# *Model-Independent Constraints on Inflation from the CMB*

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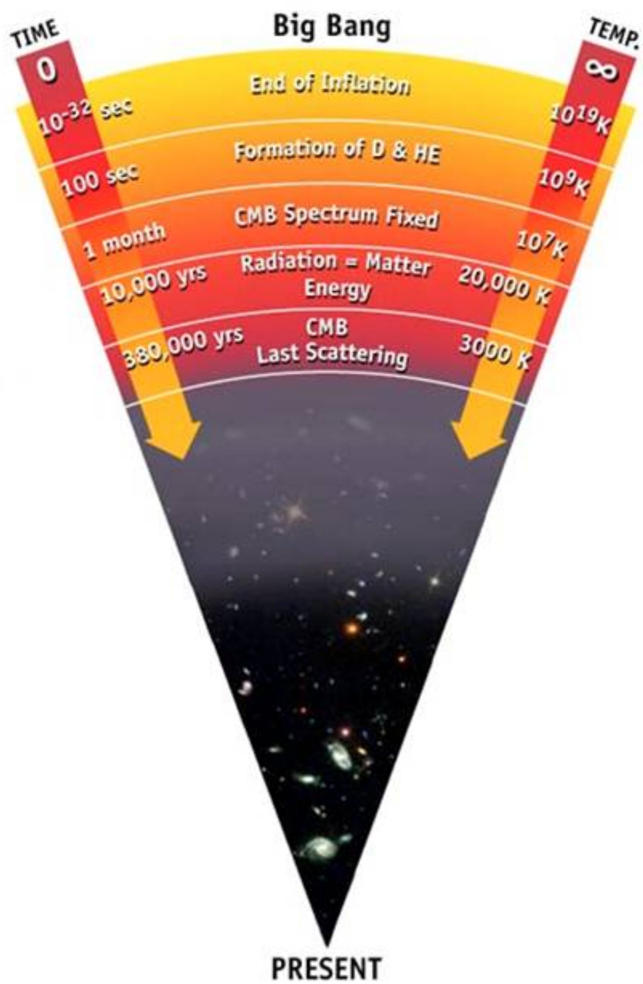
# Outline

- CMB and Inflation overview.
- General method to constrain the inflationary potential from CMB observations allowing for features.
  - Theoretical framework.
  - Analysis of data.
- Conclusions and future directions.

# Cosmic History



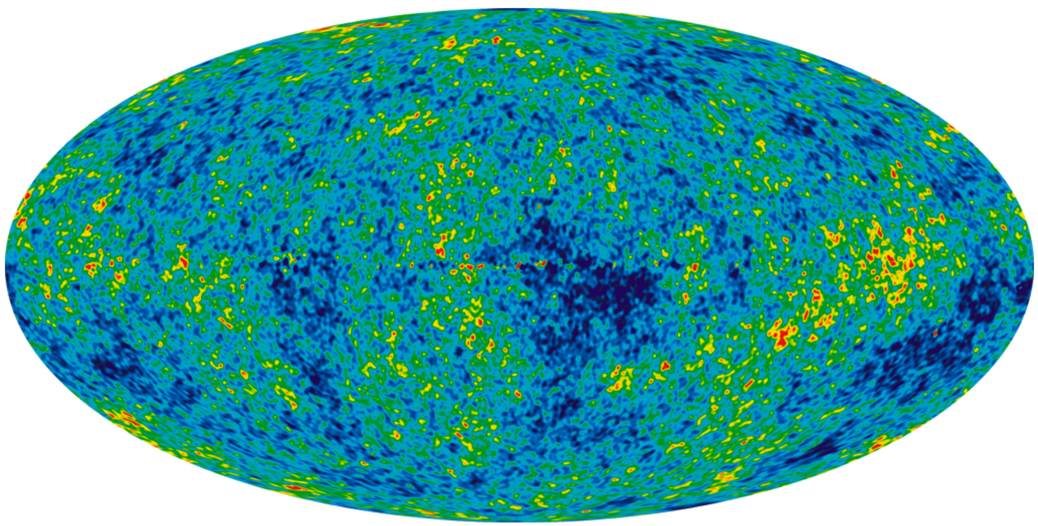

***WMAP collaboration***

- The universe began as a hot and dense plasma of particles in thermal equilibrium.
- Recombination (z $\approx$1100): $p^+ + e^- \to H$
  Universe becomes transparent to CMB photons.

  Photons mainly freestream.
- Radiation from first stars and quasars reionizes the universe (z $\approx$10) and $\sim$10% of the photons re-scatter.
- We observe these photons at T $\approx$ 2.725 K.

# CMB Anisotropies

“Snapshot” of the Early Universe



***WMAP collaboration***

Gaussian random fluctuations: $\Delta T \approx 100 \mu K$

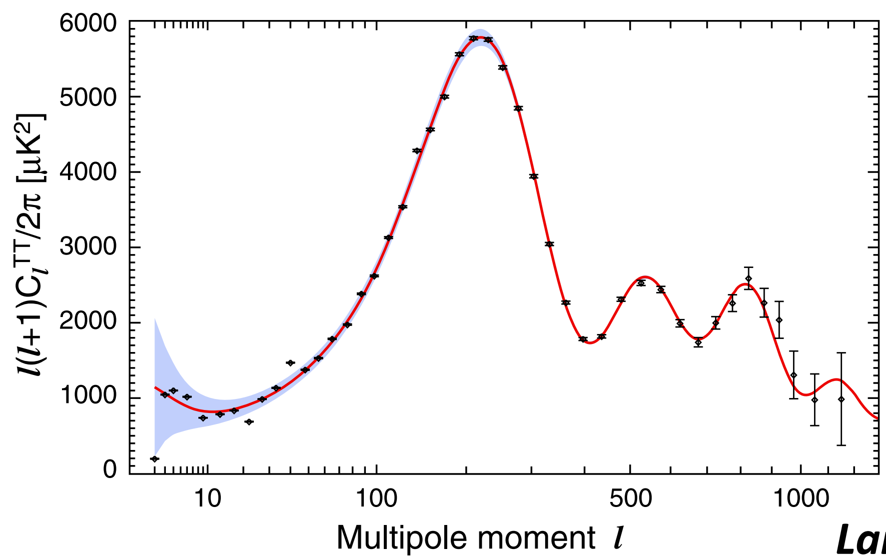
# CMB Power Spectrum

Power spectrum: contains all the information for a Gaussian, isotropic field.

$$\Delta T\left(\hat{\mathbf{n}}\right) = \sum_{\ell m} T_{\ell m} Y_{\ell m}\left(\hat{\mathbf{n}}\right)$$

$$\langle T_{\ell m} T^{*}_{\ell' m'} \rangle = C_{\ell}^{TT} \delta_{\ell \ell'} \delta_{m m'}$$

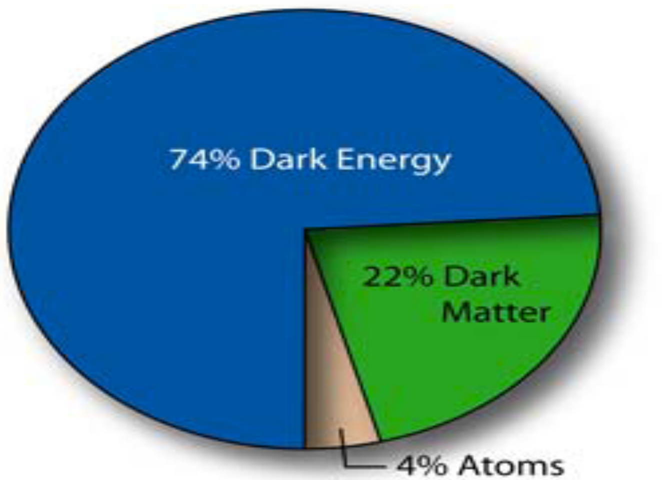
It has been **predicted** and **measured** with good precision.




***Larson et al., (2010)***

# ΛCDM: the "Standard" Model of Cosmology

Homogeneous background

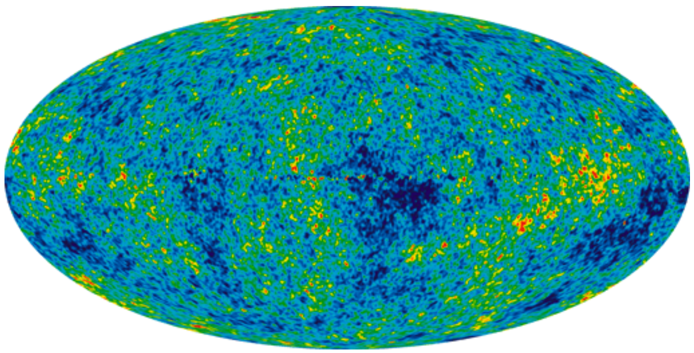



$$\Omega_b h^2, \Omega_c h^2, \Omega_\Lambda, \tau, \theta$$

- Baryonic matter: 4%
- Cold dark matter: 22%
- Dark energy: 74%

**Λ? CDM?**

Perturbations



$$A_s, n_s$$

- Nearly-scale invariant
- Gaussian

**Origin?**

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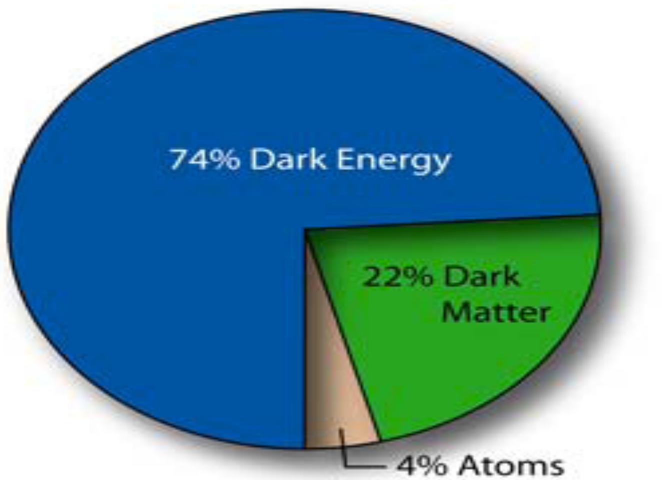


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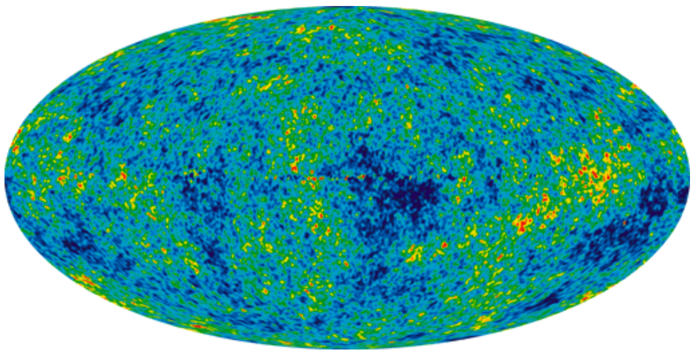

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# Inflation

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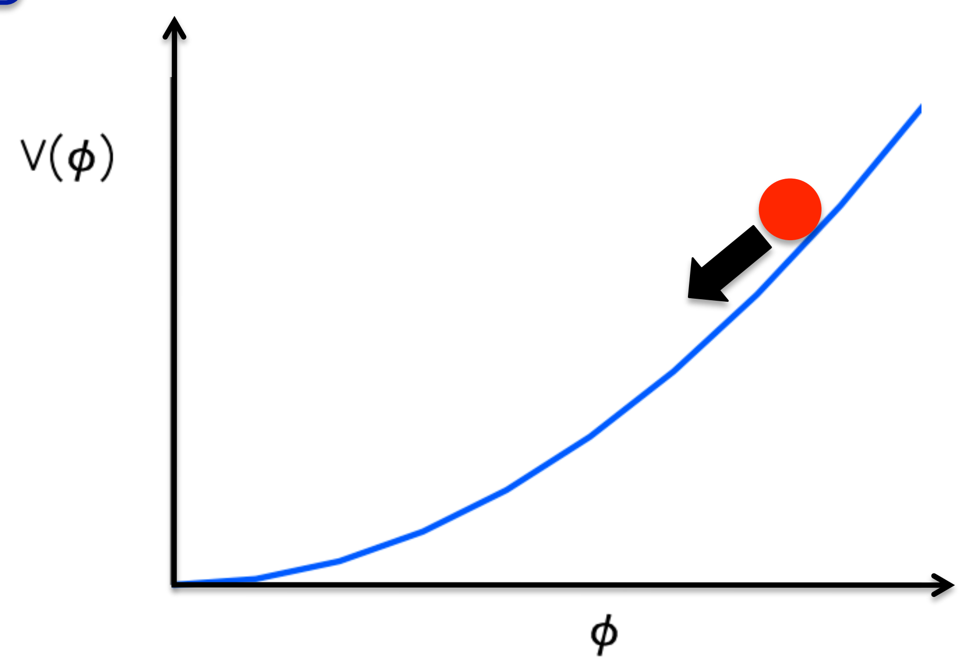
*What causes inflation?*

# The Dynamics of Inflation

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2$$ ← expansion rate

$$= \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

↑ energy density

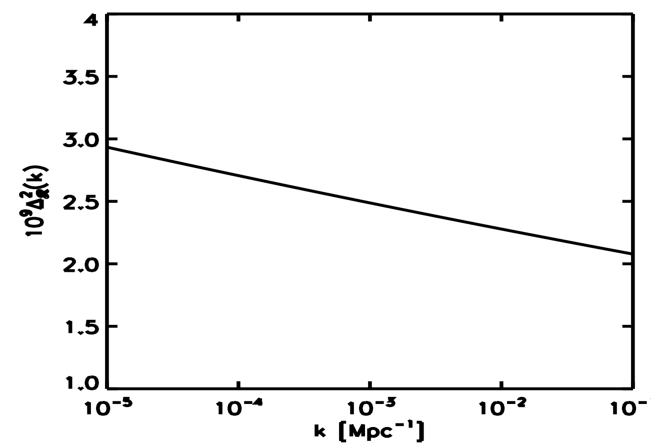



$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

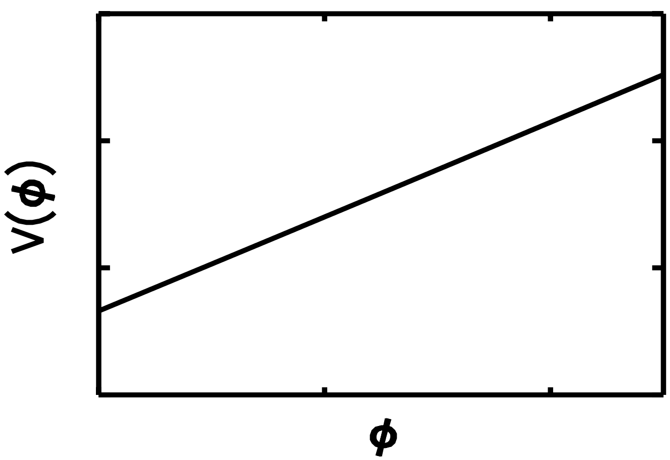
→ friction

# Connecting Theory with Observations

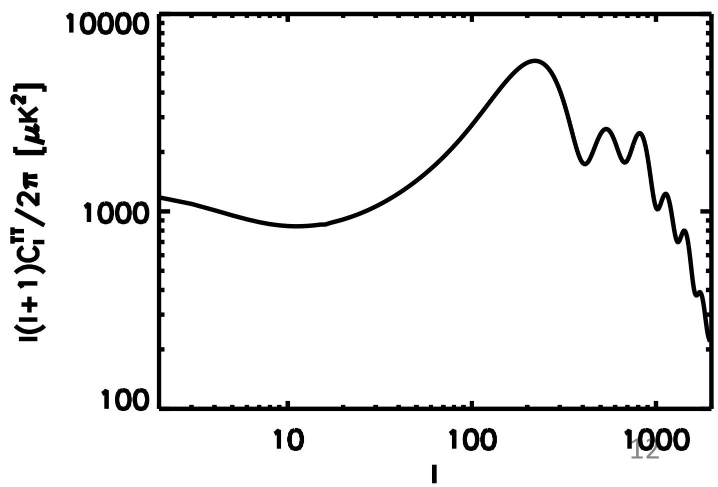
**POWER SPECTRUM**




**INFLATIONARY POTENTIAL**




**CMB TEMPERATURE POWER SPECTRUM**



# Connecting Theory with Observations

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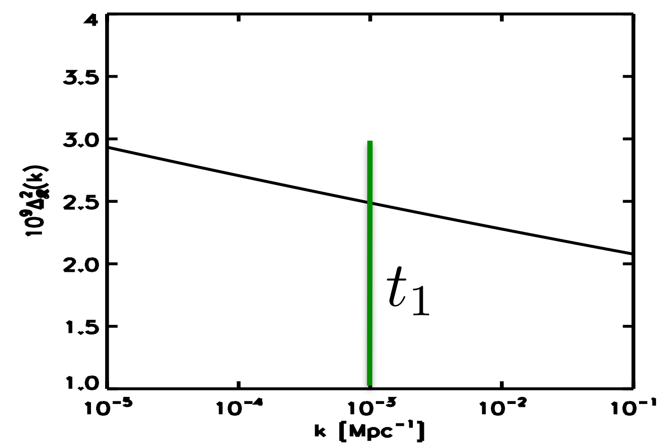


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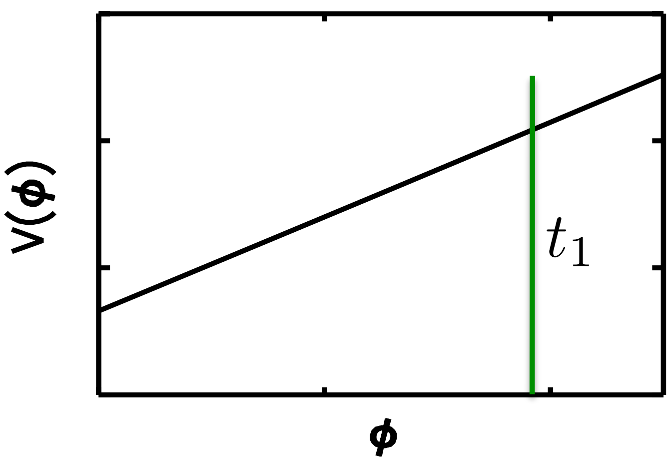


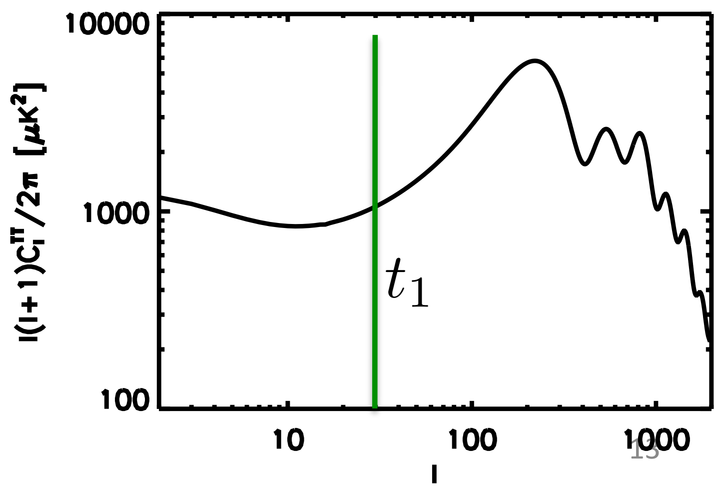

# Connecting Theory with Observations

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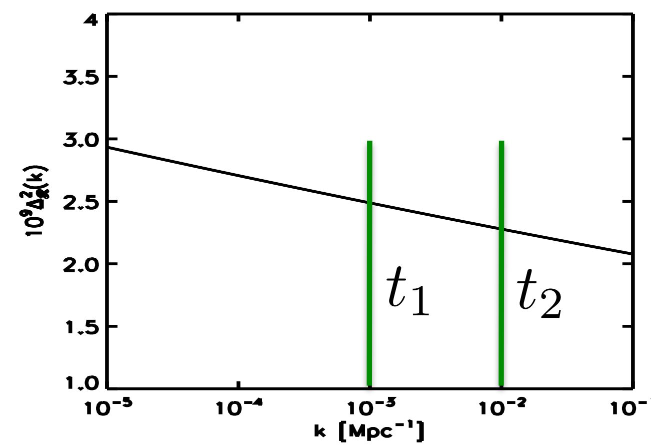


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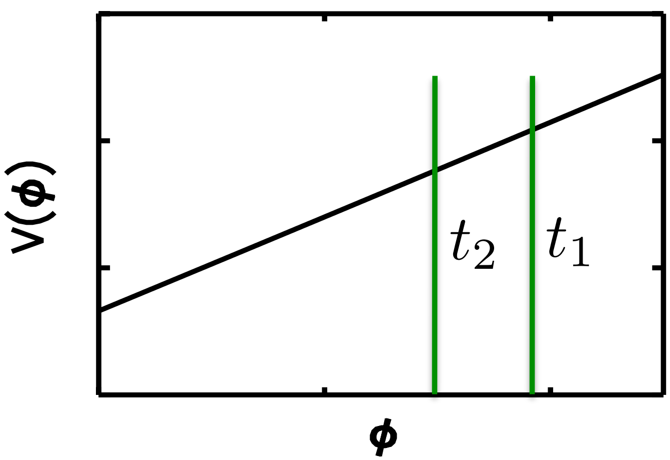


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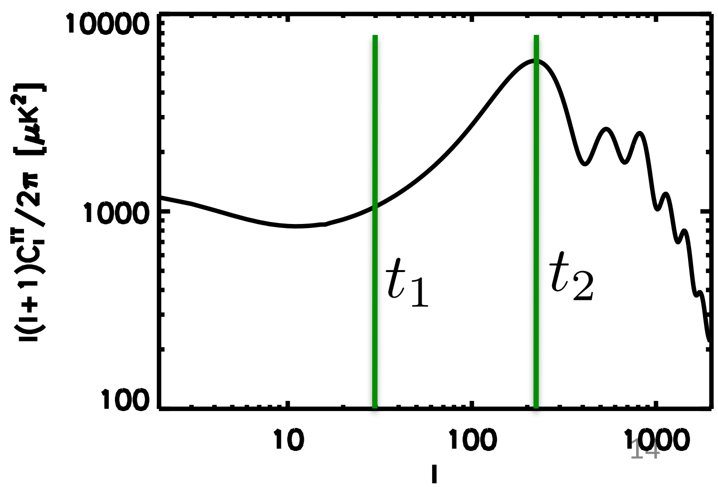


# Connecting Theory with Observations

**Goal: to shed light on the physics of inflation by using CMB observations**

# Standard Slow Roll

Technique for computing the initial curvature power spectrum for inflationary models where the scalar field potential is sufficiently flat and slowly varying.

$$\epsilon_H \equiv \frac{1}{2}\left(\frac{\dot{\phi}}{H}\right)^2$$

$$\eta_H \equiv -\left(\frac{\ddot{\phi}}{H\dot{\phi}}\right)$$

$$\delta_2 \equiv \frac{\dddot{\phi}}{H^2\dot{\phi}}$$

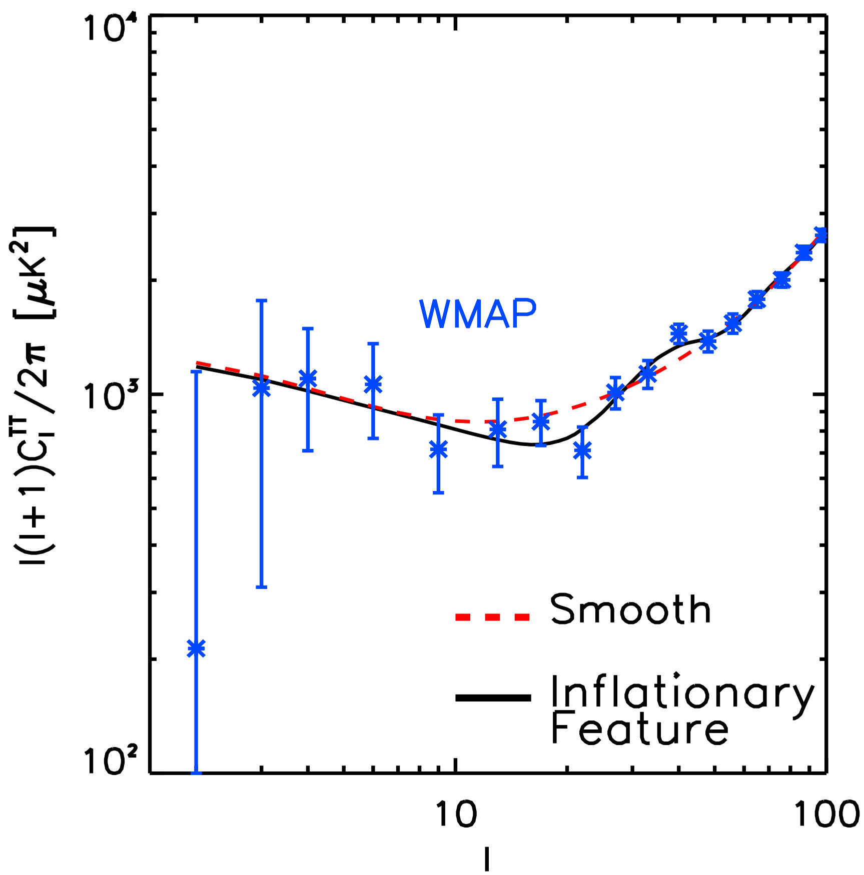
Slow-roll parameters

<< 1 and slowly varying

Linked to the **shape of the potential**

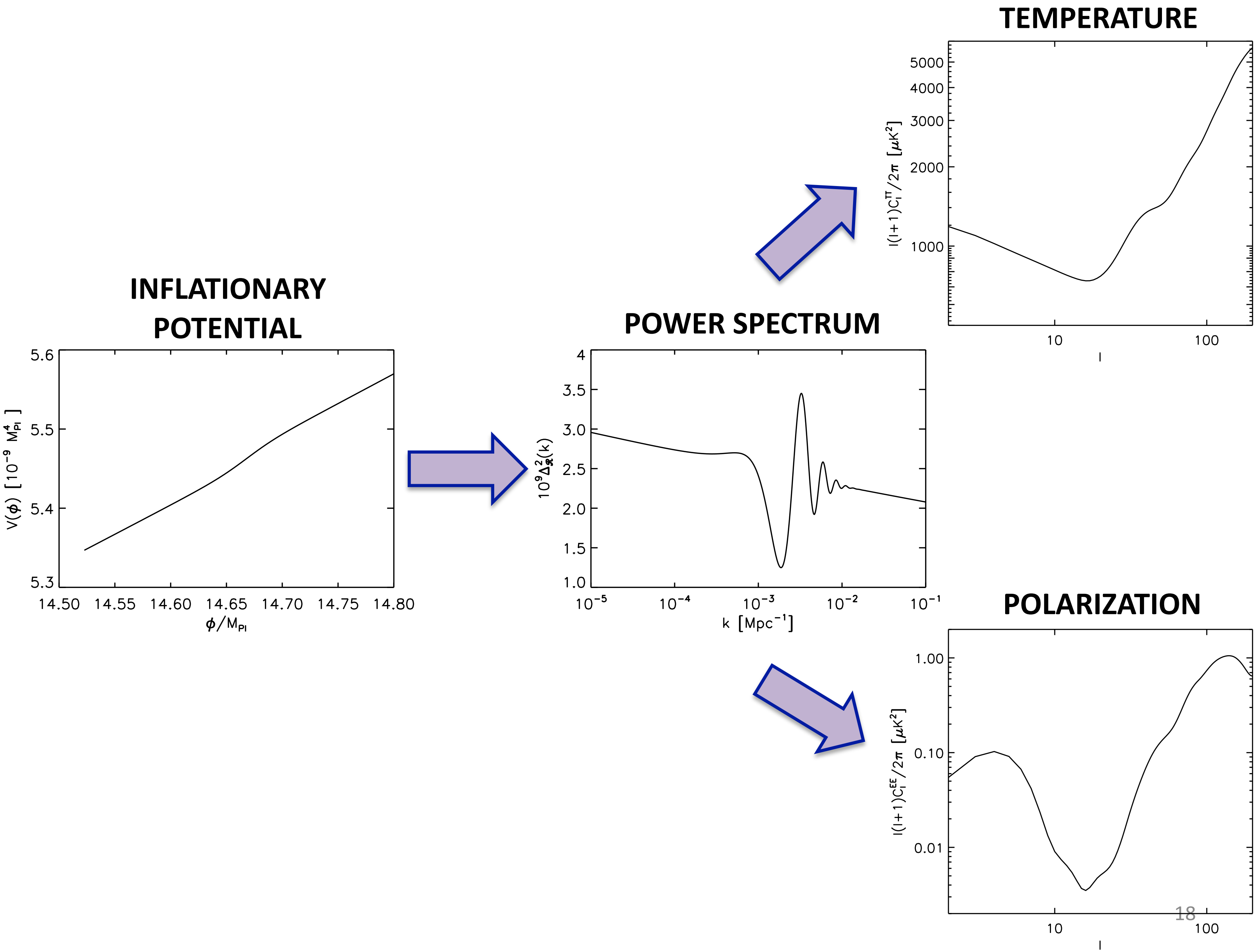
Slow roll approximation: $\Delta^2_{\mathcal{R}} \approx \left[(1-(2C+1)\epsilon_H - C\eta_H)\frac{H^2}{2\pi|\dot{\phi}|}\right]^2_{k\eta\approx 1}$

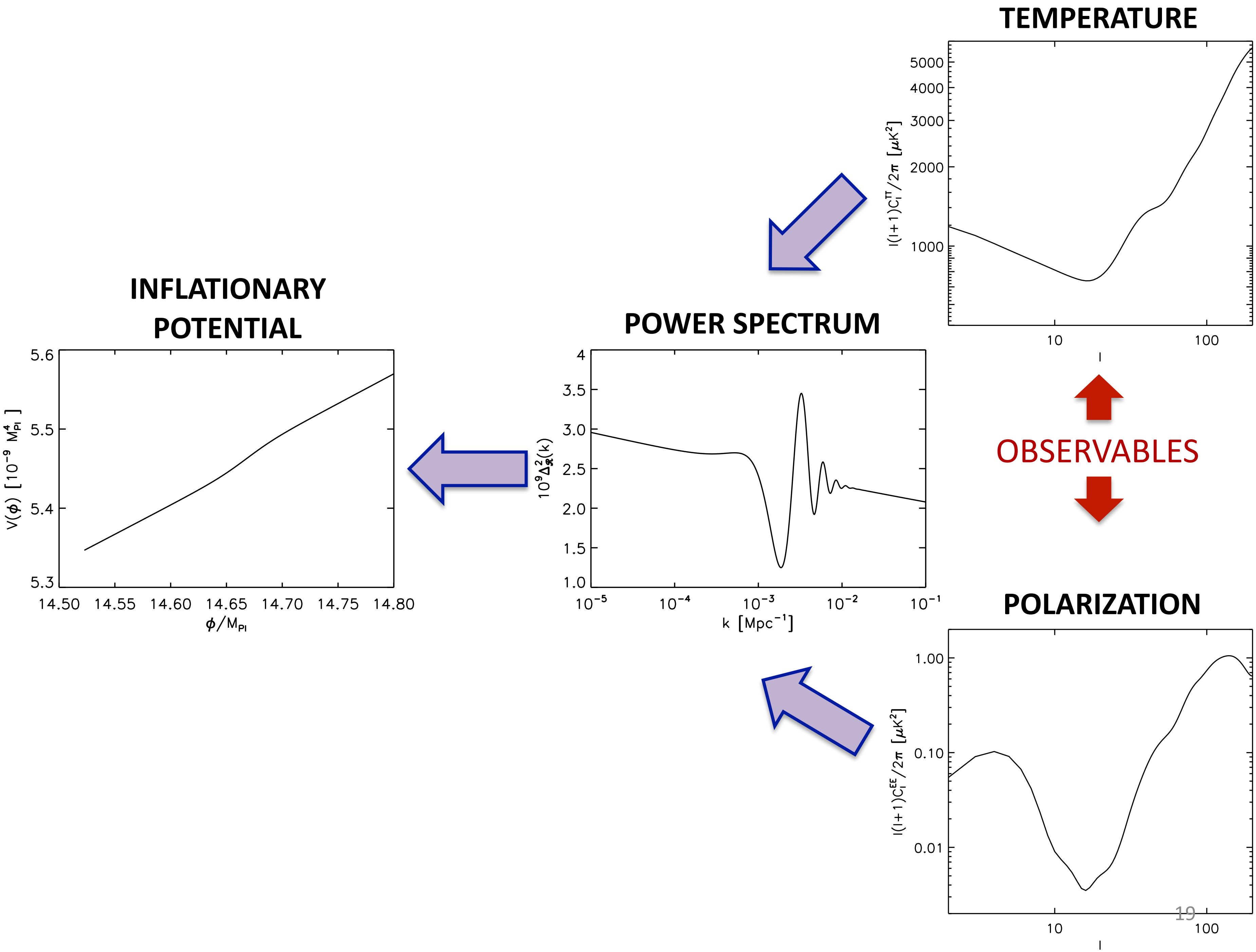
# Inflationary Features




- Glitches in the temperature power spectrum of the CMB have led to interest in exploring models with features in the inflaton potential.

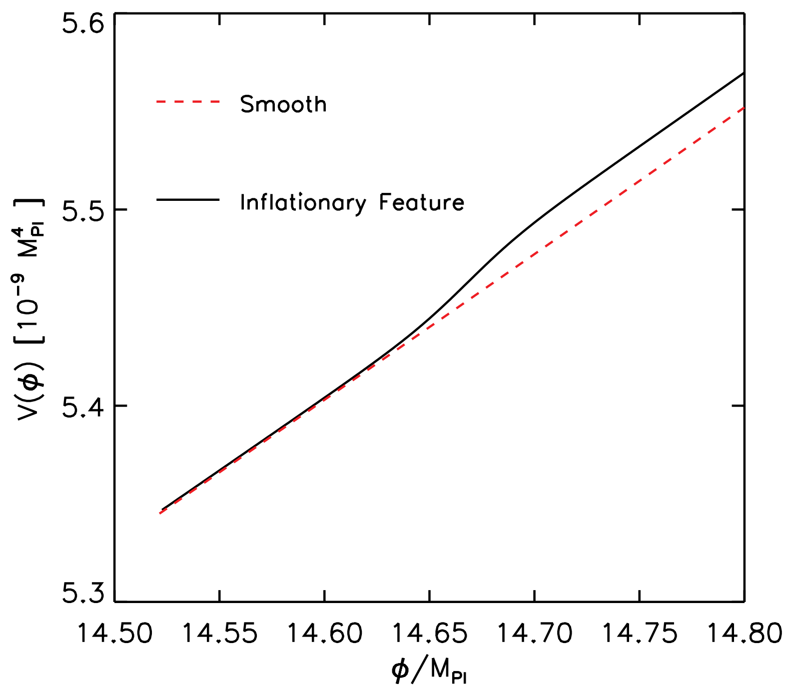
✧ *L. Covi, J.Hamann, A. Melchiorri, A. Slozar and I. Sorbera, (2006)*

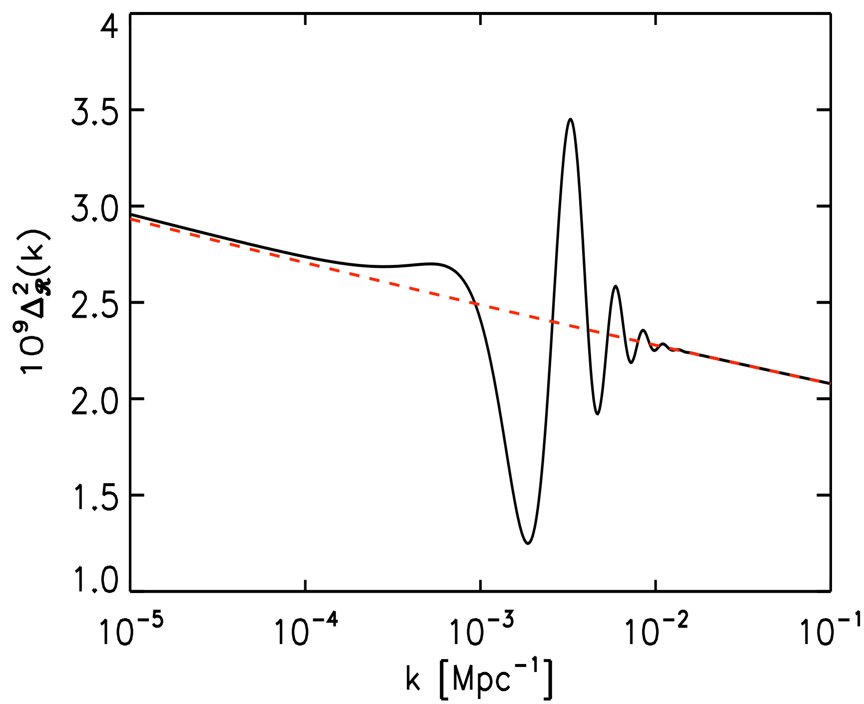

INFLATIONARY POTENTIAL
V(φ) [10⁻⁹ M_Pl⁴]
φ/M_Pl
5.3
5.4
5.5
5.6
14.50
14.55
14.60
14.65
14.70
14.75
14.80
POWER SPECTRUM
10⁹Δ²_R(k)
k [Mpc⁻¹]
1.0
1.5
2.0
2.5
3.0
3.5
4
10⁻⁵
10⁻⁴
10⁻³
10⁻²
10⁻¹
TEMPERATURE
l(l+1)C_l^TT/2π [μK²]
l
1000
2000
3000
4000
5000
10
100
POLARIZATION
l(l+1)C_l^EE/2π [μK²]
l
0.01
0.10
1.00
10
100


TEMPERATURE
POWER SPECTRUM
INFLATIONARY POTENTIAL
OBSERVABLES
POLARIZATION
$V(\phi)\ [10^{-9}\ M_{Pl}^4]$
$\phi/M_{Pl}$
$10^9 \Delta_{\mathcal{R}}^2(k)$
k [$Mpc^{-1}$]
$l(l+1)C_l^{TT}/2\pi\ [\mu K^2]$
$l(l+1)C_l^{EE}/2\pi\ [\mu K^2]$
l

# Inflationary Features

The rolling of the inflaton across the feature produces ringing in the power spectrum.

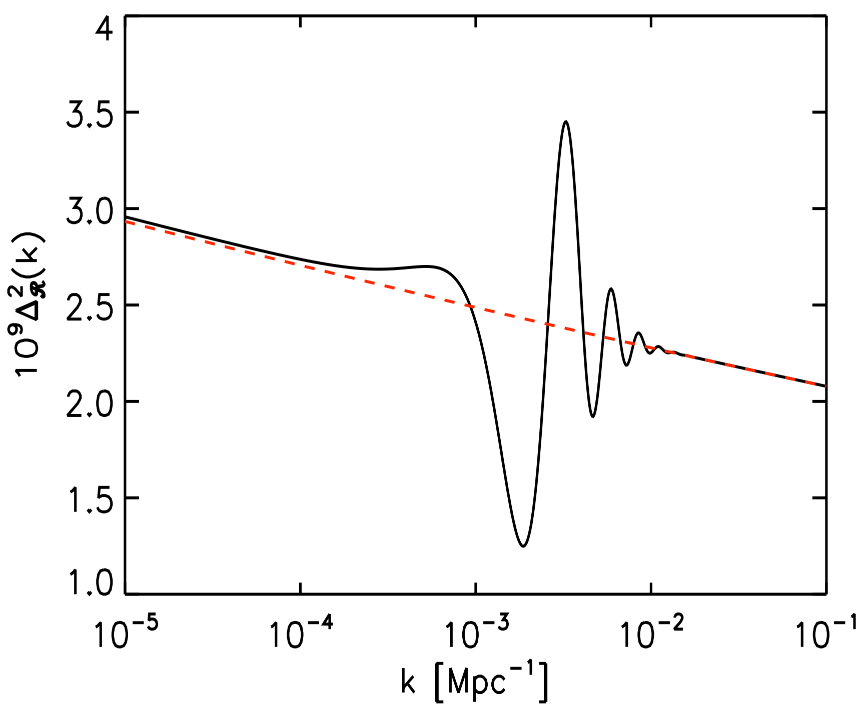


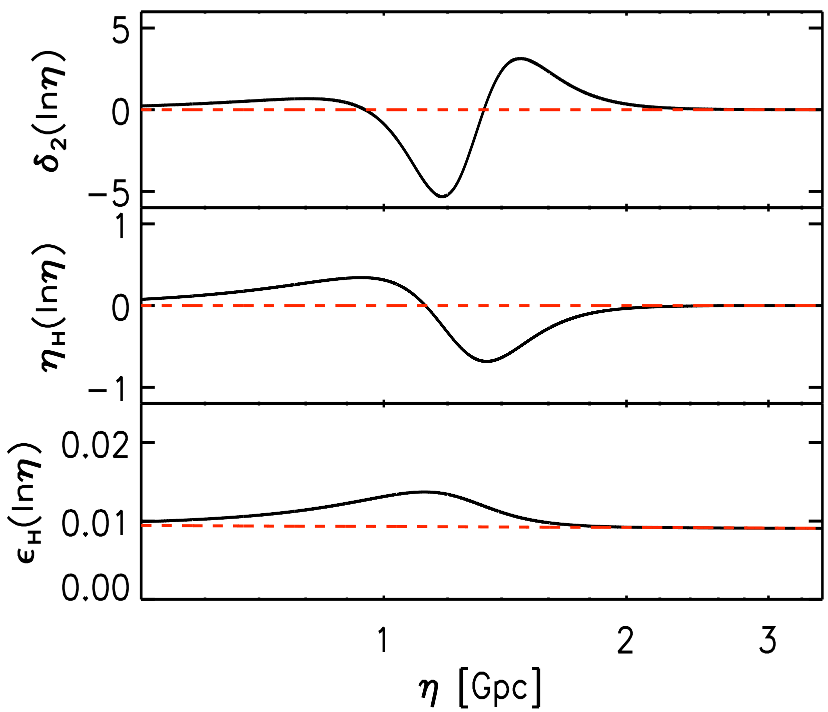

***M. Mortonson, C. Dvorkin, H.V. Peiris and W. Hu, PRD (2009)***

# Breaking Slow Roll

- These models require order unity variations in the curvature power spectrum: slow-roll parameters are not necessarily small or slowly varying.




Standard slow roll:
not a valid approximation!




***C. Dvorkin, W.Hu, PRD (2009)***

# Generalized Slow Roll

*E. Stewart, PRD (2002)*

- Field equation: $$\frac{d^2 y}{dx^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{g(\ln x)}{x^2} y$$

  $(y = \sqrt{2k} u_k;\ x = k\eta)$

- "Perfect" slow roll: $$\frac{d^2 y_0}{dx^2} + \left(1 - \frac{2}{x^2}\right) y_0 = 0$$

- GSR approximation: $$\frac{d^2 y}{dx^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{g(\ln x)}{x^2} y_0$$

Source function (linear in slow-roll parameters)

Solution can be constructed with a Green function approach.

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$$\frac{d^2y}{dx^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{g(\ln x)}{x^2} y_0$$

Solution can be constructed with a Green function approach.

***BUT…***

- Nodes in the power spectrum.
- Curvature is not constant for modes outside the horizon.

# Our GSR solution for large features

- The curvature power spectrum depends on a single source function:

$$\ln \Delta^2_{\mathcal{R}}(k) = G(\ln \eta_{\min}) + \int_{\eta_{\min}}^{\eta_{\max}} \frac{d\eta}{\eta} W(k\eta) G'(\ln \eta) + \ln\left[1 + \frac{1}{2}\left(\int_{\eta_{\min}}^{\eta_{\max}} \frac{d\eta}{\eta} X(k\eta) G'(\ln \eta)\right)^2\right]$$

***C. Dvorkin, W. Hu, PRD (2009)***

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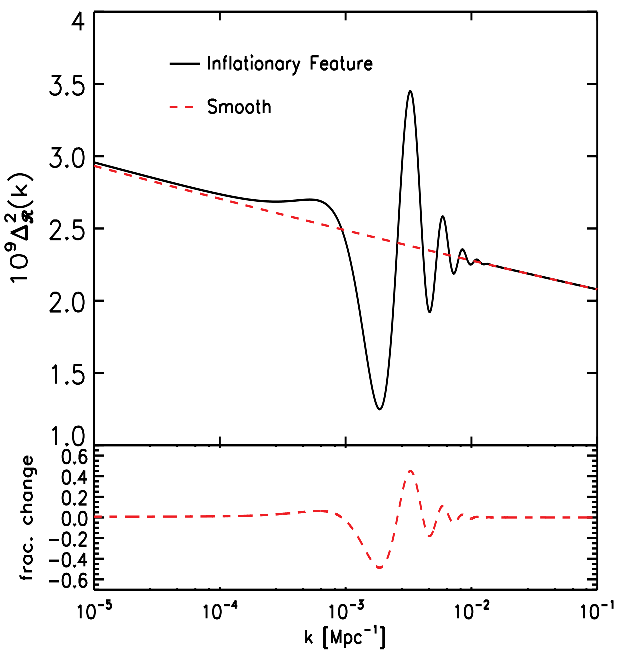
***C. Dvorkin, W. Hu, PRD (2009)***

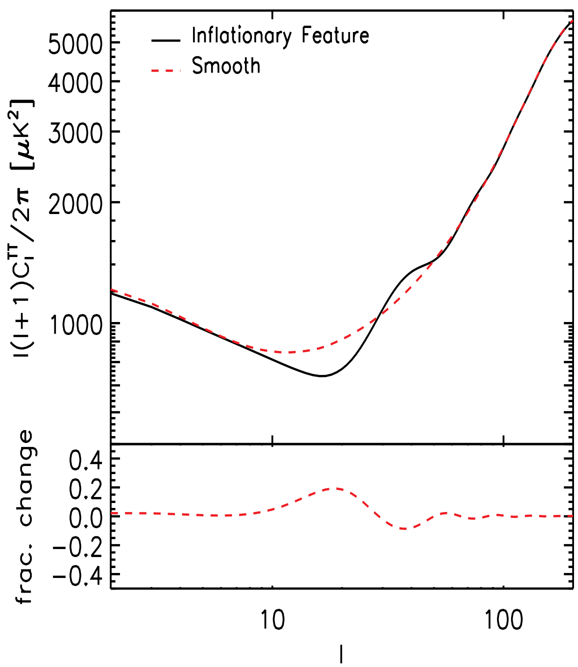
✓ Constant curvature for modes outside the horizon.
✓ We recover the slow-roll result for a constant source.
✓ Well controlled for time varying and order unity slow-roll parameters: percent level errors.
✓ Simple to relate to the inflaton potential: $G' \approx 3 \left( \frac{V_{,\phi}}{V} \right)^2 - 2 \left( \frac{V_{,\phi\phi}}{V} \right)$

# Second order Generalized Slow Roll: Well controlled

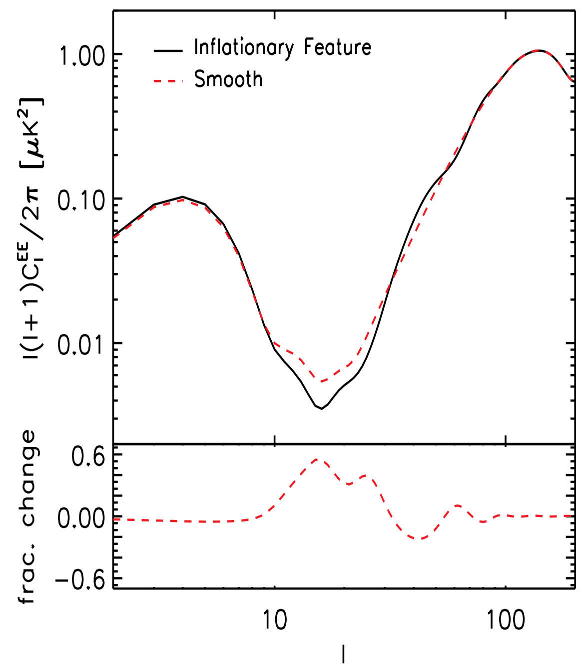
**POWER SPECTRUM**




**TEMPERATURE**




**POLARIZATION**



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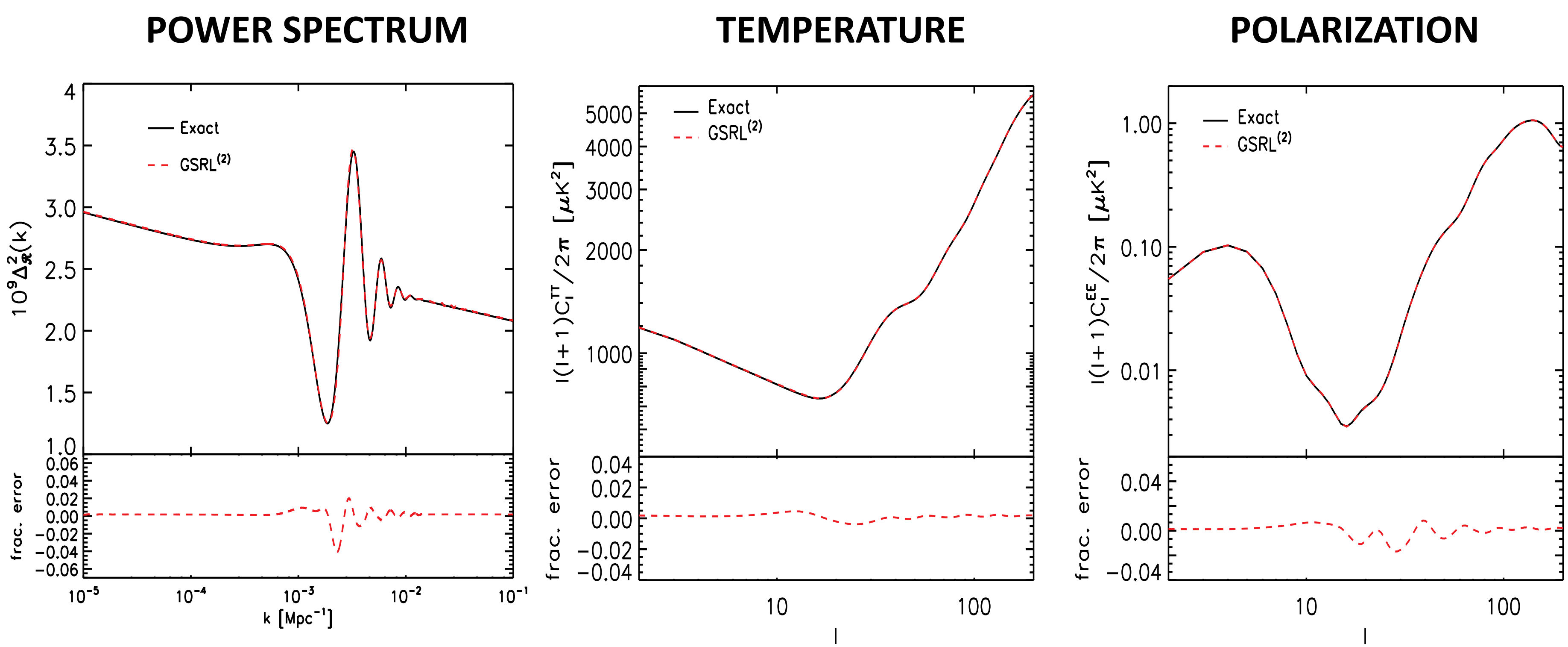
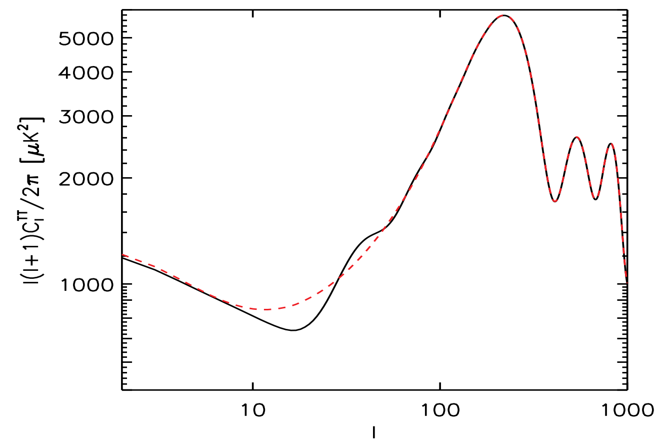



***C. Dvorkin, W. Hu, PRD (2009)***

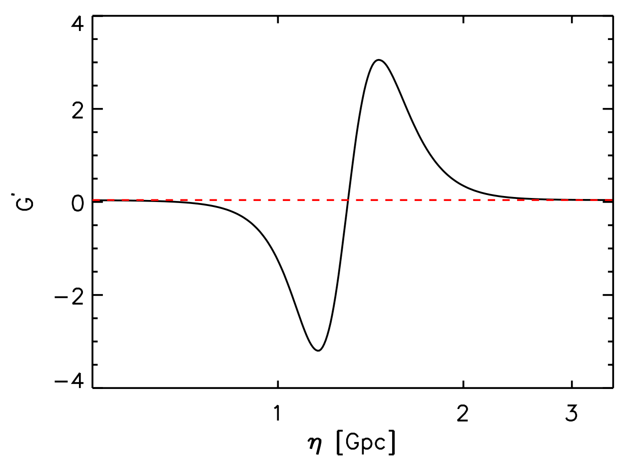
**Accurate at <1% level for order unity features!**

# *We can map observational constraints from the CMB onto constraints on the source…*

- Power spectrum

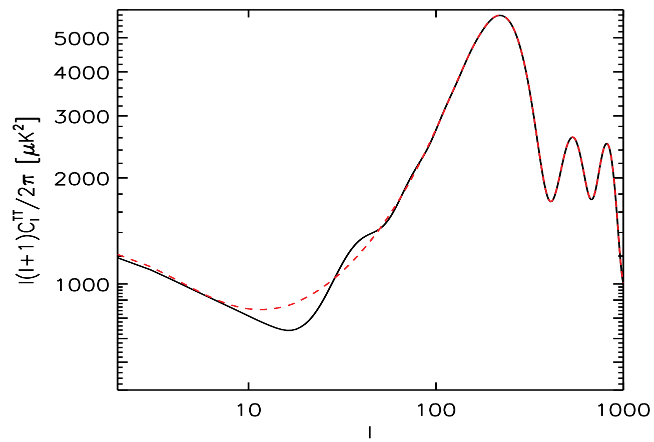


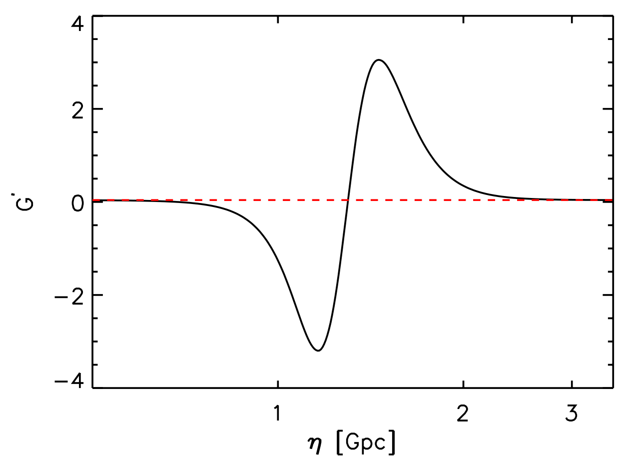

- Source

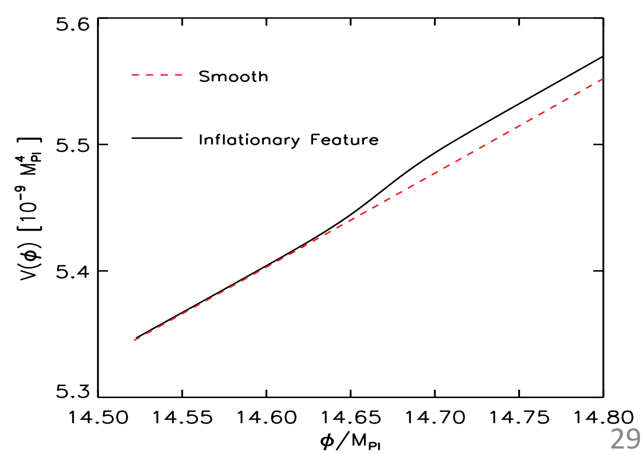


# *...and use these empirical constraints to test any model of single-field inflation.*

- Power spectrum
- Source
- Inflationary Model



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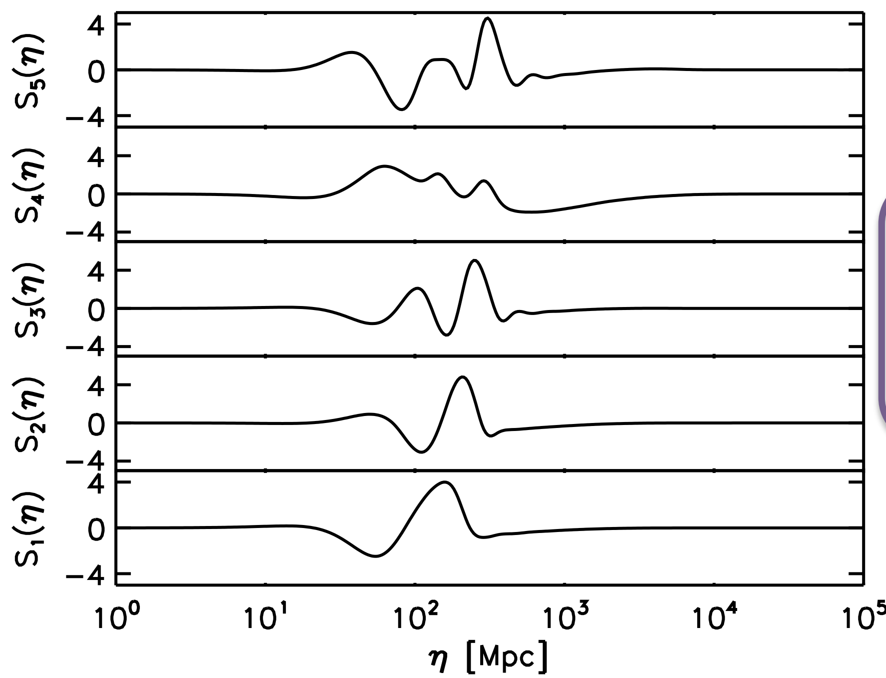
# Model-independent constraints

***Principal components*** *(of covariance matrix of perturbations in the source)*: basis for a complete representation of observable properties of the source function.

$$G' = 1 - n_s + \sum_{a=1}^{N} m_a S_a(\ln \eta)$$

***C. Dvorkin and W. Hu, PRD (2010)***




**Defined a priori from covariance matrix: avoids *a posteriori* bias when looking at the data.**

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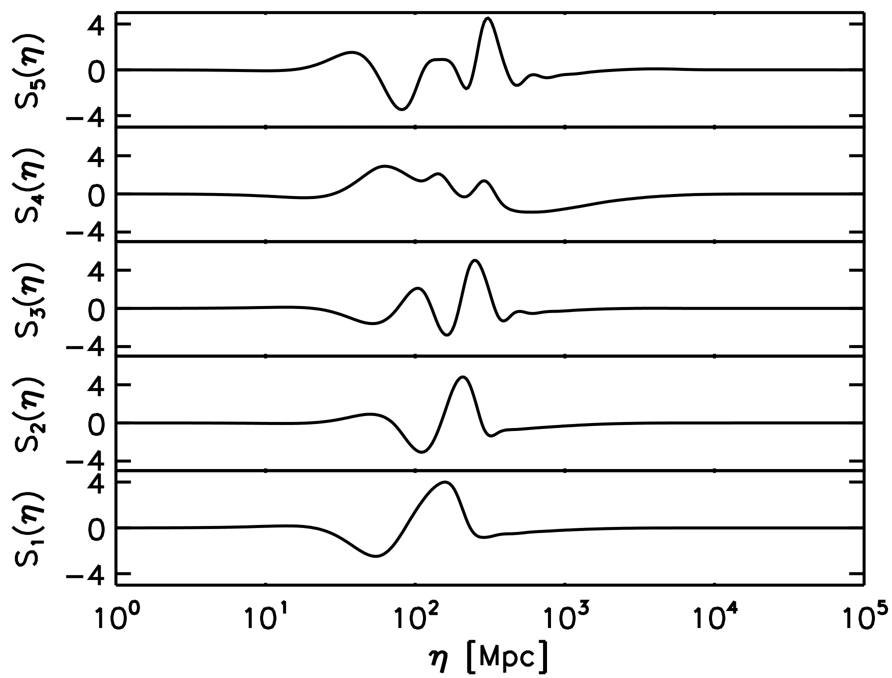
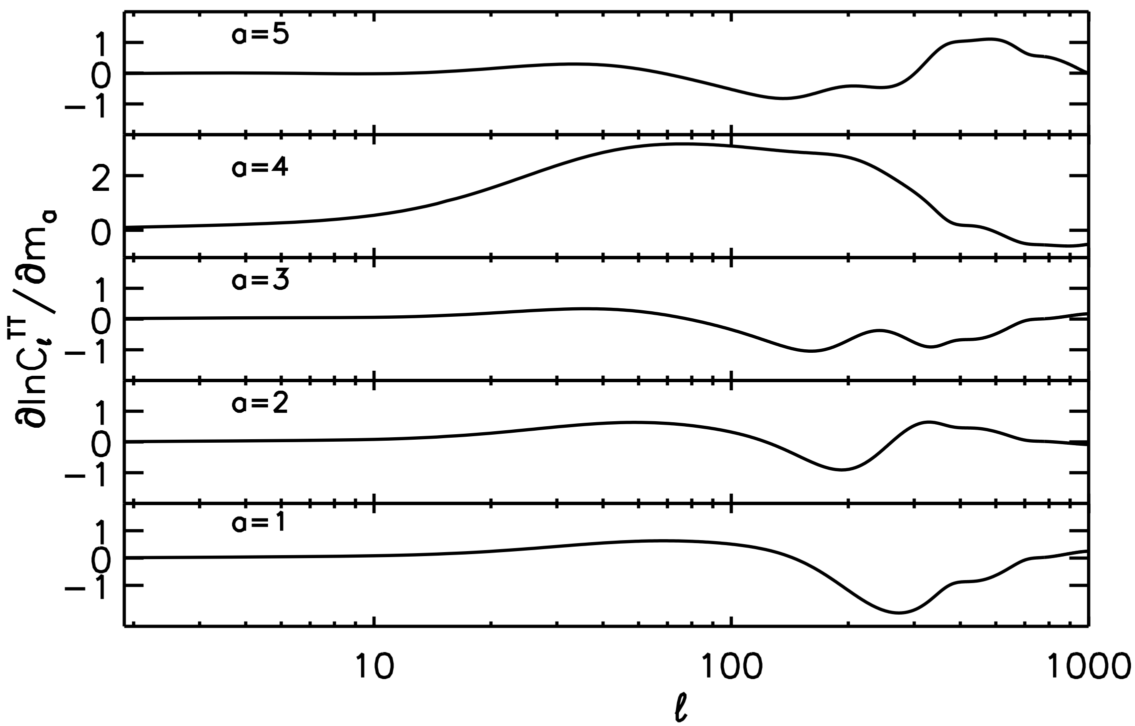



- **Ranked in order of observability.**
- **Keep 5 best measured modes.**

# Lower order PC's in WMAP

- Have their weight in the region best measured by the data (angular scales around the first acoustic peak, l $\approx$ 200).




***C. Dvorkin, W. Hu, PRD (2010)***

# Implementing GSR to the data

GSR allows efficient computation: $\Delta^2_{\mathcal{R}} = F(A_s, n_s, m_1, ..., m_N)$

- COSMOMC "Fast parameters": do not require computation of the CMB radiation transfer function.

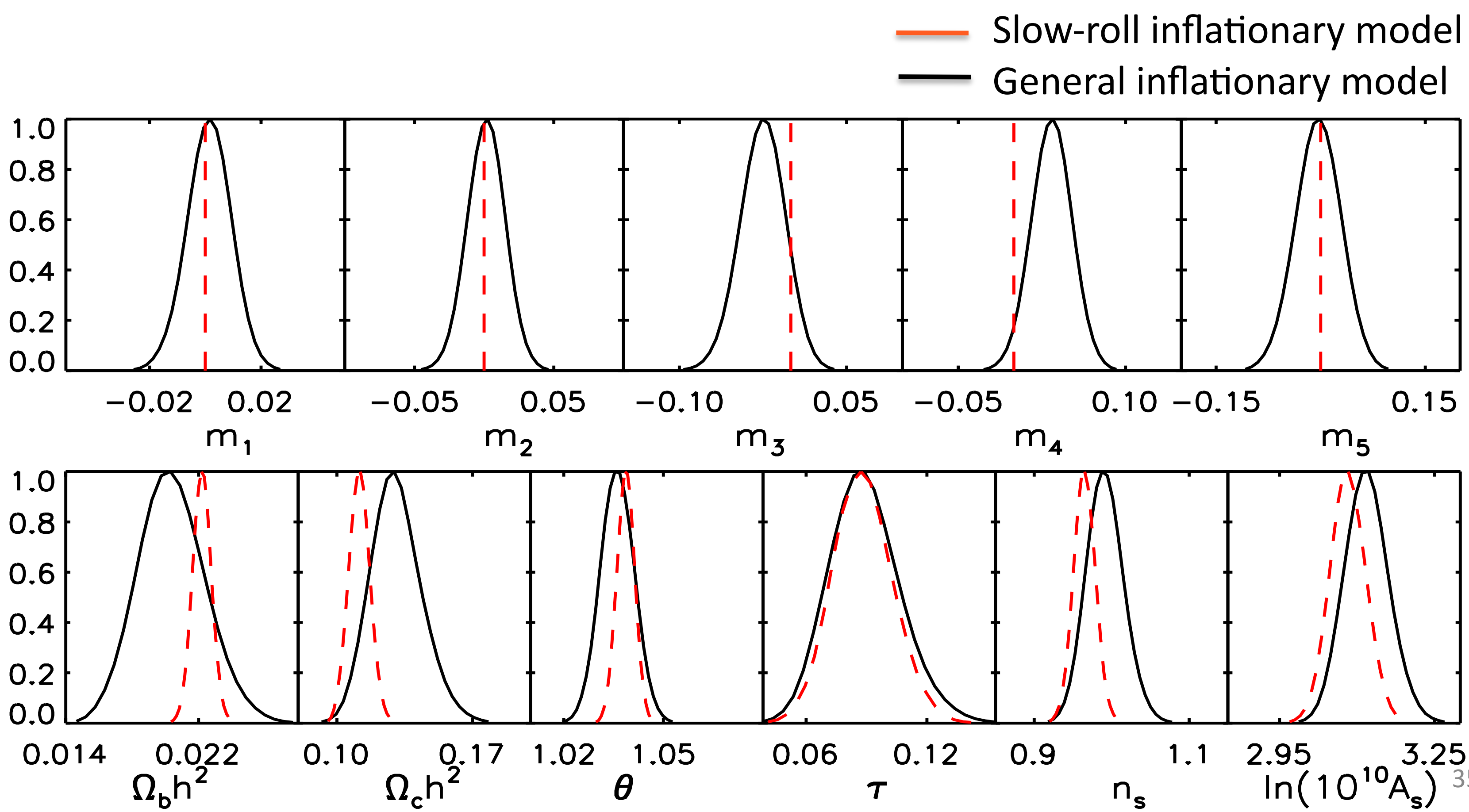
➡ Main bottleneck in the likelihood code:

- OMP parallelized **WMAP likelihood code** and improved its speed by ~ **5*Ncore**

**Publicly available: http://background.uchicago.edu/wmap_fast/**

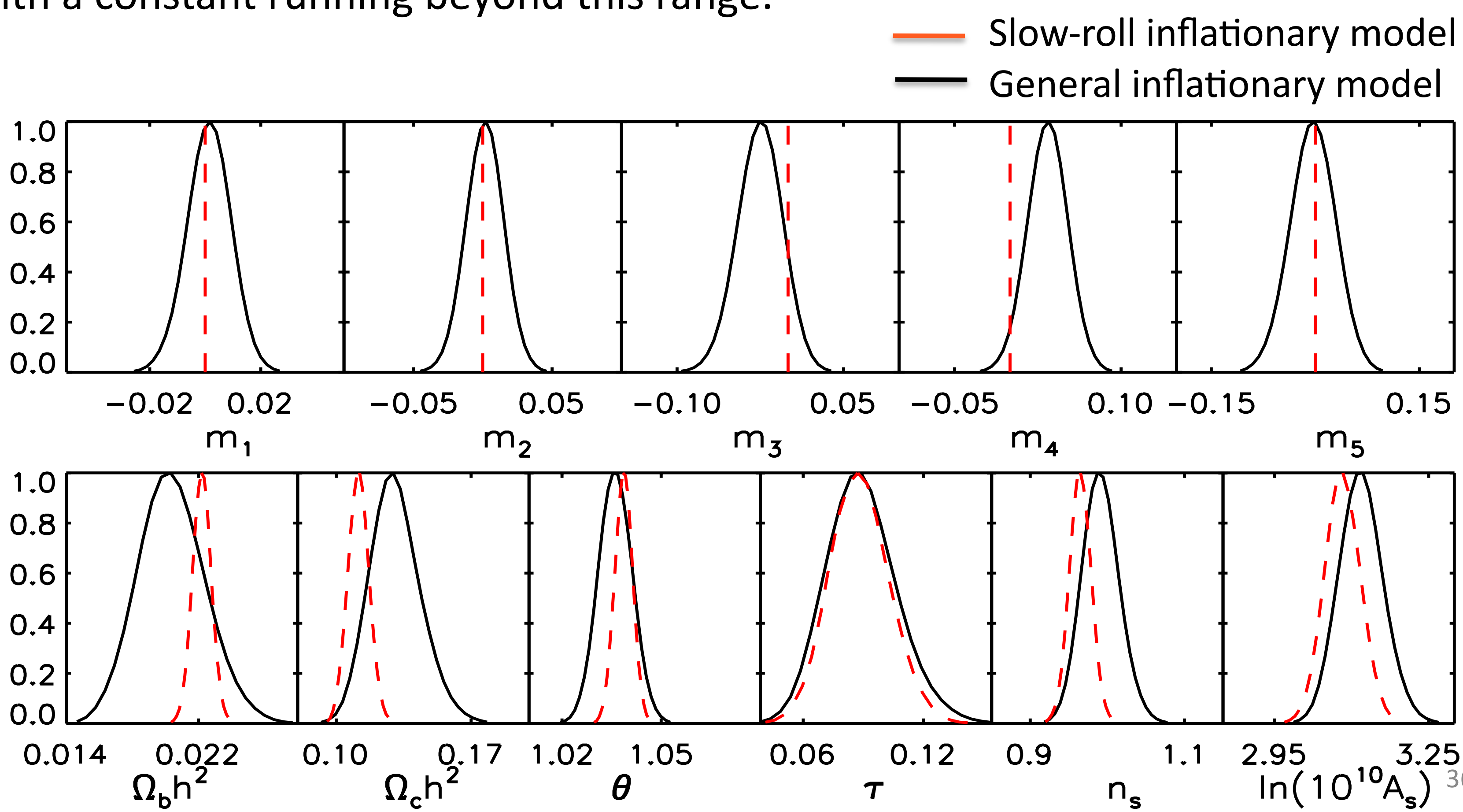
# WMAP7 constraints from MCMC's

- Non-zero values represent deviations from slow-roll and power-law spectrum.
- 1 out of 5 shows a 95% CL preference for a non-zero value, but only with a high cold dark matter density (which is disfavored by current SN and H0 data).



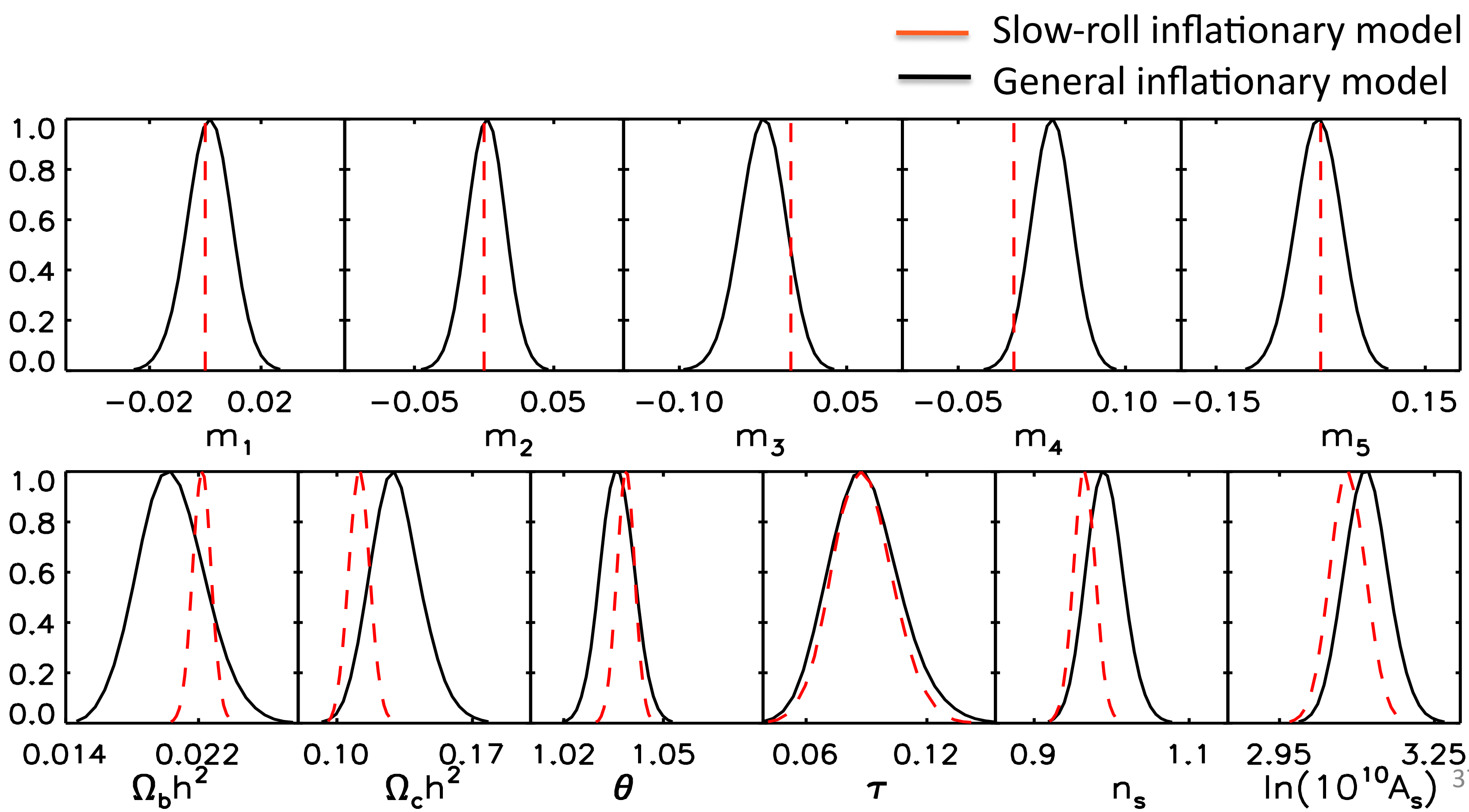
# WMAP7 constraints from MCMC's

- Interestingly, the 4th component carries most of the information about running of the tilt.
- It resembles a local running of the tilt for l~30-800, but it is marginally consistent with a constant running beyond this range.



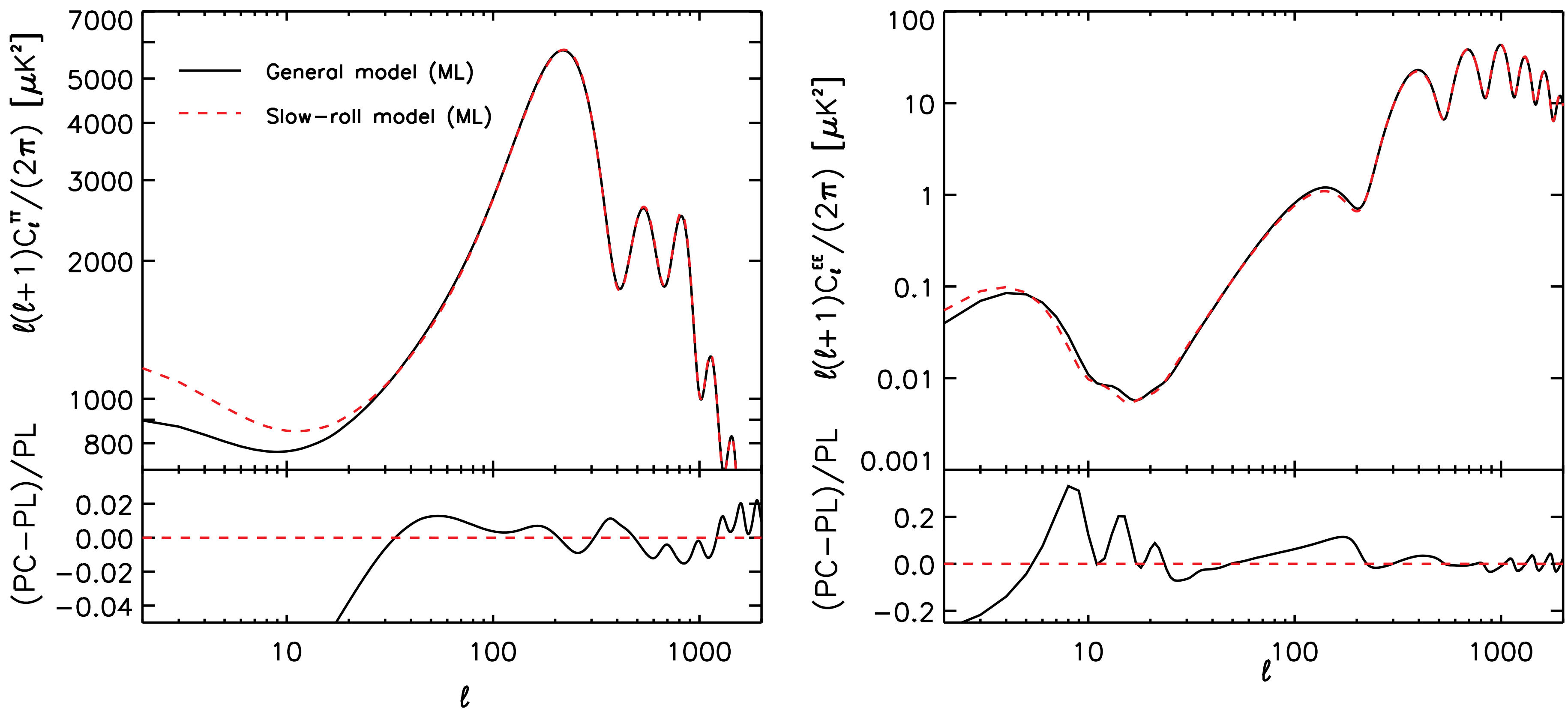
# WMAP7 constraints from MCMC's

- Consistency with a smooth inflationary potential: $\Delta\chi^2 \approx 5$ (with 5 additional parameters); robust to inclusion of tensor modes, spatial curvature and SZ emission.



# Future data: better constraints!

- Small-scale temperature measurements at l>1000 and future polarization data at better than 10% at l>100 (Planck) will improve inflationary constraints.

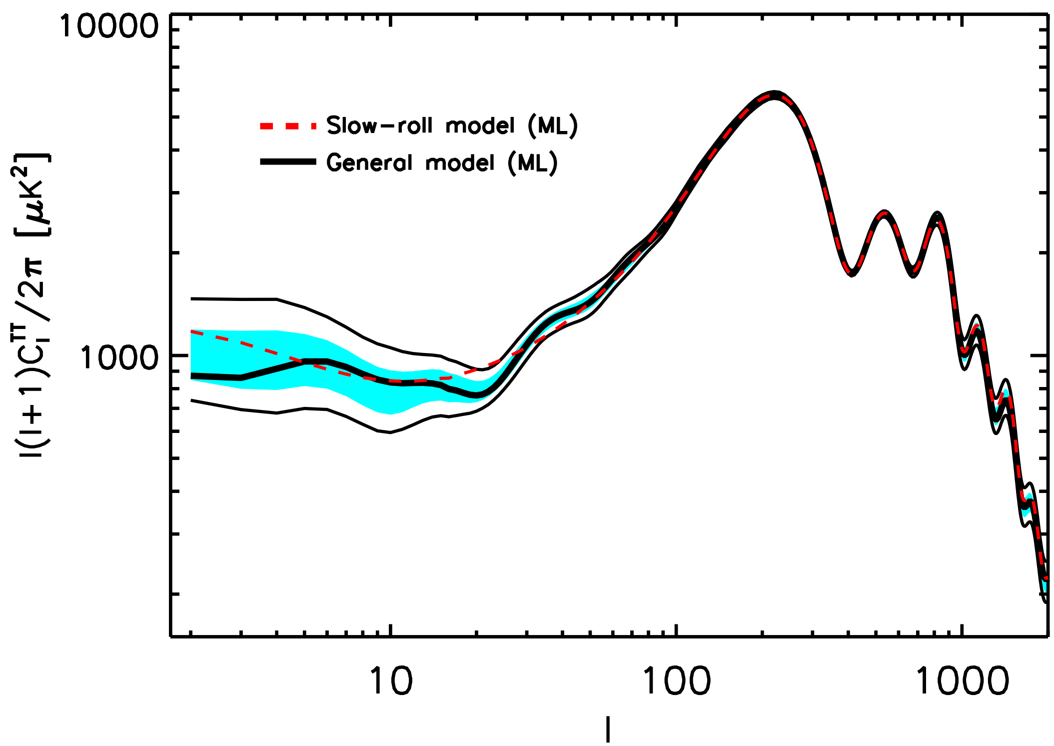



*C. Dvorkin, W. Hu, PRD (2009)*

# Work in progress

# Constraining the entire observable range of scales

- A complete basis of 20 PCs is required to account for large features in poorly constrained regions of the data.

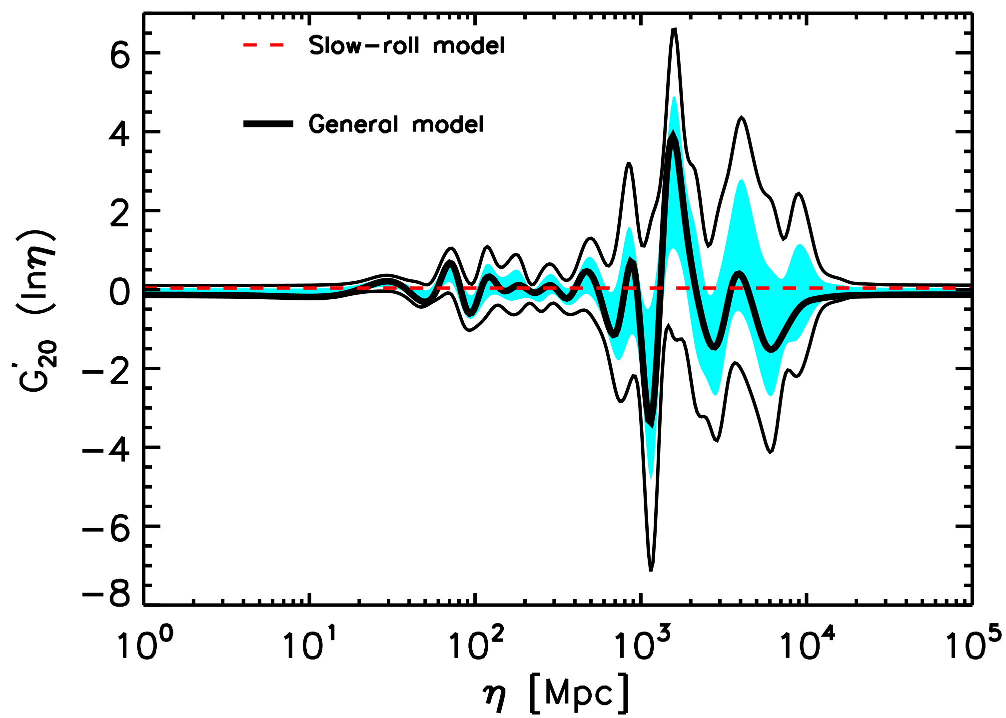



**WMAP7 + BICEP + QUAD data; SN + H0 + BBN constraints.**

***C. Dvorkin and W. Hu, in preparation***

# Observational constraints on the source function

- Inflationary models outside these bounds are in tension with the data.

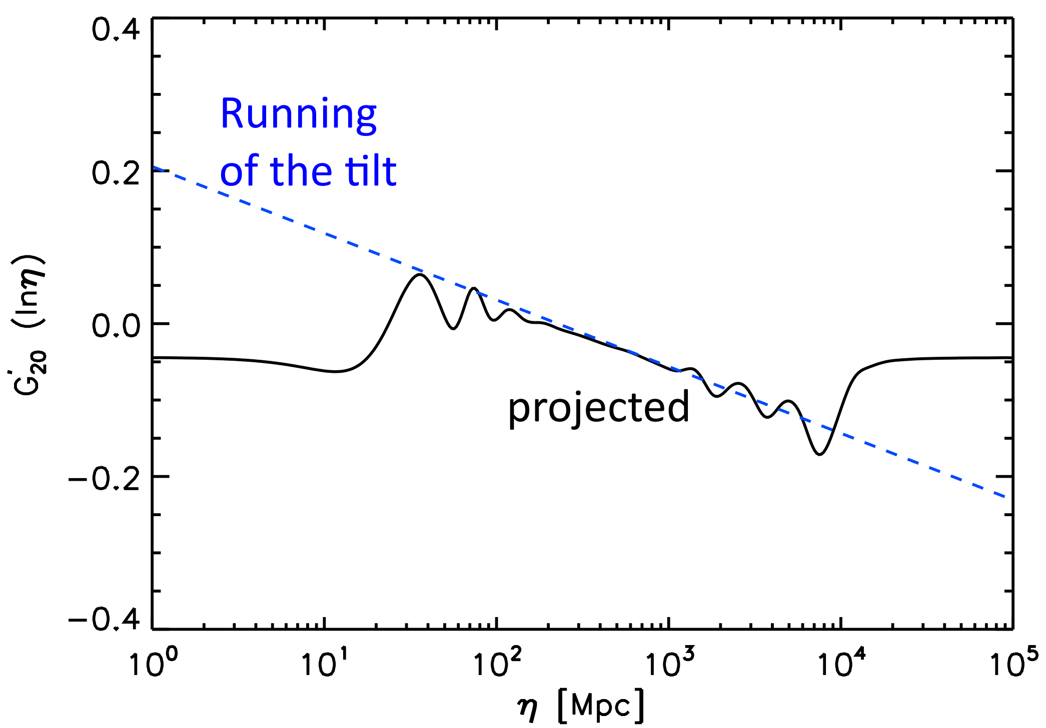



**WMAP7 + BICEP + QUAD data; SN + H0 + BBN constraints.**

***C. Dvorkin and W. Hu, in preparation***

# Testing models of inflation

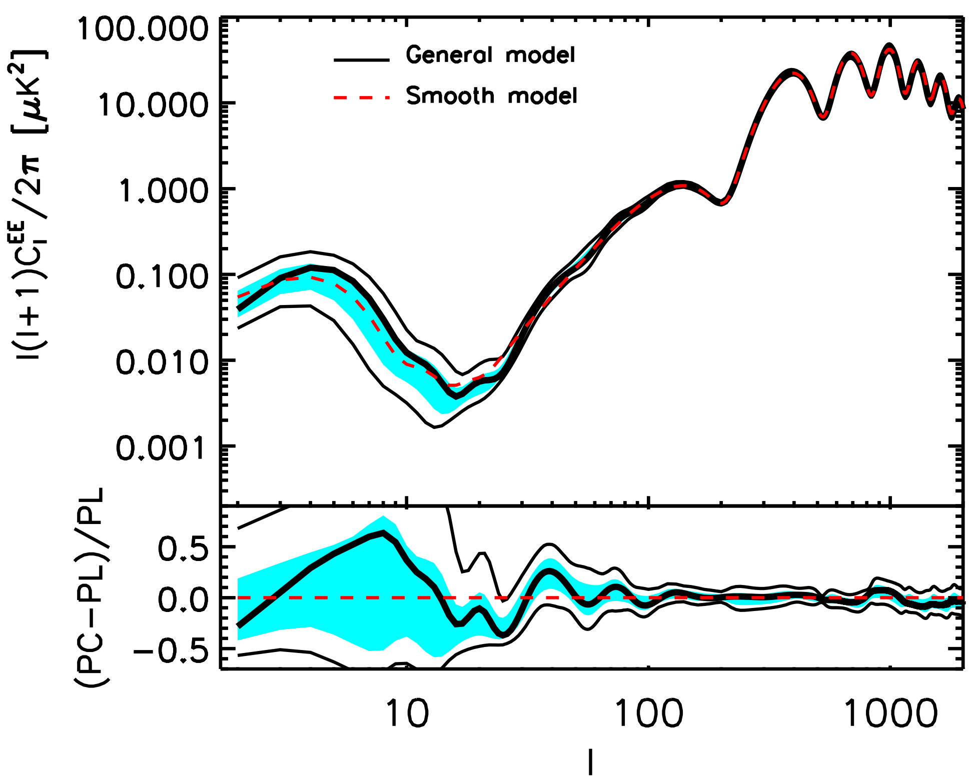
In practice, one can project any inflationary model onto the PC basis, and assess its significance using our posteriors.



# The Predictive Power of Polarization

- Measurements at l=20-40 (at the 40% level) will test the feature hypothesis.
  Caveat: confusion with reionization features.

***M. Mortonson, C. Dvorkin, H.V.Peiris, W. Hu, PRD (2009)***

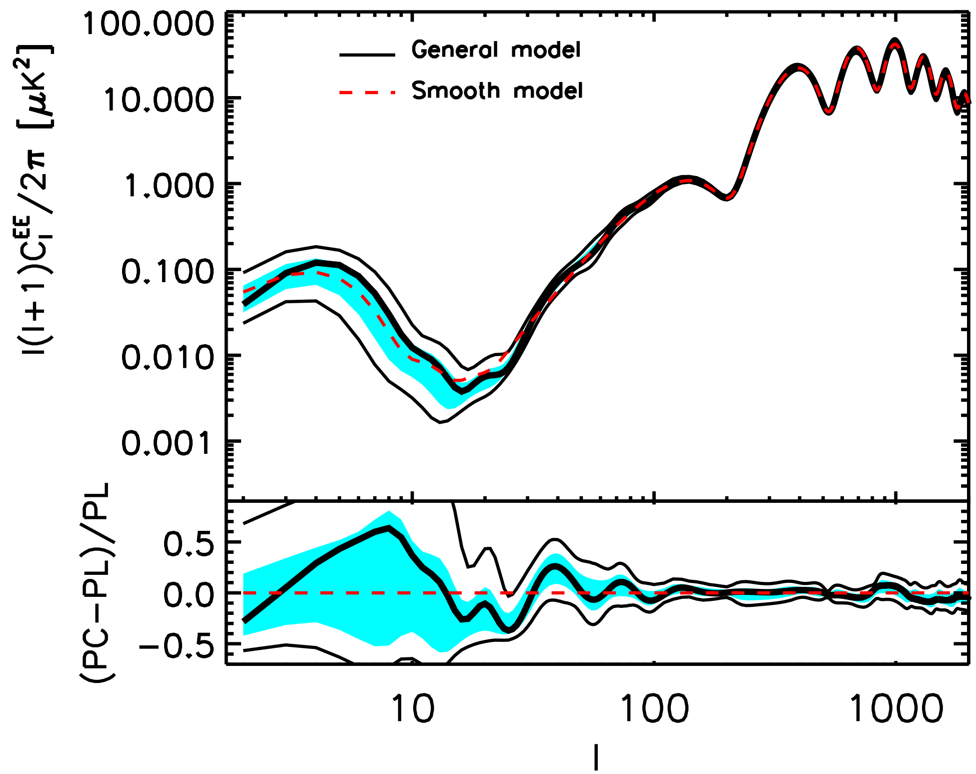



***C. Dvorkin and W. Hu, in preparation***

WMAP7 + BICEP + QUAD data;
SN + H0 + BBN constraints.

# Model-independent test of single-field inflation

- Measurements lying outside these bounds could potentially rule-out single field inflation.




***C. Dvorkin and W. Hu, in preparation***

# Conclusions and future directions

- Introduced a general formalism to constrain the inflationary potential from the data allowing for large amplitude and rapidly varying deviations from slow roll.

- Constraints around the first acoustic peak are consistent with a smooth inflationary potential.

➢ Empirical constraints can be used to test any single-field inflationary model.

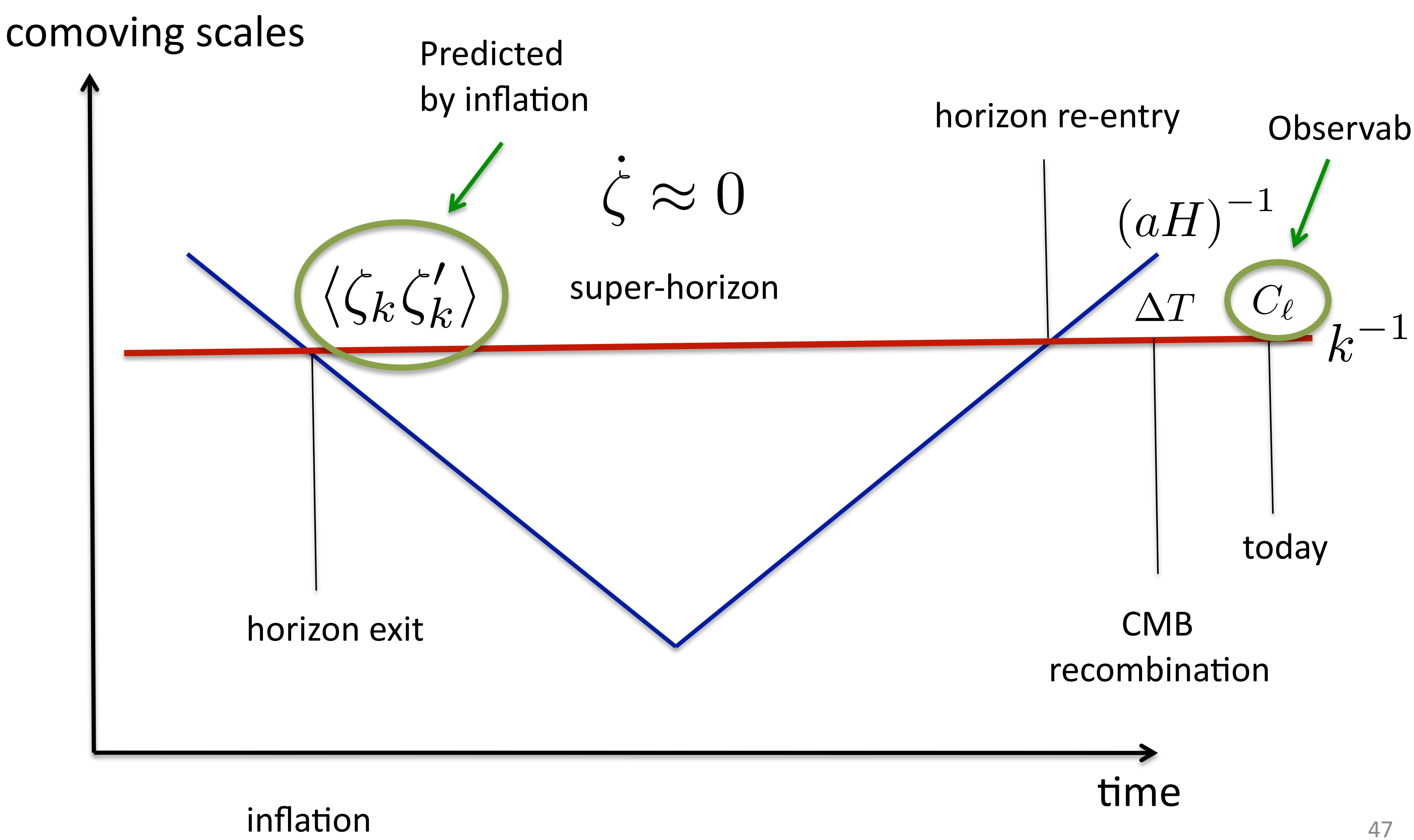
➢ Model-independent test of single-field inflation.

Future work:

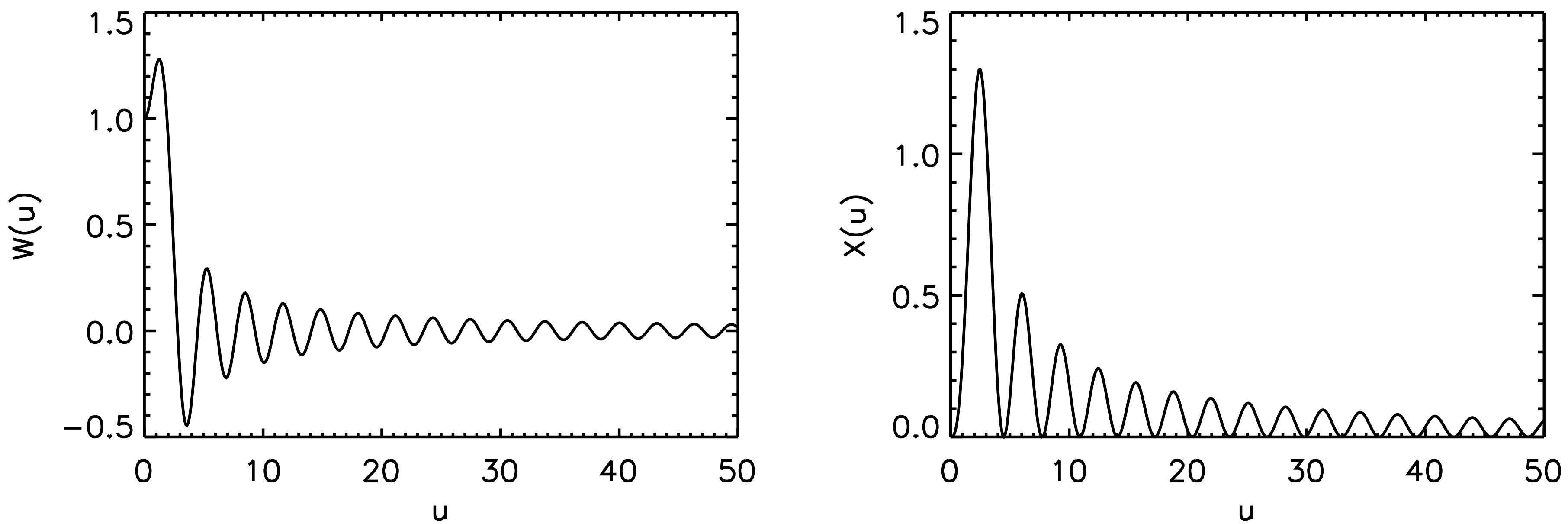
- Extend analysis to the entire range of observable CMB scales.

- Construct analogous formalism for calculating the bispectrum from the shape of the $V(\phi)$ potential.

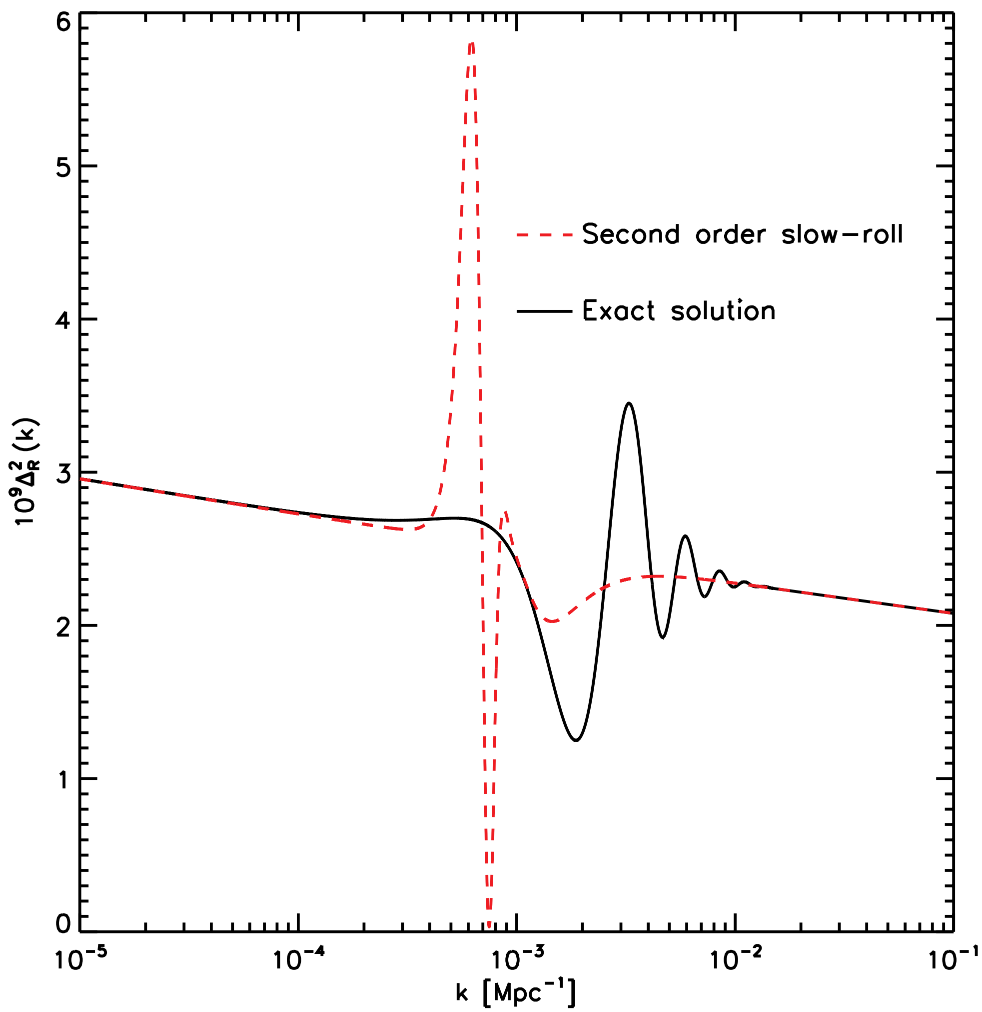
# Extra slides


comoving scales
Predicted
by inflation
$\dot{\zeta} \approx 0$
horizon re-entry
Observable
$(aH)^{-1}$
$\langle \zeta_k \zeta'_k \rangle$
super-horizon
$\Delta T$
$C_\ell$
$k^{-1}$
today
horizon exit
CMB
recombination
time
inflation

# GSR Green functions

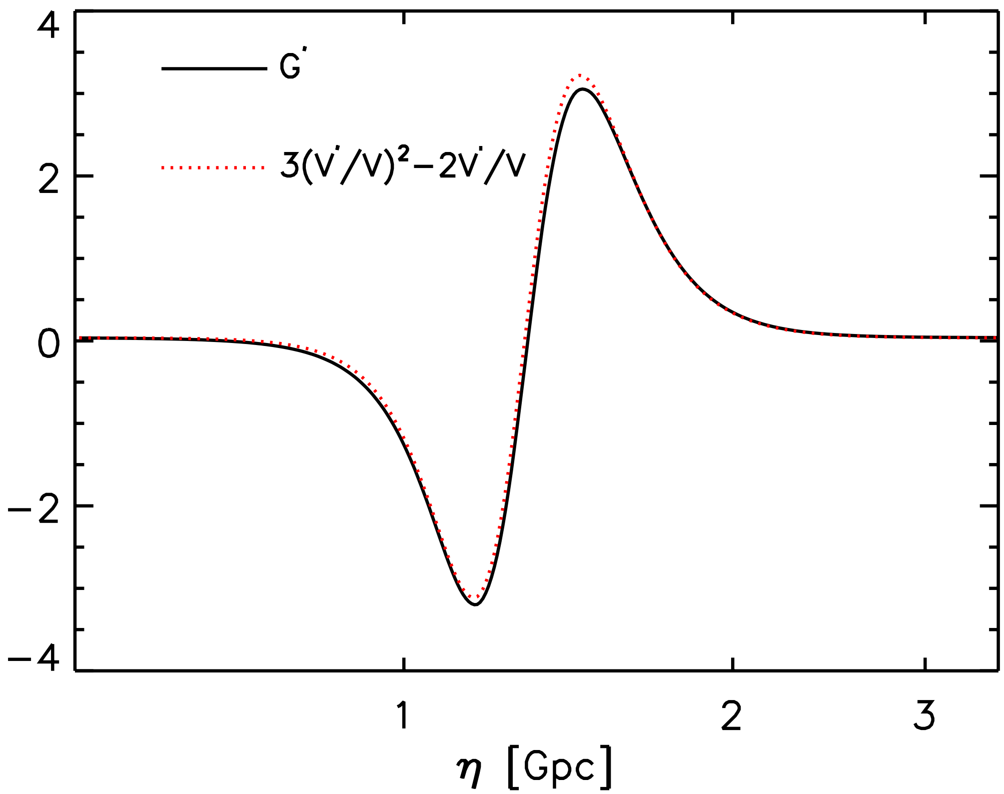


# Breaking Slow Roll



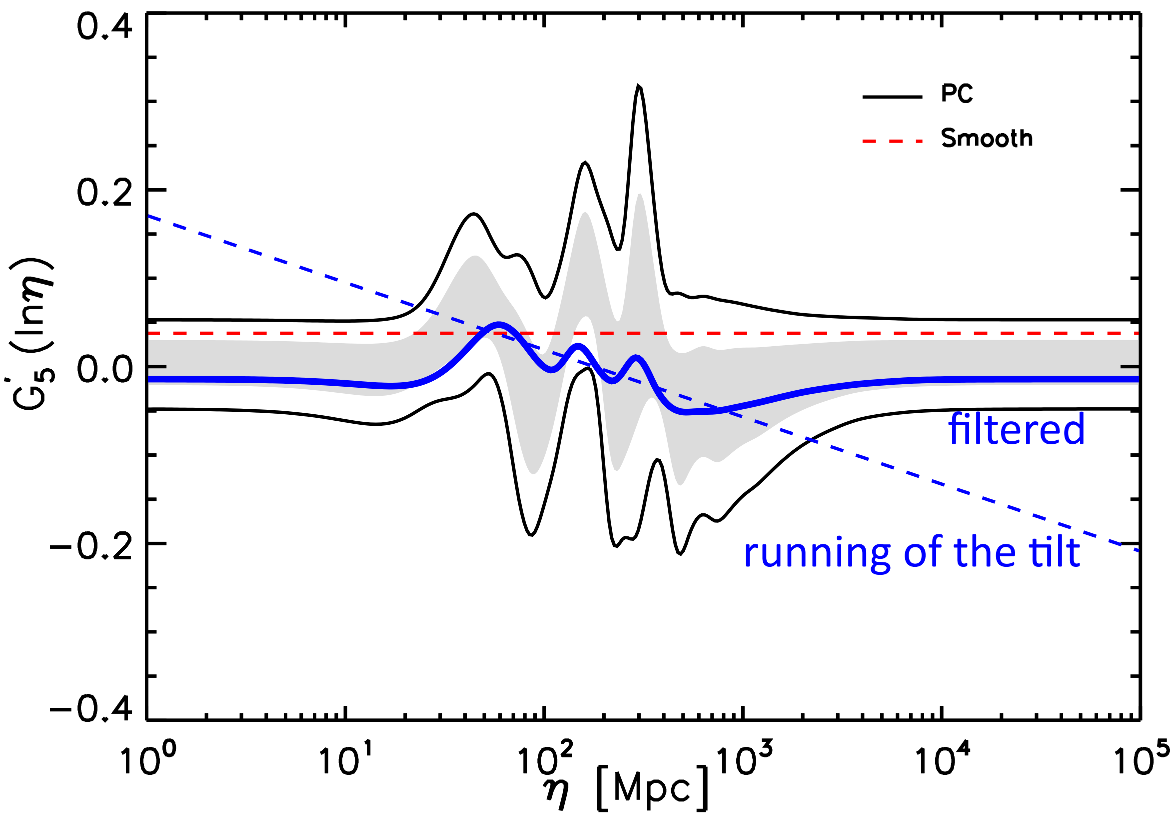
# The source function and the Potential

- Same functional dependence on the potential as the tilt in standard slow roll if features are crossed for an e-fold or less.
- Source has information on deviations from perfect slow roll.




*C. Dvorkin and W. Hu, PRD (2010)*

# Constraints on the source function with 5 PCs




**WMAP7, BICEP, QUAD; SN, H0, BBN constraints; flat universe.**

***C. Dvorkin, W. Hu, PRD (2009)***