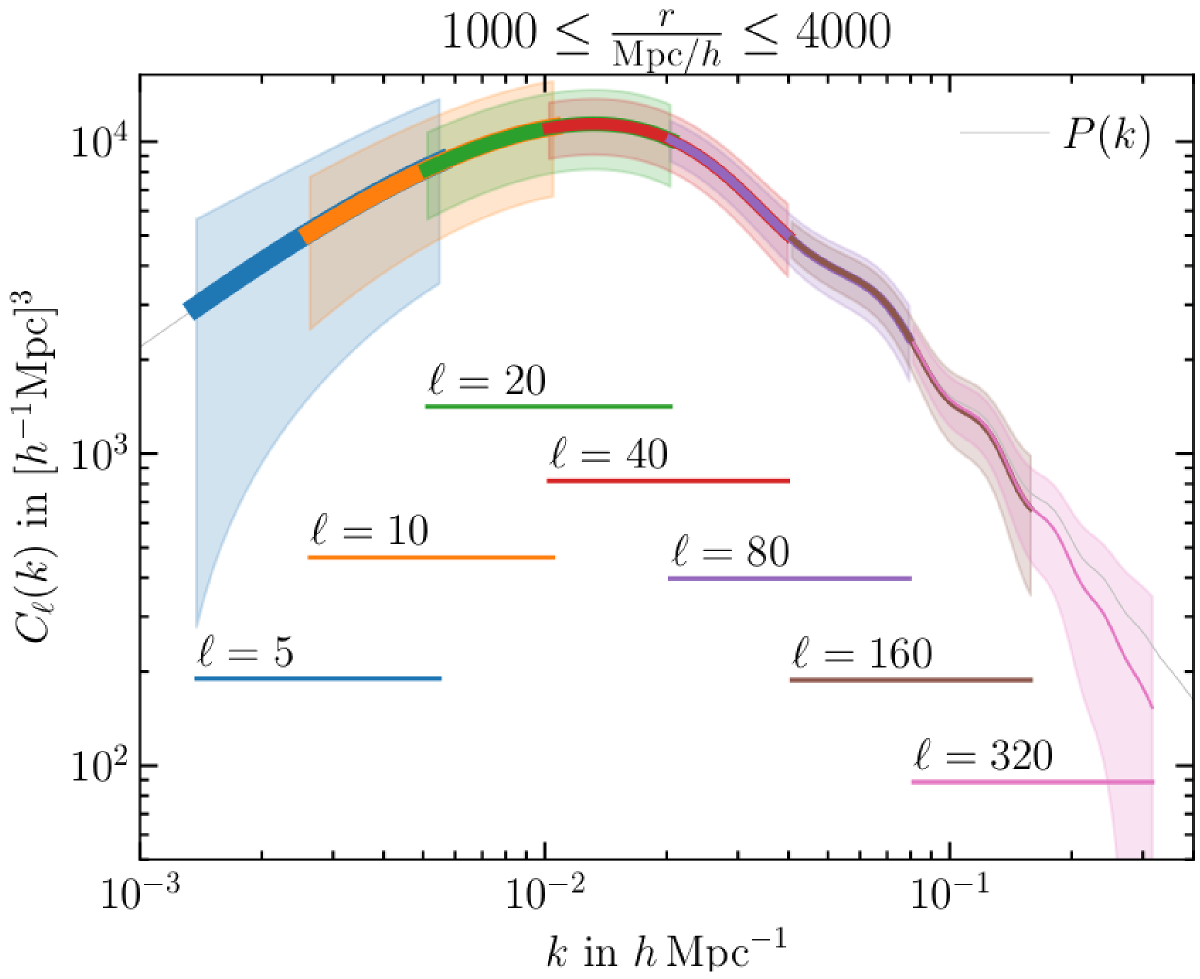
Linear Kaiser effect

The SFB power spectrum $C_\ell(k,k')$

- $b(z) \propto \frac{1}{D(z)}$

- Limber distance:

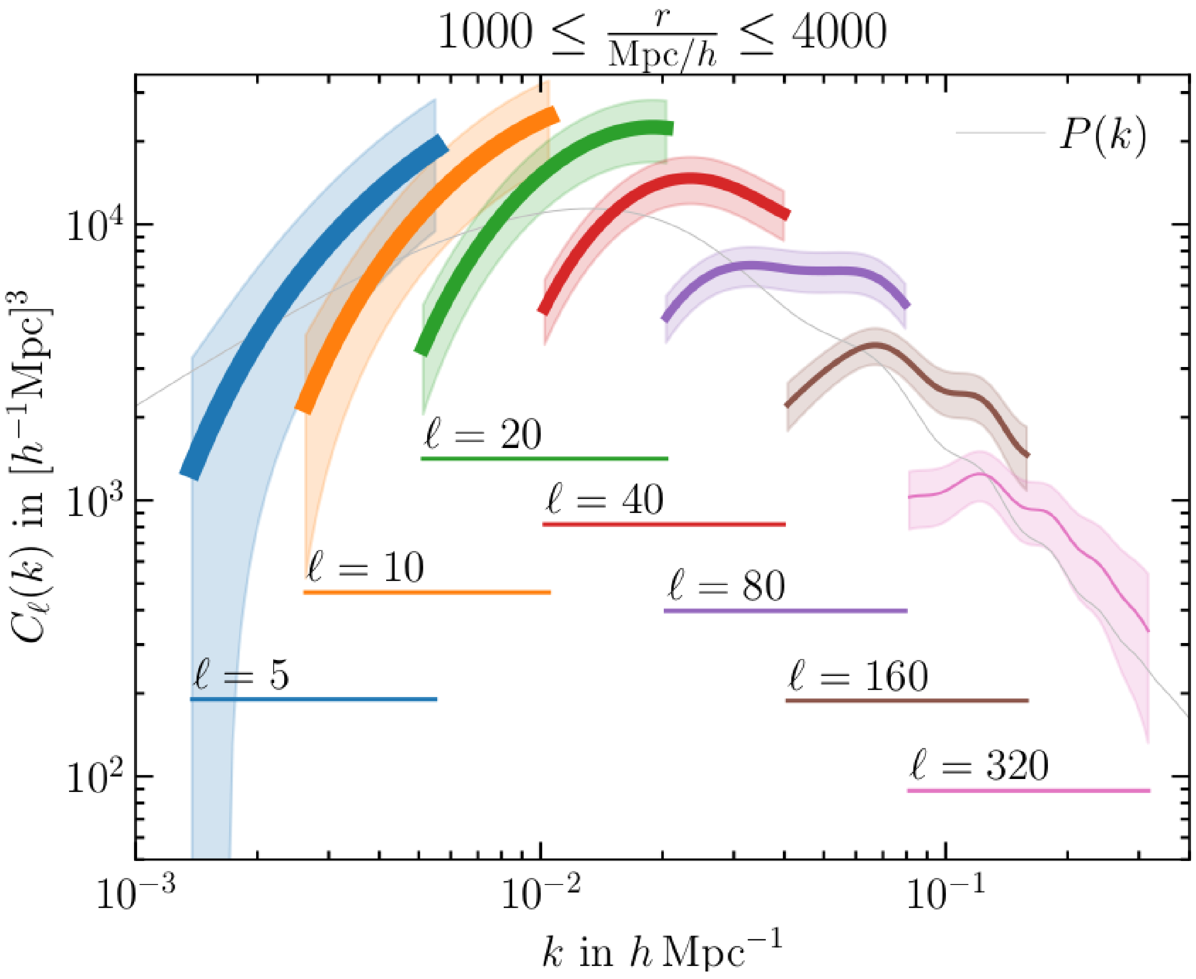
$$r = \frac{\ell + \frac{1}{2}}{k}$$

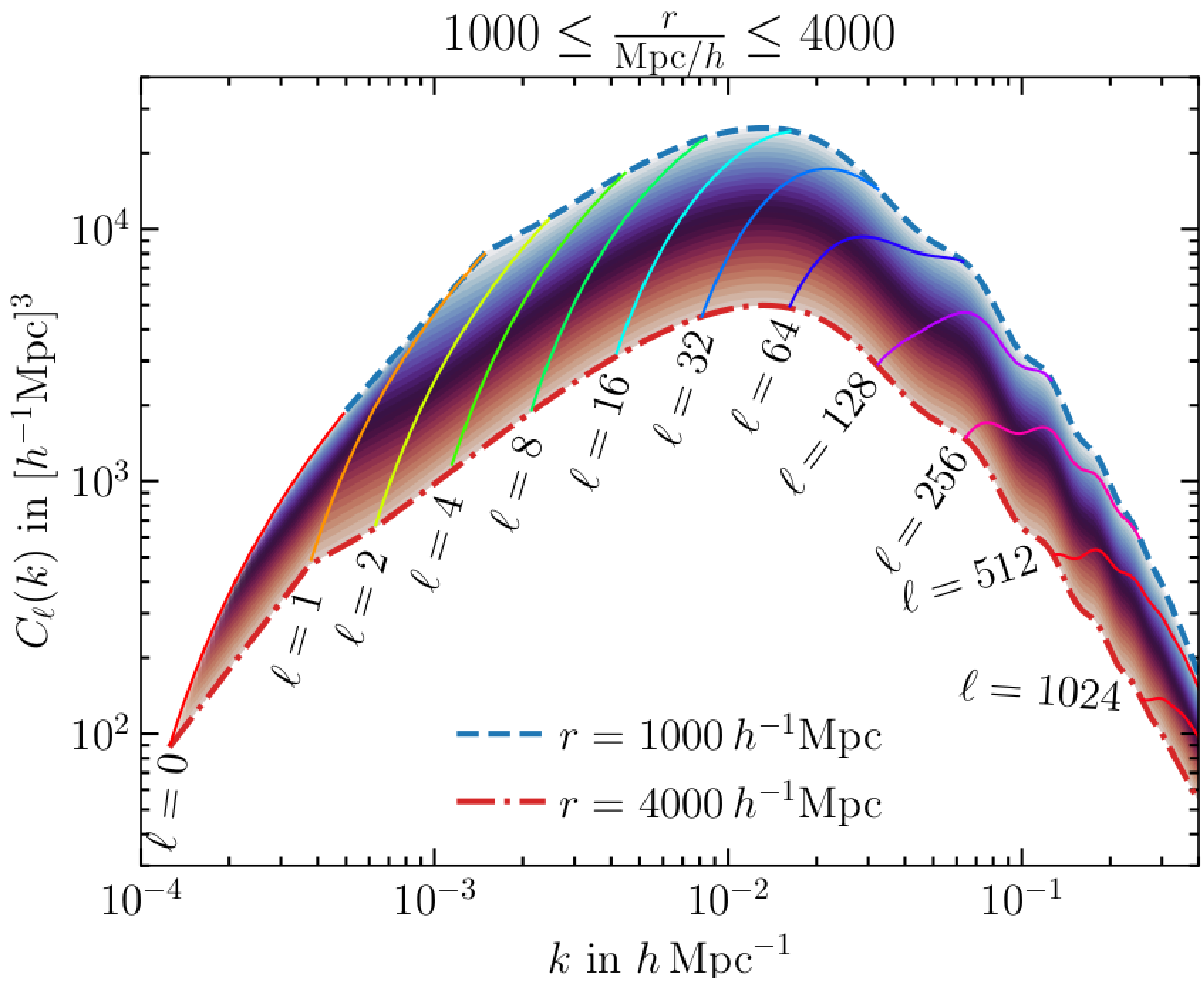


The SFB power spectrum $C_\ell(k, k')$

- $b(z) = const$
- Limber distance:

$$r = \frac{\ell + \frac{1}{2}}{k}$$





Henry S. Grasshorn Gebhardt, JPL/Caltech

Why a new code?

- We follow Samushia (2019) to introduce boundary conditions at both r_{min} and r_{max}

$$\nabla^2 f = -k^2 f$$

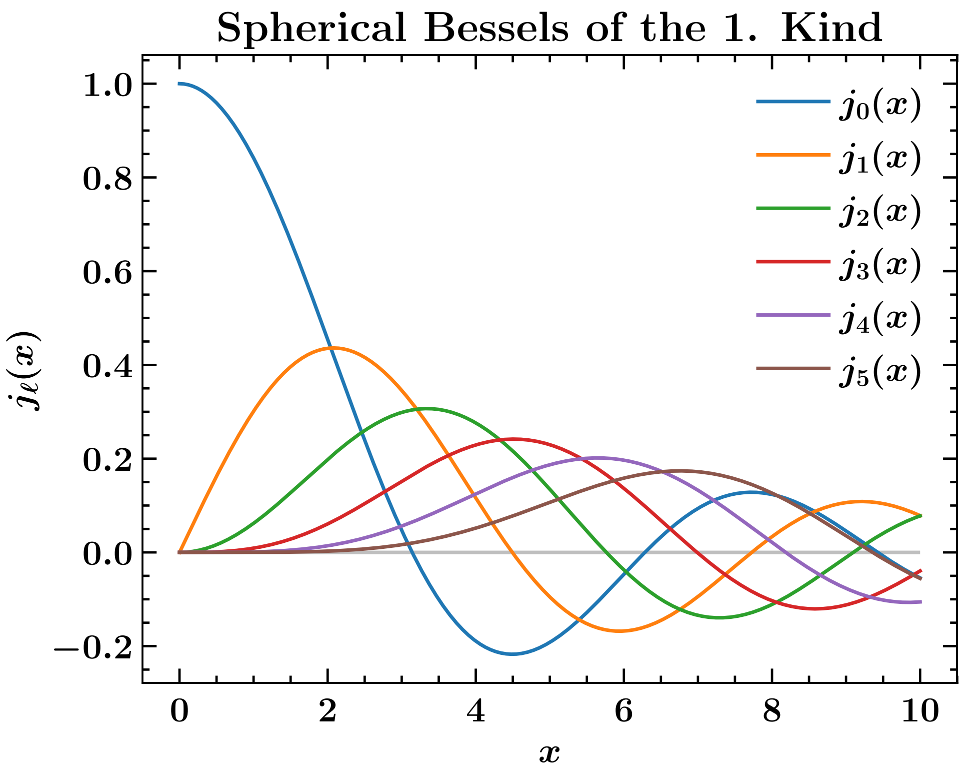
Spherical Bessels of 1. and 2. kind

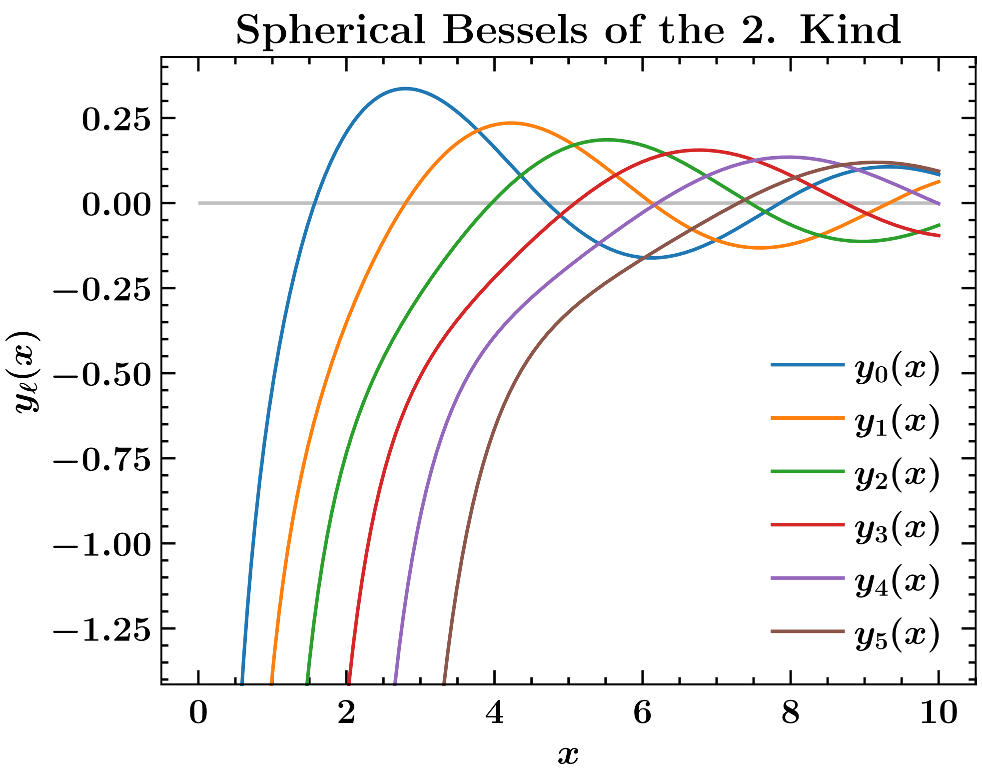
$$f_{\ell\mu}(k;r,\theta,\phi) = \left[c_j\, j_\ell(kr) + c_y\, y_\ell(kr)\right] \times \left[c_p\, P_\ell^\mu(\cos\theta) + c_q\, Q_\ell^\mu(\cos\theta)\right] \times \left[c_+\, e^{i\mu\phi} + c_-\, e^{-i\mu\phi}\right],$$

Spherical harmonics $Y_{\ell m}$

- Numerically more stable
- We use potential boundary conditions

Spherical Bessel and Neumann functions





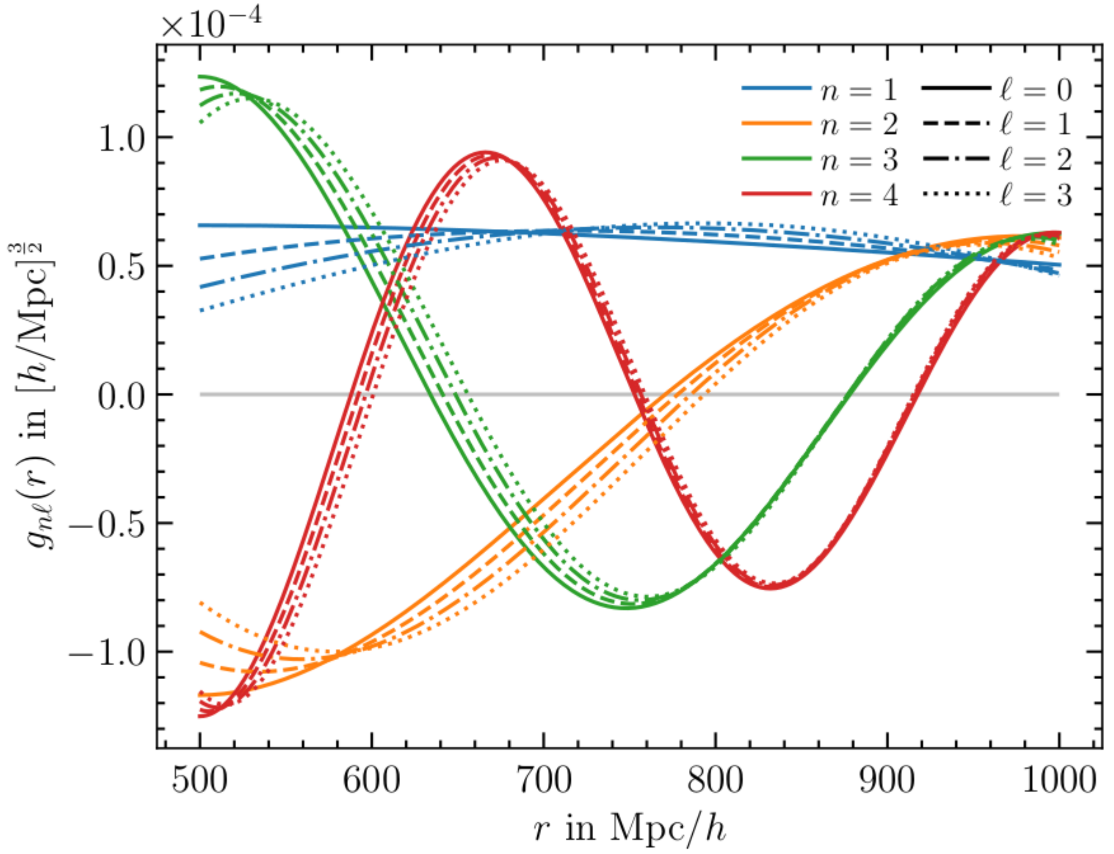
$$
\begin{aligned}
f_{\ell\mu}(k; r, \theta, \phi) = & \left[c_j\, j_\ell(kr) + c_y\, y_\ell(kr)\right] \\
& \times \left[c_p\, P_\ell^\mu(\cos\theta) + c_q\, Q_\ell^\mu(\cos\theta)\right] \\
& \times \left[c_+\, e^{i\mu\phi} + c_-\, e^{-i\mu\phi}\right],
\end{aligned}
$$

Radial basis functions

- Boundary conditions at both r_{min} and r_{max}
- Spherical Bessels of the second kind

$$g_{n\ell}(r) = c_{n\ell}\, j_\ell(k_{n\ell}\, r) + d_{n\ell}\, y_\ell(k_{n\ell}\, r)$$

➢Orthogonal basis where the survey is sensitive (Samushia 2019)

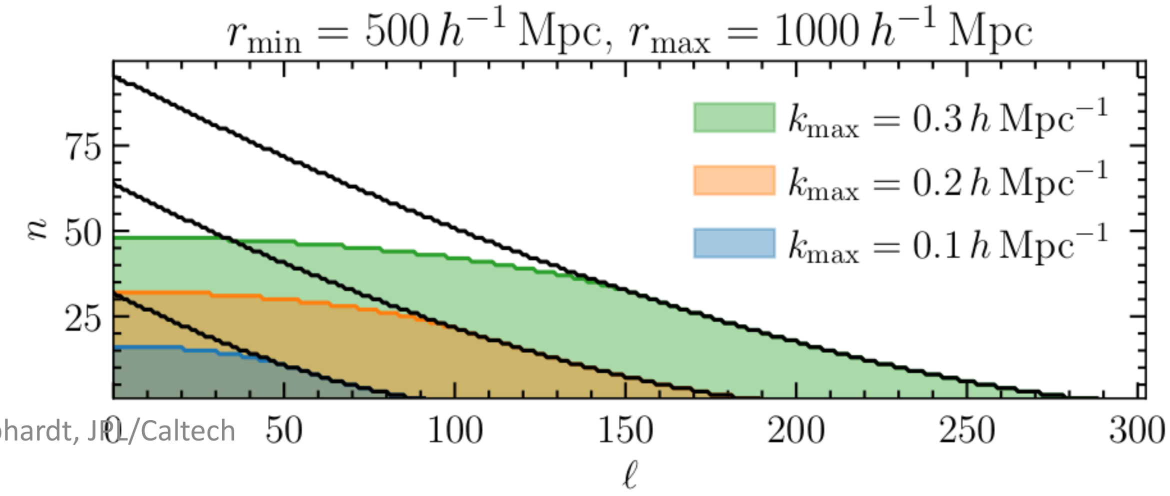


Basic Algorithm

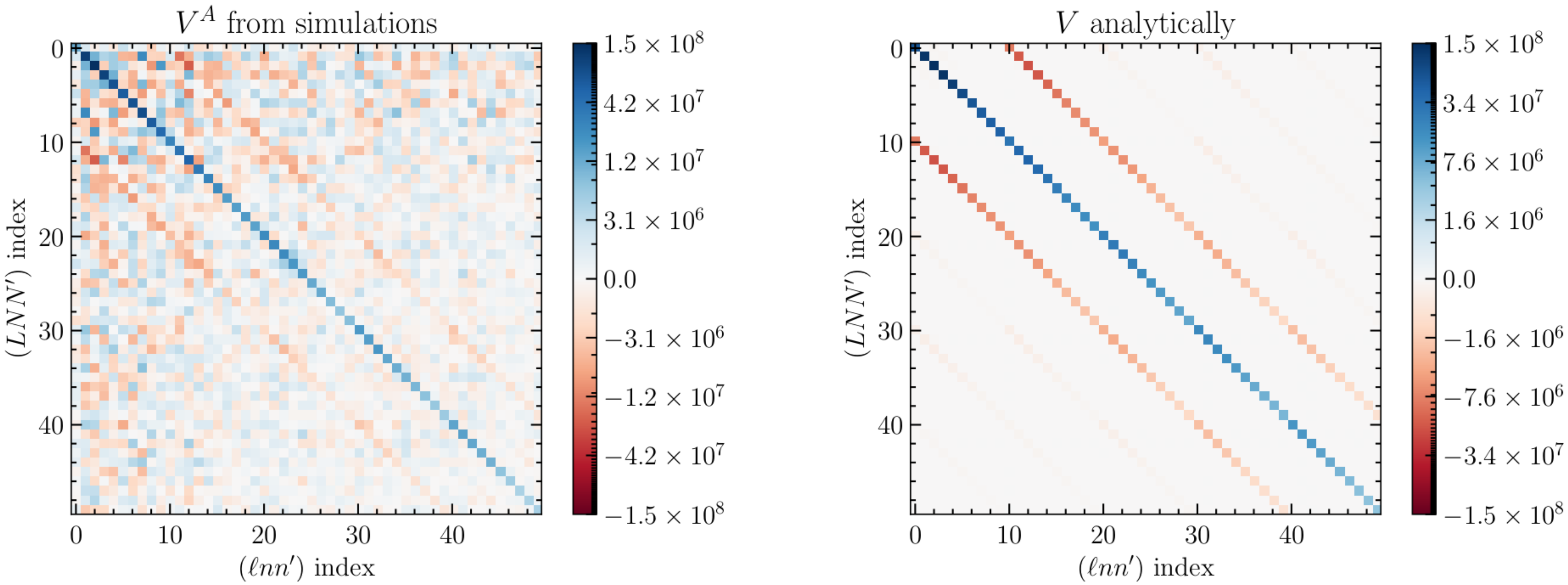
1. Radial Bessel transform with individual galaxies (Leistedt *et al* 2012)
2. Spherical harmonic transform with HEALPix (Gorski *et al* 2005)
3. Pseudo-SFB power spectrum bandpowers with window corrections (Hivon *et al* 2002)

$$\hat{C}^{\mathrm{obs}}_{\ell n n'} = \frac{1}{2\ell+1} \sum_m \delta^{\mathrm{obs}}_{n\ell m} \delta^{\mathrm{obs},*}_{n'\ell m}$$

4. Subtract shot noise

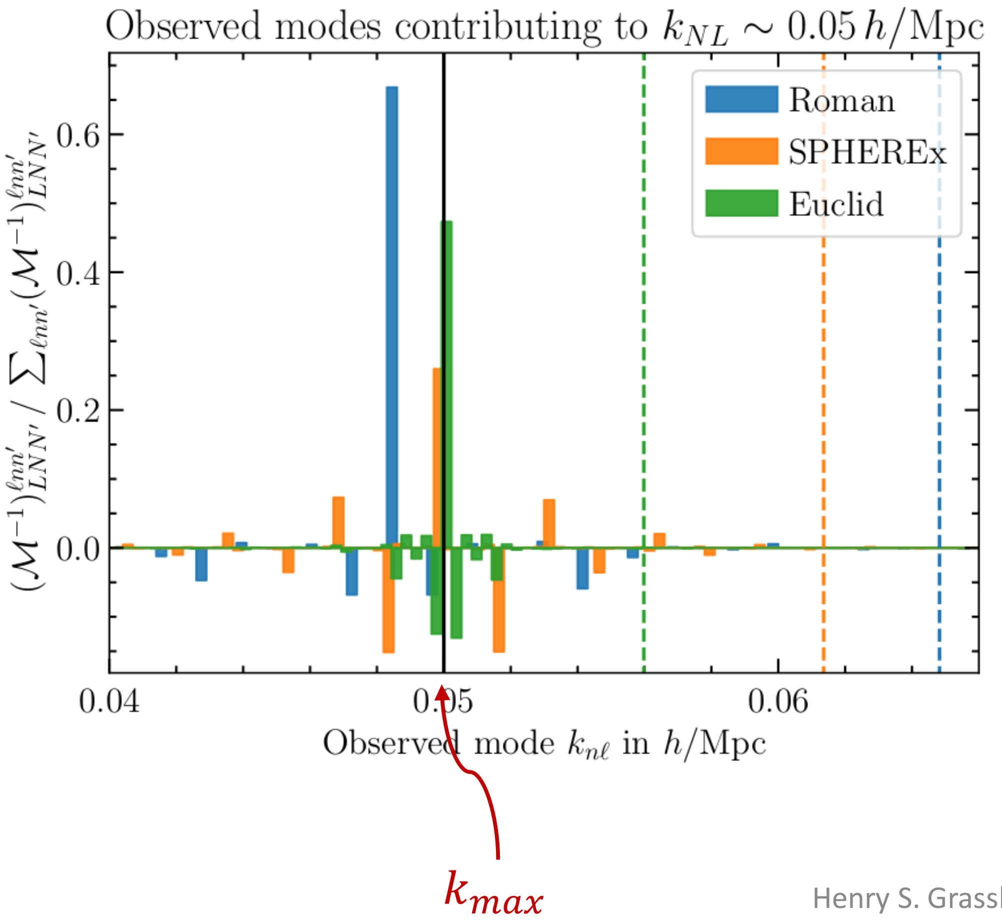


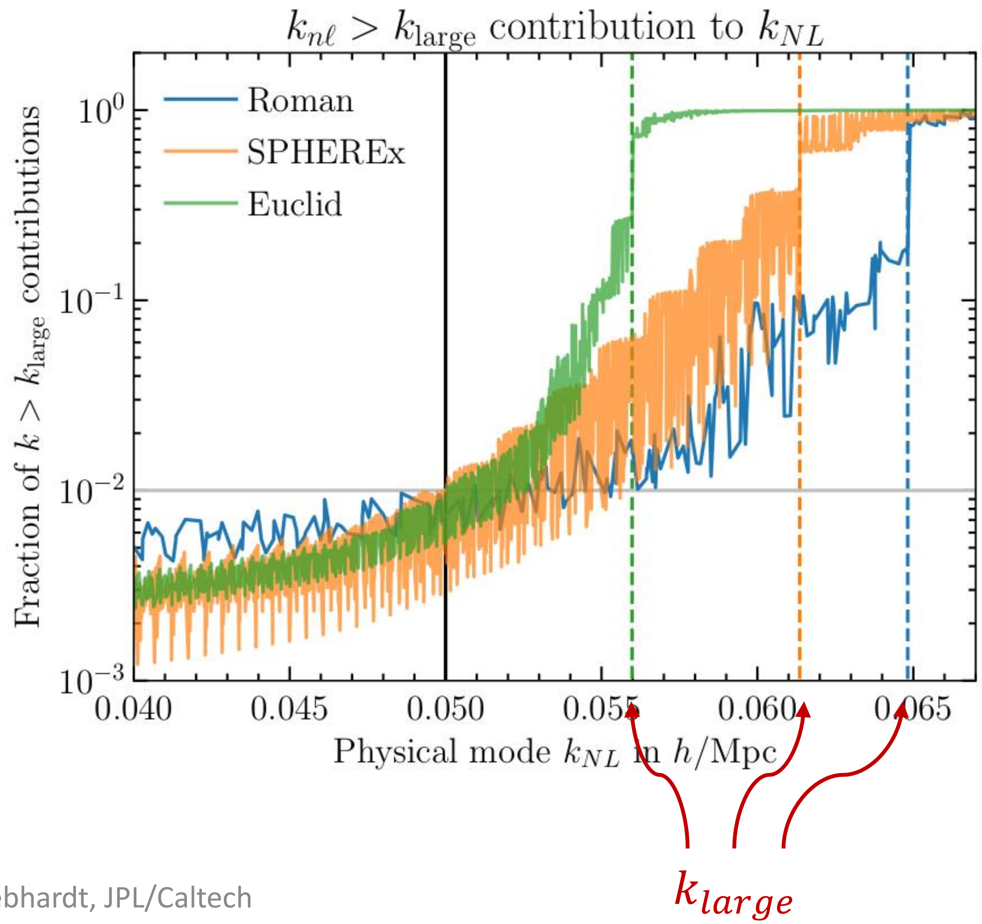
Covariance matrix V



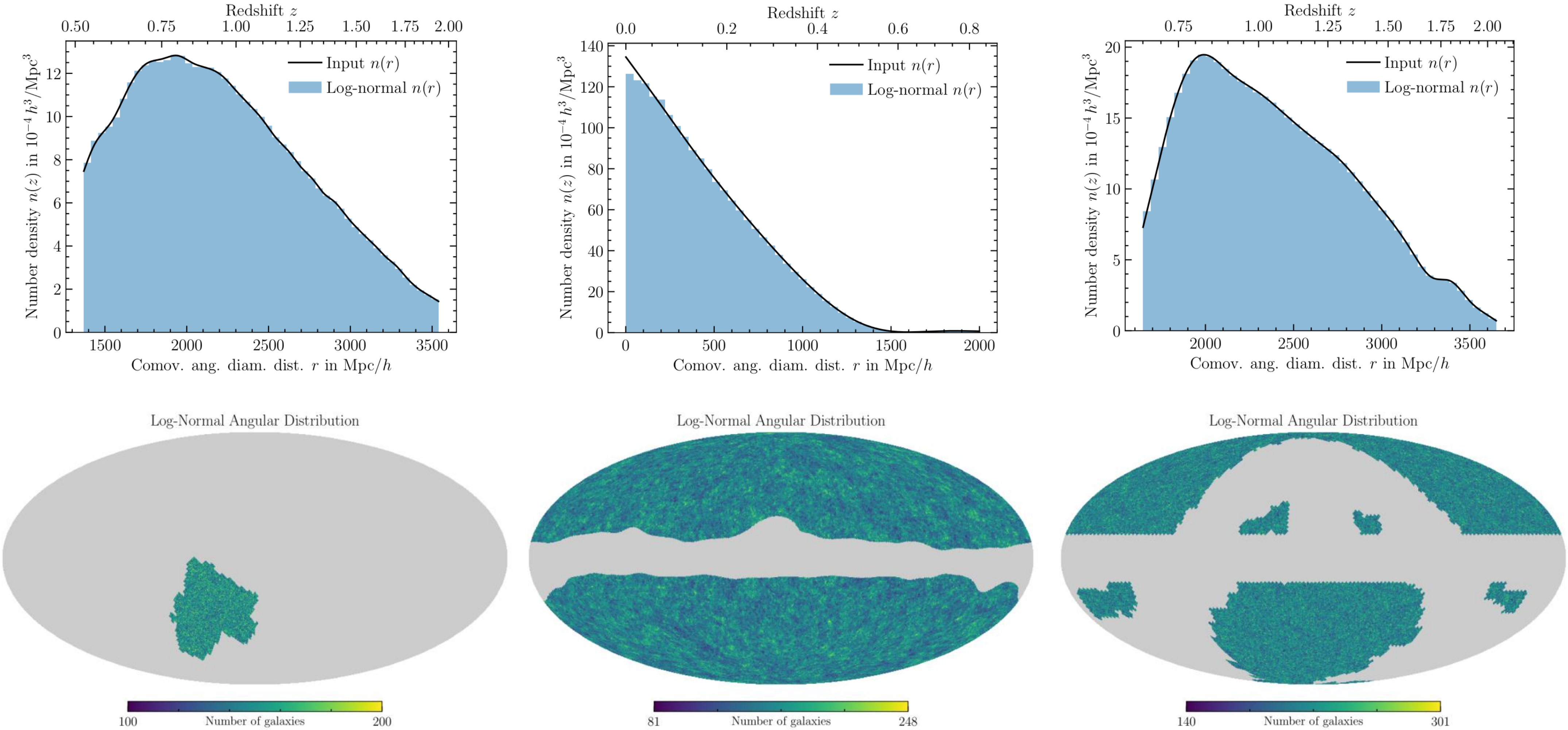
Full window deconvolution requires modes above k_{max}

$$\hat{C}^{\text{obs}}_{\ell n n'} = \sum_{LNN'} \mathcal{M}^{LNN'}_{\ell n n'} \hat{C}^{A}_{LNN'}$$



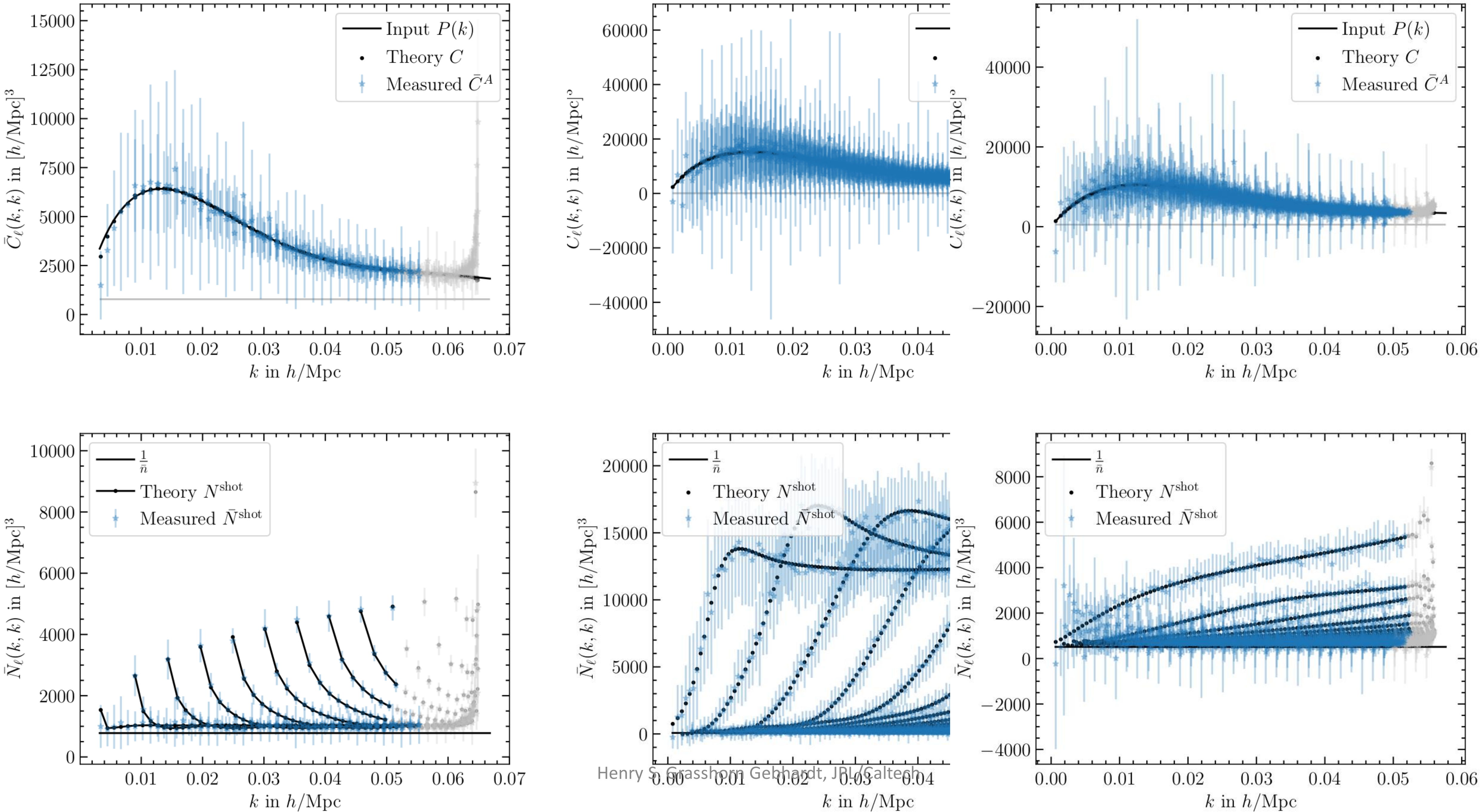


Examples: *Roman*-, *SPHEREx*-, *Euclid*-like



(Special thanks to Katarina Markovič)

Roman-like SPHEREx-like Euclid-like



Summary & Conclusion

- SFB is theoretically optimal for **deep** and **wide** surveys
- Boundary conditions at r_{min} and r_{max} make SFB analysis more **numerically stable**
- SuperFaB is feasible for large surveys
- Future:
  - Need fast $P(k) \to C_\ell(k, k')$ code
  - Apply to data!