Parameter estimation in cosmology

The *Planck* likelihood

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01/14/16

Outlook

The *Planck* mission

Data products

Cosmological parameter estimation from CMB data

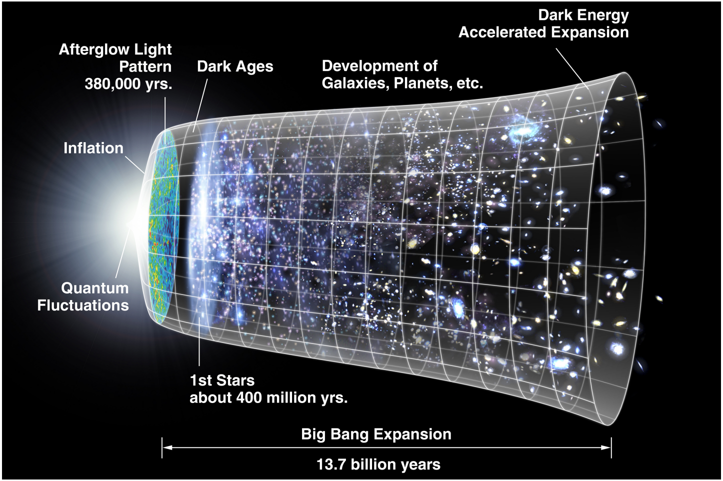
- Methodology
- The *Planck* likelihood
- Results

Future S4 experiments

1) Introduction

The CMB as a cosmological probe

There is a tight connection between the most fundamental properties of the Universe and the CMB.

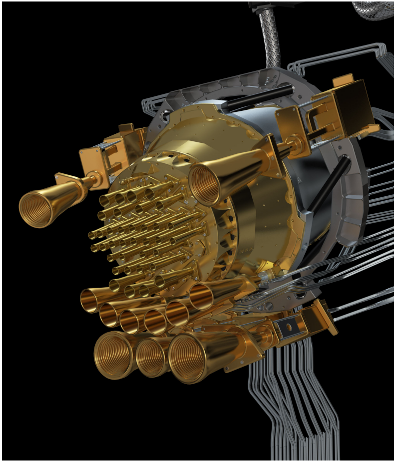


The *Planck* mission

©ESA

- Satellite mission at L2
- 1.5 meter mirror
- Two instruments
- Cryogenically cooled
- Launched in '09
- 2.5 / 4 year mission
- About 1 bn. Euros

The focal plane



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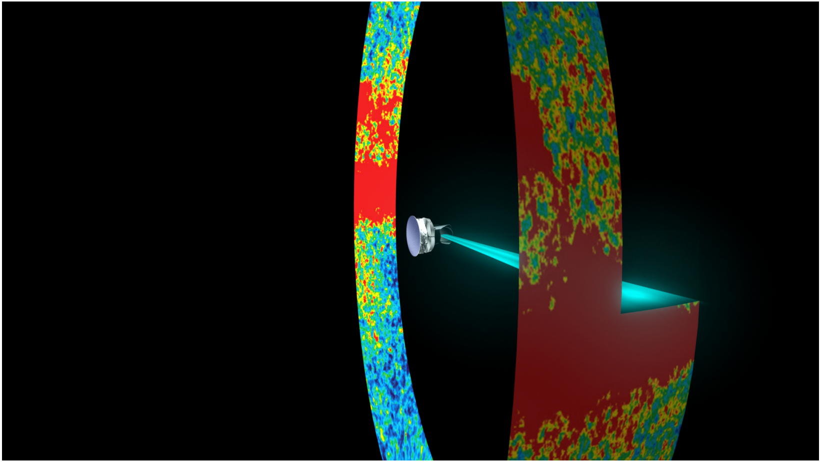
Low frequency instrument (LFI):

- Radiometers at 20 K
- Three channels at **30**, **44**, and **70** GHz
- Resolution from 33' to 14'

High frequency instrument (HFI):

- Bolometers at 0.1 K
- Six channels at **100**, **143**, **217**, **353**, 545, and 857 GHz
- Resolution from 10' to 5'

Planck scanning strategy



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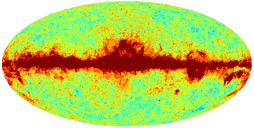
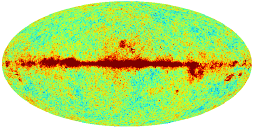
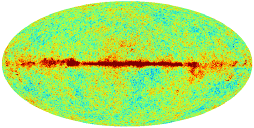
The 2015 maps – temperature



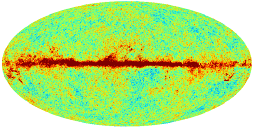
30 GHz



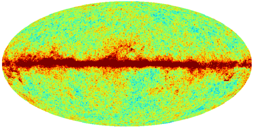
44 GHz



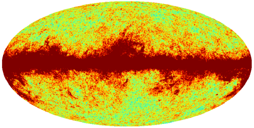
70 GHz



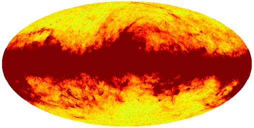
100 GHz



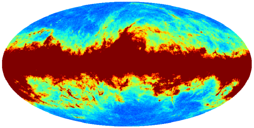
143 GHz



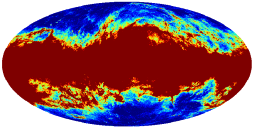
217 GHz



353 GHz

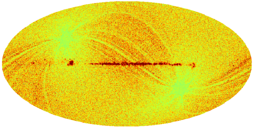


545 GHz

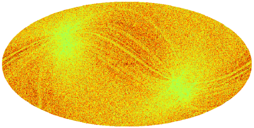


857 GHz

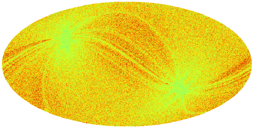
The 2015 maps – polarization intensity



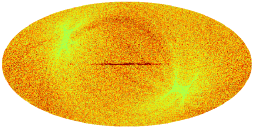
30 GHz



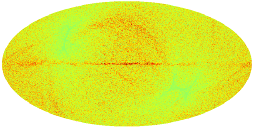
44 GHz



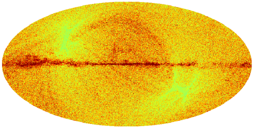
70 GHz



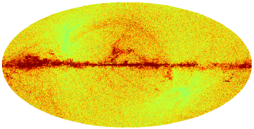
100 GHz



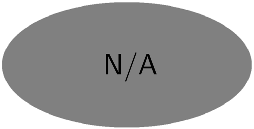
143 GHz



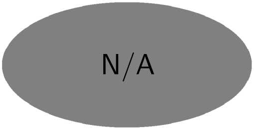
217 GHz



353 GHz



545 GHz



857 GHz

2) Parameter inference from CMB experiments

Parameter estimation from CMB data

We want to infer a parameter vector θ in agreement with the statistical properties of our data d.

We make use of Bayes' theorem:

$$P(\theta|d) \propto \mathcal{L}\left(d \,\middle|\, C_\ell^{\mathrm{theory}}(\theta)\right) P(\theta)$$

to solve the equation in terms of a forward modeling problem for C_ℓ^{theory} using a Monte Carlo algorithm.

Construction of the likelihood function

We want to calculate the likelihood as a function of C_ℓ given the data for a Gaussian random field:

$$\mathcal{L}\left(d \middle| C_\ell^{\mathrm{theory}}(\theta)\right) = \frac{1}{\sqrt{(2\pi)^N |\mathbf{S}(C_\ell) + \mathbf{N}|}}\, \mathrm{e}^{-1/2\, d^\dagger (\mathbf{S}(C_\ell)+\mathbf{N})^{-1} d}$$

In general, $(\mathbf{S} + \mathbf{N})$ is a dense $N \times N$ matrix, where $N \sim \mathcal{O}(10^7)$.

$\Rightarrow$ A direct evaluation is currently computationally prohibitive.

The *Planck* hybrid likelihood approach

At low multipoles, $\ell < 30$:

Exact pixel space likelihood,

$$\mathcal{L}\left(d \middle| C_\ell^{\text{theory}}(\theta)\right) \propto \exp\left(-1/2\, d^\dagger \mathbf{C}^{-1} d\right) .$$

Numerically expensive, evaluations take $\mathcal{O}(\ell_{\max}^6)$ operations.

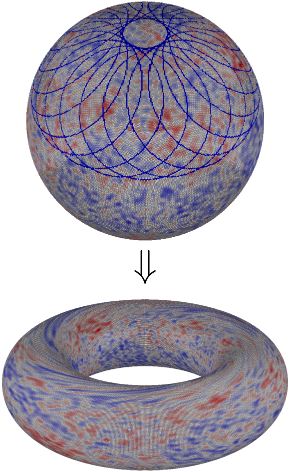
At high multipoles, $\ell \geq 30$:

We approximate the exact expression with a Gaussian C_ℓ likelihood

$\Rightarrow$ We work with a pre-compressed data vector: the empirical power spectrum coefficients

Check: Gaussian C_ℓ likelihood I

We can verify this approximation using simulations with a idealized properties:



For a regular scanning strategy on iso-latitude circles, we can map the data onto a torus ($S^1 \times S^1 \cong T^2$).

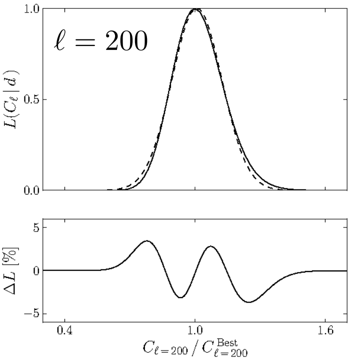
On the torus, $\mathbf{S} + \mathbf{N}$ becomes block diagonal in Fourier space, i.e. an exact likelihood analysis becomes possible.

This test allows us to verify our approximation.

Elsner & Wandelt (2012)

Check: Gaussian C_ℓ likelihood II

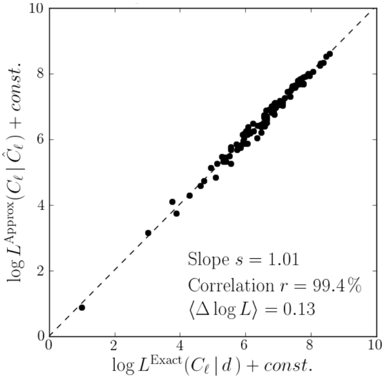
Indeed, the C_ℓ posterior distributions are non-Gaussian up to the highest multipole moments:



Elsner & Wandelt (2012)

Check: Gaussian C_ℓ likelihood III

Still, the likelihood values of a typical MCMC chain exploring a small number of cosmological parameters are in excellent agreement:



$\Rightarrow$ We can justify using a Gaussian C_ℓ likelihood at high multipoles.

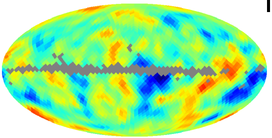
3) The likelihood for *Planck*

Low-ℓ likelihood: data

Temperature:

We use component separation with *Planck*, *WMAP*, and 408 MHz observations as input,
$f_{\mathrm{SKY}} = 94\%$.

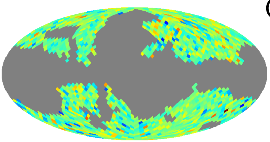
I



Polarization:

We use the *Planck* 70 GHz full mission map without surveys 2, 4,
cleaned with 30 and 353 GHz maps,
$f_{\mathrm{SKY}} = 46\%$.

Q



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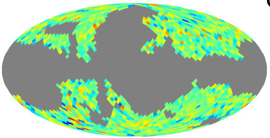
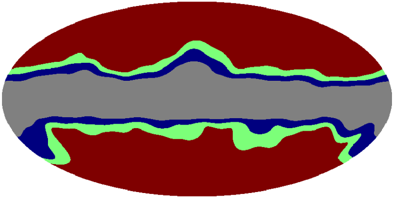
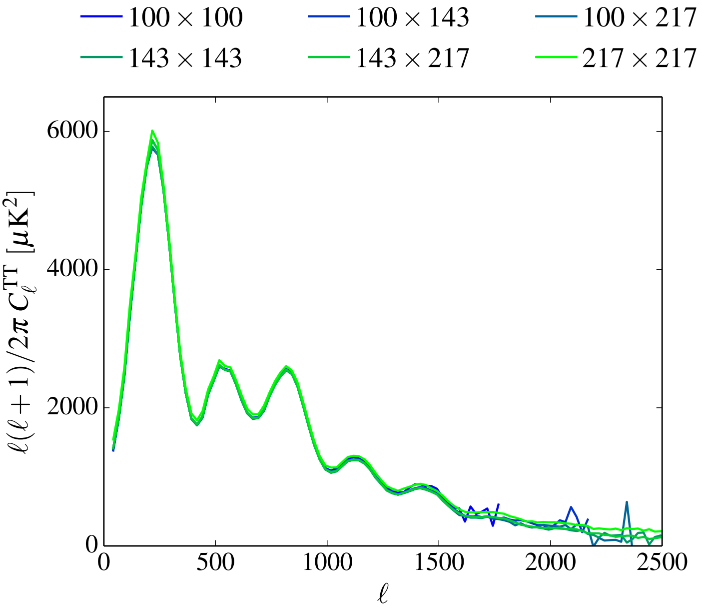


High-ℓ likelihood: masks

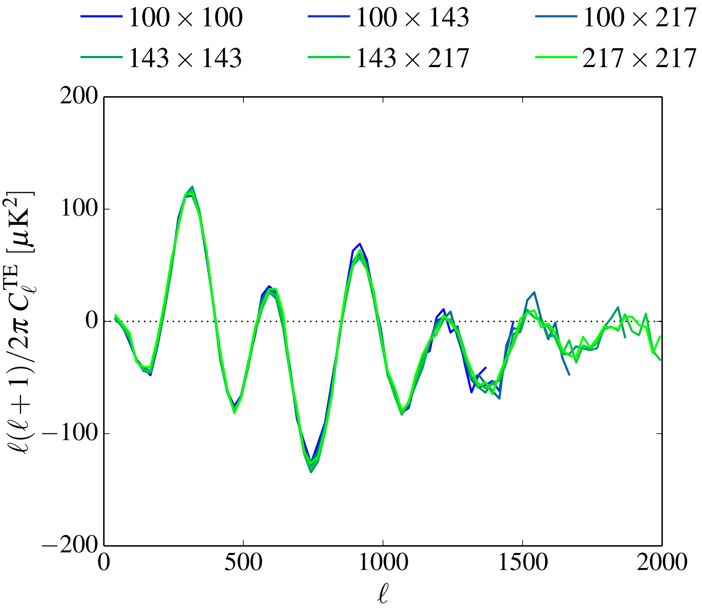


Frequency	Temperature	Polarization
100 GHz	galactic + point source + CO $f_{\mathrm{SKY}} \approx 66\%$	galactic $f_{\mathrm{SKY}} \approx 70\%$
143 GHz	galactic + point source $f_{\mathrm{SKY}} \approx 57\%$	galactic $f_{\mathrm{SKY}} \approx 50\%$
217 GHz	galactic + point source + CO $f_{\mathrm{SKY}} \approx 47\%$	galactic $f_{\mathrm{SKY}} \approx 41\%$

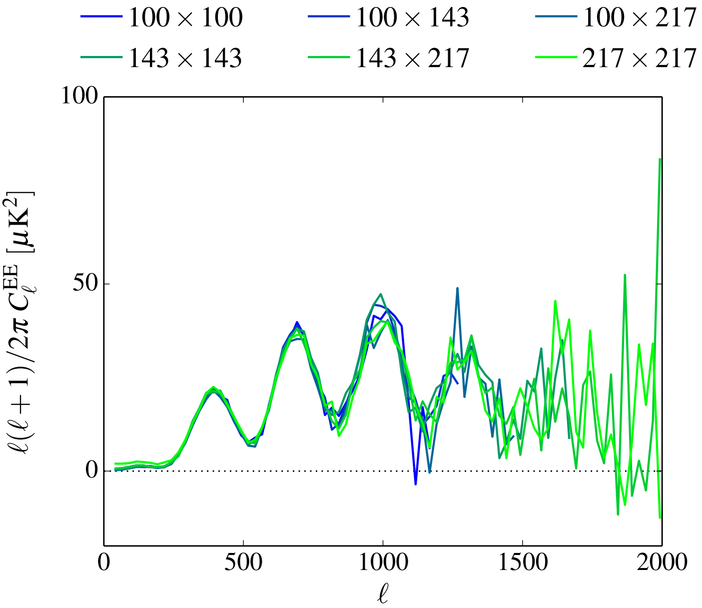
High-ℓ likelihood: TT power spectra



High-ℓ likelihood: TE power spectra



High-ℓ likelihood: EE power spectra



Data selection for the high-ℓ likelihood

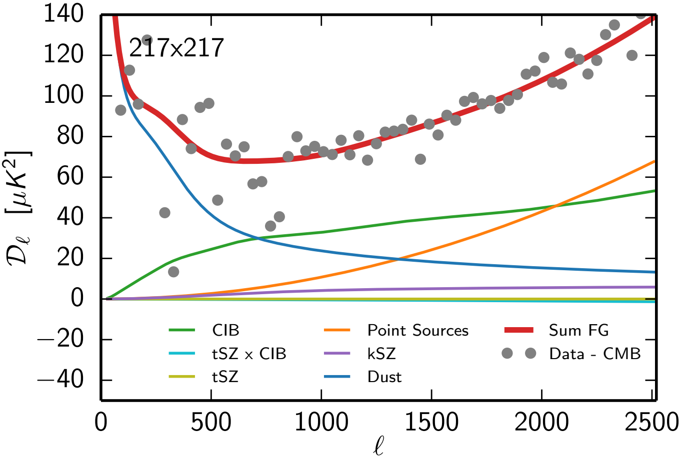
Frequency	beam [arcmin]	noise $[\mu K^2]$[1]	ℓ-range
100 GHz	9	$\frac{D^{\mathrm{TT}}_{\ell=1800}}{b^2_{\ell=1800}} \approx 20000$	T: $30 \leq \ell \leq 1200$ P: $30 \leq \ell \leq 1000$
143 GHz	7	$\frac{D^{\mathrm{TT}}_{\ell=1800}}{b^2_{\ell=1800}} \approx 700$	T: $30 \leq \ell \leq 2000$ P: $30 \leq \ell \leq 2000$
217 GHz	5	$\frac{D^{\mathrm{TT}}_{\ell=1800}}{b^2_{\ell=1800}} \approx 400$	T: $30 \leq \ell \leq 2500$ P: $500 \leq \ell \leq 2000$
100×143			T: $\emptyset$ P: $30 \leq \ell \leq 1000$
100×217			T: $\emptyset$ P: $500 \leq \ell \leq 1000$
143×217			T: $30 \leq \ell \leq 2500$ P: $500 \leq \ell \leq 2000$

[1] $D_\ell = \ell(\ell+1)/2\pi\, C_\ell$, b_ℓ: beam

The high-ℓ likelihood

We construct a fiducial Gaussian likelihood, using

- a parametric foreground model to marginalize over (12 parameters)

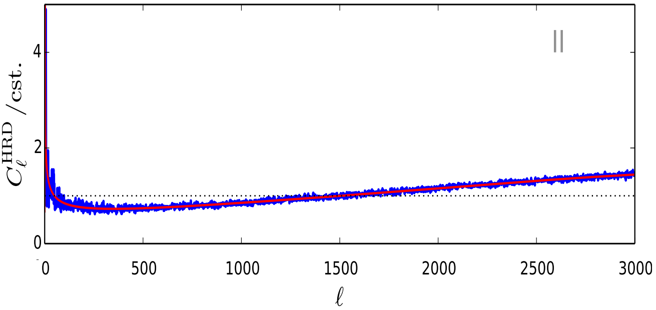


Planck XI (2015)

The high-ℓ likelihood

We construct a fiducial Gaussian likelihood, using

- a parametric foreground model to marginalize over
- noise estimates of the data, obtained from half-ring difference maps, corrected for bias using the difference between auto and cross spectra

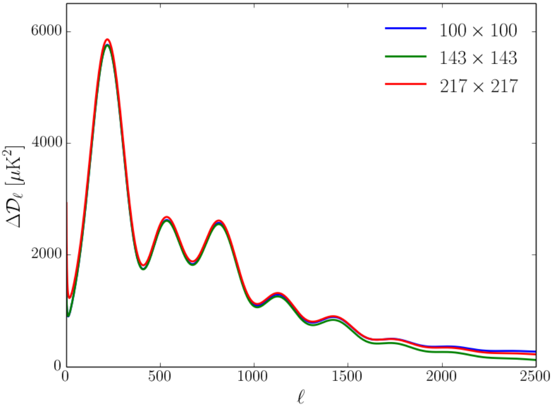


Planck XI (2015)

The high-ℓ likelihood

We construct a fiducial Gaussian likelihood, using

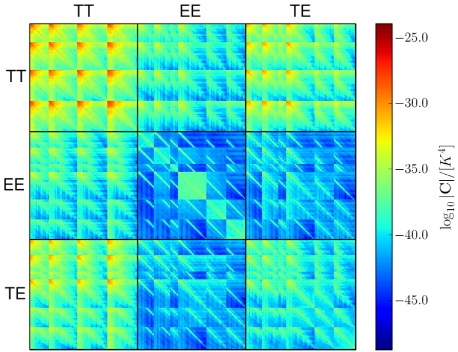
- a parametric foreground model to marginalize over
- noise estimates of the data, obtained from half-ring difference maps, corrected for bias
- a set of best fit power spectra at each frequency



The high-ℓ likelihood

We construct a fiducial Gaussian likelihood, using

- a parametric foreground model to marginalize over
- noise estimates of the data, obtained from half-ring difference maps, corrected for bias
- a set of best fit power spectra at each frequency
- analytical approximations to compute C_ℓ covariance matrices

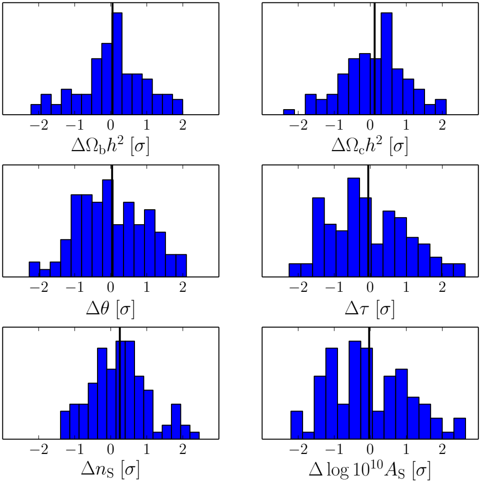


Binned matrix with 2300×2300 elements

Condition number: $\mathcal{O}(10^{11})$

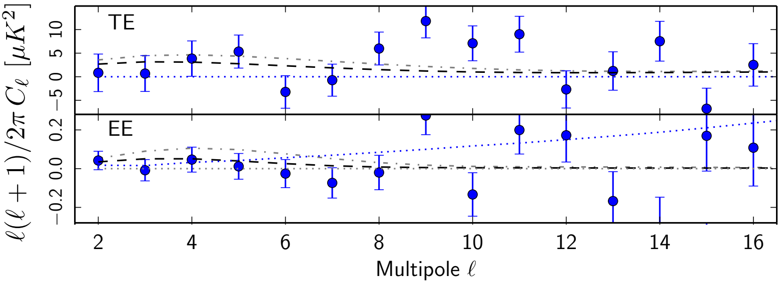
Likelihood verification on simulations

We computed cosmological parameters from 100 simulated HFI data sets, marginalizing over 12 foreground parameters.



4) Results

Measurements of the optical depth at low-ℓ



353 GHz dust cleaning changed τ from $\tau = 0.089$ to $\tau = 0.067$.
New reionization redshift has decreased to $z_{\mathrm{RE}} = 8.8$.

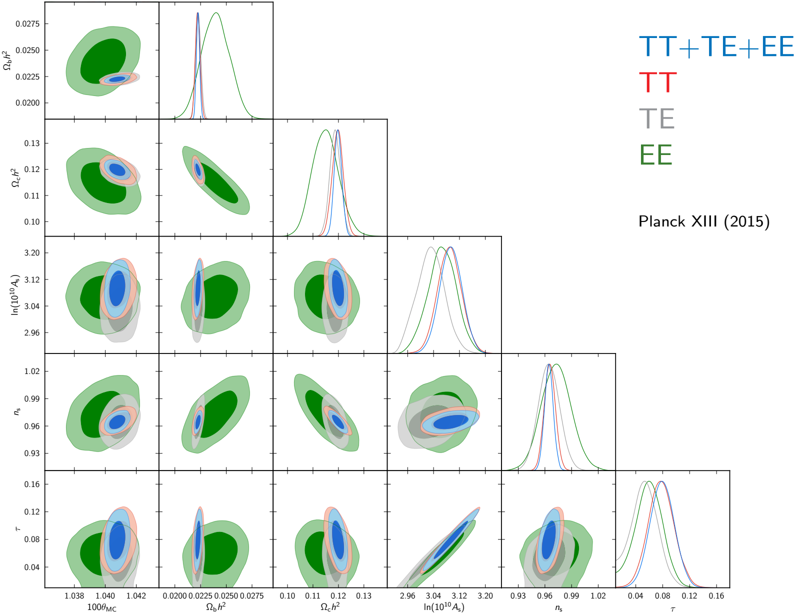
Planck XI (2015)

Cosmological parameters from temperature data

The 2015 cosmological parameters are in good agreement with 2013,

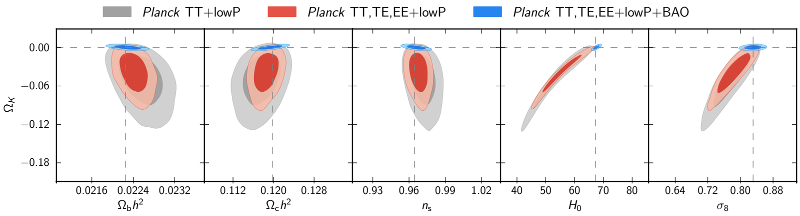
Parameter	2013	2015	('13 - '15) / std('13)
$\Omega_b h^2$	0.02217	0.02226	$-0.3\,\sigma$
$\Omega_c h^2$	0.1186	0.1186	$0.0\,\sigma$
$100 \cdot \theta_{MC}$	1.0414	1.0410	$0.6\,\sigma$
τ	0.089	0.066	$0.7\,\sigma$
n_s	0.9635	0.9677	$-0.4\,\sigma$
$\log(10^{10} A_s)$	3.085	3.062	$0.4\,\sigma$
σ_8	0.823	0.815	$0.4\,\sigma$
H_0	67.9	67.8	$0.1\,\sigma$

Parameter consistency temperature/polarization



ΛCDM extensions: Ω_K

For CMB alone, curvature is strongly correlated with H_0. BAO measurements break degeneracy.

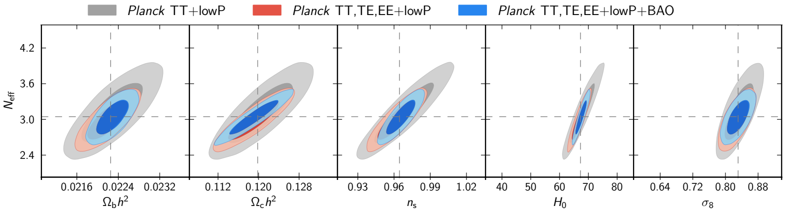


$\Omega_K = 0.000 \pm 0.005$ (95% *Planck* TT + BAO)

Planck XIII (2015)

ΛCDM extensions: N_{eff}

The number of relativistic species is consistent with $N_{eff} = 3.046$,

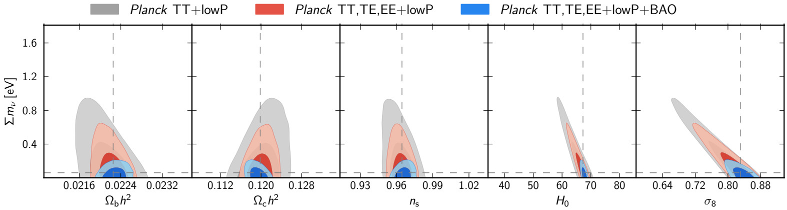


Planck T+P rules out $N_{eff} = 2, 4$ at high significance.

Planck XIII (2015)

ΛCDM extensions: Σm_ν

Upper limits on Neutrino masses have improved,



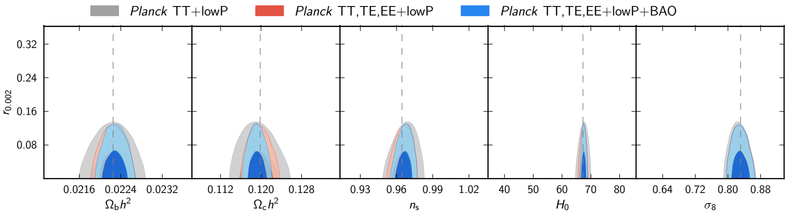
$\Sigma m_\nu < 0.21$ eV (95% *Planck* TT + BAO)

Constraints are approaching the lower bound of 0.06 eV.

Planck XIII (2015)

Tensor-to-scalar ratio – TT constraints

From temperature alone, we obtain upper limits on the tensor-to-scalar ratio,

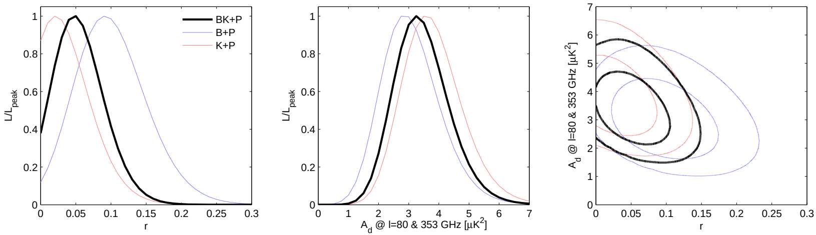


$r < 0.11$ (95% *Planck* TT)

Planck XIII (2015)

Tensor-to-scalar ratio – BB constraints

In a joint effort with BICEP2/Keck, we obtain upper limits from BB power spectra,

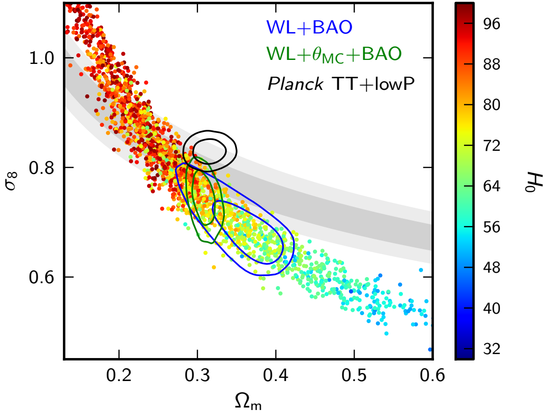


$r < 0.12$ (95% BICEP2/Keck + *Planck* BB)

BICEP2/Keck & Planck (2015)

The σ_8/Ω_m inconsistency

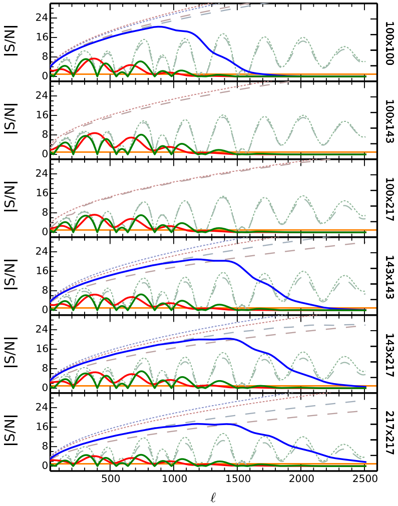
The CFHTLenS survey provides parameter constraints from weak lensing (Kitching+ 2014).



Results are in tension once small scale information is included in CFHTlenS analysis. Baryon feedback?

Planck XIII (2015)

Power spectrum signal-to-noise ratios



Planck XI (2015)

The signal-to-noise ratio of the Planck channels used:

- Close to cosmic variance limit in TT
- Still much information in EE and TE left to explore

Outlook: S4 experiment example

Error bars on parameters for the *COrE* mission computed from 105, 135, 165, 195, and 225 GHz channels in comparison to *Planck*:

Parameter	Relative error [%] *Planck*	Relative error [%] *COrE*
$\Omega_{\mathrm{b}}\ h^2$	0.70	0.19
$\Omega_{\mathrm{DM}}\ h^2$	1.20	0.56
$\theta \times 10^2$	0.03	8.2×10^{-3}
n_{S}	0.50	0.21
$\log(10^{10} A_{\mathrm{S}})$	0.85	0.31
τ	16.5	6.8

Elsner et al. in prep.

Summary

Cosmological parameters can be robustly computed from CMB data using a hybrid likelihood approach.

Analyzing *Planck* data, we find:

- Temperature and polarization data are consistent
- The Universe appears to be simple:

  - Flat geometry
  - Adiabatic fluctuations
  - Near scale invariant primordial power spectrum, no running
  - Only upper limits on r

$\rightarrow$ ΛCDM cosmology remains an excellent fit to the data.

Acknowledgments