Relativistic effects on LSS: power spectrum and bispectrum

Enea Di Dio

ED, Durrer, Marozzi, Montanari [arXiv:1510.04202]
Irsic , ED, Viel [arXiv:1510.03436]
ED, Montanari, Raccanelli, Durrer, Kamionkowski, Lesgourgues [arXiv:1603.09073]
ED [arXiv:1609.09044]

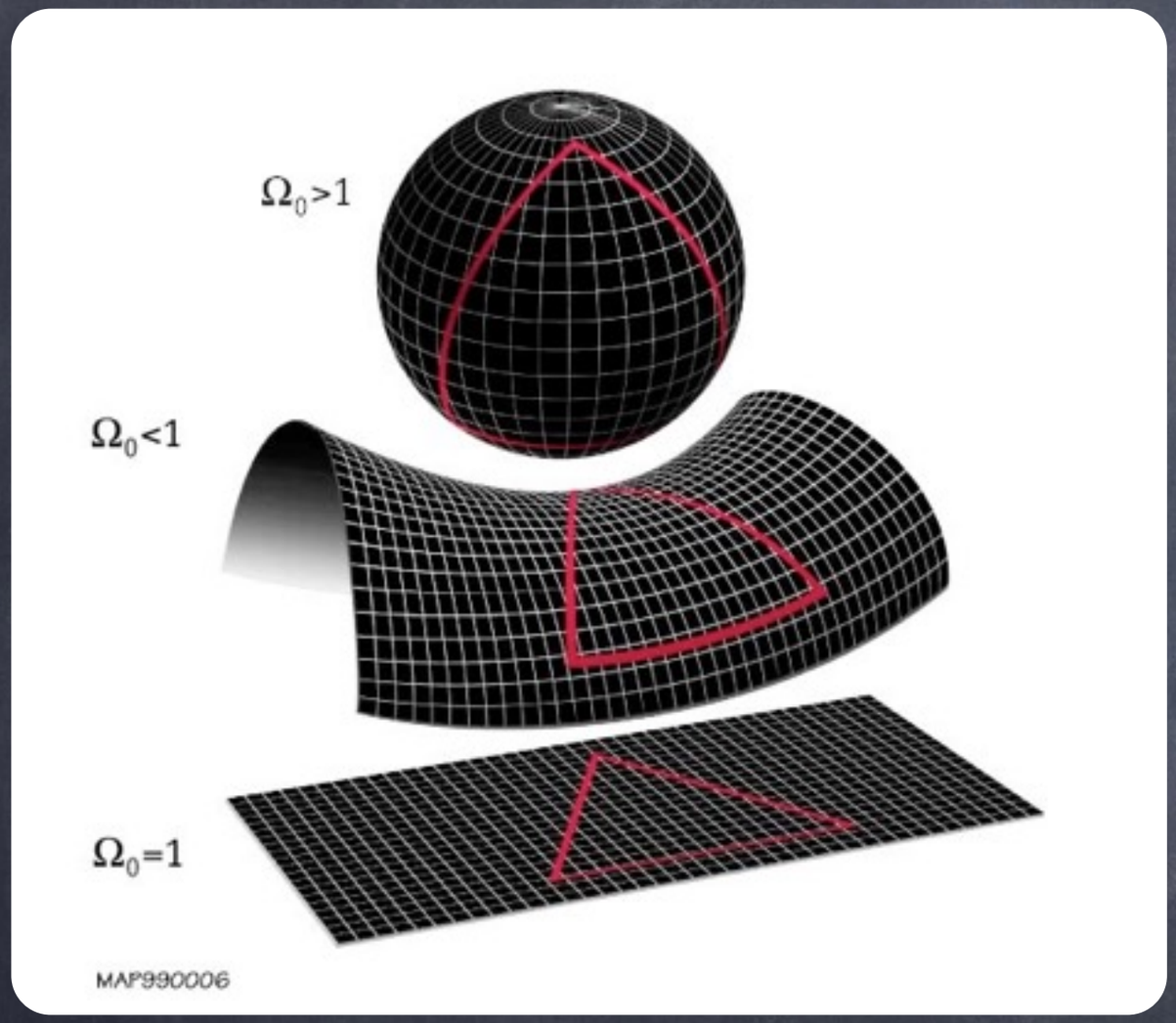
INPA Seminar

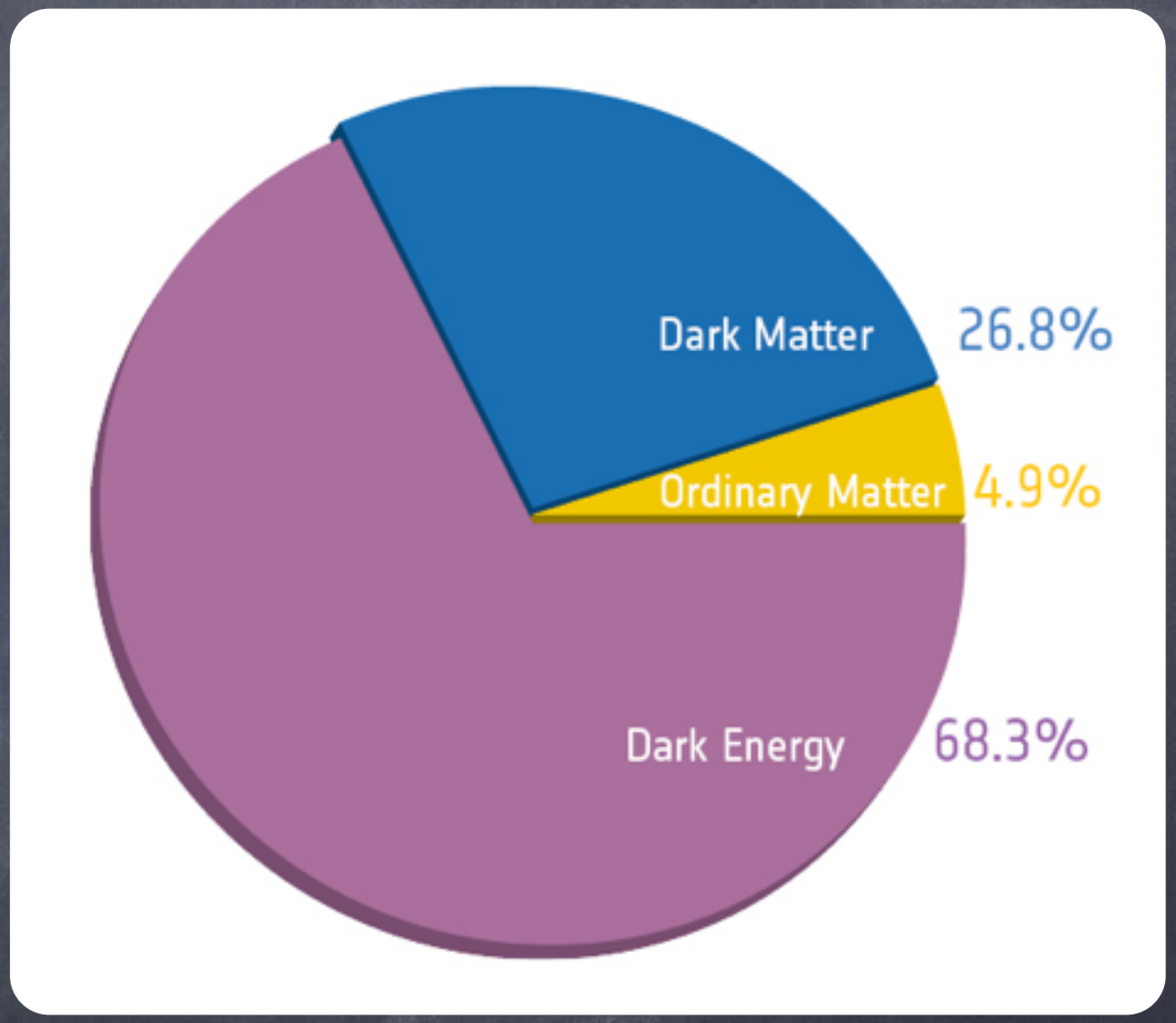
INAF
ISTITUTO NAZIONALE DI ASTROFISICA
NATIONAL INSTITUTE FOR ASTROPHYSICS

Berkeley, October 21st

erc
European Research Council
Established by the European Commission

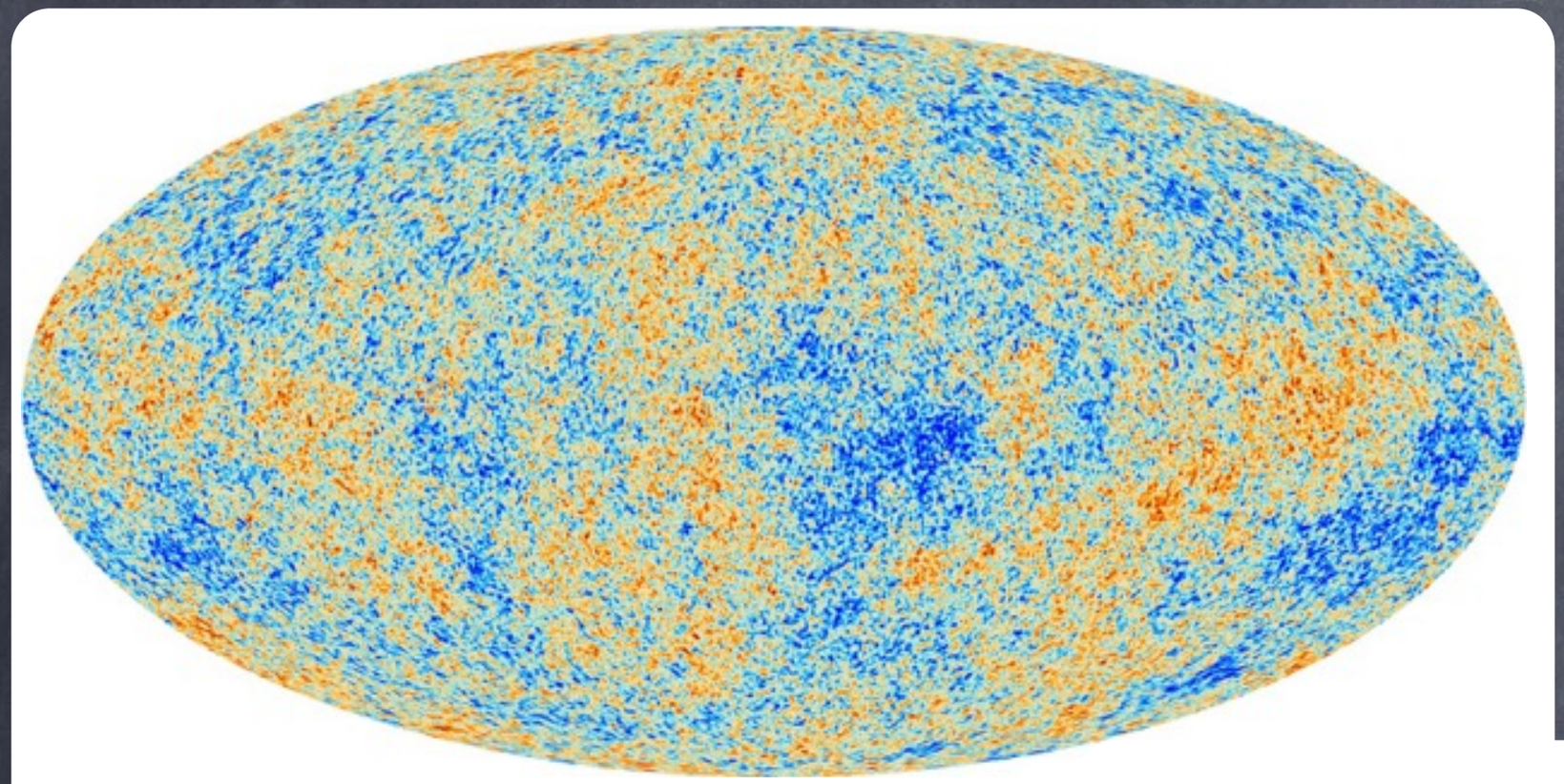
Standard Cosmological Model



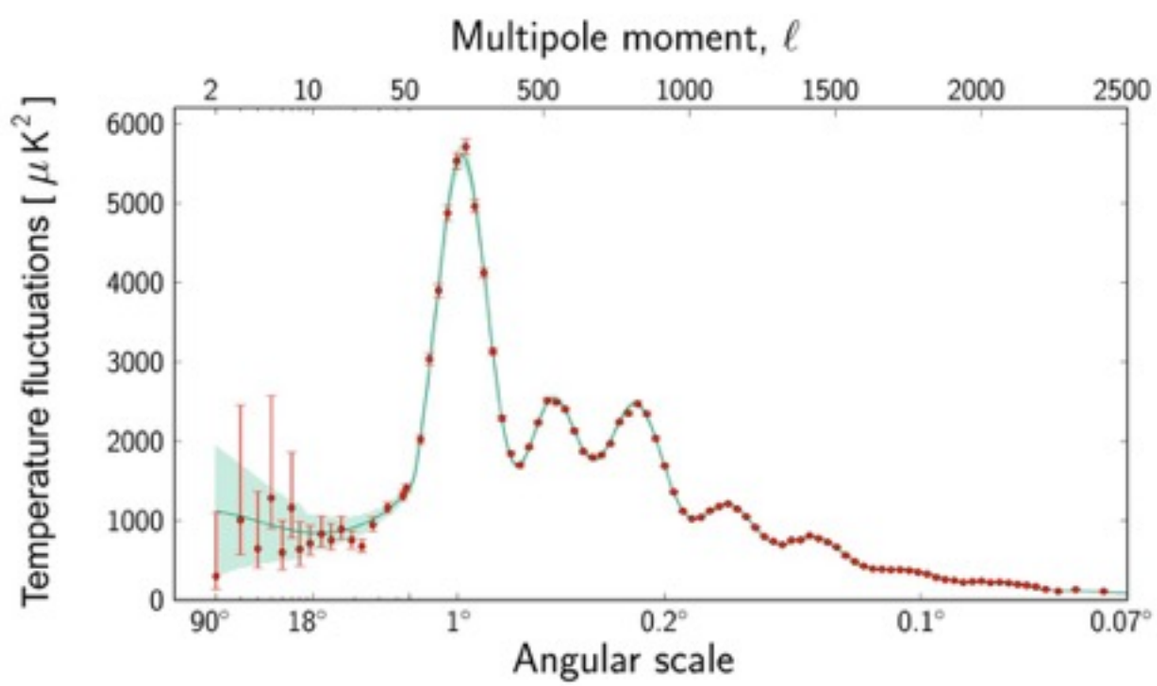


$$G_{\mu\nu} = \kappa T_{\mu\nu}$$

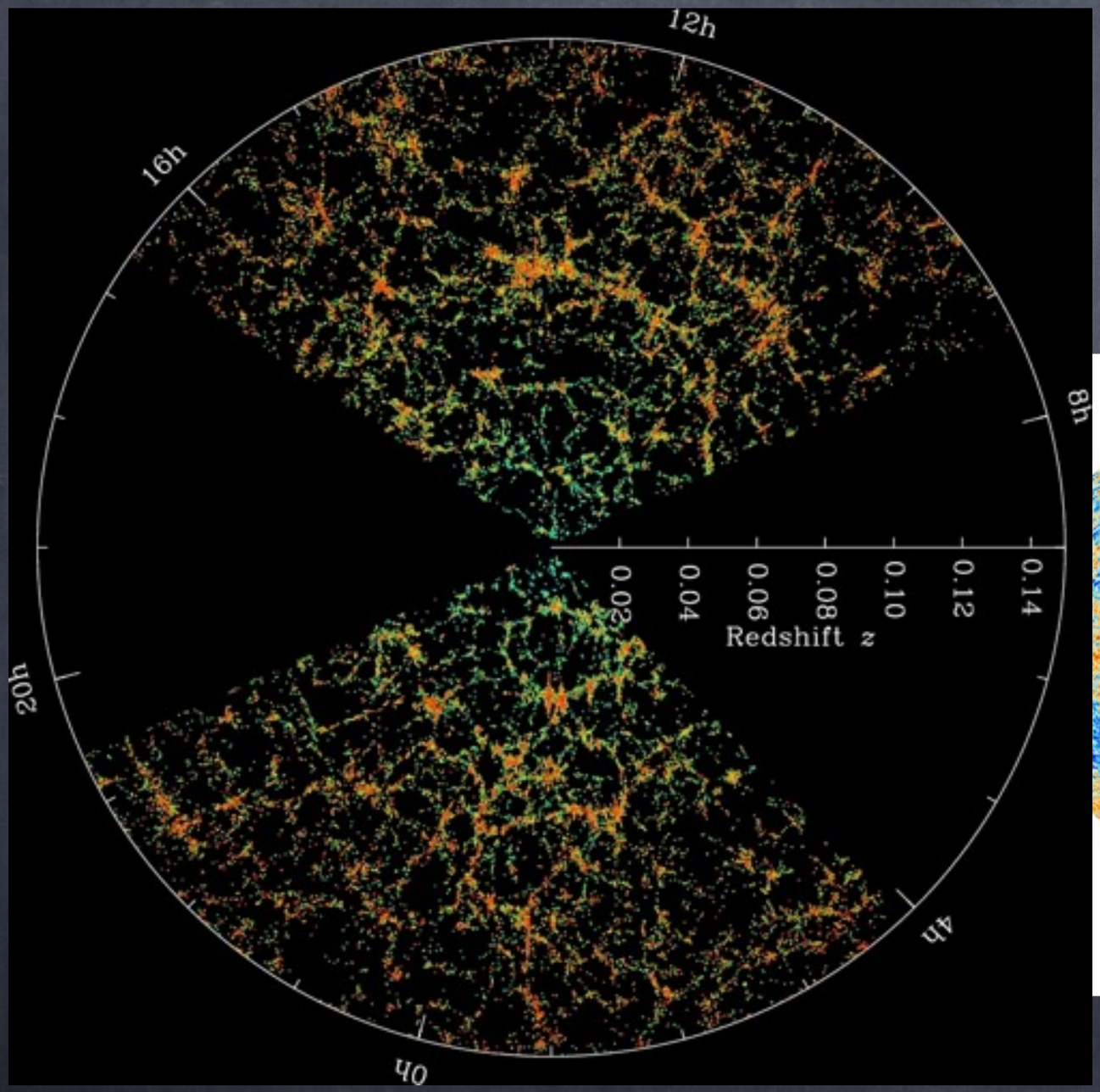
CMB



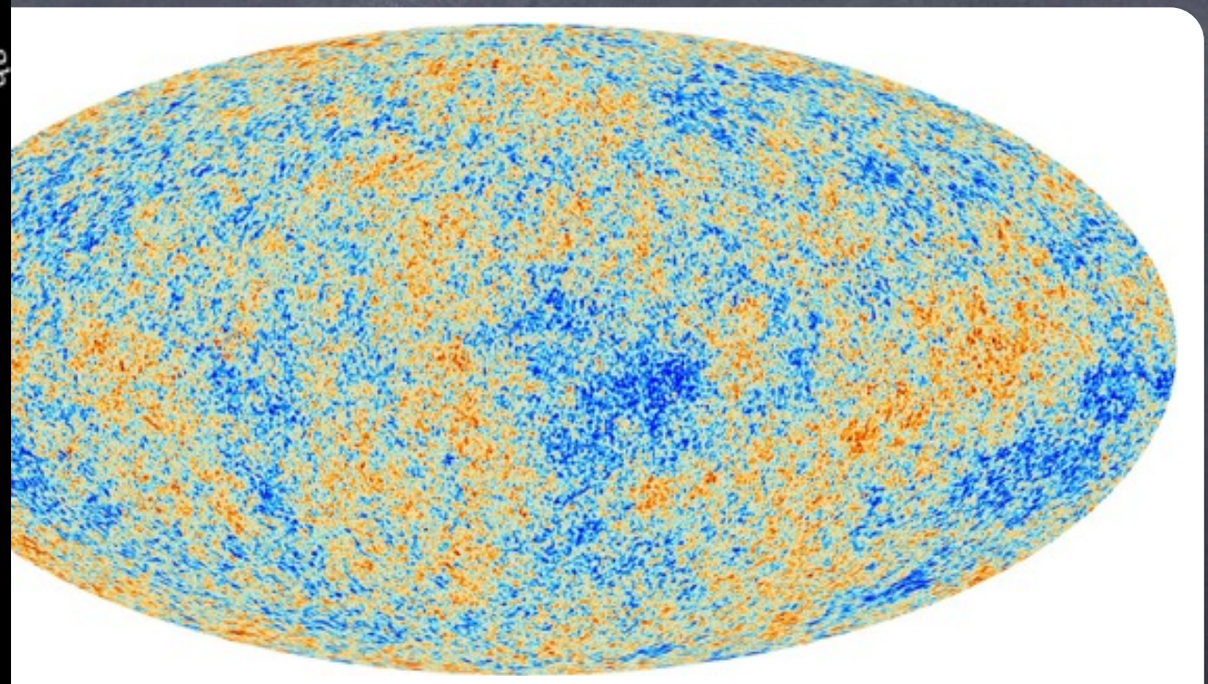
Planck Collaboration



Large Scale Structures



$$2\sum_{\ell=2}^{2500}(2\ell+1)\sim 10^7$$



Planck Collaboration

$$(3000)^3 = 2.7\times 10^{10}$$

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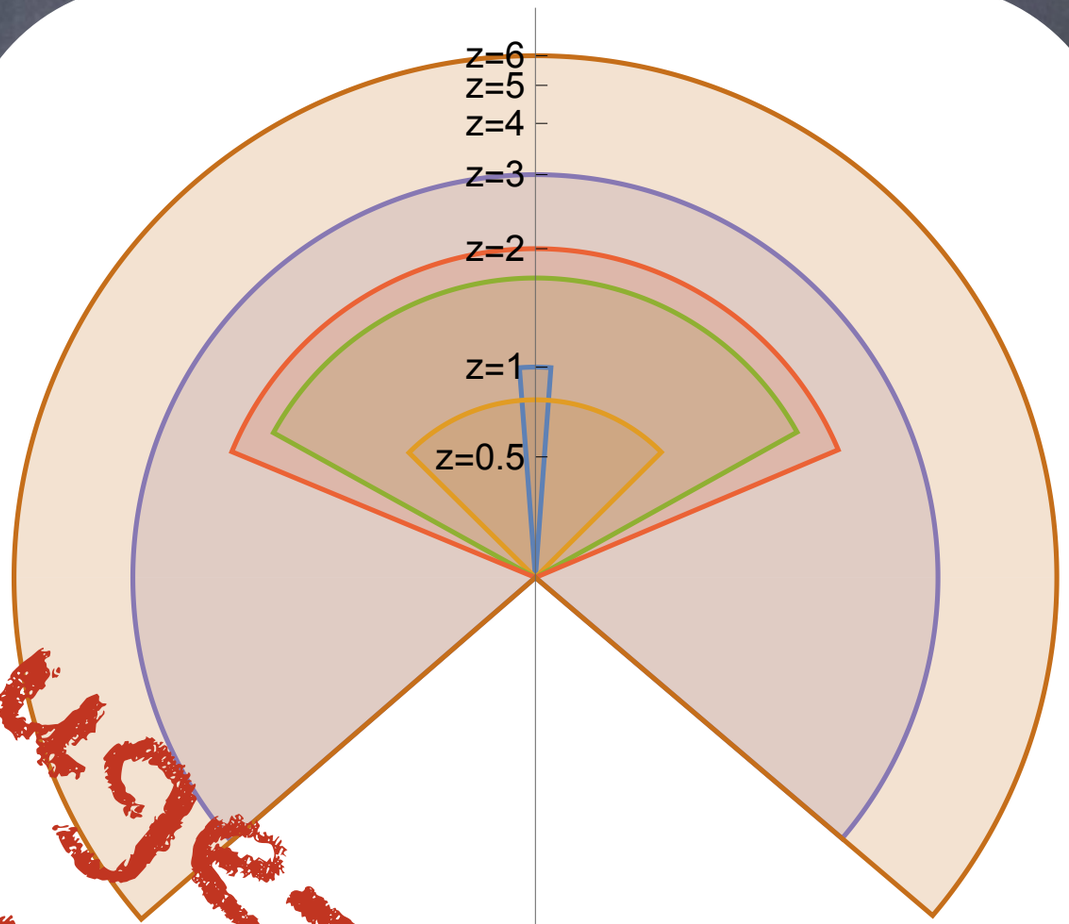
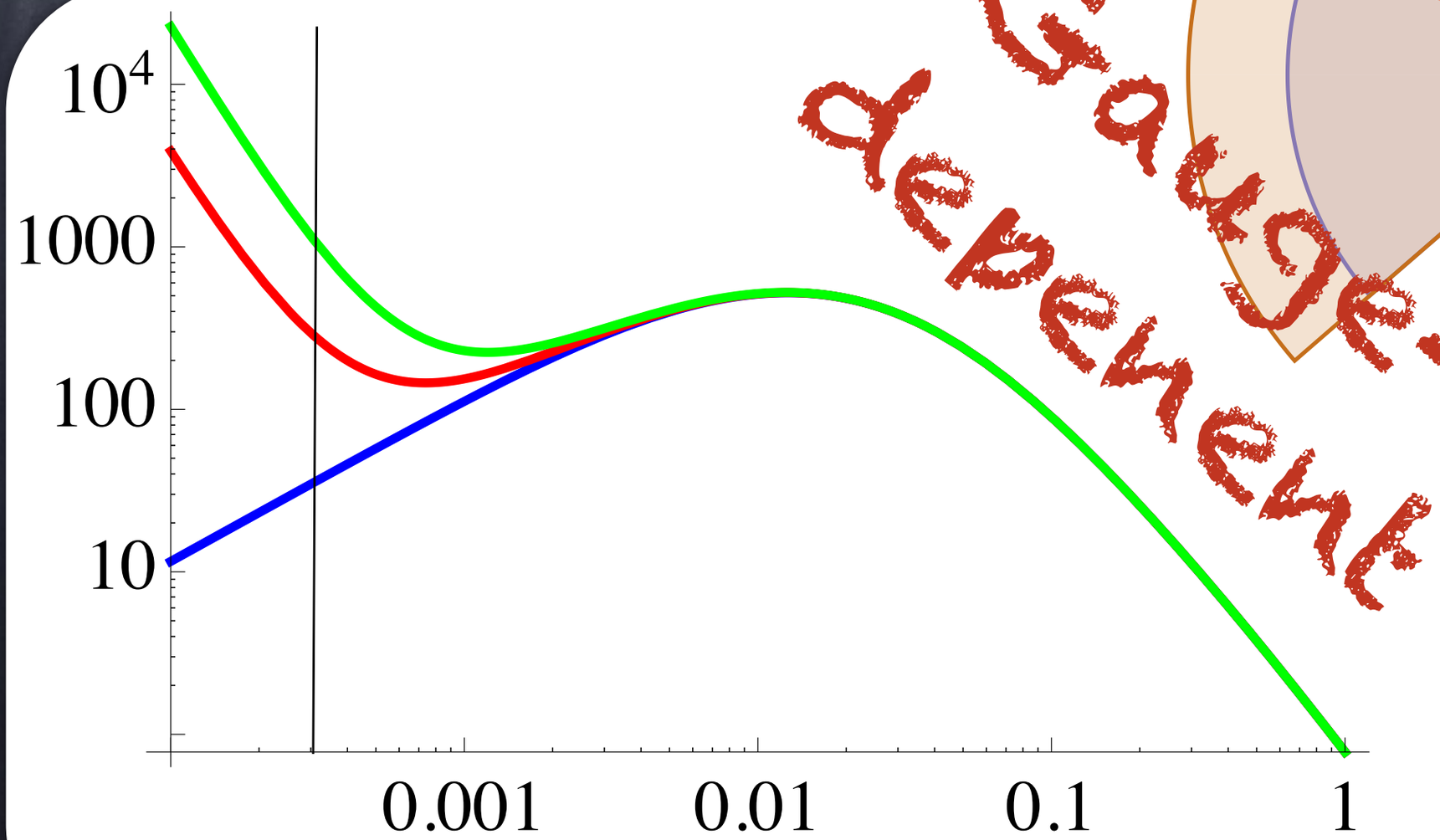
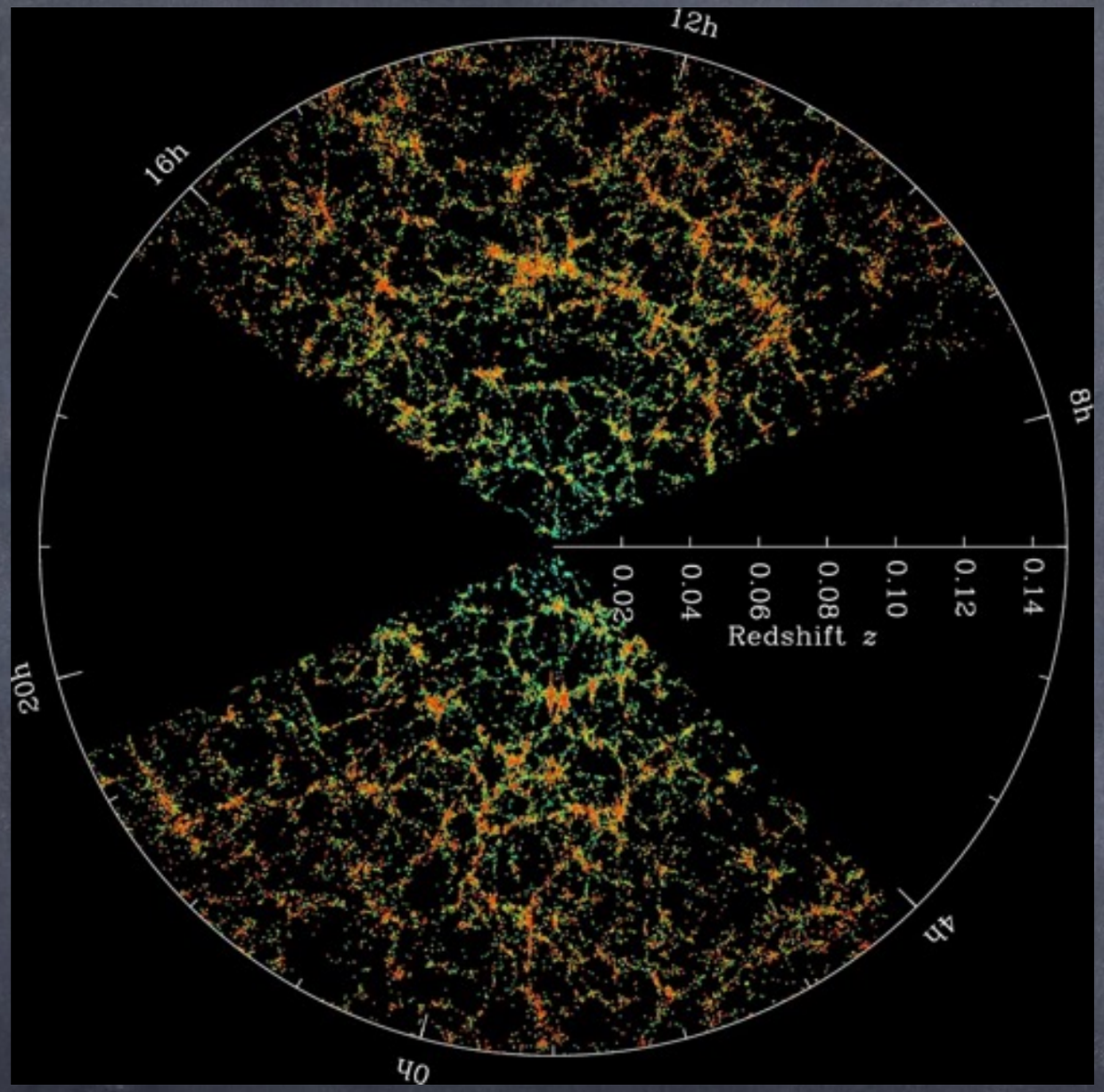




Gauge-dependent

Bonvin & Durrer [arXiv:1105.5280]

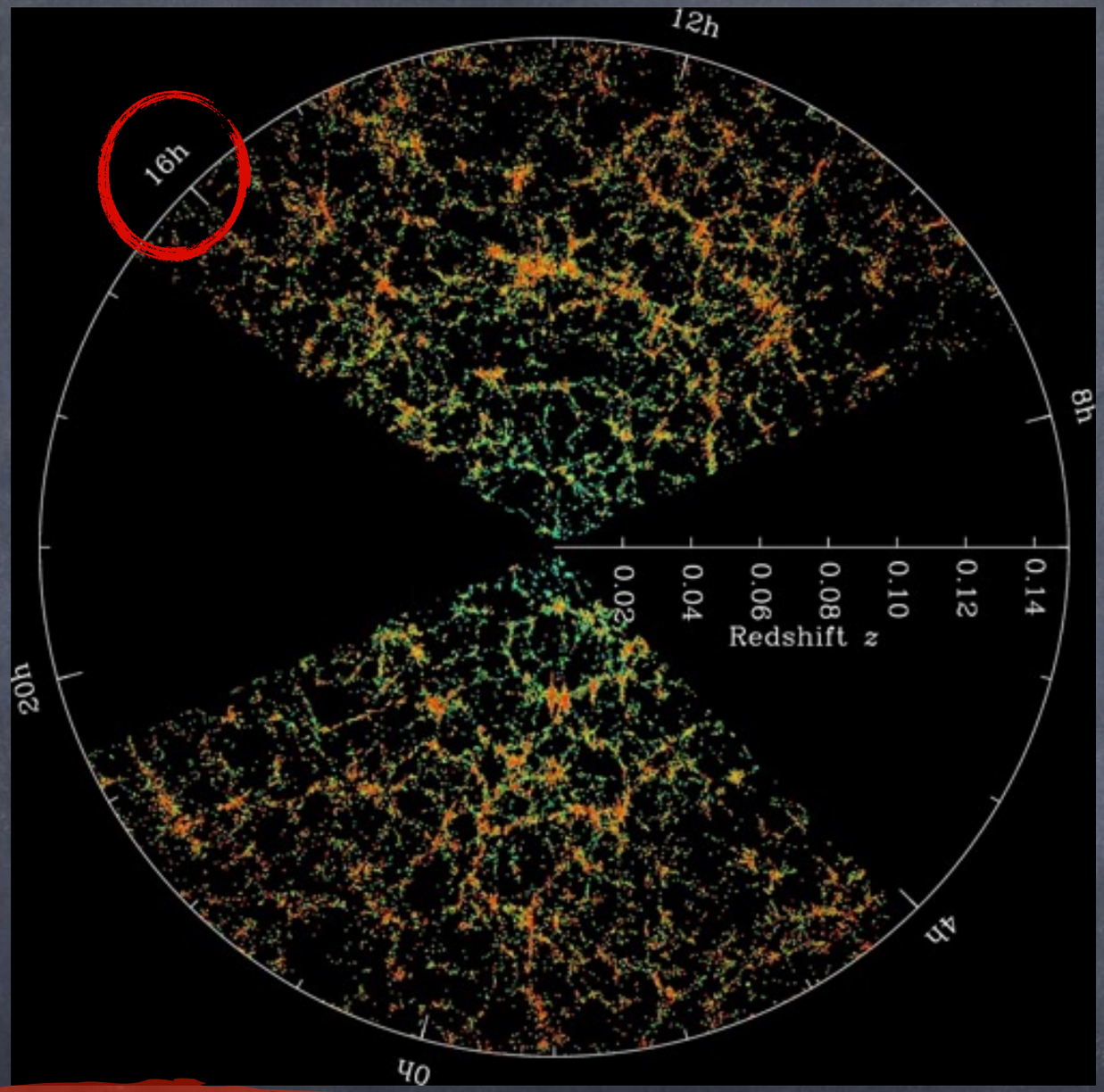
What do we really observe?



Angular position $\mathbf{n}$ Redshift z

$$N\left(\mathbf{n}, z\right) d\Omega_{\mathbf{n}} dz$$

What do we really observe?



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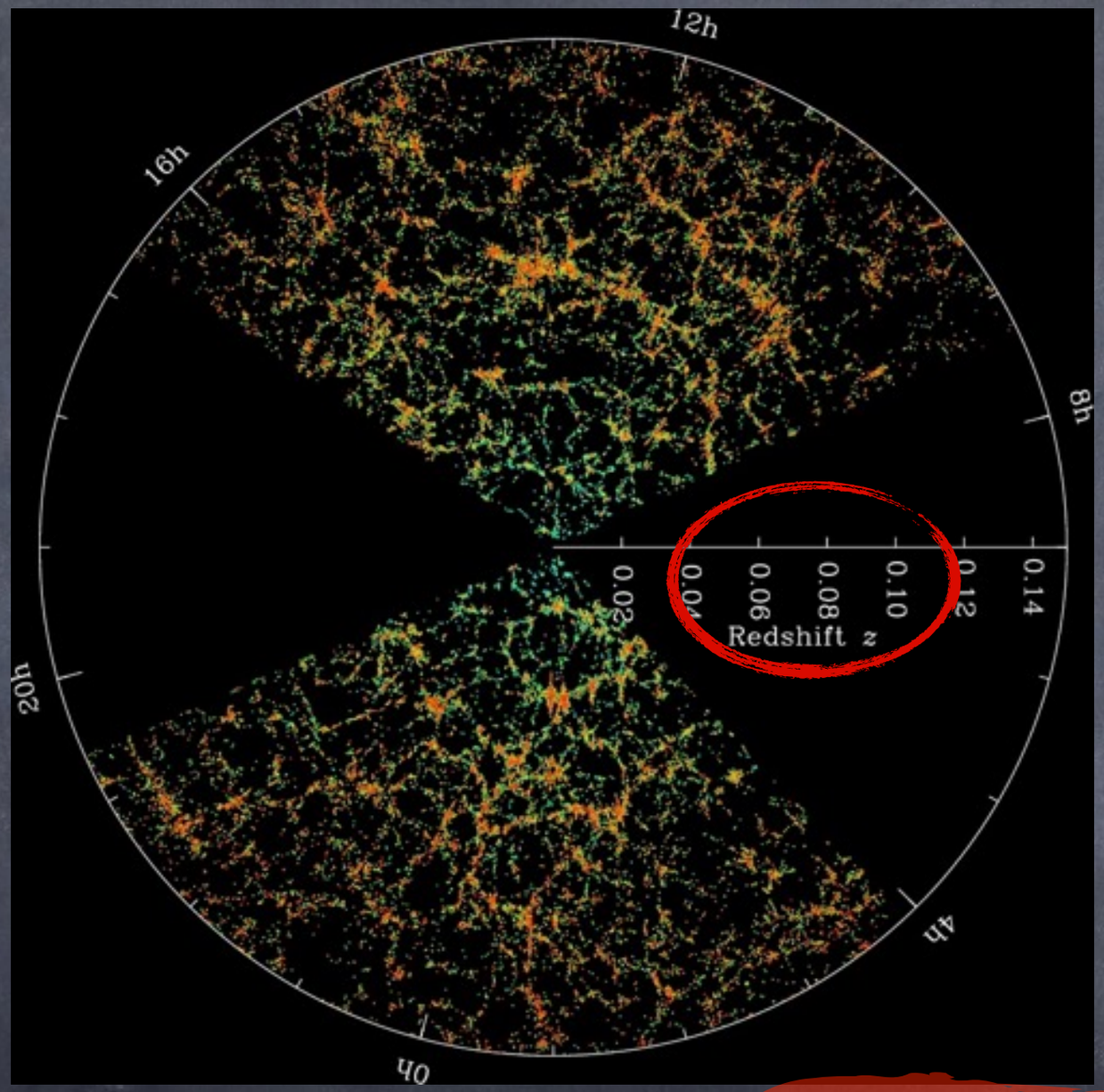

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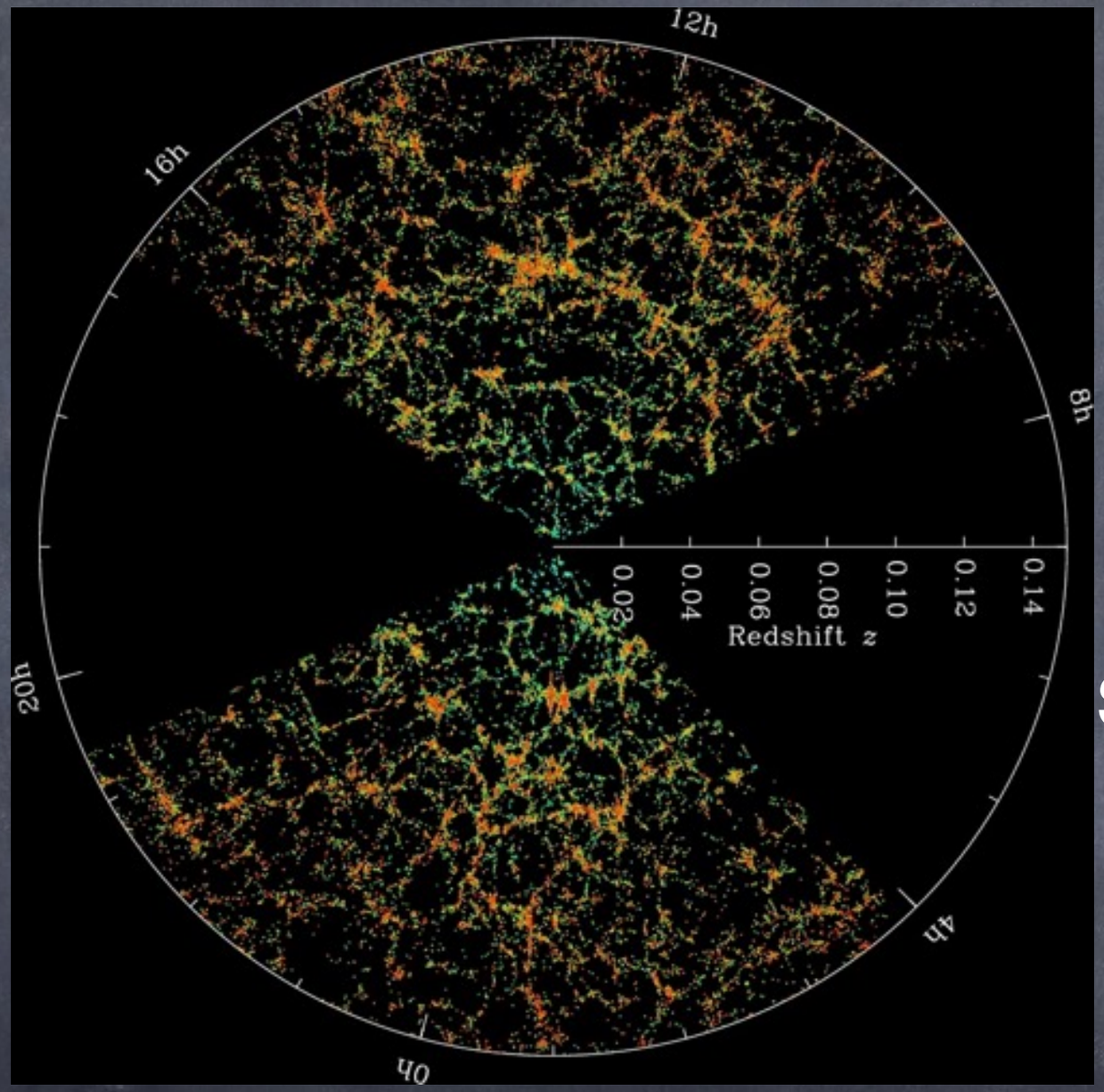

Info about mass, spectral type, …

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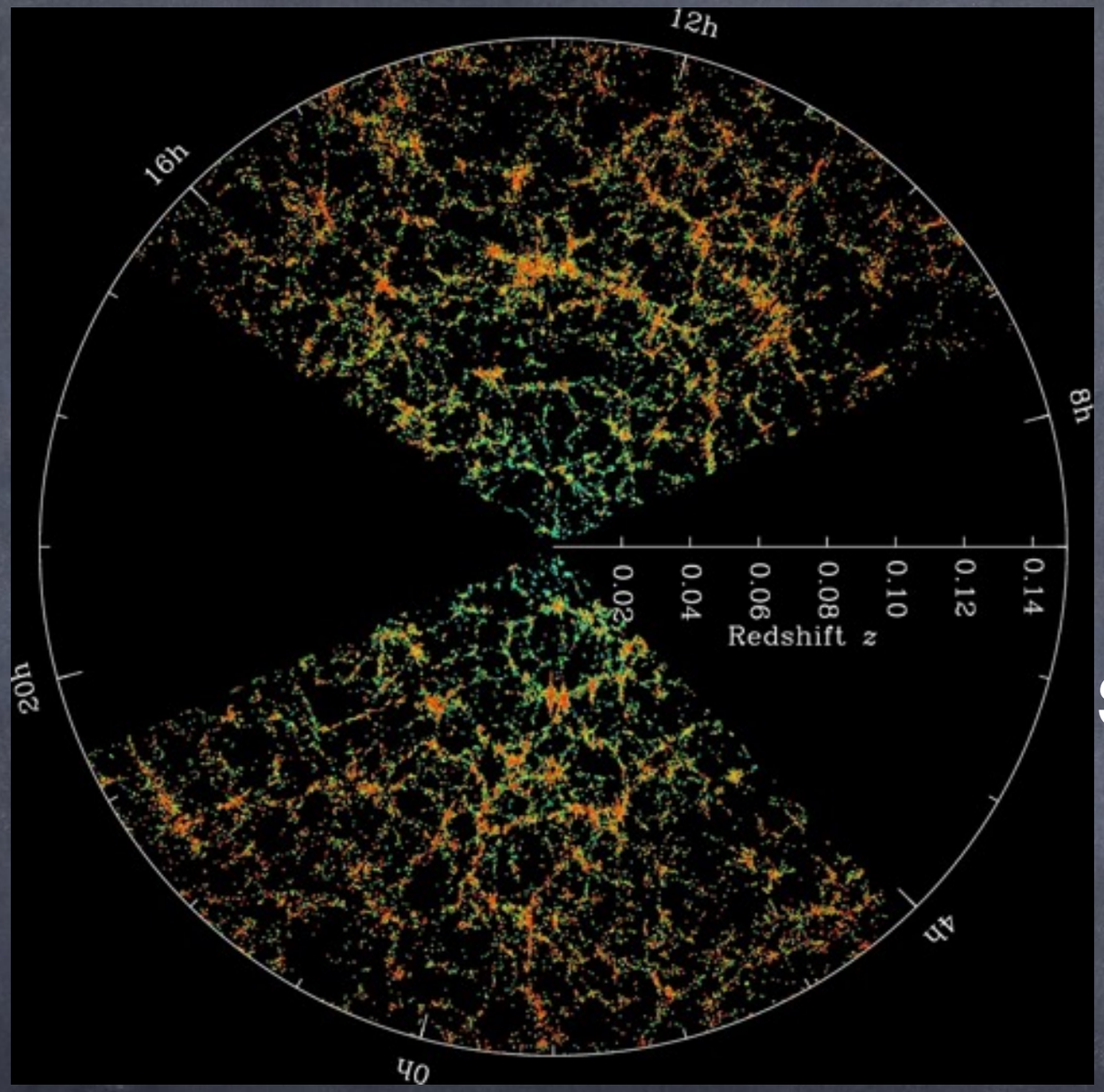

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$$\Delta(\mathbf{n}, z) \equiv \frac{N(\mathbf{n}, z) - <N>(z)}{<N>(z)}$$

Large Scale Structures

$$\Delta_N(\mathbf{n}, z, m_*) = b(z) D_{cm}\left(L > \bar{L}_*\right) + \mathcal{H}^{-1}\partial_r \mathbf{n}\cdot\mathbf{v}$$

Standard

$$- \frac{2-5s}{2}\int_0^r \frac{r-r'}{rr'}\Delta_\Omega\left(\Psi+\Phi\right)dr'$$

$$+ \left(5s + \frac{2-5s}{\mathcal{H}r} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - f^N_{\rm evo}\right)\mathbf{n}\cdot\mathbf{v} + (f^N_{\rm evo} - 3)\mathcal{H}v$$

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Relativistic Effects

Bonvin & Durrer [arXiv:1105.5280],
Challinor & Lewis [arXiv:1105.5292],
Yoo [arXiv:1009.3021]

$$P_{\rm obs}\left(k,\mu,z\right)=b\left(z\right)^2\left(1+\beta\left(z\right)\mu^2\right)^2P\left(k,z\right)$$

Standard

Relativistic Effects

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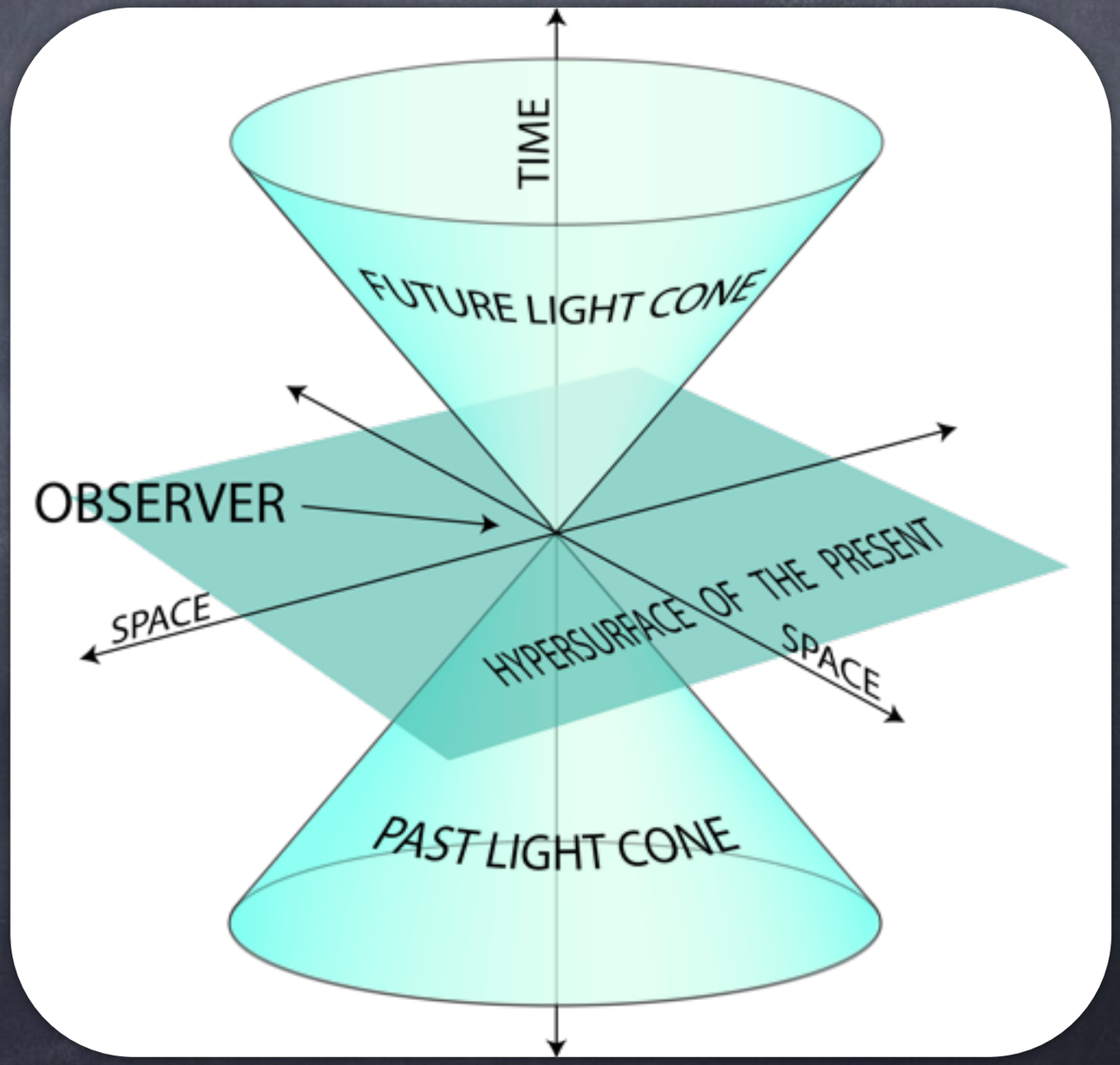

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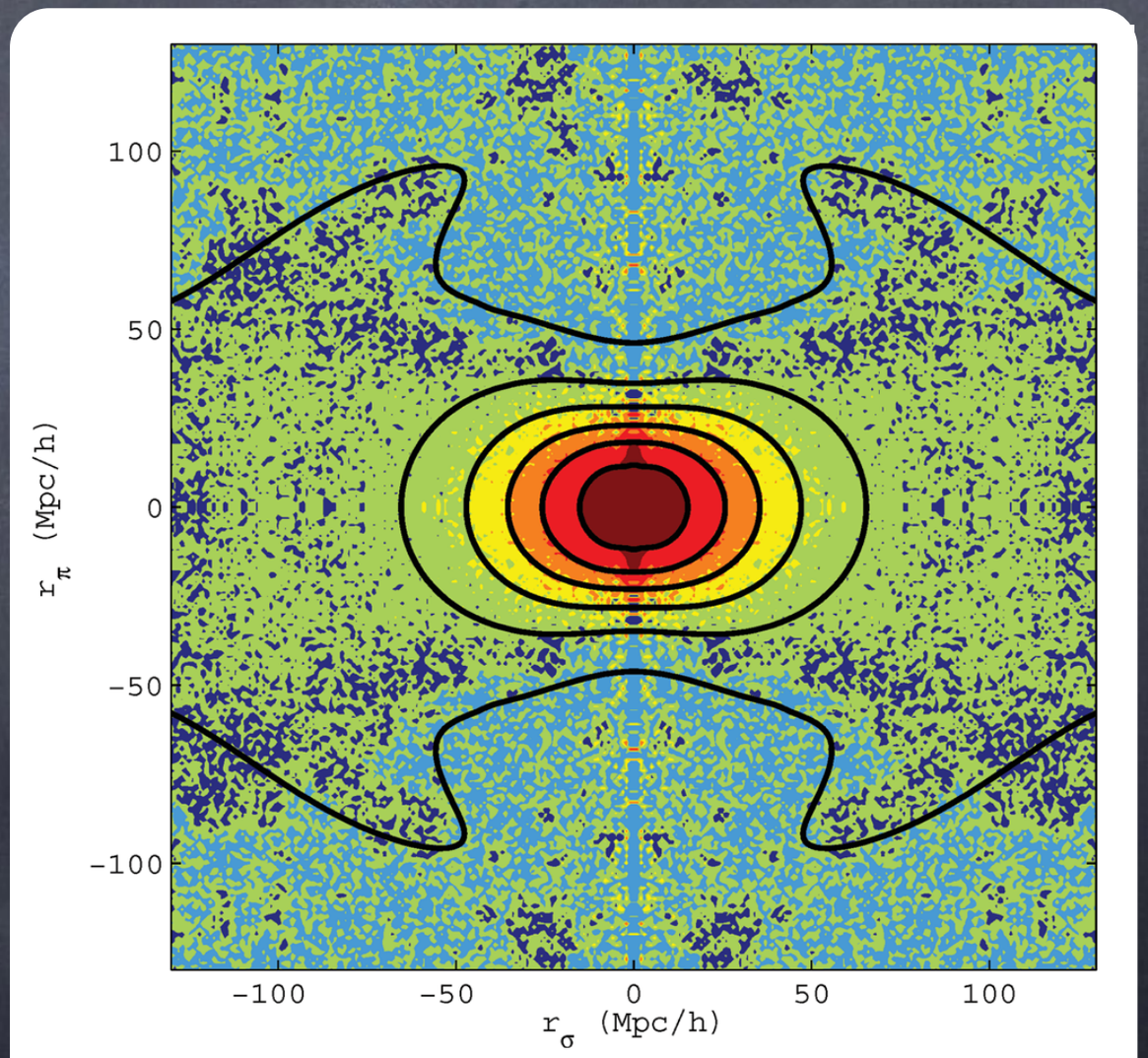
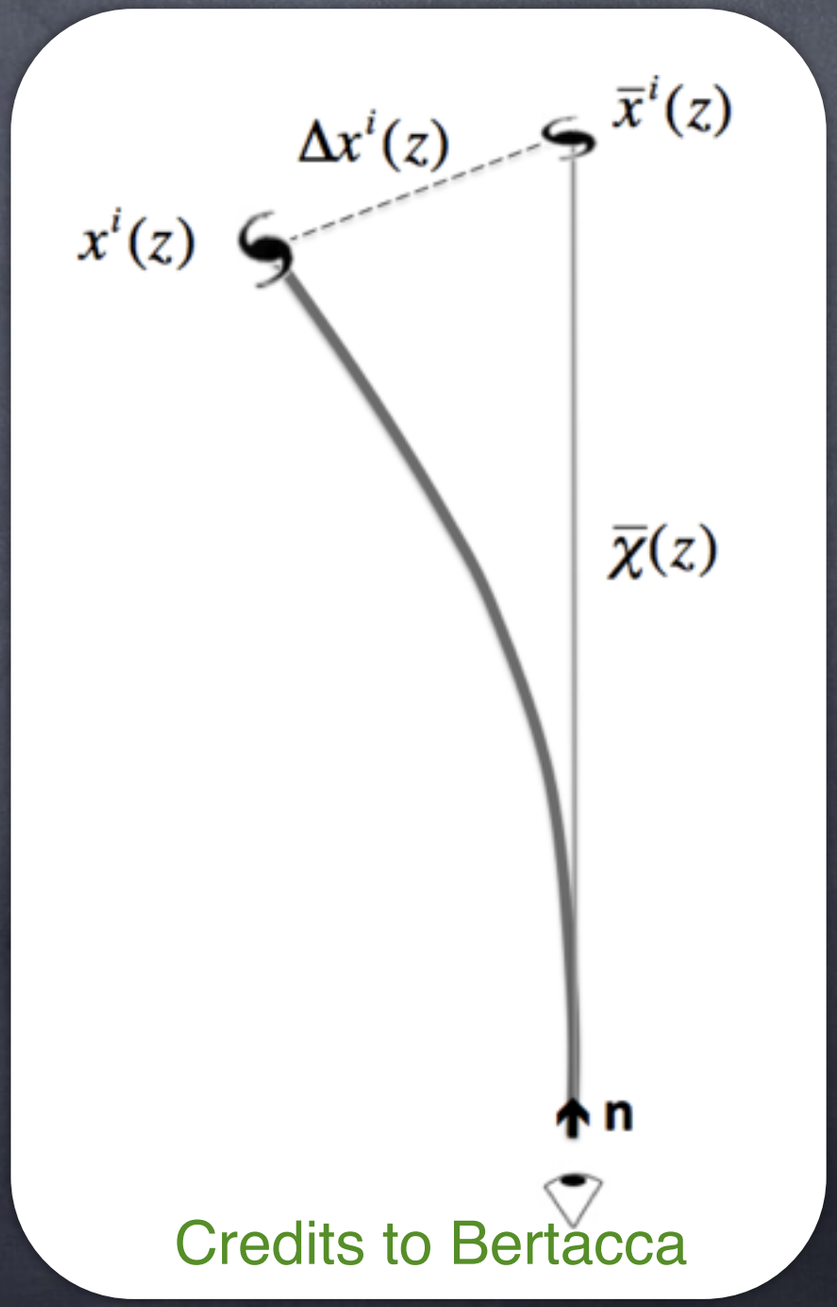


Reid et al ’12 [arXiv:1203.6641]

What do we really observe?

To compute $\Delta(\mathbf{n}, z) \equiv \dfrac{N(\mathbf{n}, z) - < N > (z)}{< N > (z)}$
we have to consider:

- observation on the past lightcone
- redshift perturbed by peculiar velocity
- light deflection



Credits to Bertacca

Large Scale Structures

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$\sim D$

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$$\sim \frac{\mathcal{H}}{k}D \quad + \left(5s + \frac{2-5s}{\mathcal{H}r} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - f^N_{\rm evo}\right)\mathbf{n}\cdot\mathbf{v} + \left(f^N_{\rm evo} - 3\right)\mathcal{H}v$$

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$$\sim \left(\frac{\mathcal{H}}{k}\right)^2 D$$

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$$

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Cosmic Magnification

DETECTION OF COSMIC MAGNIFICATION WITH THE SLOAN DIGITAL SKY SURVEY

Ryan Scranton[1], Brice Ménard[2], Gordon T. Richards[3], Robert C. Nichol[4], Adam D. Myers[5], Bhuvnesh Jain[6], Alex Gray[7], Matthias Bartelmann[8], Robert J. Brunner[5], Andrew J. Connolly[1], James E. Gunn[3], Ravi K. Sheth[6], Neta A. Bahcall[3], John Brinkman[9], Jon Loveday[10], Donald P. Schneider[11], Aniruddha Thakar[12], Donald G. York[13,14]

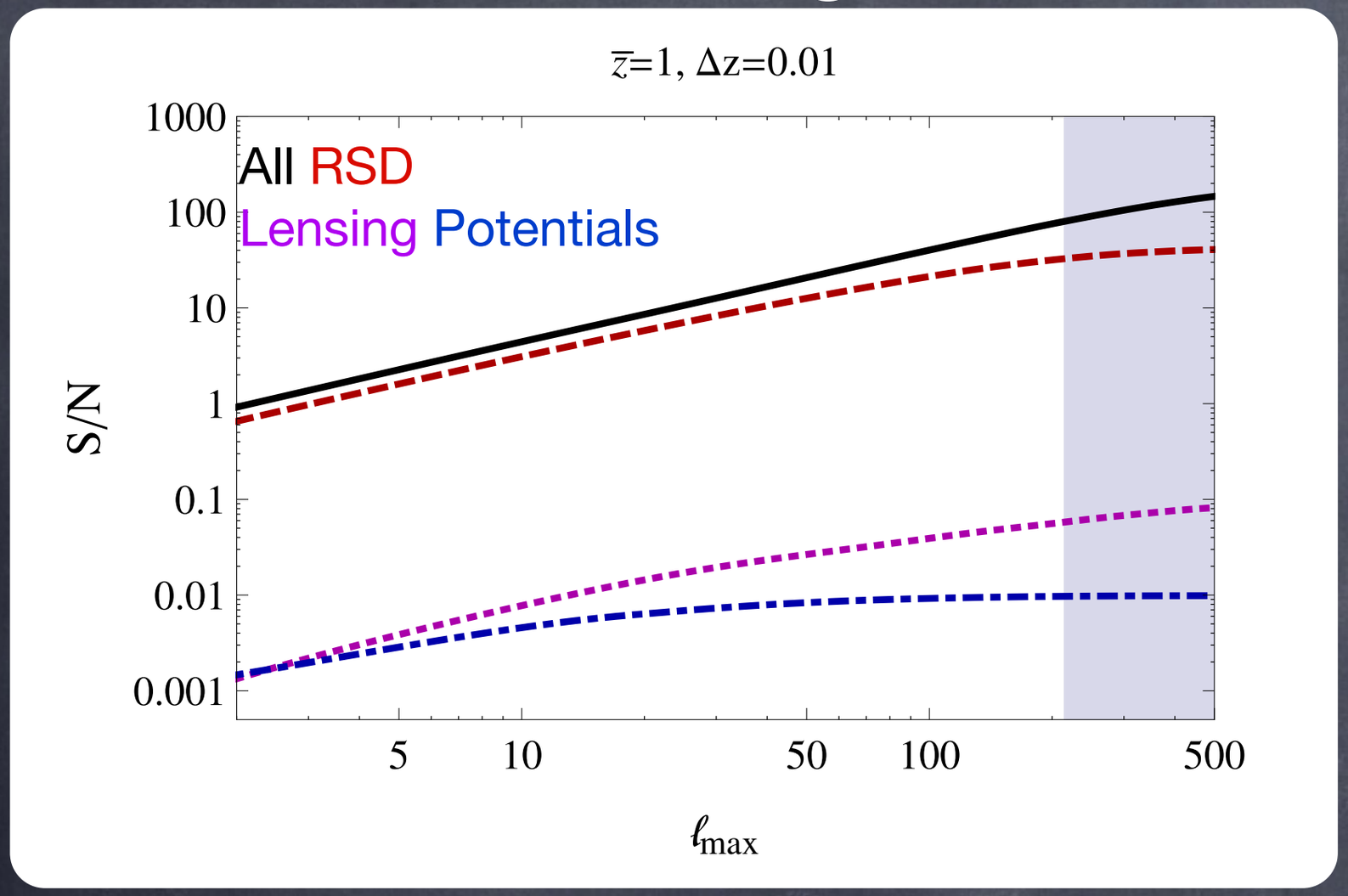
Draft: February 2, 2008

ABSTRACT

We present an 8σ detection of cosmic magnification measured by the variation of quasar density due to gravitational lensing by foreground large scale structure. To make this measurement we used 3800 square degrees of photometric observations from the Sloan Digital Sky Survey (SDSS) containing $\sim 200,000$ quasars and 13 million galaxies. Our measurement of the galaxy-quasar cross-correlation function exhibits the amplitude, angular dependence and change in sign as a function of the slope of the observed quasar number counts that is expected from magnification bias due to weak gravitational lensing. We show that observational uncertainties (stellar contamination, Galactic dust extinction, seeing variations and errors in the photometric redshifts) are well controlled and do not significantly affect the lensing signal. By weighting the quasars with the number count slope, we combine the cross-correlation of quasars for our full magnitude range and detect the lensing signal at $> 4\sigma$ in all five SDSS filters. Our measurements of cosmic magnification probe scales ranging from $60\,h^{-1}$ kpc to $10\,h^{-1}$ Mpc and are in good agreement with theoretical predictions based on the WMAP concordance cosmology. As with galaxy-galaxy lensing, future measurements of cosmic magnification will provide useful constraints on the galaxy-mass power spectrum.

arXiv:astro-ph/0504510

Lensing Potential - Euclid



ED, Montanari, Durrer, Lesgourgues [arXiv:1308.6186]

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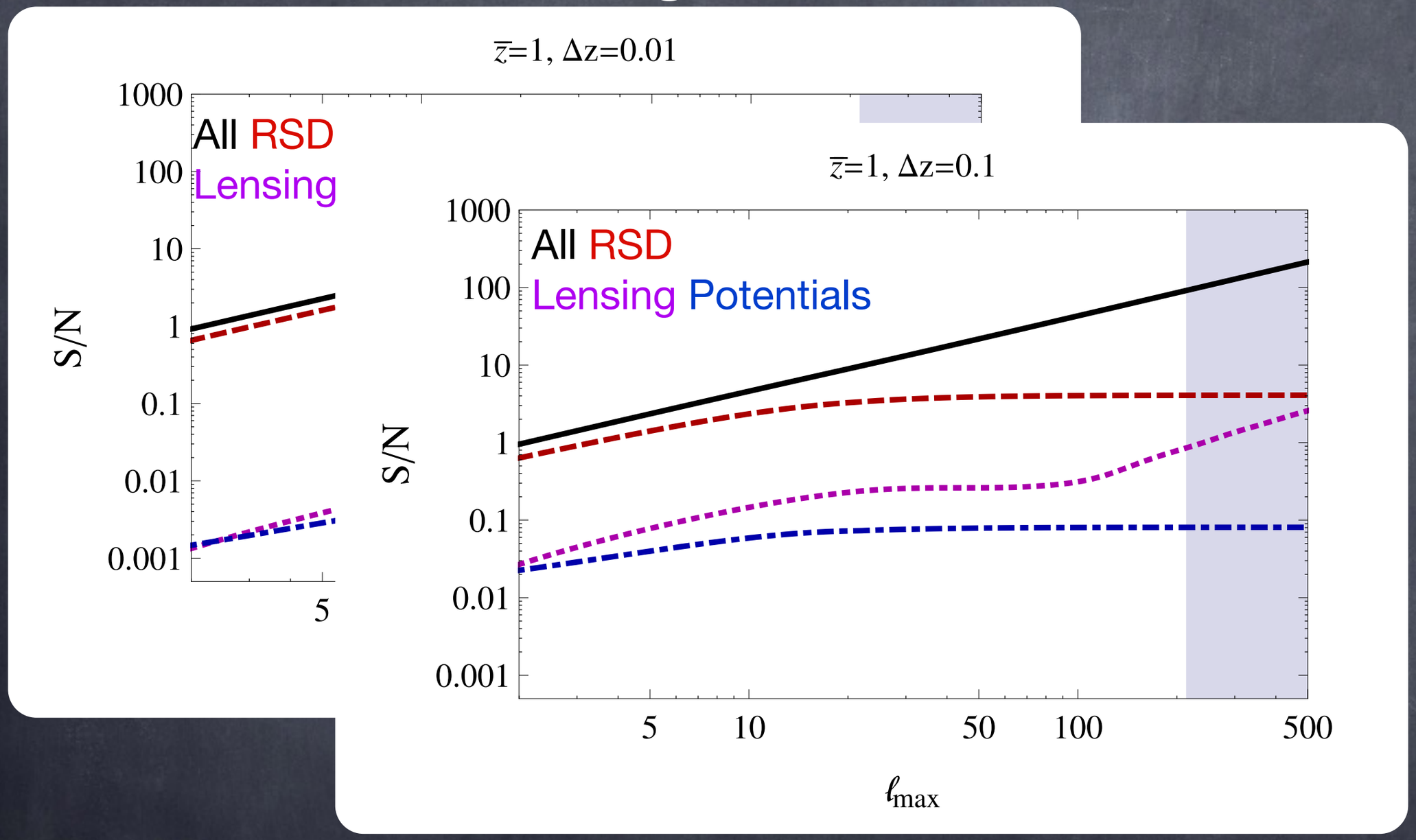

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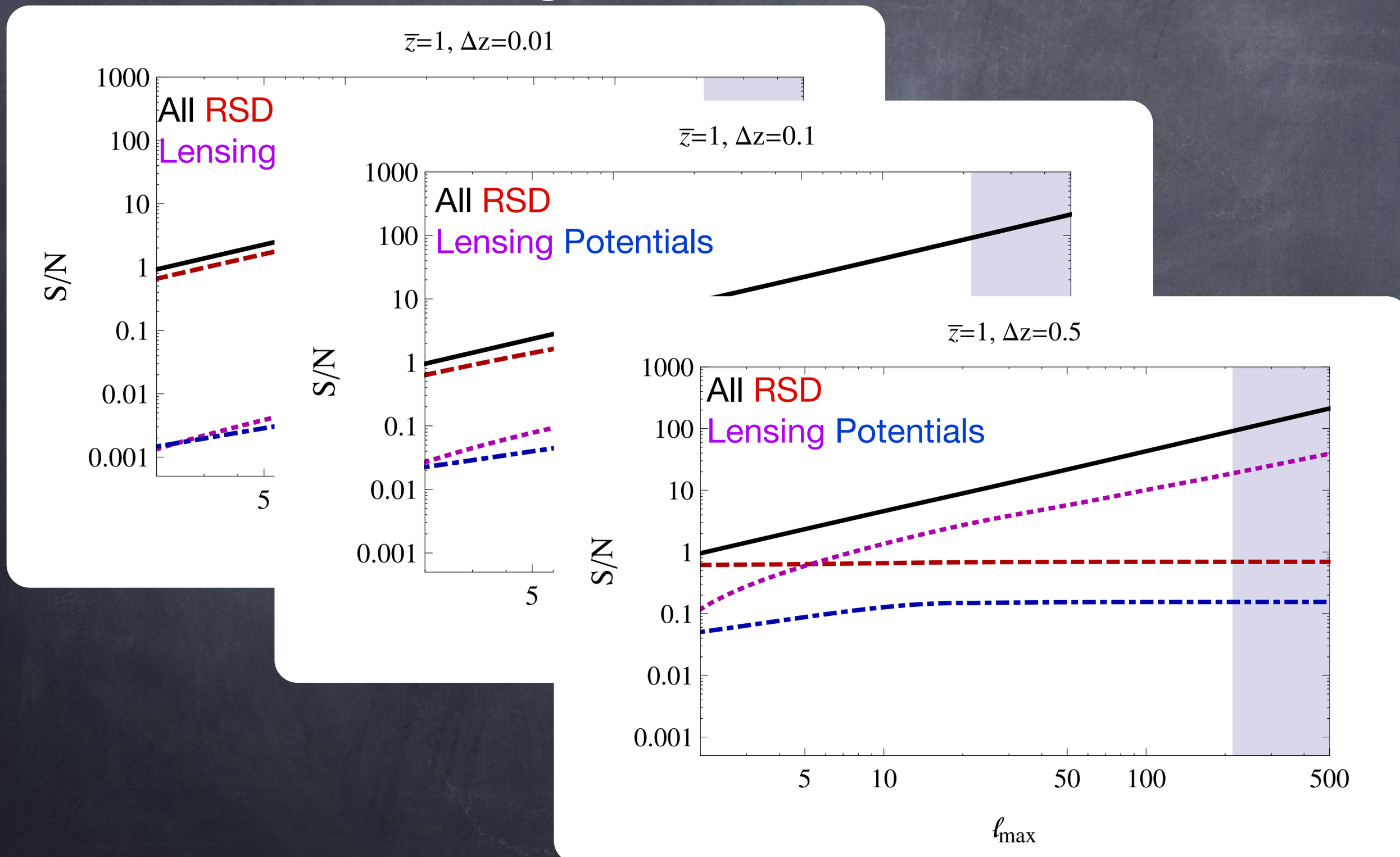

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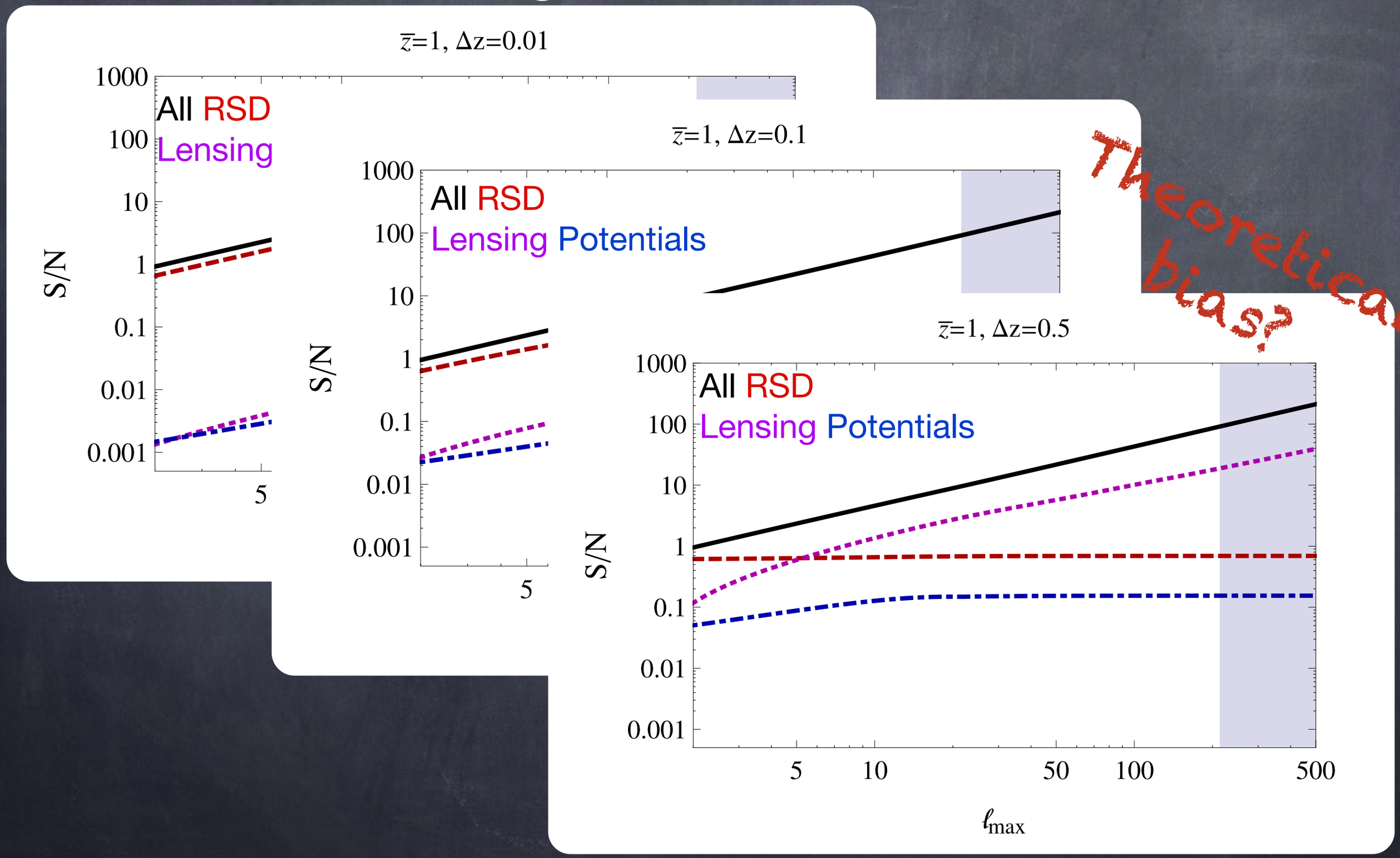

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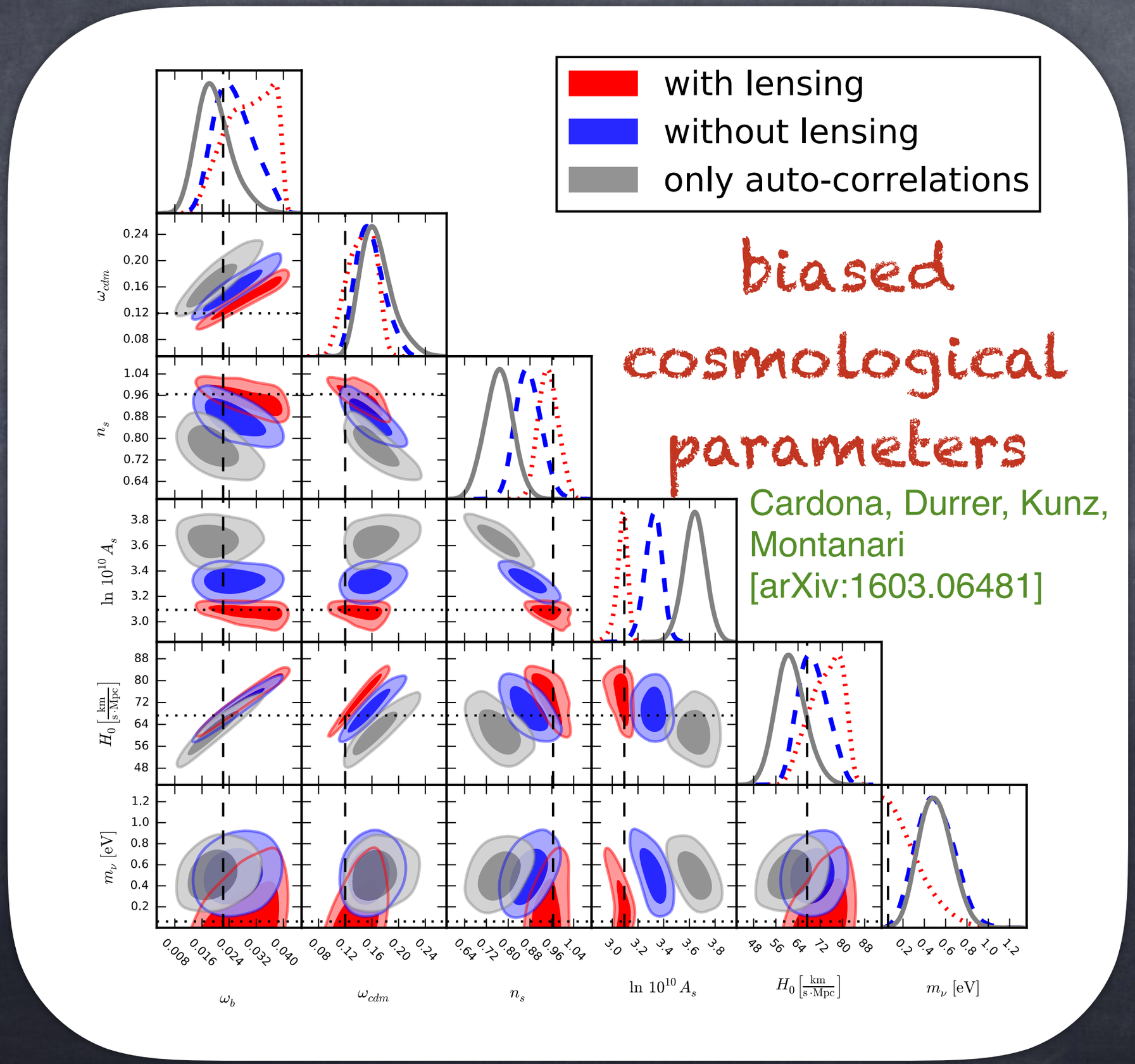

Large Scale Structures

$$
\begin{aligned}
\Delta_N(\mathbf{n}, z, m_*) = {} & b(z) D_{cm}\left(L > \bar{L}_*\right) + \mathcal{H}^{-1}\partial_r \mathbf{n}\cdot\mathbf{v} \\
& - \frac{2-5s}{2}\int_0^r \frac{r-r'}{rr'}\Delta_\Omega\left(\Psi+\Phi\right)dr' \\
& + \left(5s + \frac{2-5s}{\mathcal{H}r} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - f^N_{\rm evo}\right)\mathbf{n}\cdot\mathbf{v} + (f^N_{\rm evo} - 3)\mathcal{H}v \\
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Doppler term $$+\left(5s + \frac{2-5s}{\mathcal{H}r} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - f^N_{\rm evo}\right)\mathbf{n}\cdot\mathbf{v} + (f^N_{\rm evo} - 3)\mathcal{H}v$$

- imaginary power spectrum
  (McDonald [arXiv:0907.5220])
- dipole and octupole in galaxy correlation functions
  (Raccanelli et al [arXiv:1306.6646], Bonvin et al [arxiv:1309.1321])
- multitracer and shot-noise canceling techniques
  (Yoo et al [arXiv:1206.5809])

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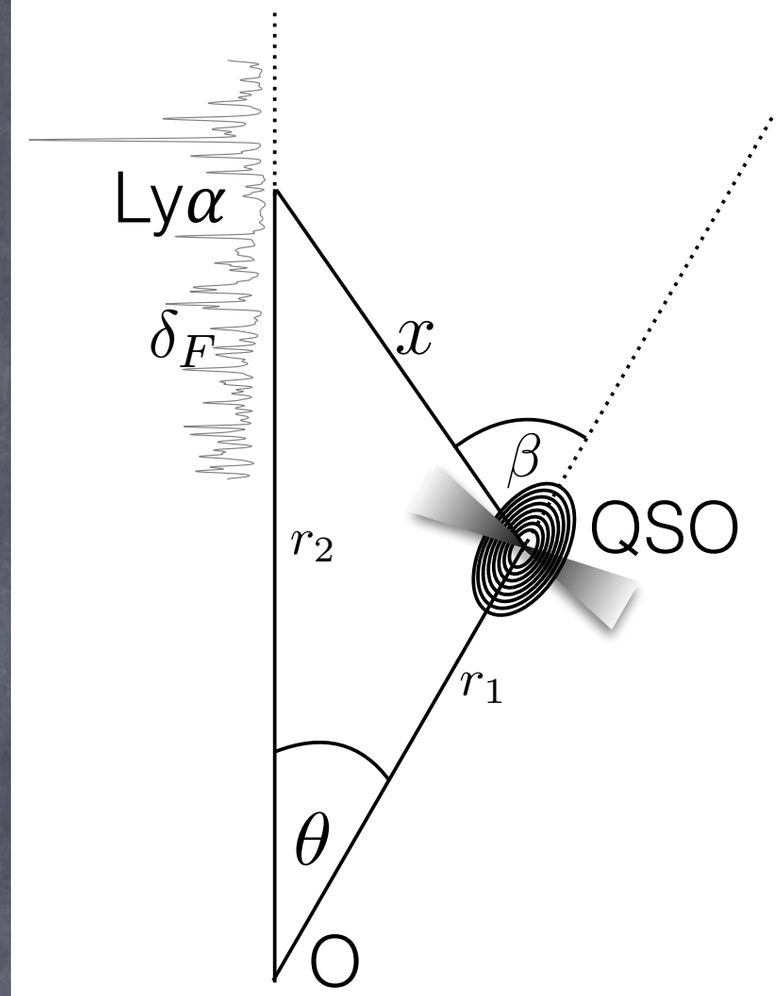
Quasars x Ly-alpha correlation

Irsic , ED, Viel [arXiv:1510.03436]

$$\xi_{Q\alpha}\left(z_1, z_2, \theta\right) = \left\langle \Delta_Q\left(\mathbf{n}_1, z_1\right) \delta_F\left(\mathbf{n}_2, z_2\right)\right\rangle$$

Large bias factor difference

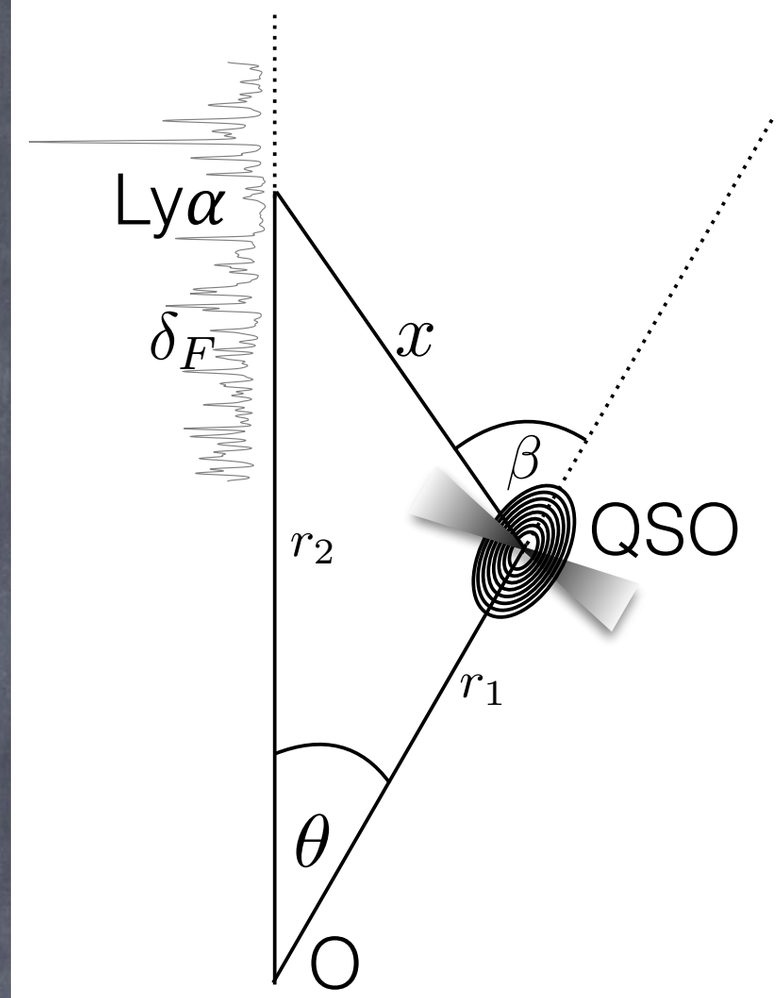


Quasars x Ly-alpha correlation

Irsic , ED, Viel [arXiv:1510.03436]

$$\xi_{Q\alpha}\left(z_1, z_2, \theta\right) = \left\langle \Delta_Q\left(\mathbf{n}_1, z_1\right) \delta_F\left(\mathbf{n}_2, z_2\right)\right\rangle$$

Large bias factor difference

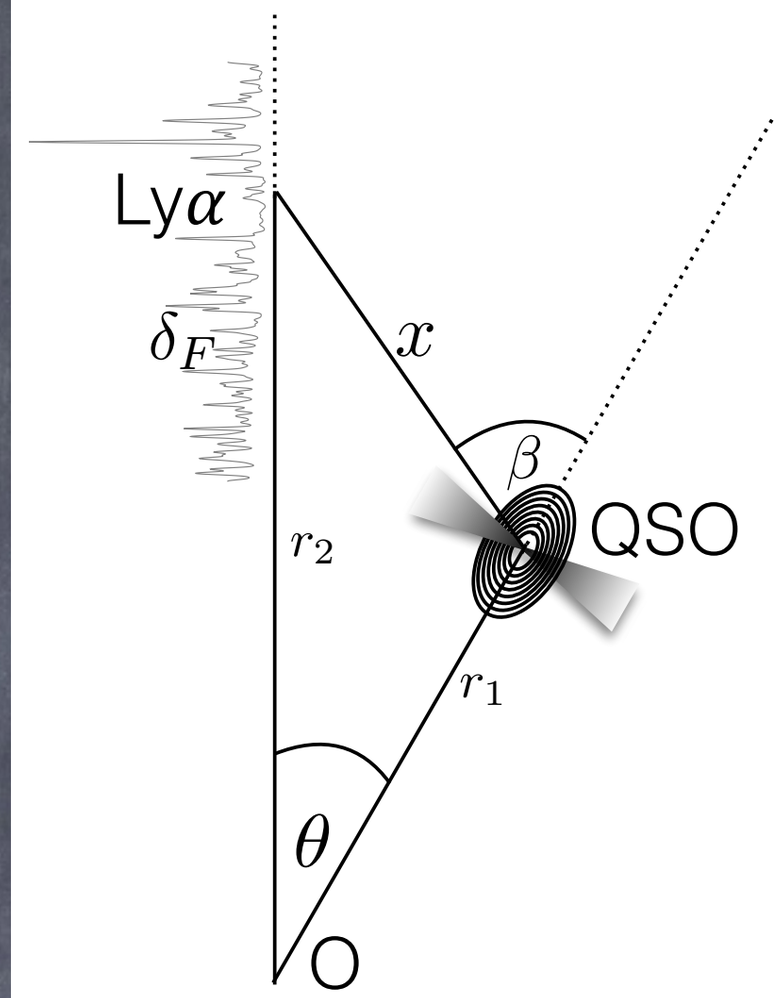


$$\delta_F\left(\mathbf{n}, z\right) = b_\alpha \delta^{\mathrm{sync}} + b_v \mathcal{H}^{-1} \partial_r \mathbf{n}\cdot\mathbf{v} - \bar{\tau}\left(z\right)\left[-\left(2 + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right)\frac{\delta z}{1+z} + \mathbf{n}\cdot\mathbf{v} + \Psi + \mathcal{H}^{-1}\dot{\Phi} - 3\mathcal{H}v\right]$$

Quasars x Ly-alpha correlation

Irsic , ED, Viel [arXiv:1510.03436]

$$\xi_{Q\alpha}\left(z_1, z_2, \theta\right) = \left\langle \Delta_Q\left(\mathbf{n}_1, z_1\right) \delta_F\left(\mathbf{n}_2, z_2\right)\right\rangle$$



Large bias factor difference

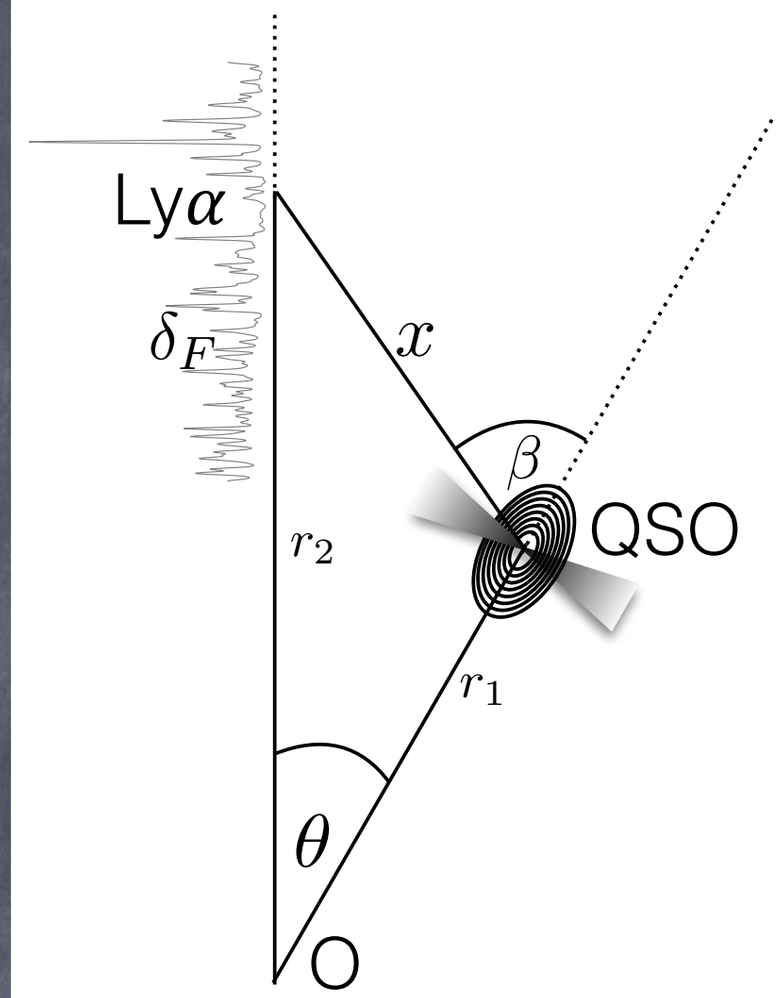
$$\delta_F\left(\mathbf{n}, z\right) = b_\alpha \delta^{\mathrm{sync}} + b_v \mathcal{H}^{-1} \partial_r \mathbf{n} \cdot \mathbf{v} - \bar{\tau}\left(z\right)\left[-\left(2 + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right)\frac{\delta z}{1+z} + \mathbf{n}\cdot\mathbf{v} + \Psi + \mathcal{H}^{-1}\dot{\Phi} - 3\mathcal{H}v\right]$$

Standard

Quasars x Ly-alpha correlation

Irsic , ED, Viel [arXiv:1510.03436]

$$\xi_{Q\alpha}\left(z_1, z_2, \theta\right) = \left\langle \Delta_Q\left(\mathbf{n}_1, z_1\right) \delta_F\left(\mathbf{n}_2, z_2\right)\right\rangle$$



Large bias factor difference

$$\delta_F\left(\mathbf{n}, z\right) = b_\alpha \delta^{\mathrm{sync}} + b_v \mathcal{H}^{-1} \partial_r \mathbf{n} \cdot \mathbf{v} - \bar{\tau}\left(z\right) \left[-\left(2 + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right) \frac{\delta z}{1+z} + \mathbf{n} \cdot \mathbf{v} + \Psi + \mathcal{H}^{-1} \dot{\Phi} - 3\mathcal{H} v \right]$$

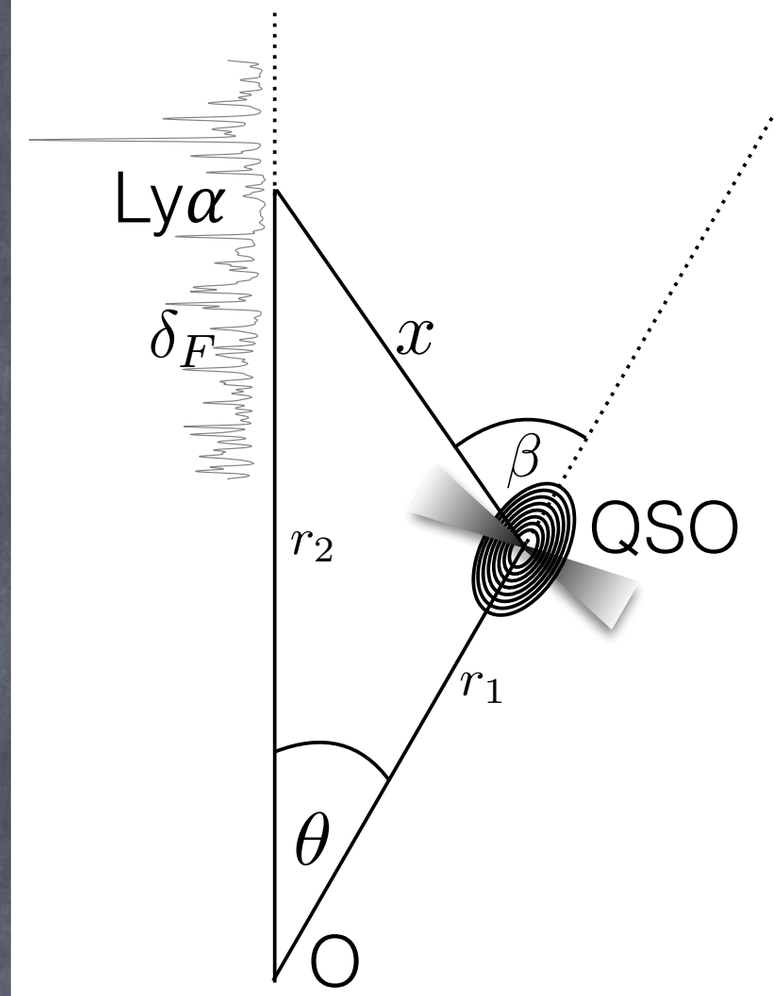
Relativistic

Quasars x Ly-alpha correlation

Irsic , ED, Viel [arXiv:1510.03436]

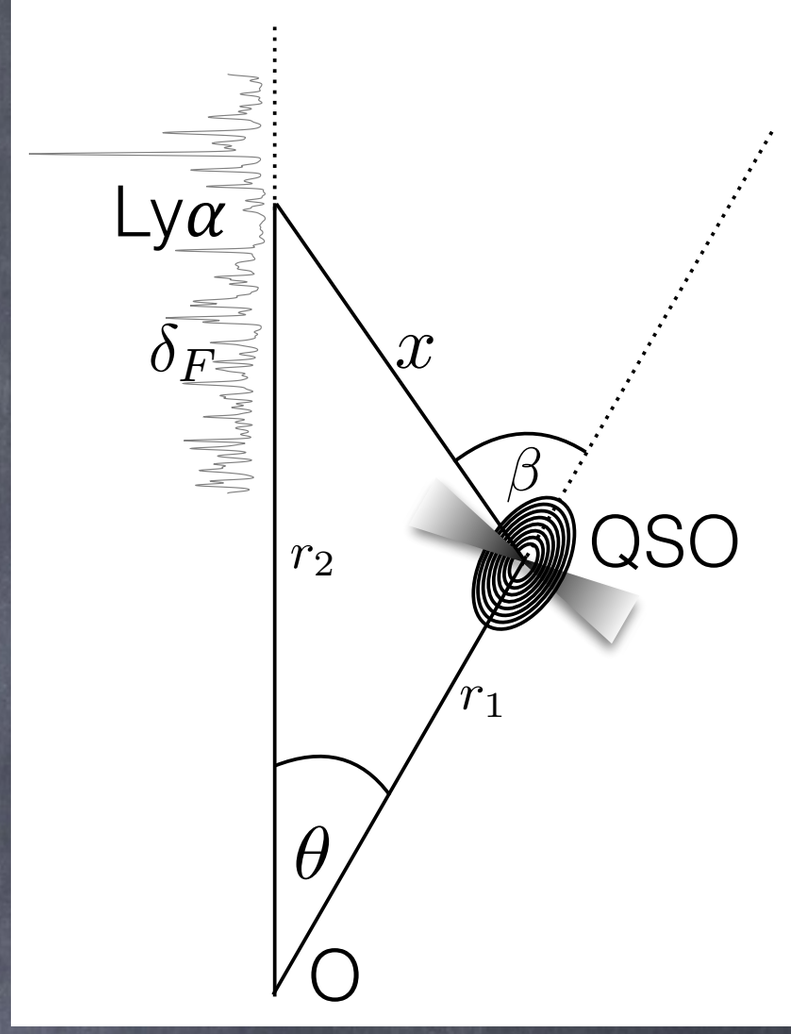
$$\xi_{Q\alpha}\left(z_1, z_2, \theta\right) = \left\langle \Delta_Q\left(\mathbf{n}_1, z_1\right) \delta_F\left(\mathbf{n}_2, z_2\right)\right\rangle$$

$$\xi_{Q\alpha} = \xi_{Q\alpha}^{\text{newt}} + \xi_{Q\alpha}^{\text{magnification}} + \xi_{Q\alpha}^{\text{relativistic}}$$



Quasars x Ly-alpha correlation

Irsic , ED, Viel [arXiv:1510.03436]



$$\xi_{Q\alpha}\left(z_1, z_2, \theta\right) = \left\langle \Delta_Q\left(\mathbf{n}_1, z_1\right) \delta_F\left(\mathbf{n}_2, z_2\right)\right\rangle$$

$$\xi_{Q\alpha} = \xi_{Q\alpha}^{\text{newt}} + \xi_{Q\alpha}^{\text{magnification}} + \xi_{Q\alpha}^{\text{relativistic}}$$

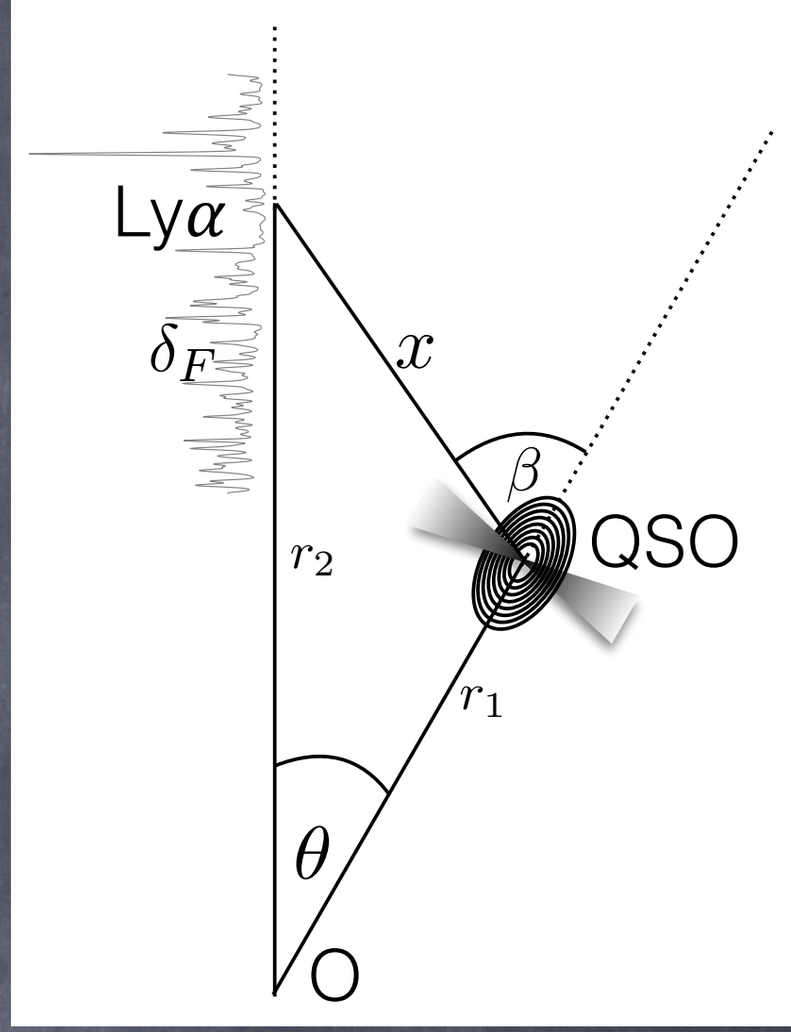
$$\begin{aligned}
\xi_{Q\alpha}^{\text{newt}} \sim\ & b_Q b_\alpha \int \frac{dk}{2\pi^2} k^2 P\left(k\right) j_0\left(kx\right) \\
& + b_v \int \frac{dk}{2\pi^2} k^2 f^2 P\left(k\right) \left[\frac{1}{5} j_0\left(kx\right) - \frac{4}{7} j_2\left(kx\right) + \frac{8}{35} j_4\left(kx\right)\right] \\
& + \left(b_Q b_v + b_\alpha\right) \int \frac{dk}{2\pi^2} k^2 f P\left(k\right) \left[\frac{1}{3} j_0\left(kx\right) - \frac{2}{3} j_2\left(kx\right)\right]
\end{aligned}$$

Order $\mathcal{O}(1)$

Even spherical Bessel functions

Quasars x Ly-alpha correlation

Irsic , ED, Viel [arXiv:1510.03436]



$$\xi_{Q\alpha}\left(z_1, z_2, \theta\right) = \left\langle \Delta_Q\left(\mathbf{n}_1, z_1\right) \delta_F\left(\mathbf{n}_2, z_2\right)\right\rangle$$

$$\xi_{Q\alpha} = \xi_{Q\alpha}^{\text{newt}} + \xi_{Q\alpha}^{\text{magnification}} + \xi_{Q\alpha}^{\text{relativistic}}$$

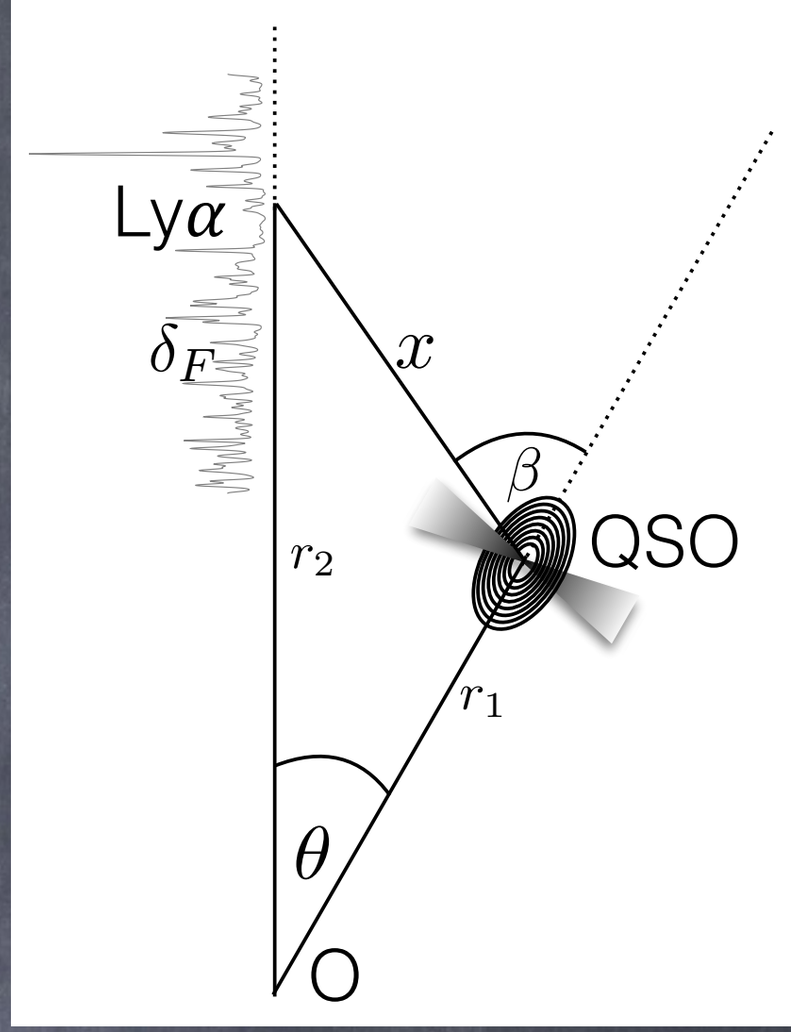
$$\begin{aligned}
\xi_{Q\alpha}^{\text{Doppler}} \sim & \left(-b_Q \mathcal{R}_\alpha + \mathcal{R}_Q b_\alpha\right) \int \frac{dk}{2\pi^2} k^2 f P\left(k\right) j_1\left(kx\right) \frac{\mathcal{H}}{k} \\
& + \left(-\mathcal{R}_\alpha + \mathcal{R}_Q b_v\right) \int \frac{dk}{2\pi^2} k^2 f^2 P\left(k\right) \left[\frac{3}{5} j_1\left(kx\right) - \frac{2}{5} j_3\left(kx\right)\right] \frac{\mathcal{H}}{k} \\
& + \mathcal{R}_\alpha \mathcal{R}_Q \int \frac{dk}{2\pi^2} k^2 f^2 P\left(k\right) \left[\frac{1}{3} j_0\left(kx\right) - \frac{2}{3} j_2\left(kx\right)\right] \left(\frac{\mathcal{H}}{k}\right)^2
\end{aligned}$$

Order $\mathcal{O}\left(\mathcal{H}/k\right)$

Odd spherical Bessel functions

Quasars x Ly-alpha correlation

Irsic , ED, Viel [arXiv:1510.03436]



$$\xi_{Q\alpha}\left(z_1, z_2, \theta\right) = \left\langle \Delta_Q\left(\mathbf{n}_1, z_1\right) \delta_F\left(\mathbf{n}_2, z_2\right)\right\rangle$$

$$\xi_{Q\alpha} = \xi_{Q\alpha}^{\text{newt}} + \xi_{Q\alpha}^{\text{magnification}} + \xi_{Q\alpha}^{\text{relativistic}}$$

single tracer

$$\xi_{Q\alpha}^{\text{Doppler}} \sim \cancel{\left(-b_Q \mathcal{R}_\alpha + \mathcal{R}_Q b_\alpha\right)} \int \frac{dk}{2\pi^2} k^2 f P\left(k\right) j_1\left(kx\right) \frac{\mathcal{H}}{k}$$

$$+ \cancel{\left(-\mathcal{R}_\alpha + \mathcal{R}_Q b_v\right)} \int \frac{dk}{2\pi^2} k^2 f^2 P\left(k\right) \left[\frac{3}{5} j_1\left(kx\right) - \frac{2}{5} j_3\left(kx\right)\right] \frac{\mathcal{H}}{k}$$

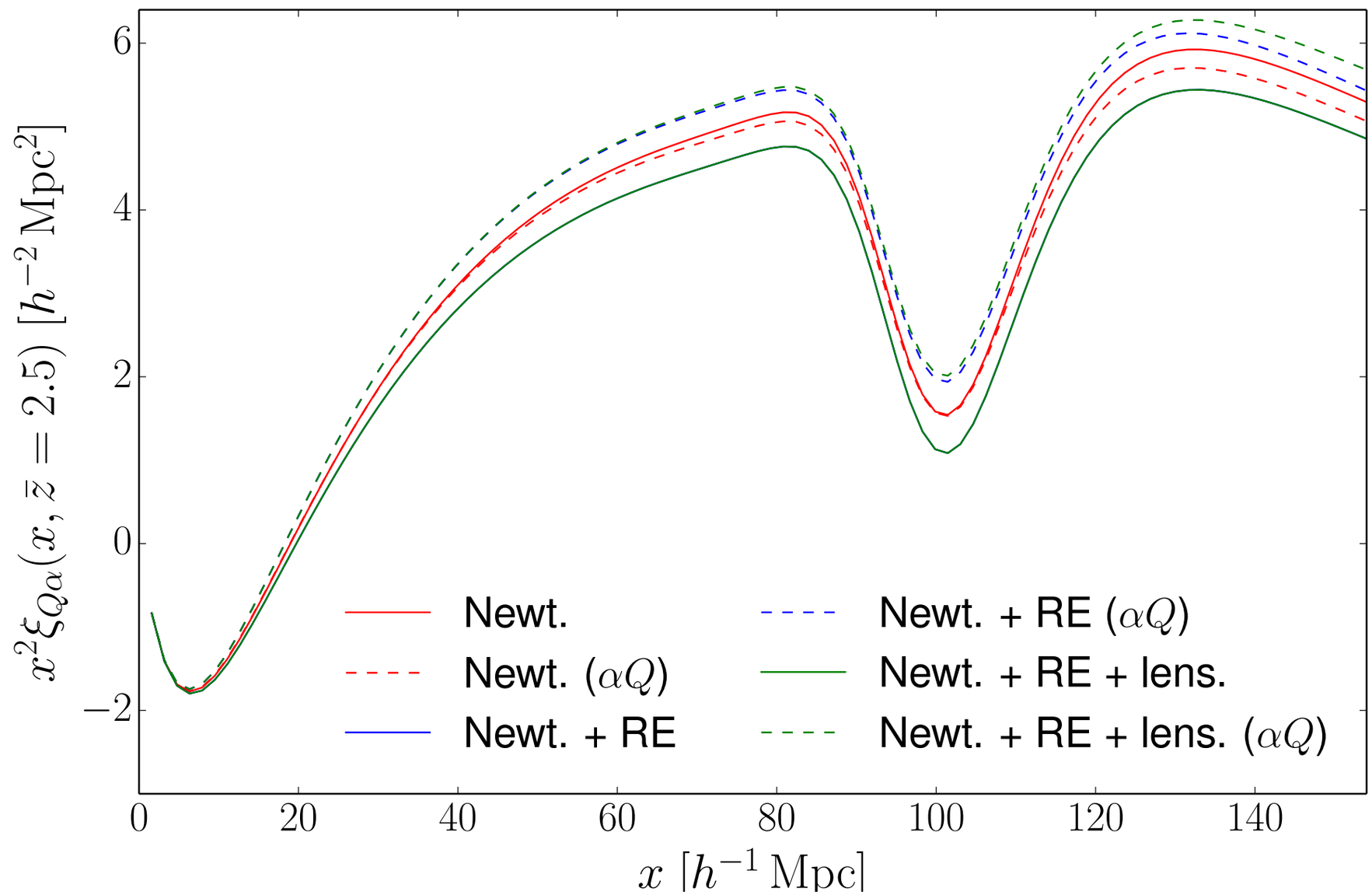
$$+ \mathcal{R}_\alpha \mathcal{R}_Q \int \frac{dk}{2\pi^2} k^2 f^2 P\left(k\right) \left[\frac{1}{3} j_0\left(kx\right) - \frac{2}{3} j_2\left(kx\right)\right] \left(\frac{\mathcal{H}}{k}\right)^2$$

Order $\mathcal{O}\left(\cancel{\mathcal{H}/k}\right)$ $\mathcal{O}\left(\mathcal{H}^2/k^2\right)$

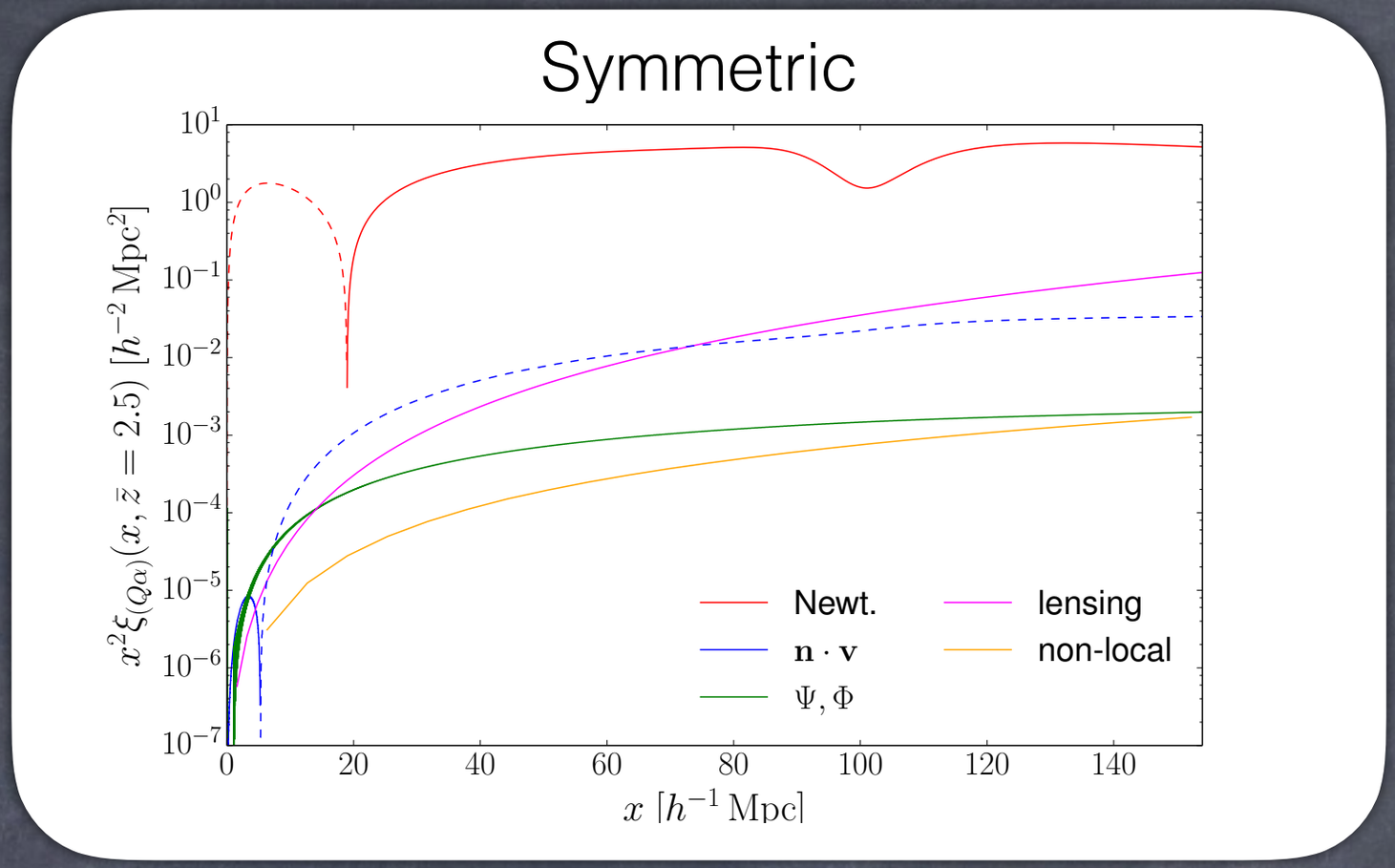
even

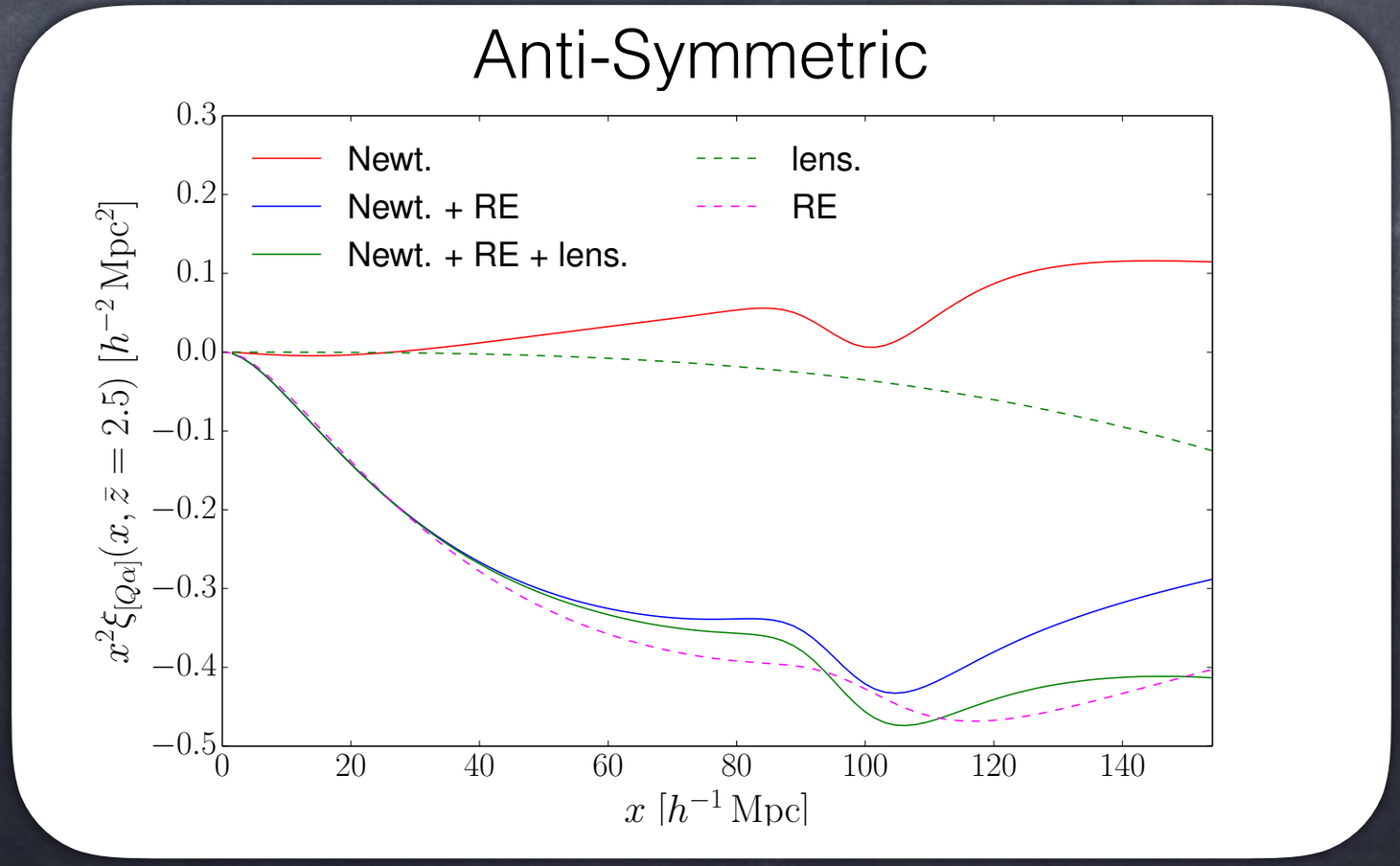
~~Odd~~ spherical Bessel functions

Relativistic effects on Lyman-α forest



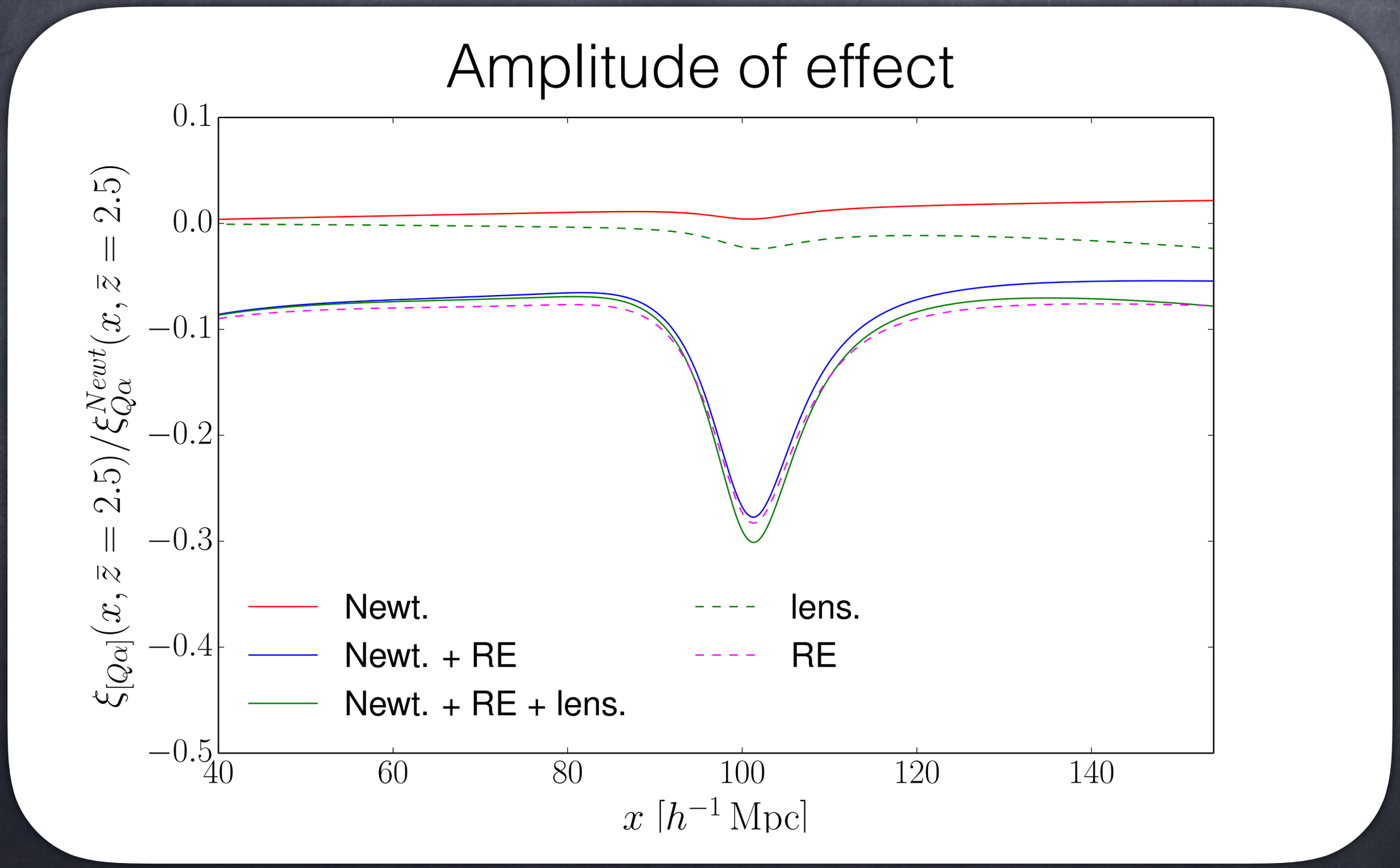
Irsic , ED, Viel [arXiv:1510.03436]





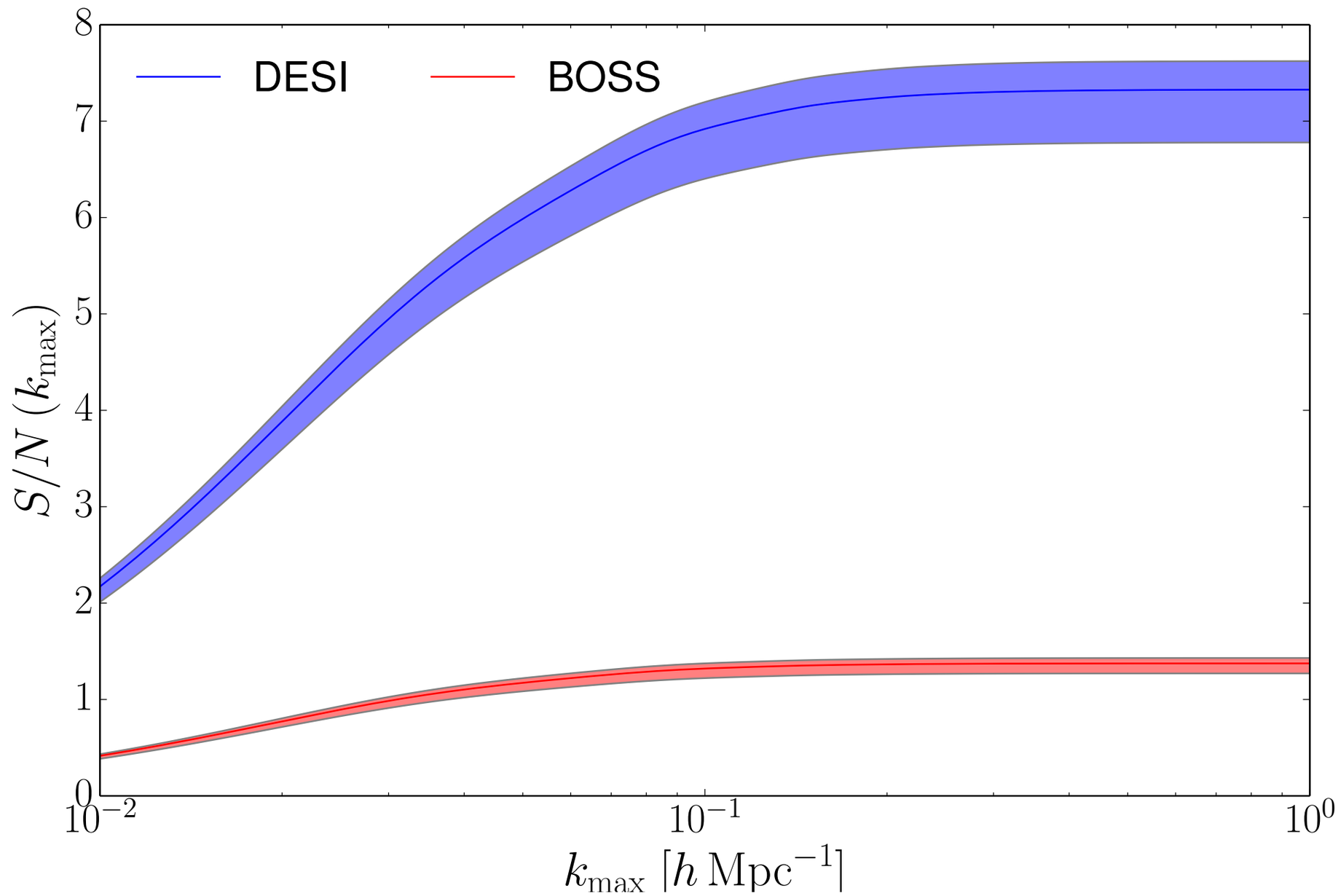
Irsic , ED, Viel [arXiv:1510.03436]

Relativistic effects on Lyman-α forest



Irsic , ED, Viel [arXiv:1510.03436]

Relativistic effects on Lyman-α forest



Irsic , ED, Viel [arXiv:1510.03436]

Single tracer vs multi-tracers

	single	multi
Leading order correction	$\mathcal{O}\left(\mathcal{H}^2/k^2\right)$	$\mathcal{O}\left(\mathcal{H}/k\right)$
Parity	even	odd
Relevant scales	super-Hubble	all
Limited by	cosmic variance	shot noise
Forecasted detection	No	Yes

Large Scale Structures

$$
\begin{aligned}
\Delta_N(\mathbf{n}, z, m_*) = {} & b(z) D_{cm}\left(L > \bar{L}_*\right) + \mathcal{H}^{-1}\partial_r \mathbf{n}\cdot\mathbf{v} \\
& - \frac{2-5s}{2}\int_0^r \frac{r-r'}{rr'}\Delta_\Omega\left(\Psi+\Phi\right)dr' \\
& + \left(5s + \frac{2-5s}{\mathcal{H}r} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - f^N_{\rm evo}\right)\mathbf{n}\cdot\mathbf{v} + (f^N_{\rm evo} - 3)\mathcal{H}v \\
& + \left(5s + \frac{2-5s}{\mathcal{H}r} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - f^N_{\rm evo}\right)\int_0^r \left(\dot{\Psi} + \dot{\Phi}\right)dr' \\
& + \left(5s + \frac{2-5s}{\mathcal{H}r} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - f^N_{\rm evo}\right)\Psi \\
& + \frac{2-5s}{r}\int_0^r \left(\Psi+\Phi\right)dr \\
& + \left(5s-2\right)\Phi + \Psi + \mathcal{H}^{-1}\dot{\Phi}
\end{aligned}
$$

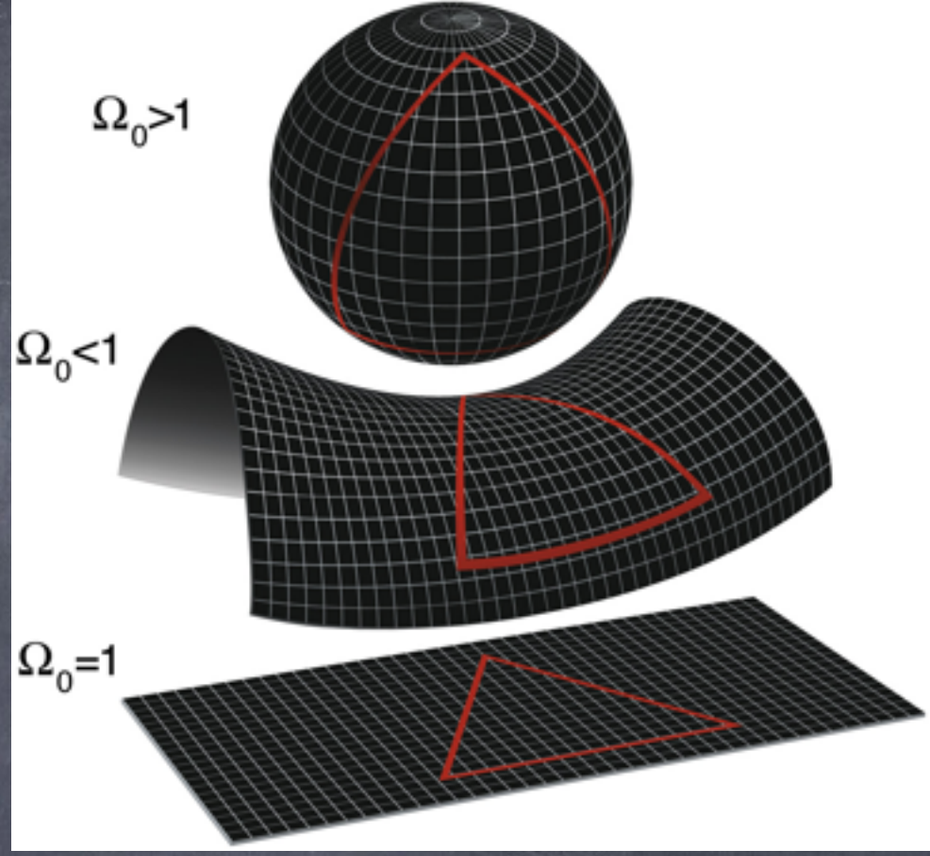
Bonvin & Durrer [arXiv:1105.5280],
Challinor & Lewis [arXiv:1105.5292],
Yoo [arXiv:1009.3021]

Curvature constraints from LSS

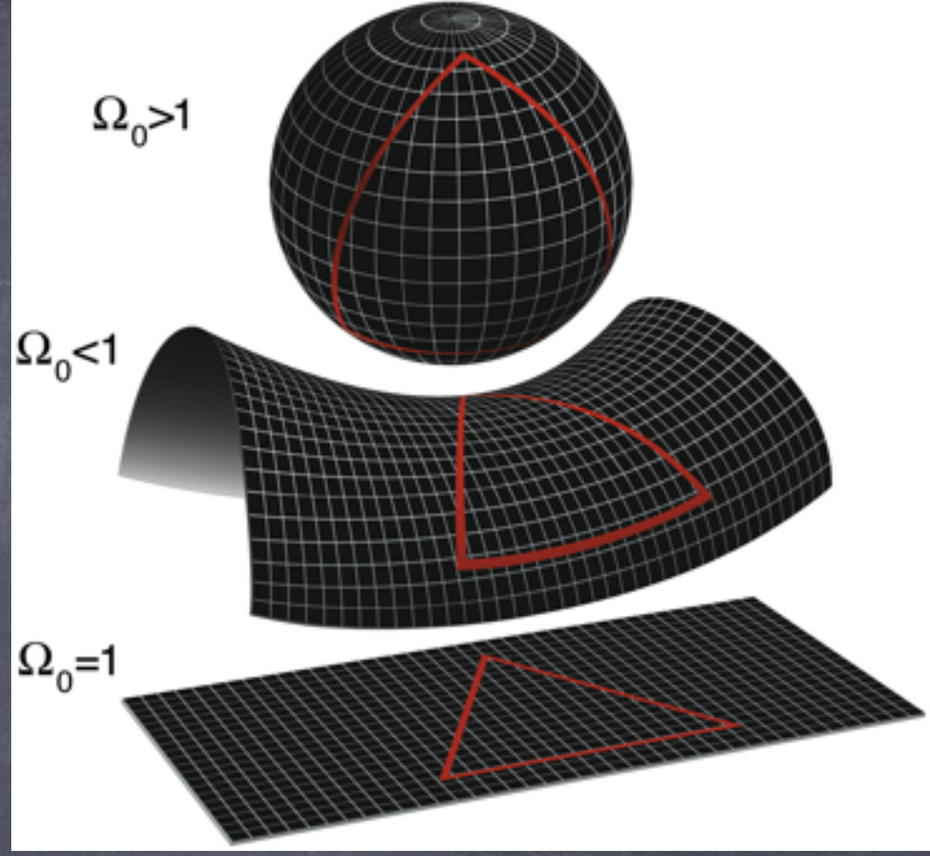
Tightest constraints $|\Omega_K| \leq 0.003$

Predicted limit of detectability $\Omega_K \sim \mathcal{O}\left(10^{-4}\right)$

Cosmic variance limit $\Omega_K \sim \mathcal{O}\left(10^{-5}\right)$



Curvature constraints from LSS



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Predicted limit of detectability $\Omega_K \sim \mathcal{O}\left(10^{-4}\right)$

Cosmic variance limit $\Omega_K \sim \mathcal{O}\left(10^{-5}\right)$

$$d\tilde{s}^2 = a^2\left(-\left(1+2\Psi\right)d\tau^2 + \left(1-2\Phi\right)\gamma_{ij}dx^i dx^j\right)$$

where $$\gamma_{ij}dx^i dx^j = \left[dr^2 + \left\{S_K^2\left(r\right)\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right]$$

and $$S_K\left(r\right) = \begin{cases} \frac{1}{\sqrt{K}}\sin\left(\sqrt{K}r\right) & \text{for } K > 0 \\ r & \text{for } K = 0 \\ \frac{1}{\sqrt{|K|}}\sinh\left(\sqrt{|K|}r\right) & \text{for } K < 0 \end{cases}$$

Curvature constraints from LSS

$$
\begin{aligned}
\Delta\left(\mathbf{n}, z, m_*\right) = {} & b D_{cm} + \frac{1}{\mathcal{H}} \partial_r \left(\mathbf{n} \cdot \mathbf{v}\right) \\
& - \frac{2-5s}{2} \int_\tau^{\tau_o} \frac{S_K\left(r-\tilde{r}\right)}{S_K\left(r\right) S_K\left(\tilde{r}\right)} \Delta_\Omega \left(\Psi + \Phi\right) d\tilde{\tau} \\
& + \left(5s + \frac{2-5s}{\mathcal{H}} \frac{S'_K}{S_K} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - f_{\mathrm{evo}}\right) \\
& \times \left(\Psi + \mathbf{n} \cdot \mathbf{v} + \int_\tau^{\tau_o} \left(\dot{\Psi} + \dot{\Phi}\right) d\tilde{\tau}\right) \\
& + \left(5s - 2\right) \Phi + \Psi + \frac{1}{\mathcal{H}} \dot{\Phi} + \left(f_{\mathrm{evo}} - 3\right) \mathcal{H} v \\
& + \left(2 - 5s\right) \frac{S'_K}{S_K} \int_\tau^{\tau_o} \left(\Psi + \Phi\right) d\tilde{\tau}
\end{aligned}
$$

ED, Montanari, Raccanelli, Durrer, Kamionkowski, Lesgourgues [arXiv:1603.09073]

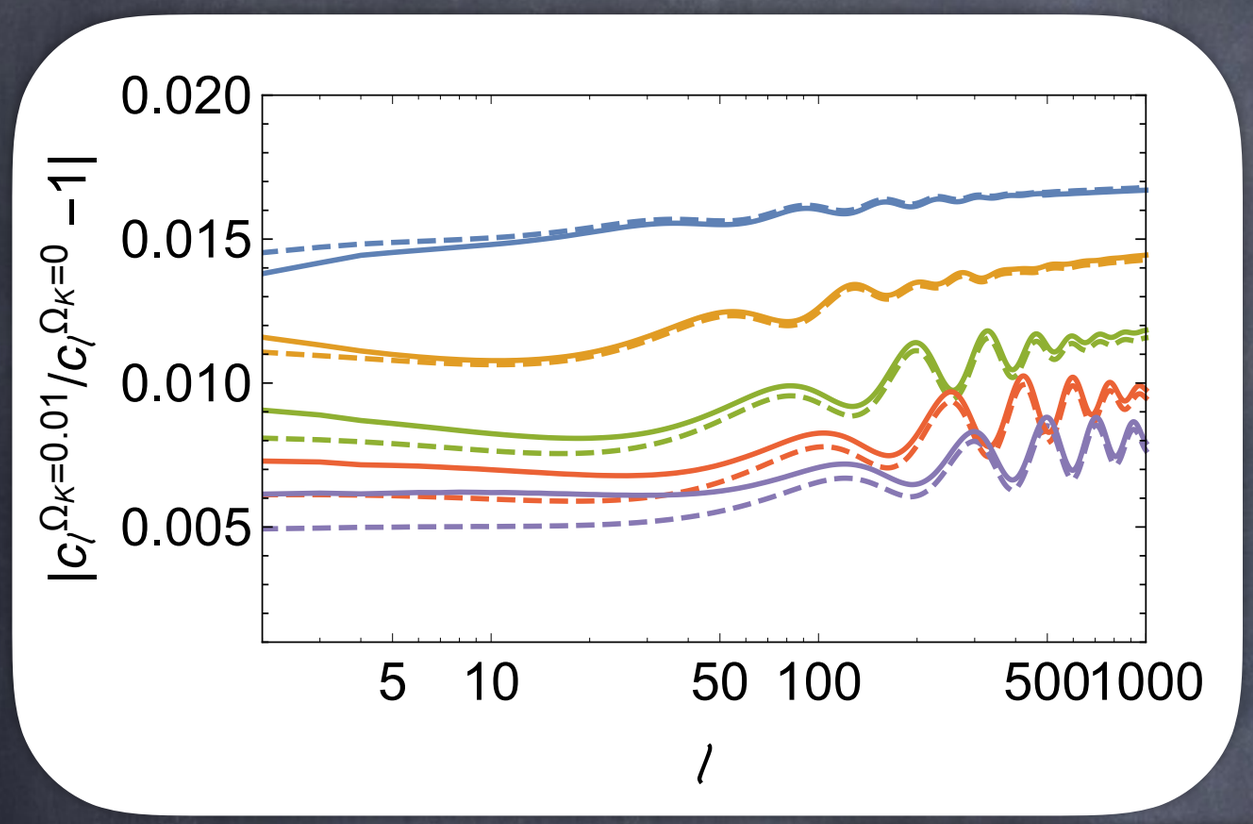
Curvature constraints from LSS

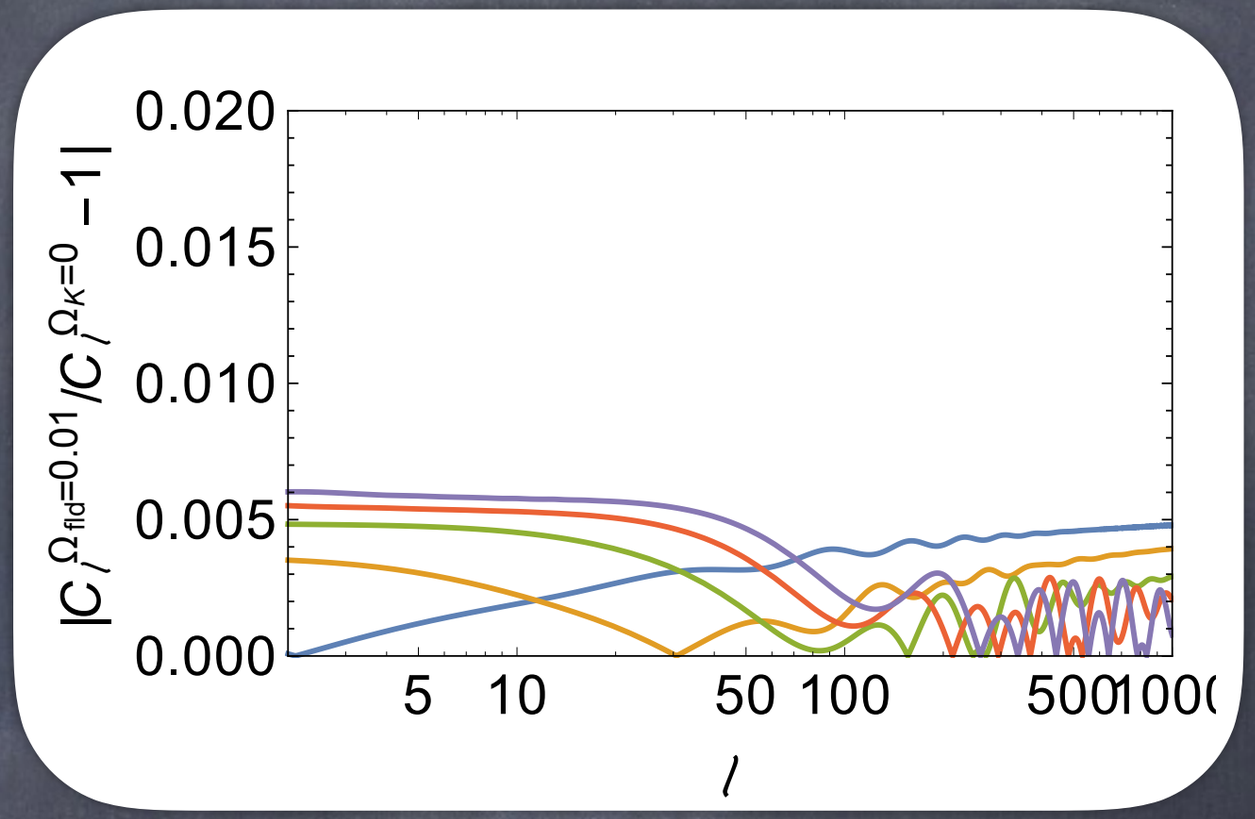
$$\Delta\left(\mathbf{n}, z, m_*\right) = bD_{cm} + \frac{1}{\mathcal{H}}\partial_r\left(\mathbf{n}\cdot\mathbf{v}\right)$$
$$- \frac{2-5s}{2}\int_\tau^{\tau_o} \frac{S_K\left(r-\tilde{r}\right)}{S_K\left(r\right)S_K\left(\tilde{r}\right)}\Delta_\Omega\left(\Psi+\Phi\right)d\tilde{\tau}$$
$$- \left(\frac{2-5s\,S'_K}{} \qquad \frac{\dot{\mathcal{H}}}{} f_{\rm evo}\right)$$
$$\times\left(\Psi + \mathbf{n}\cdot\mathbf{v} + \int_\tau \left(\Psi+\Phi\right)d\tilde{\tau}\right)$$
$$+ \left(5s-2\right)\Phi + \Psi + \frac{1}{\mathcal{H}}\dot{\Phi} + \left(f_{\rm evo}-3\right)\mathcal{H}v$$
$$+ \left(2-5s\right)\frac{S'_K}{S_K}\int_\tau^{\tau_o}\left(\Psi+\Phi\right)d\tilde{\tau}$$

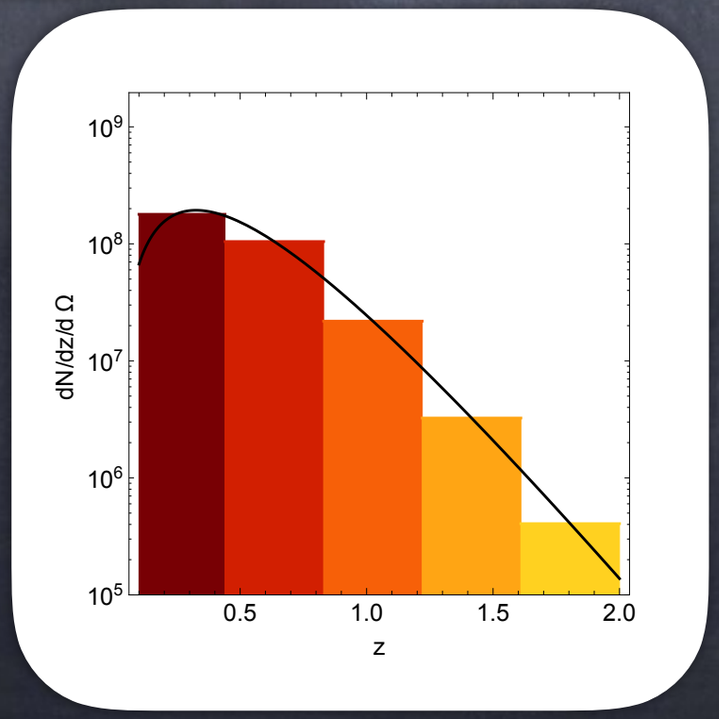
CLASS v.2.5
http://class-code.net

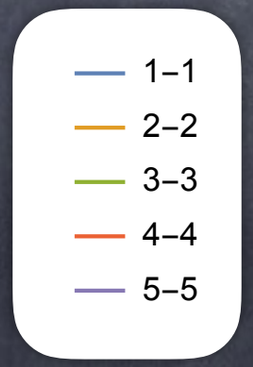
ED, Montanari, Raccanelli, Durrer, Kamionkowski, Lesgourgues [arXiv:1603.09073]

Curvature constraints from LSS



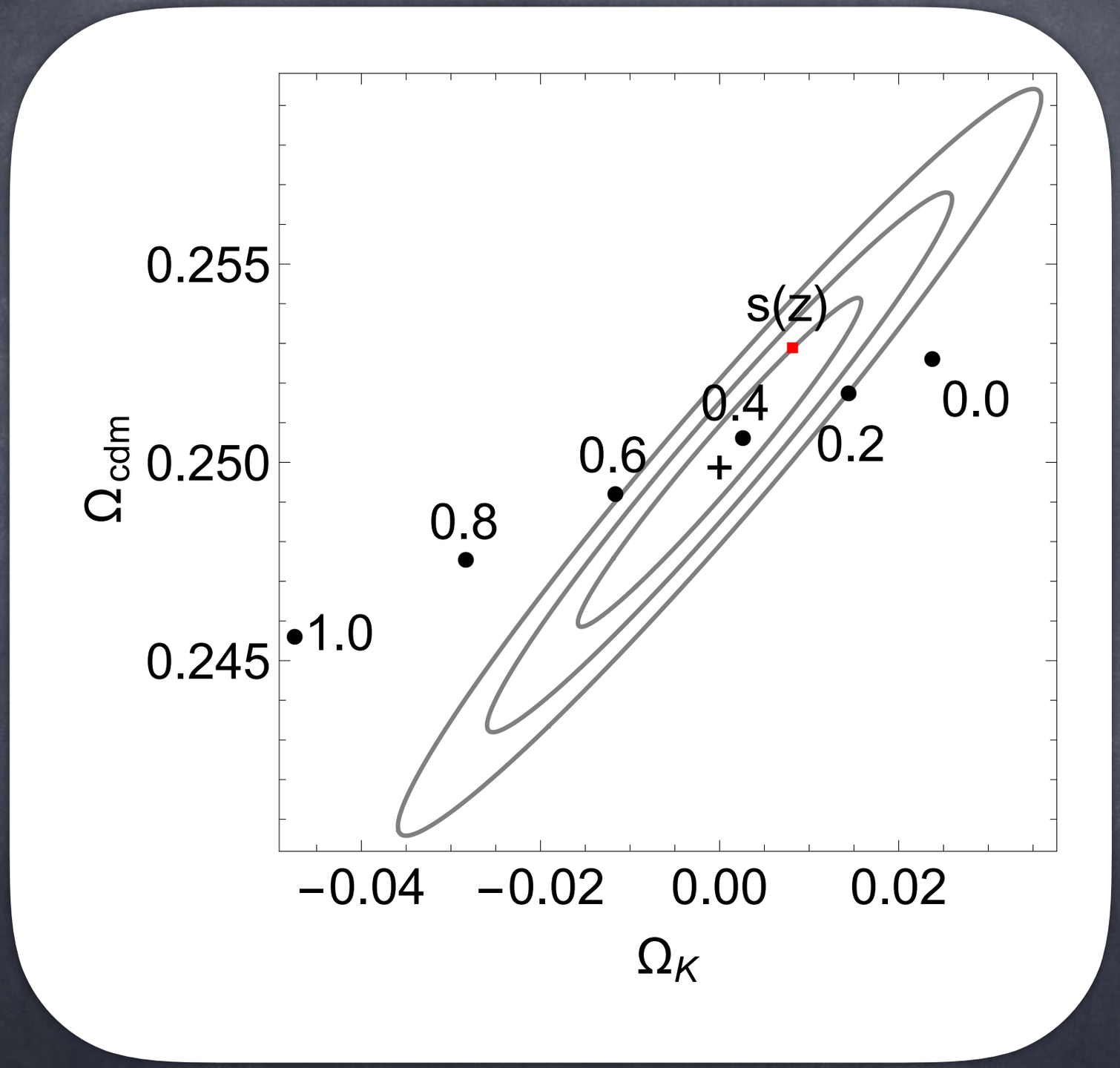






ED, Montanari, Raccanelli, Durrer, Kamionkowski, Lesgourgues [arXiv:1603.09073]

Curvature constraints from LSS



ED, Montanari, Raccanelli, Durrer, Kamionkowski, Lesgourgues [arXiv:1603.09073]

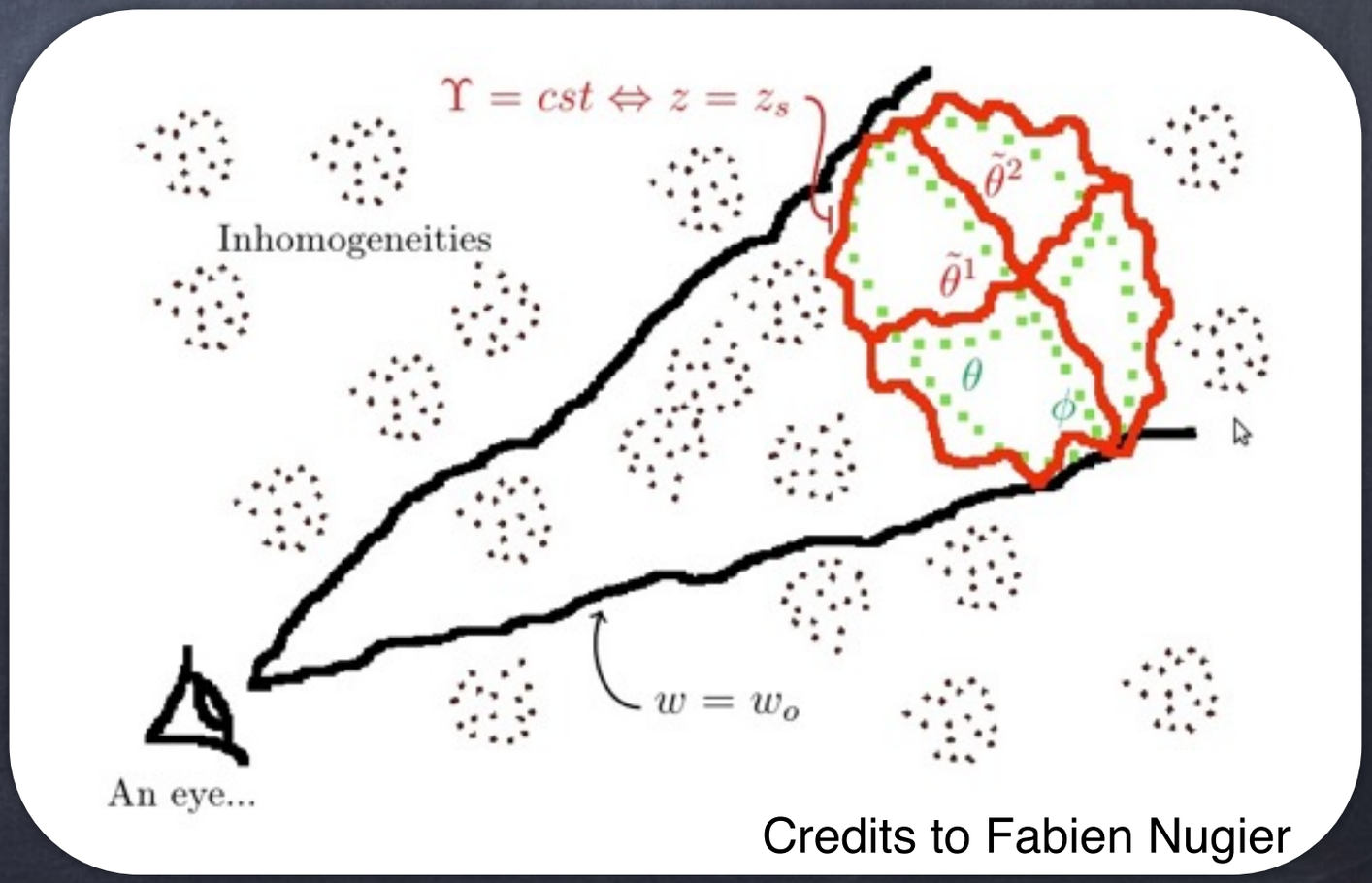
Second Order

In the weakly non-linear regime second order perturbation theory can be applied

Because of non-linear gravitational effects the power spectrum does not encode all the statistical information. → Bi-spectrum

Born approximation fails
→ Perturbed geodesics

Geodesic light-cone gauge



Credits to Fabien Nugier

Gasperini, Marozzi, Nugier, Veneziano [arXiv:1104.1167]

$$
\begin{aligned}
\Sigma_{IS} = & \left(-\frac{2}{\mathcal{H}_s r_s} - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\Bigg\{-v_{||s}^{(2)} - \frac{1}{2}\phi_s^{(2)} - \frac{1}{2}\int_{\eta_s}^{\eta_0} d\eta' \partial_{\eta'}\left[\phi^{(2)}(\eta') + \psi^{(2)}(\eta')\right] + \frac{1}{2}\left(v_{||s}\right)^2 \\
& + \frac{1}{2}\left(\psi_s^I\right)^2 + \left(-v_{||s} - \psi_s^I\right)\left(-\psi_s^I - 2\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta')\right) + \frac{1}{2} v_{\perp s}^a v_{\perp a\, s} \\
& + 2a\, v_{\perp s}^a \partial_a \int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta') + 4\int_{\eta_s}^{\eta_0} d\eta' \Bigg[\psi^I(\eta')\,\partial_{\eta'}\psi^I(\eta') + \partial_{\eta'}\psi^I(\eta')\int_{\eta'}^{\eta_o} d\eta'' \partial_{\eta''}\psi^I(\eta'') \\
& + \psi^I(\eta')\int_{\eta'}^{\eta_o} d\eta'' \partial_{\eta''}^2 \psi^I(\eta'') - \gamma_0^{ab}\partial_a\left(\int_{\eta'}^{\eta_o} d\eta'' \psi^I(\eta'')\right)\partial_b\left(\int_{\eta'}^{\eta_o} d\eta'' \partial_{\eta''}\psi^I(\eta'')\right)\Bigg] \\
& + 2\partial_a\left(v_{||s} + \psi_s^I\right)\int_{\eta_s}^{\eta_o} d\eta' \gamma_0^{ab}\partial_b \int_{\eta'}^{\eta_o} d\eta'' \psi^I(\eta'') \\
& + 4\int_{\eta_s}^{\eta_o} d\eta' \partial_a\left(\partial_{\eta'}\psi^I(\eta')\right)\int_{\eta'}^{\eta_o} d\eta'' \gamma_0^{ab}\partial_b \int_{\eta''}^{\eta_o} d\eta''' \psi^I(\eta''')\Bigg\} \\
& + \left[\frac{1}{2}\frac{\mathcal{H}_s'}{\mathcal{H}_s^2} + \frac{3}{2}\left(\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)^2 - \frac{1}{2}\frac{\mathcal{H}_s''}{\mathcal{H}_s^3} + \frac{1}{\mathcal{H}_s r_s}\left(1 + 3\frac{\mathcal{H}_s'}{\mathcal{H}_s^2} + \frac{1}{\mathcal{H}_s r_s}\right)\right]\left[\left(v_{||s}\right)^2 + \left(\psi_s^I\right)^2 + 2\psi_s^I v_{||s}\right. \\
& \left. + 4\left(v_{||s} + \psi_s^I\right)\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta') + 4\left(\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta')\right)^2\right] - \psi_s^{(2)} + \frac{1}{2}\phi_s^{(2)} + \frac{1}{2\mathcal{H}_s}\partial_\eta \psi_s^{(2)} \\
& + \frac{1}{\mathcal{H}_s}\partial_r v_{||s}^{(2)} - \frac{1}{2}\frac{1}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2\left[\psi^{(2)} + \phi^{(2)}\right](\eta') + \frac{1}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \left[\psi^{(2)} + \phi^{(2)}\right](\eta') \\
& + 2\left(1 - \frac{1}{\mathcal{H}_s r_s}\right)\Bigg\{-\frac{2}{\mathcal{H}_s} v_{||s}\partial_r v_{||s} - \left(v_{||s}\right)^2 - v_{\perp a\, s} v_{\perp\, s}^a + \Bigg[-\frac{1}{\mathcal{H}_s}\partial_\eta \psi_s^I - 2\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta') \\
& - \frac{2}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta') + \frac{2}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2\psi^I(\eta')\Bigg] v_{||s} + \Bigg[-2\psi_s^I - 4\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta') \\
& - 2\mathcal{H}_s\int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta')\Bigg]\frac{1}{\mathcal{H}_s}\partial_r v_{||s} - a v_{\perp\, s}^a \partial_a \int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta') + \Bigg[\partial_r \psi_s^I + 2\partial_\eta \psi_s^I \\
& + 2\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}^2 \psi^I(\eta')\Bigg]\left(-2\int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta')\right) \\
& - \left(-\psi_s^I - 2\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta')\right)\Bigg[\frac{2}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2\psi^I(\eta') - \frac{2}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta') \\
& - \frac{1}{\mathcal{H}_s}\partial_\eta \psi_s^I - \psi_s^I\Bigg]\Bigg\} + \frac{3}{2} v_{\perp a\, s} v_{\perp\, s}^a - \frac{2}{\mathcal{H}_s} a v_{\perp\, s}^a \partial_a v_{||s} + \left(\frac{5}{2} + \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\left(v_{||s}\right)^2 \\
& + \left(5 + 3\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\frac{1}{\mathcal{H}_s} v_{||s}\partial_r v_{||s} + \frac{1}{\mathcal{H}_s^2}\left[v_{||s}\partial_r^2 v_{||s} + \left(\partial_r v_{||s}\right)^2\right] + \Bigg[-\frac{1}{\mathcal{H}_s^2}\left(\partial_r^2\psi_s^I + \partial_\eta^2\psi_s^I - \partial_\eta\partial_r\psi_s^I\right) \\
& - \frac{1}{\mathcal{H}_s}\partial_r \psi_s^I - \frac{3}{\mathcal{H}_s}\left(-1 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2} - \frac{2}{3}\frac{1}{\mathcal{H}_s r_s}\right)\partial_\eta \psi_s^I - \frac{4}{r_s}\left(-1 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta') \\
& + \left(-2 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\frac{2}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2\psi^I(\eta') - 2\left(-2 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta') \\
& + \frac{4}{\mathcal{H}_s r_s}\psi_s^I - \frac{2}{\mathcal{H}_s r_s^2}\int_{\eta_s}^{\eta_o} d\eta' \Delta_2\psi^I(\eta')\Bigg] v_{||s} + \Bigg[2\left(2 + \frac{\mathcal{H}_s'}{\mathcal{H}_s^2} + \frac{2}{\mathcal{H}_s r_s}\right)\partial_r v_{||s} \\
& + \frac{2}{\mathcal{H}_s}\partial_r^2 v_{||s}\Bigg]\int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta') + \left[\frac{2}{\mathcal{H}_s}\left(5 + 3\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\partial_r v_{||s} + \frac{2}{\mathcal{H}_s^2}\partial_r^2 v_{||s}\right]\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta') \\
& - \frac{2}{\mathcal{H}_s}\partial_r v_{||s}\frac{1}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2\psi^I(\eta') + \frac{2}{\mathcal{H}_s^2}\partial_\eta \psi_s^I \partial_r v_{||s} + \frac{1}{\mathcal{H}_s}\Bigg[\frac{1}{\mathcal{H}_s}\partial_r^2 v_{||s} \\
& + \left(6 + 3\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\partial_r v_{||s}\Bigg]\psi_s^I + \frac{2}{\mathcal{H}_s r_s} a v_{\perp\, s}^a \partial_a \int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta') - \frac{1}{\mathcal{H}_s} a v_{\perp\, s}^a \partial_a \psi_s^I \\
& - \frac{4}{\mathcal{H}_s}\gamma_{0s}^{ab}\partial_a v_{||s}\partial_b \int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta') + \frac{4}{r_s^2}\left(\int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta')\right)^2 + \Bigg\{\Bigg[2\left(-2 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\psi_s^I \\
& + 4\left(-2 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta') - \frac{2}{\mathcal{H}_s}\partial_\eta \psi_s^I\Bigg]\frac{1}{r_s} + 2\left(-2 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}^2\psi^I(\eta') \\
& + 2\left(-2 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\partial_\eta \psi_s^I + \left(-1 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\partial_r \psi_s^I - \frac{1}{\mathcal{H}_s}\partial_\eta\partial_r \psi_s^I\Bigg\}\left(-2\int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta')\right) \\
& + \Bigg[\frac{3}{\mathcal{H}_s}\left(-1 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2} - \frac{2}{3}\frac{1}{\mathcal{H}_s r_s}\right)\partial_\eta \psi_s^I + \frac{1}{\mathcal{H}_s}\partial_r \psi_s^I - \left(2 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\psi_s^I \\
& + \frac{1}{\mathcal{H}_s^2}\left(\partial_r^2\psi_s^I + \partial_\eta^2\psi_s^I - \partial_\eta\partial_r\psi_s^I\right)\Bigg]\left(-2\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta')\right) + \Bigg[\left(-2 - 2\frac{\mathcal{H}_s'}{\mathcal{H}_s^2} - \frac{2}{\mathcal{H}_s r_s}\right)\psi_s^I \\
& + 4\left(-2 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2} - \frac{1}{\mathcal{H}_s r_s}\right)\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta') - \frac{10}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta') \\
& - \frac{2}{\mathcal{H}_s}\partial_\eta \psi_s^I\Bigg]\frac{1}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2\psi^I(\eta') + 2\left(\frac{1}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2\psi^I(\eta')\right)^2 \\
& + \left(-\frac{1}{2} - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\left(\psi_s^I\right)^2 + \frac{1}{\mathcal{H}_s^2}\left(\partial_\eta \psi_s^I\right)^2 + \Bigg[-\frac{1}{\mathcal{H}_s^2}\left(\partial_r^2\psi_s^I + \partial_\eta^2\psi_s^I - \partial_\eta\partial_r\psi_s^I\right) \\
& + \frac{1}{\mathcal{H}_s}\left(4 + 3\frac{\mathcal{H}_s'}{\mathcal{H}_s^2} + \frac{2}{\mathcal{H}_s r_s}\right)\partial_\eta \psi_s^I - \frac{1}{\mathcal{H}_s}\partial_r \psi_s^I\Bigg]\psi_s^I \\
& + \frac{4}{\mathcal{H}_s}\gamma_{0\,s}^{ab}\partial_a\left(\int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta')\right)\partial_b\left(\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta')\right) + 2\partial_a\Bigg[\left(1 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)v_{||s} - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\psi_s^I \\
& - \frac{1}{\mathcal{H}_s}\left(\partial_\eta \psi_s^I + \partial_r v_{||s}\right)\Bigg]\int_{\eta_s}^{\eta_o} d\eta' \gamma_0^{ab}\partial_b \int_{\eta'}^{\eta_o} d\eta'' \psi^I(\eta'') \\
& + 4\left(1 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\int_{\eta_s}^{\eta_o} d\eta' \partial_a\left(\partial_{\eta'}\psi^I(\eta')\right)\int_{\eta'}^{\eta_o} d\eta'' \gamma_0^{ab}\partial_b \int_{\eta''}^{\eta_o} d\eta''' \psi^I(\eta''') \\
& + 2a v_{\perp\, s}^a \partial_a \int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta') + \frac{4}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \Bigg[\psi^I(\eta')\left(-\psi^I(\eta') - 2\int_{\eta'}^{\eta_o} d\eta'' \partial_{\eta''}\psi^I(\eta'')\right) \\
& + \gamma_0^{ab}\partial_a\left(\int_{\eta'}^{\eta_o} d\eta'' \psi^I(\eta'')\right)\partial_b\left(\int_{\eta'}^{\eta_o} d\eta'' \psi^I(\eta'')\right)\Bigg] + 4\partial_a \psi_s^I \int_{\eta_s}^{\eta_o} d\eta' \gamma_0^{ab}\partial_b \int_{\eta'}^{\eta_o} d\eta'' \psi^I(\eta'') \\
& + \frac{4}{r_s}\Bigg[\int_{\eta_s}^{\eta_o} d\eta' \partial_a\left(\frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2\psi^I(\eta') - 2\psi^I(\eta')\right)\int_{\eta'}^{\eta_o} d\eta'' \gamma_0^{ab}\partial_b \int_{\eta''}^{\eta_o} d\eta''' \psi^I(\eta''')\Bigg] \\
& + 2\partial_a\left(\int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta')\right)\Bigg[-\gamma_{0s}^{ab}\partial_b \int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta') - 2\int_{\eta_s}^{\eta_o} d\eta' \gamma_0^{ab}\partial_b \int_{\eta'}^{\eta_o} d\eta'' \partial_{\eta''}\psi^I(\eta'')\Bigg] \\
& + 2\partial_a\left(\int_{\eta_s}^{\eta_o} d\eta' \gamma_0^{bc}\partial_c \int_{\eta'}^{\eta_o} d\eta'' \psi^I(\eta'')\right)\partial_b\left(\int_{\eta_s}^{\eta_o} d\eta' \gamma_0^{ad}\partial_d \int_{\eta'}^{\eta_o} d\eta'' \psi^I(\eta'')\right) \\
& - 4\left(\int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta')\right)\int_{\eta_s}^{\eta_o} d\eta' \Bigg[-\frac{1}{(\eta_o - \eta')^3}\int_{\eta'}^{\eta_o} d\eta'' \Delta_2\psi^I(\eta'') + \frac{1}{(\eta_o - \eta')^2}\Bigg(\frac{1}{2}\Delta_2\psi^I(\eta') \\
& + \int_{\eta'}^{\eta_o} d\eta'' \partial_{\eta''}\left(\Delta_2\psi^I(\eta'')\right)\Bigg)\Bigg] + \frac{2}{(\sin\theta_o)^2}\left[\frac{1}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \frac{\eta' - \eta_s}{\eta_o - \eta'}\partial_{\theta_o}\psi^I(\eta')\right]^2 \\
& + \frac{2}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2\Bigg[\psi^I(\eta')\left(-\psi^I(\eta') - 2\int_{\eta'}^{\eta_o} d\eta'' \partial_{\eta''}\psi^I(\eta'')\right) \\
& + \gamma_0^{ab}\partial_a\left(\int_{\eta'}^{\eta_o} d\eta'' \psi^I(\eta'')\right)\partial_b\left(\int_{\eta'}^{\eta_o} d\eta'' \psi^I(\eta'')\right)\Bigg] \\
& + 2\int_{\eta_s}^{\eta_o} d\eta' \Bigg\{-2\psi^I(\eta')\frac{1}{\eta_o - \eta'}\int_{\eta'}^{\eta_o} d\eta'' \frac{\eta'' - \eta'}{\eta_o - \eta''}\Delta_2\partial_{\eta''}\psi^I(\eta'') \\
& + 2\gamma_0^{ab}\partial_b\left(\int_{\eta'}^{\eta_o} d\eta'' \psi^I(\eta'')\right)\frac{1}{\eta_o - \eta'}\int_{\eta'}^{\eta_o} d\eta'' \frac{\eta'' - \eta'}{\eta_o - \eta''}\partial_a\Delta_2\psi^I(\eta'') \\
& - \left(-2\psi^I(\eta') - 2\int_{\eta'}^{\eta_o} d\eta'' \partial_{\eta''}\psi^I(\eta'')\right)\frac{1}{(\eta_o - \eta')^2}\int_{\eta'}^{\eta_o} d\eta'' \Delta_2\psi^I(\eta'') \\
& - 2\partial_a \psi^I(\eta')\int_{\eta'}^{\eta_o} d\eta'' \gamma_0^{ab}\partial_b \int_{\eta''}^{\eta_o} d\eta''' \partial_{\eta'''}\psi^I(\eta''') \\
& + 2\partial_a\left[\gamma_0^{db}\partial_b \int_{\eta'}^{\eta_o} d\eta'' \psi^I(\eta'')\right]\int_{\eta'}^{\eta_o} d\eta'' \partial_d\left[\gamma_0^{ac}\partial_c \int_{\eta''}^{\eta_o} d\eta''' \psi^I(\eta''')\right] \\
& + 2\gamma_0^{ab}\partial_a\left(\psi^I(\eta') + \int_{\eta'}^{\eta_o} d\eta'' \partial_{\eta''}\psi^I(\eta'')\right)\partial_b \int_{\eta'}^{\eta_o} d\eta'' \psi^I(\eta'')\Bigg\} \\
& + \Bigg[\left(\frac{2}{\mathcal{H}_s r_s} + \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\left(v_{||s} + \psi_s^I + 2\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta')\right) - 3v_{||s} + \frac{1}{\mathcal{H}_s}\partial_r v_{||s} - 4\psi_s^I \\
& - 6\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta') + \frac{4}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta') - \frac{2}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2\psi^I(\eta') + \frac{1}{\mathcal{H}_s}\partial_\eta \psi_s^I\Bigg]\delta_\rho^{(1)} \\
& + \left[\frac{1}{\bar\rho}\partial_\eta\left(\bar\rho\,\delta_\rho^{(1)}\right) - \partial_r \delta_\rho^{(1)}\right]\frac{1}{\mathcal{H}_s}\left(-v_{||s} - \psi_s^I - 2\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta')\right) \\
& + 2\partial_r \delta_\rho^{(1)}\int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta') - 2\partial_a \delta_\rho^{(1)}\int_{\eta_s}^{\eta_o} d\eta' \gamma_0^{ab}\partial_b \int_{\eta'}^{\eta_o} d\eta'' \psi^I(\eta'') + \delta_\rho^{(2)}\,. \qquad (4.43)
\end{aligned}
$$

ED, Durrer, Marozzi, Montanari [arXiv:1407.0376]

$$
\begin{aligned}
\Sigma_{IS} = {} & \left(-\frac{2}{\mathcal{H}_s r_s} - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\Bigg\{-v_{||s}^{(2)} - \frac{1}{2}\phi_s^{(2)} - \frac{1}{2}\int_{\eta_s}^{\eta_0} d\eta' \partial_{\eta'}\left[\phi^{(2)}(\eta') + \psi^{(2)}(\eta')\right] + \frac{1}{2}\left(v_{||s}\right)^2 \\
& + \frac{1}{2}\left(\psi_s^I\right)^2 + \left(-v_{||s} - \psi_s^I\right)\left(-\psi_s^I - 2\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta')\right) + \frac{1}{2} v_{\perp s}^a v_{\perp a\, s} \\
& + 2a\, v_{\perp s}^a \partial_a \int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta') + 4\int_{\eta_s}^{\eta_0} d\eta' \Bigg[\psi^I(\eta')\,\partial_{\eta'}\psi^I(\eta') + \partial_{\eta'}\psi^I(\eta')\int_{\eta'}^{\eta_o} d\eta'' \partial_{\eta''}\psi^I(\eta'') \\
& + \psi^I(\eta')\int_{\eta'}^{\eta_o} d\eta'' \partial_{\eta''}^2 \psi^I(\eta'') - \gamma_0^{ab}\partial_a\left(\int_{\eta'}^{\eta_o} d\eta'' \psi^I(\eta'')\right)\partial_b\left(\int_{\eta'}^{\eta_o} d\eta'' \partial_{\eta''}\psi^I(\eta'')\right)\Bigg] \\
& + 2\partial_a\left(v_{||s} + \psi_s^I\right)\int_{\eta_s}^{\eta_o} d\eta' \gamma_0^{ab}\partial_b \int_{\eta'}^{\eta_o} d\eta'' \psi^I(\eta'') \\
& + 4\int_{\eta_s}^{\eta_o} d\eta' \partial_a\left(\partial_{\eta'}\psi^I(\eta')\right)\int_{\eta'}^{\eta_o} d\eta'' \gamma_0^{ab}\partial_b \int_{\eta''}^{\eta_o} d\eta''' \psi^I(\eta''')\Bigg\} \\
& + \left[\frac{1}{2}\frac{\mathcal{H}_s'}{\mathcal{H}_s^2} + \frac{3}{2}\left(\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)^2 - \frac{1}{2}\frac{\mathcal{H}_s''}{\mathcal{H}_s^3} + \frac{1}{\mathcal{H}_s r_s}\left(1 + 3\frac{\mathcal{H}_s'}{\mathcal{H}_s^2} + \frac{1}{\mathcal{H}_s r_s}\right)\right]\left[\left(v_{||s}\right)^2 + \left(\psi_s^I\right)^2 + 2\psi_s^I v_{||s}\right. \\
& \left. + 4\left(v_{||s} + \psi_s^I\right)\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta') + 4\left(\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta')\right)^2\right] - \psi_s^{(2)} + \frac{1}{2}\phi_s^{(2)} + \frac{1}{2\mathcal{H}_s}\partial_\eta \psi_s^{(2)} \\
& + \frac{1}{\mathcal{H}_s}\partial_r v_{||s}^{(2)} - \frac{1}{2}\frac{1}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2\left[\psi^{(2)} + \phi^{(2)}\right](\eta') + \frac{1}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \left[\psi^{(2)} + \phi^{(2)}\right](\eta') \\
& + 2\left(1 - \frac{1}{\mathcal{H}_s r_s}\right)\Bigg\{-\frac{2}{\mathcal{H}_s} v_{||s}\partial_r v_{||s} - \left(v_{||s}\right)^2 - \\
& - \frac{2}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta') + \frac{2}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2 \\
& - 2\mathcal{H}_s\int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta')\Bigg]\frac{1}{\mathcal{H}_s}\partial_r v_{||s} - a v_{\perp s}^a \partial_a \\
& + 2\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}^2 \psi^I(\eta')\Bigg]\left(-2\int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta'\right. \\
& - \left(-\psi_s^I - 2\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta')\right)\left[\frac{2}{r_s}\int_{\eta_s}^{\eta_o}\right. \\
& - \frac{1}{\mathcal{H}_s}\partial_\eta \psi_s^I - \psi_s^I\Bigg]\Bigg\} + \frac{3}{2} v_{\perp a\, s} v_{\perp s}^a - \frac{2}{\mathcal{H}_s} a v \\
& + \left(5 + 3\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\frac{1}{\mathcal{H}_s} v_{||s}\partial_r v_{||s} + \frac{1}{\mathcal{H}_s^2}\left[v_{||s}\partial_r^2 v\right. \\
& - \frac{1}{\mathcal{H}_s}\partial_r \psi_s^I - \frac{3}{\mathcal{H}_s}\left(-1 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2} - \frac{2}{3}\frac{1}{\mathcal{H}_s r_s}\right)\partial_\eta \psi_s \\
& + \left(-2 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\frac{2}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2 \psi^I(\eta') - 2\left(-2 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta') \\
& + \frac{4}{\mathcal{H}_s r_s}\psi_s^I - \frac{2}{\mathcal{H}_s r_s^2}\int_{\eta_s}^{\eta_o} d\eta' \Delta_2 \psi^I(\eta')\Bigg] v_{||s} + \left[2\left(2 + \frac{\mathcal{H}_s'}{\mathcal{H}_s^2} + \frac{2}{\mathcal{H}_s r_s}\right)\partial_r v_{||s}\right. \\
& + \frac{2}{\mathcal{H}_s}\partial_r^2 v_{||s}\Bigg]\int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta') + \left[\frac{2}{\mathcal{H}_s}\left(5 + 3\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\partial_r v_{||s} + \frac{2}{\mathcal{H}_s^2}\partial_r^2 v_{||s}\right]\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta') \\
& - \frac{2}{\mathcal{H}_s}\partial_r v_{||s}\frac{1}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2 \psi^I(\eta') + \frac{2}{\mathcal{H}_s^2}\partial_\eta \psi_s^I \partial_r v_{||s} + \frac{1}{\mathcal{H}_s}\left[\frac{1}{\mathcal{H}_s}\partial_r^2 v_{||s}\right. \\
& + \left(6 + 3\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\partial_r v_{||s}\Bigg]\psi_s^I + \frac{2}{\mathcal{H}_s r_s} a v_{\perp s}^a \partial_a \int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta') - \frac{1}{\mathcal{H}_s} a v_{\perp s}^a \partial_a \psi_s^I \\
& - \frac{4}{\mathcal{H}_s}\gamma_{0s}^{ab}\partial_a v_{||s}\partial_b \int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta') + \frac{4}{r_s^2}\left(\int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta')\right)^2 + \Bigg\{\Bigg[2\left(-2 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\psi_s^I \\
& + 4\left(-2 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta') - \frac{2}{\mathcal{H}_s}\partial_\eta \psi_s^I\Bigg]\frac{1}{r_s} + 2\left(-2 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}^2 \psi^I(\eta') \\
& + 2\left(-2 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\partial_\eta \psi_s^I + \left(-1 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\partial_r \psi_s^I - \frac{1}{\mathcal{H}_s}\partial_\eta \partial_r \psi_s^I\Bigg\}\left(-2\int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta')\right) \\
& + \left[\frac{3}{\mathcal{H}_s}\left(-1 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2} - \frac{2}{3}\frac{1}{\mathcal{H}_s r_s}\right)\partial_\eta \psi_s^I + \frac{1}{\mathcal{H}_s}\partial_r \psi_s^I - \left(2 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\psi_s^I\right. \\
& + \frac{1}{\mathcal{H}_s^2}\left(\partial_r^2 \psi_s^I + \partial_\eta^2 \psi_s^I - \partial_\eta \partial_r \psi_s^I\right)\Bigg]\left(-2\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta')\right) + \left[\left(-2 - 2\frac{\mathcal{H}_s'}{\mathcal{H}_s^2} - \frac{2}{\mathcal{H}_s r_s}\right)\psi_s^I\right. \\
& + 4\left(-2 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2} - \frac{1}{\mathcal{H}_s r_s}\right)\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta') - \frac{10}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta') \\
& - \frac{2}{\mathcal{H}_s}\partial_\eta \psi_s^I\Bigg]\frac{1}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2 \psi^I(\eta') + 2\left(\frac{1}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2 \psi^I(\eta')\right)^2 \\
& + \left(-\frac{1}{2} - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\left(\psi_s^I\right)^2 + \frac{1}{\mathcal{H}_s^2}\left(\partial_\eta \psi_s^I\right)^2 + \Bigg[-\frac{1}{\mathcal{H}_s^2}\left(\partial_r^2 \psi_s^I + \partial_\eta^2 \psi_s^I - \partial_\eta \partial_r \psi_s^I\right) \\
& + \frac{1}{\mathcal{H}_s}\left(4 + 3\frac{\mathcal{H}_s'}{\mathcal{H}_s^2} + \frac{2}{\mathcal{H}_s r_s}\right)\partial_\eta \psi_s^I - \frac{1}{\mathcal{H}_s}\partial_r \psi_s^I\Bigg]\psi_s^I \\
& + \frac{4}{\mathcal{H}_s}\gamma_{0s}^{ab}\partial_a\left(\int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta')\right)\partial_b\left(\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta')\right) + 2\partial_a\Bigg[\left(1 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)v_{||s} - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\psi_s^I \\
& - \frac{1}{\mathcal{H}_s}\left(\partial_\eta \psi_s^I + \partial_r v_{||s}\right)\Bigg]\int_{\eta_s}^{\eta_o} d\eta' \gamma_0^{ab}\partial_b \int_{\eta'}^{\eta_o} d\eta'' \psi^I(\eta'') \\
& + 4\left(1 - \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\int_{\eta_s}^{\eta_o} d\eta' \partial_a\left(\partial_{\eta'}\psi^I(\eta')\right)\int_{\eta'}^{\eta_o} d\eta'' \gamma_0^{ab}\partial_b \int_{\eta''}^{\eta_o} d\eta''' \psi^I(\eta''') \\
& + 2a v_{\perp s}^a \partial_a \int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta') + \frac{4}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \Bigg[\psi^I(\eta')\left(-\psi^I(\eta') - 2\int_{\eta'}^{\eta_o} d\eta'' \partial_{\eta''}\psi^I(\eta'')\right) \\
& \left.\left.\right)\right)\Bigg] + 4\partial_a \psi_s^I \int_{\eta_s}^{\eta_o} d\eta' \gamma_0^{ab}\partial_b \int_{\eta'}^{\eta_o} d\eta'' \psi^I(\eta'') \\
& \left.\left(\eta'\right)\right)\int_{\eta'}^{\eta_o} d\eta'' \gamma_0^{ab}\partial_b \int_{\eta''}^{\eta_o} d\eta''' \psi^I(\eta''')\Bigg] \\
& \left.(\eta') - 2\int_{\eta_s}^{\eta_o} d\eta' \gamma_0^{ab}\partial_b \int_{\eta'}^{\eta_o} d\eta'' \partial_{\eta''}\psi^I(\eta'')\Bigg] \right. \\
& \left.\int_{\eta_s}^{\eta_o} d\eta' \gamma_0^{ad}\partial_d \int_{\eta'}^{\eta_o} d\eta'' \psi^I(\eta'')\right) \\
& \int_{\eta'}^{\eta_o} d\eta'' \Delta_2 \psi^I(\eta'') + \frac{1}{(\eta_o - \eta')^2}\left(\frac{1}{2}\Delta_2 \psi^I(\eta')\right. \\
& \left[\frac{1}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \frac{\eta' - \eta_s}{\eta_o - \eta'}\partial_{\theta_o}\psi^I(\eta')\right]^2 \\
& - 2\int_{\eta'}^{\eta_o} d\eta'' \partial_{\eta''}\psi^I(\eta'')\Bigg) \\
& + \gamma_0^{ab}\partial_a\left(\int_{\eta'}^{\eta_o} d\eta' \psi^I(\eta')\right)\partial_b\left(\int_{\eta'}^{\eta_o} d\eta'' \psi^I(\eta'')\right)\Bigg] \\
& + 2\int_{\eta_s}^{\eta_o} d\eta' \Bigg\{-2\psi^I(\eta')\frac{1}{\eta_o - \eta'}\int_{\eta'}^{\eta_o} d\eta'' \frac{\eta'' - \eta'}{\eta_o - \eta''}\Delta_2 \partial_{\eta''}\psi^I(\eta'') \\
& + 2\gamma_0^{ab}\partial_b\left(\int_{\eta'}^{\eta_o} d\eta'' \psi^I(\eta'')\right)\frac{1}{\eta_o - \eta'}\int_{\eta'}^{\eta_o} d\eta'' \frac{\eta'' - \eta'}{\eta_o - \eta''}\partial_a \Delta_2 \psi^I(\eta'') \\
& - \left(-2\psi^I(\eta') - 2\int_{\eta'}^{\eta_o} d\eta'' \partial_{\eta''}\psi^I(\eta'')\right)\frac{1}{(\eta_o - \eta')^2}\int_{\eta'}^{\eta_o} d\eta'' \Delta_2 \psi^I(\eta'') \\
& - 2\partial_a \psi^I(\eta')\int_{\eta'}^{\eta_o} d\eta'' \gamma_0^{ab}\partial_b \int_{\eta''}^{\eta_o} d\eta''' \partial_{\eta'''}\psi^I(\eta''') \\
& + 2\partial_a\left[\gamma_0^{db}\partial_b \int_{\eta'}^{\eta_o} d\eta'' \psi^I(\eta'')\right]\int_{\eta'}^{\eta_o} d\eta'' \partial_d\left[\gamma_0^{ac}\partial_c \int_{\eta''}^{\eta_o} d\eta''' \psi^I(\eta''')\right] \\
& + 2\gamma_0^{ab}\partial_a\left(\psi^I(\eta') + \int_{\eta'}^{\eta_o} d\eta'' \partial_{\eta''}\psi^I(\eta'')\right)\partial_b \int_{\eta'}^{\eta_o} d\eta'' \psi^I(\eta'')\Bigg\} \\
& + \Bigg[\left(\frac{2}{\mathcal{H}_s r_s} + \frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\left(v_{||s} + \psi_s^I + 2\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta')\right) - 3v_{||s} + \frac{1}{\mathcal{H}_s}\partial_r v_{||s} - 4\psi_s^I \\
& - 6\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta') + \frac{4}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta') - \frac{2}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2 \psi^I(\eta') + \frac{1}{\mathcal{H}_s}\partial_\eta \psi_s^I\Bigg]\delta_\rho^{(1)} \\
& + \left[\frac{1}{\bar{\rho}}\partial_\eta\left(\bar{\rho}\,\delta_\rho^{(1)}\right) - \partial_r \delta_\rho^{(1)}\right]\frac{1}{\mathcal{H}_s}\left(-v_{||s} - \psi_s^I - 2\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta')\right) \\
& + 2\partial_r \delta_\rho^{(1)}\int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta') - 2\partial_a \delta_\rho^{(1)}\int_{\eta_s}^{\eta_o} d\eta' \gamma_0^{ab}\partial_b \int_{\eta'}^{\eta_o} d\eta'' \psi^I(\eta'') + \delta_\rho^{(2)}\,. \qquad (4.43)
\end{aligned}
$$

See also
Yoo and Zaldarriaga [arXiv:1406.4140]
Bertacca, Maartens, Clarkson [arXiv:1405.4403]

and for a comparison
Trøst Nielsen and Durrer [arXiv:1606.02113]

ED, Durrer, Marozzi, Montanari [arXiv:1407.0376]

Beyond linear order

$$\Delta(\mathbf{n}, z) \equiv \frac{N(\mathbf{n}, z) - <N>(z)}{<N>(z)}$$

Leading terms

$$\begin{aligned}\Sigma^{(2)}(\mathbf{n}, z) = {} & \delta^{(2)} + \mathcal{H}^{-1}\partial_r^2 v^{(2)} + \mathcal{H}^{-2}\left(\partial_r^2 v\right)^2 + \mathcal{H}^{-2}\partial_r v \partial_r^3 v + \mathcal{H}^{-1}\left(\partial_r v \partial_r \delta + \partial_r^2 v\, \delta\right) \\ & - 2\kappa^{(2)} - 2\delta\kappa + \nabla_a \delta \nabla^a \psi + \mathcal{H}^{-1}\left[-2(\partial_r^2 v)\kappa + \nabla_a(\partial_r^2 v)\nabla^a \psi\right] + 2\kappa^2 \\ & - 2\nabla_b \kappa \nabla^b \psi - \frac{1}{2r(z)}\int_0^{r(z)} dr \frac{r(z) - r}{r} \Delta_2\left(\nabla^b \Psi_1 \nabla_b \Psi_1\right) - 2\int_0^{r(z)} \frac{dr}{r} \nabla^a \Psi_1 \nabla_a \kappa\end{aligned}$$

ED, Durrer, Marozzi, Montanari [arXiv:1510.04202]

Beyond linear order

$$\Delta(\mathbf{n}, z) \equiv \frac{N(\mathbf{n}, z) - <N>(z)}{<N>(z)}$$

Leading terms

density-RSD

$$\begin{aligned}\Sigma^{(2)}(\mathbf{n}, z) = & \boxed{\delta^{(2)} + \mathcal{H}^{-1}\partial_r^2 v^{(2)} + \mathcal{H}^{-2}\left(\partial_r^2 v\right)^2 + \mathcal{H}^{-2}\partial_r v \partial_r^3 v + \mathcal{H}^{-1}\left(\partial_r v \partial_r \delta + \partial_r^2 v\, \delta\right)} \\ & - 2\kappa^{(2)} - 2\delta\kappa + \nabla_a \delta \nabla^a \psi + \mathcal{H}^{-1}\left[-2(\partial_r^2 v)\kappa + \nabla_a(\partial_r^2 v)\nabla^a \psi\right] + 2\kappa^2 \\ & - 2\nabla_b \kappa \nabla^b \psi - \frac{1}{2r(z)}\int_0^{r(z)} dr \frac{r(z) - r}{r} \Delta_2\left(\nabla^b \Psi_1 \nabla_b \Psi_1\right) - 2\int_0^{r(z)} \frac{dr}{r} \nabla^a \Psi_1 \nabla_a \kappa\end{aligned}$$

ED, Durrer, Marozzi, Montanari [arXiv:1510.04202]

Beyond linear order

$$\Delta(\mathbf{n}, z) \equiv \frac{N(\mathbf{n}, z) - <N>(z)}{<N>(z)}$$

$$Z_2(\mathbf{k}_1, \mathbf{k}_2) = \frac{b_2}{2} + b_1 F_2(\mathbf{k}_1, \mathbf{k}_2) + f\mu^2 G_2(\mathbf{k}_1, \mathbf{k}_2) + \frac{f\mu k}{2}\left[\frac{\mu_1}{k_1}\left(b_1 + f\mu_2^2\right) + \frac{\mu_2}{k_2}\left(b_1 + f\mu_1^2\right)\right]$$

density-RSD

$$\begin{aligned}\Sigma^{(2)}(\mathbf{n}, z) = {} & \delta^{(2)} + \mathcal{H}^{-1}\partial_r^2 v^{(2)} + \mathcal{H}^{-2}\left(\partial_r^2 v\right)^2 + \mathcal{H}^{-2}\partial_r v \partial_r^3 v + \mathcal{H}^{-1}\left(\partial_r v \partial_r \delta + \partial_r^2 v\, \delta\right) \\ & - 2\kappa^{(2)} - 2\delta\kappa + \nabla_a \delta \nabla^a \psi + \mathcal{H}^{-1}\left[-2(\partial_r^2 v)\kappa + \nabla_a(\partial_r^2 v)\nabla^a \psi\right] + 2\kappa^2 \\ & - 2\nabla_b \kappa \nabla^b \psi - \frac{1}{2r(z)}\int_0^{r(z)} dr \frac{r(z) - r}{r}\Delta_2\left(\nabla^b \Psi_1 \nabla_b \Psi_1\right) - 2\int_0^{r(z)} \frac{dr}{r}\nabla^a \Psi_1 \nabla_a \kappa\end{aligned}$$

ED, Durrer, Marozzi, Montanari [arXiv:1510.04202]

Beyond linear order

$$\Delta(\mathbf{n}, z) \equiv \frac{N(\mathbf{n}, z) - <N>(z)}{<N>(z)}$$

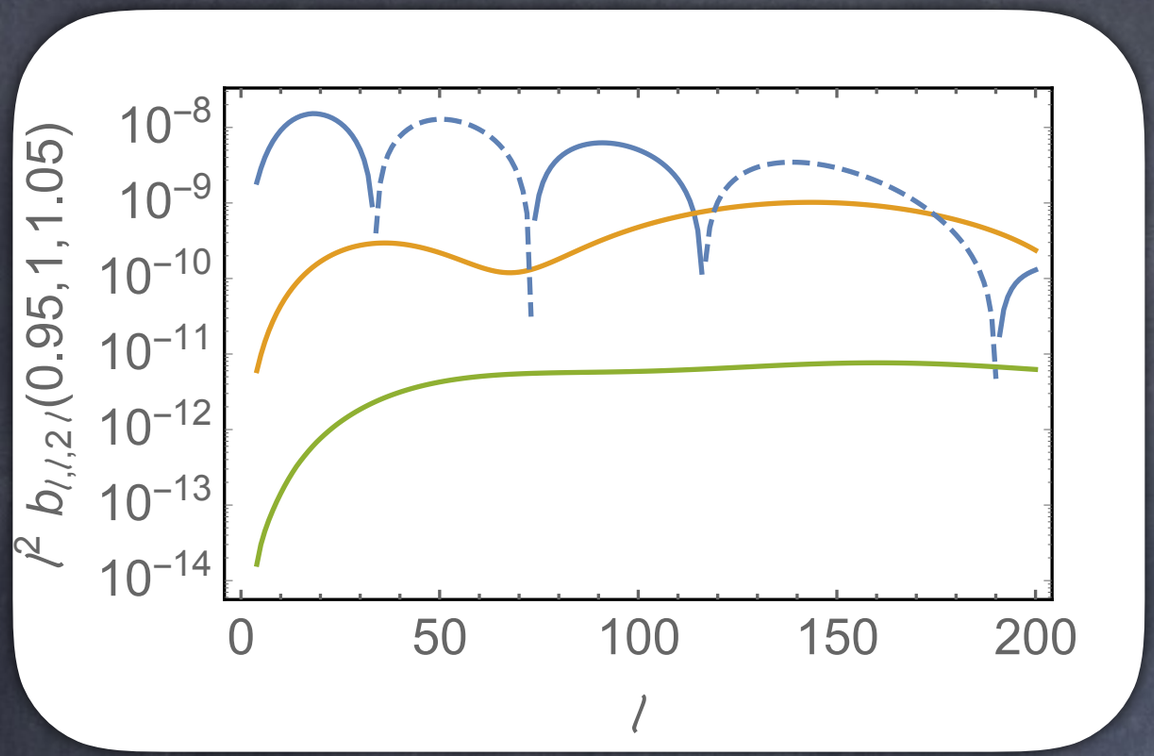
Leading terms

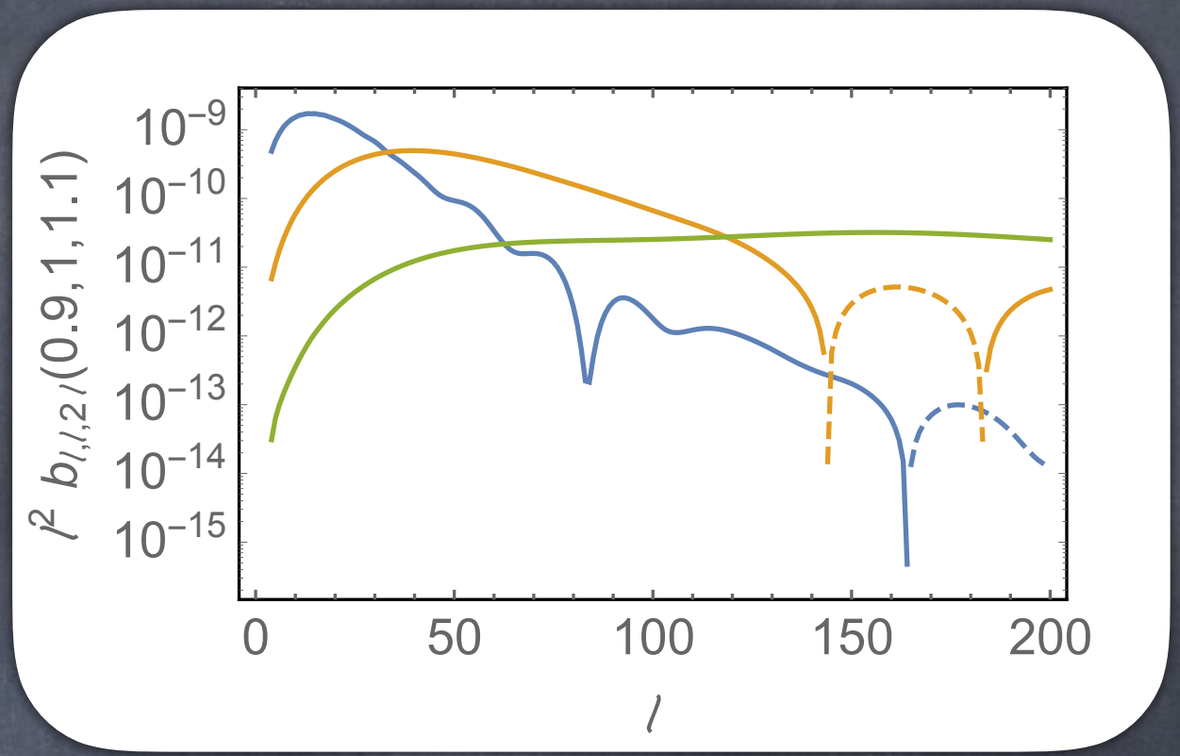
$$\Sigma^{(2)}(\mathbf{n}, z) = \delta^{(2)} + \mathcal{H}^{-1}\partial_r^2 v^{(2)} + \mathcal{H}^{-2}\left(\partial_r^2 v\right)^2 + \mathcal{H}^{-2}\partial_r v \partial_r^3 v + \mathcal{H}^{-1}\left(\partial_r v \partial_r \delta + \partial_r^2 v\, \delta\right)$$
$$- 2\kappa^{(2)} - 2\delta\kappa + \nabla_a \delta \nabla^a \psi + \mathcal{H}^{-1}\left[-2(\partial_r^2 v)\kappa + \nabla_a(\partial_r^2 v)\nabla^a \psi\right] + 2\kappa^2$$
$$- 2\nabla_b \kappa \nabla^b \psi - \frac{1}{2r(z)}\int_0^{r(z)} dr \frac{r(z) - r}{r}\Delta_2\left(\nabla^b \Psi_1 \nabla_b \Psi_1\right) - 2\int_0^{r(z)} \frac{dr}{r}\nabla^a \Psi_1 \nabla_a \kappa$$

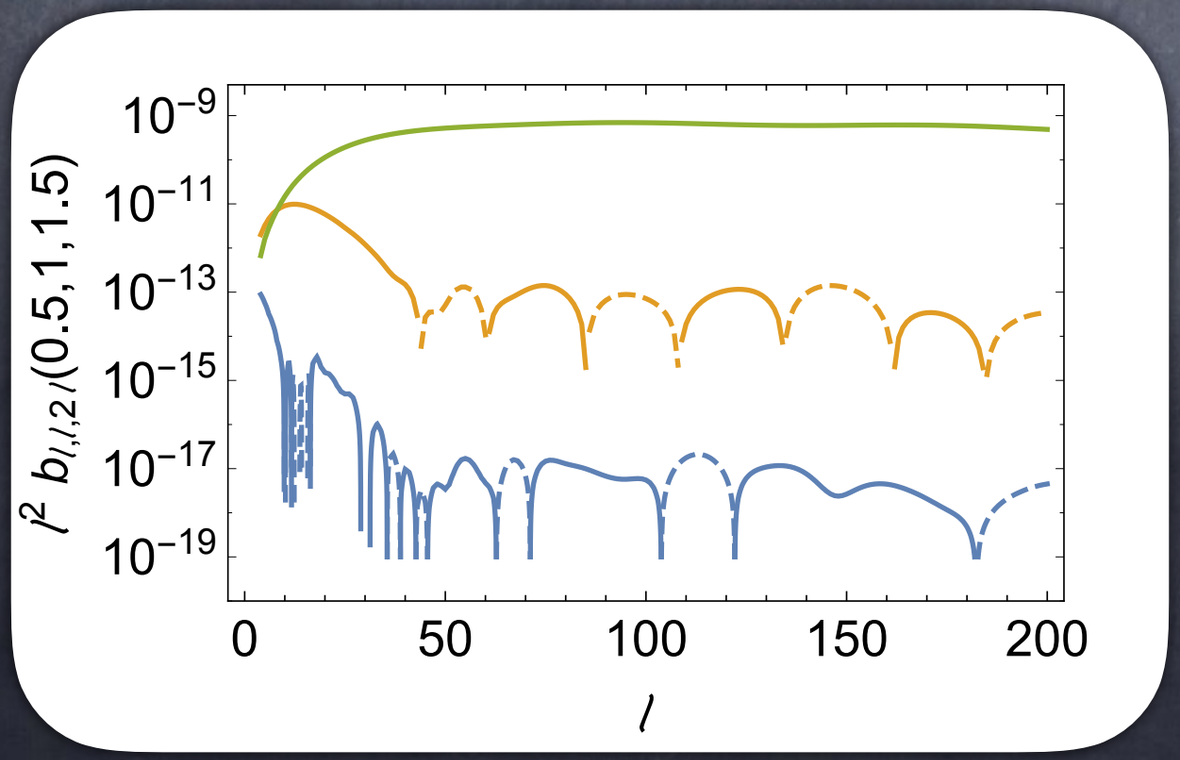
like-lensing terms

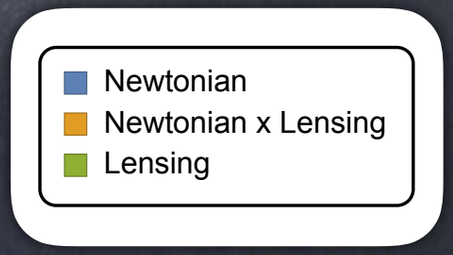
ED, Durrer, Marozzi, Montanari [arXiv:1510.04202]

Bispectrum









ED, Durrer, Marozzi, Montanari [arXiv:1510.04202]

Lensed Power Spectrum

$$\Sigma^{(2)}(\mathbf{n},z) = \delta^{(2)} + \mathcal{H}^{-1}\partial_r^2 v^{(2)} + \mathcal{H}^{-2}\left(\partial_r^2 v\right)^2 + \mathcal{H}^{-2}\partial_r v \partial_r^3 v + \mathcal{H}^{-1}\left(\partial_r v \partial_r \delta + \partial_r^2 v\, \delta\right)$$
$$- 2\kappa^{(2)} - 2\delta\kappa + \nabla_a \delta \nabla^a \psi + \mathcal{H}^{-1}\left[-2(\partial_r^2 v)\kappa + \nabla_a(\partial_r^2 v)\nabla^a\psi\right] + 2\kappa^2$$
$$- 2\nabla_b\kappa\nabla^b\psi - \frac{1}{2r(z)}\int_0^{r(z)} dr \frac{r(z)-r}{r}\Delta_2\left(\nabla^b\Psi_1\nabla_b\Psi_1\right) - 2\int_0^{r(z)}\frac{dr}{r}\nabla^a\Psi_1\nabla_a\kappa$$

Lensing induced by deflection angle

ED [arXiv:1609.09044]

Lensed Power Spectrum

$$\Sigma^{(2)}(\mathbf{n},z) = \delta^{(2)} + \mathcal{H}^{-1}\partial_r^2 v^{(2)} + \mathcal{H}^{-2}\left(\partial_r^2 v\right)^2 + \mathcal{H}^{-2}\partial_r v \partial_r^3 v + \mathcal{H}^{-1}\left(\partial_r v \partial_r \delta + \partial_r^2 v\, \delta\right)$$
$$- 2\kappa^{(2)} - 2\delta\kappa + \nabla_a \delta \nabla^a \psi + \mathcal{H}^{-1}\left[-2(\partial_r^2 v)\kappa + \nabla_a(\partial_r^2 v)\nabla^a\psi\right] + 2\kappa^2$$
$$- 2\nabla_b\kappa\nabla^b\psi - \frac{1}{2r(z)}\int_0^{r(z)} dr \frac{r(z)-r}{r}\Delta_2\left(\nabla^b\Psi_1\nabla_b\Psi_1\right) - 2\int_0^{r(z)}\frac{dr}{r}\nabla^a\Psi_1\nabla_a\kappa$$

Lensing induced by deflection angle

$$\tilde{\delta}\left(\mathbf{x}\right) = \delta\left(\mathbf{x} + \nabla\psi\right)$$

ED [arXiv:1609.09044]

Lensed Power Spectrum

$$\Sigma^{(2)}(\mathbf{n},z) = \delta^{(2)} + \mathcal{H}^{-1}\partial_r^2 v^{(2)} + \mathcal{H}^{-2}\left(\partial_r^2 v\right)^2 + \mathcal{H}^{-2}\partial_r v \partial_r^3 v + \mathcal{H}^{-1}\left(\partial_r v \partial_r \delta + \partial_r^2 v\, \delta\right)$$
$$- 2\kappa^{(2)} - 2\delta\kappa + \nabla_a \delta \nabla^a \psi + \mathcal{H}^{-1}\left[-2(\partial_r^2 v)\kappa + \nabla_a(\partial_r^2 v)\nabla^a\psi\right] + 2\kappa^2$$
$$- 2\nabla_b\kappa\nabla^b\psi - \frac{1}{2r(z)}\int_0^{r(z)} dr \frac{r(z)-r}{r}\Delta_2\left(\nabla^b\Psi_1\nabla_b\Psi_1\right) - 2\int_0^{r(z)}\frac{dr}{r}\nabla^a\Psi_1\nabla_a\kappa$$

Lensing induced by deflection angle

$$\tilde{\delta}\left(\mathbf{x}\right) = \delta\left(\mathbf{x} + \nabla\psi\right) \sim \delta\left(\mathbf{x}\right) + \nabla^a\psi\left(\mathbf{x}\right)\nabla_a\delta\left(\mathbf{x}\right)$$

ED [arXiv:1609.09044]

Lensed Power Spectrum

$$\Sigma^{(2)}(\mathbf{n},z) = \delta^{(2)} + \mathcal{H}^{-1}\partial_r^2 v^{(2)} + \mathcal{H}^{-2}\left(\partial_r^2 v\right)^2 + \mathcal{H}^{-2}\partial_r v \partial_r^3 v + \mathcal{H}^{-1}\left(\partial_r v \partial_r \delta + \partial_r^2 v\, \delta\right)$$
$$- 2\kappa^{(2)} - 2\delta\kappa + \nabla_a \delta \nabla^a \psi + \mathcal{H}^{-1}\left[-2(\partial_r^2 v)\kappa + \nabla_a(\partial_r^2 v)\nabla^a\psi\right] + 2\kappa^2$$
$$- 2\nabla_b\kappa\nabla^b\psi - \frac{1}{2r(z)}\int_0^{r(z)} dr \frac{r(z)-r}{r}\Delta_2\left(\nabla^b\Psi_1\nabla_b\Psi_1\right) - 2\int_0^{r(z)}\frac{dr}{r}\nabla^a\Psi_1\nabla_a\kappa$$

Lensing induced by deflection angle

$$\tilde{\delta}\left(\mathbf{x}\right) = \delta\left(\mathbf{x} + \nabla\psi\right) \sim \delta\left(\mathbf{x}\right) + \nabla^a\psi\left(\mathbf{x}\right)\nabla_a\delta\left(\mathbf{x}\right)$$
$$+ \frac{1}{2}\nabla^a\psi\left(\mathbf{x}\right)\nabla^b\psi\left(\mathbf{x}\right)\nabla_b\nabla_a\delta\left(\mathbf{x}\right) + \ldots$$

ED [arXiv:1609.09044]

Lensed Power Spectrum

$$\Sigma^{(2)}(\mathbf{n},z) = \delta^{(2)} + \mathcal{H}^{-1}\partial_r^2 v^{(2)} + \mathcal{H}^{-2}\left(\partial_r^2 v\right)^2 + \mathcal{H}^{-2}\partial_r v \partial_r^3 v + \mathcal{H}^{-1}\left(\partial_r v \partial_r \delta + \partial_r^2 v\, \delta\right)$$
$$- 2\kappa^{(2)} - 2\delta\kappa + \nabla_a \delta \nabla^a \psi + \mathcal{H}^{-1}\left[-2(\partial_r^2 v)\kappa + \nabla_a(\partial_r^2 v)\nabla^a\psi\right] + 2\kappa^2$$
$$- 2\nabla_b\kappa\nabla^b\psi - \frac{1}{2r(z)}\int_0^{r(z)} dr \frac{r(z)-r}{r}\Delta_2\left(\nabla^b\Psi_1\nabla_b\Psi_1\right) - 2\int_0^{r(z)}\frac{dr}{r}\nabla^a\Psi_1\nabla_a\kappa$$

non-perturbatively

ED [arXiv:1609.09044]

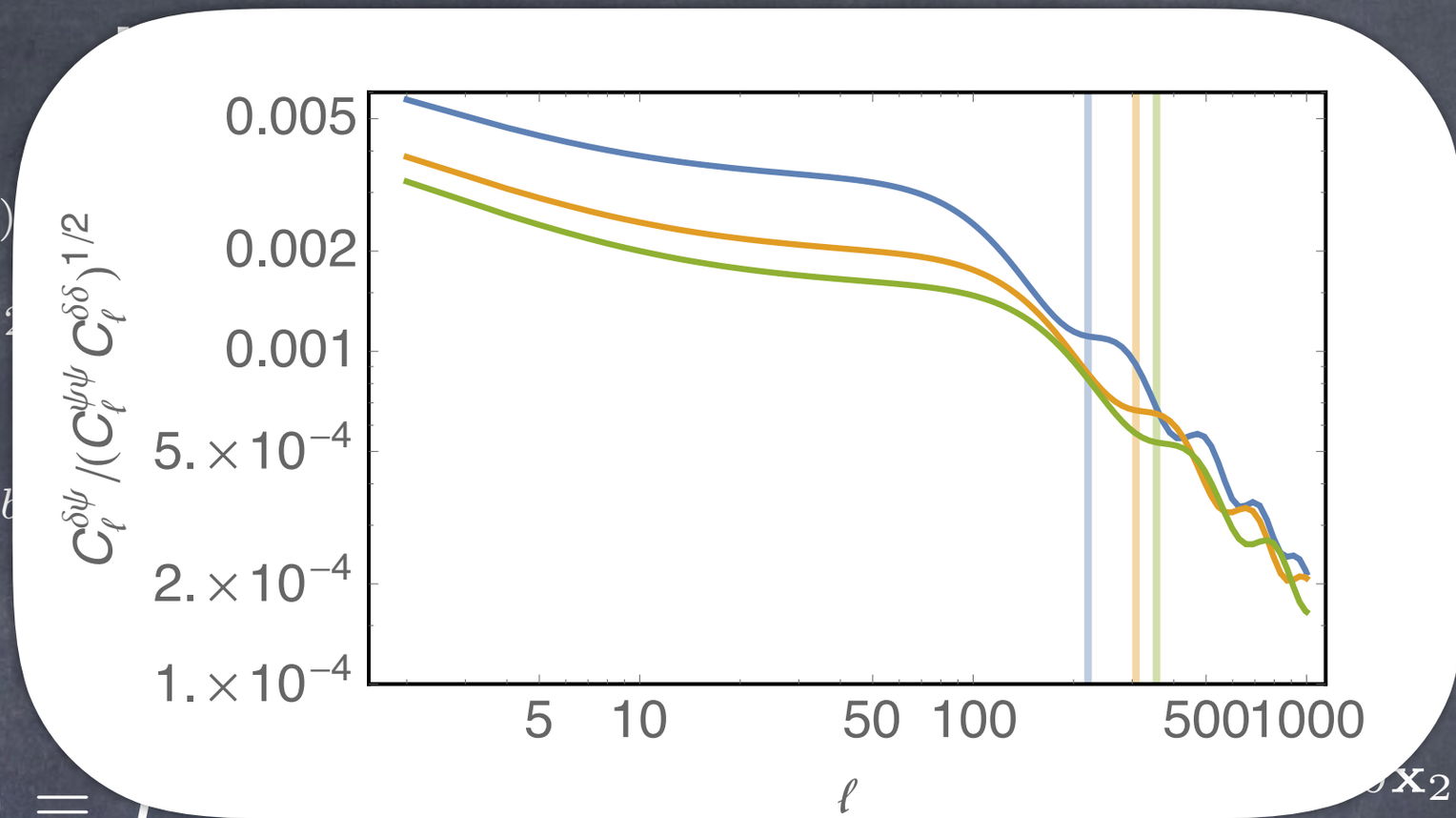
Lensed Power Spectrum

$$\Sigma^{(2)}(\mathbf{n},z) = \delta^{(2)} + \mathcal{H}^{-1}\partial_r^2 v^{(2)} + \mathcal{H}^{-2}\left(\partial_r^2 v\right)^2 + \mathcal{H}^{-2}\partial_r v \partial_r^3 v + \mathcal{H}^{-1}\left(\partial_r v \partial_r \delta + \partial_r^2 v\, \delta\right)$$
$$- 2\kappa^{(2)} - 2\delta\kappa + \nabla_a \delta \nabla^a \psi + \mathcal{H}^{-1}\left[-2(\partial_r^2 v)\kappa + \nabla_a(\partial_r^2 v)\nabla^a\psi\right] + 2\kappa^2$$
$$- 2\nabla_b\kappa\nabla^b\psi - \frac{1}{2r(z)}\int_0^{r(z)} dr \frac{r(z)-r}{r}\Delta_2\left(\nabla^b\Psi_1\nabla_b\Psi_1\right) - 2\int_0^{r(z)}\frac{dr}{r}\nabla^a\Psi_1\nabla_a\kappa$$

non-perturbatively

$$\tilde{\xi}\left(\Delta x_\perp, \Delta x_\parallel\right) = \int \frac{d^3k_1 d^3k_2}{(2\pi)^3} e^{i\mathbf{k}_1\cdot\mathbf{x}_1 + i\mathbf{k}_2\cdot\mathbf{x}_2}\left\langle e^{i\mathbf{k}_1\cdot\delta\mathbf{x}_1 + i\mathbf{k}_2\cdot\delta\mathbf{x}_2}\delta\left(\mathbf{k}_1\right)\delta\left(\mathbf{k}_2\right)\right\rangle$$

ED [arXiv:1609.09044]



$$\Sigma^{(2)}(\mathbf{n}, z) = \delta^{(2)} \qquad \partial_r \delta + \partial_r^2 v\, \delta)$$

$$- 2\kappa^{(2} \qquad + 2\kappa^2$$

$$- 2\nabla_b \qquad {}^{r(z)} \qquad \frac{dr}{r} \nabla^a \Psi_1 \nabla_a \kappa$$

$$\tilde{\xi}\left(\Delta x_\perp, \Delta x_\parallel\right) = \int \frac{}{(2\pi)^3} \qquad \mathbf{x}_2\, \delta\left(\mathbf{k}_1\right) \delta\left(\mathbf{k}_2\right)\rangle$$

$$= \int \frac{d^3 k_1 d^3 k_2}{(2\pi)^3}\; e^{i\mathbf{k}_1\cdot\mathbf{x}_1 + i\mathbf{k}_2\cdot\mathbf{x}_2} \left\langle e^{i\mathbf{k}_1\cdot\delta\mathbf{x}_1 + i\mathbf{k}_2\cdot\delta\mathbf{x}_2}\right\rangle \left\langle \delta\left(\mathbf{k}_1\right) \delta\left(\mathbf{k}_2\right)\right\rangle$$

ED [arXiv:1609.09044]

Lensed Power Spectrum

$$\Sigma^{(2)}(\mathbf{n},z) = \delta^{(2)} + \mathcal{H}^{-1}\partial_r^2 v^{(2)} + \mathcal{H}^{-2}\left(\partial_r^2 v\right)^2 + \mathcal{H}^{-2}\partial_r v \partial_r^3 v + \mathcal{H}^{-1}\left(\partial_r v \partial_r \delta + \partial_r^2 v\,\delta\right)$$
$$- 2\kappa^{(2)} - 2\delta\kappa + \nabla_a \delta \nabla^a \psi + \mathcal{H}^{-1}\left[-2(\partial_r^2 v)\kappa + \nabla_a(\partial_r^2 v)\nabla^a\psi\right] + 2\kappa^2$$
$$- 2\nabla_b\kappa\nabla^b\psi - \frac{1}{2r(z)}\int_0^{r(z)} dr \frac{r(z)-r}{r}\Delta_2\left(\nabla^b\Psi_1\nabla_b\Psi_1\right) - 2\int_0^{r(z)}\frac{dr}{r}\nabla^a\Psi_1\nabla_a\kappa$$

non-perturbatively

$$\tilde{\xi}\left(\Delta x_\perp, \Delta x_\parallel\right) = \int \frac{d^3k_1 d^3k_2}{(2\pi)^3} e^{i\mathbf{k}_1\cdot\mathbf{x}_1 + i\mathbf{k}_2\cdot\mathbf{x}_2}\left\langle e^{i\mathbf{k}_1\cdot\delta\mathbf{x}_1 + i\mathbf{k}_2\cdot\delta\mathbf{x}_2}\delta\left(\mathbf{k}_1\right)\delta\left(\mathbf{k}_2\right)\right\rangle$$
$$= \int \frac{d^3k_1 d^3k_2}{(2\pi)^3}\, e^{i\mathbf{k}_1\cdot\mathbf{x}_1 + i\mathbf{k}_2\cdot\mathbf{x}_2}\left\langle e^{i\mathbf{k}_1\cdot\delta\mathbf{x}_1 + i\mathbf{k}_2\cdot\delta\mathbf{x}_2}\right\rangle\left\langle\delta\left(\mathbf{k}_1\right)\delta\left(\mathbf{k}_2\right)\right\rangle$$
$$= \int \frac{d^3k}{(2\pi)^3}\, e^{i\mathbf{k}\cdot(\mathbf{x}_1 - \mathbf{x}_2)}\left\langle e^{i\mathbf{k}\cdot(\delta\mathbf{x}_1 - \delta\mathbf{x}_2)}\right\rangle P\left(k\right)$$

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$$= \int \frac{d^3k}{(2\pi)^3}\, e^{-i\mathbf{k}\cdot\mathbf{\Delta x}} e^{-\langle[\mathbf{k}\cdot(\delta\mathbf{x}_1 - \delta\mathbf{x}_2)]^2\rangle/2} P\left(k\right)$$

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Lensed Power Spectrum

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$$= \int \frac{d^3k}{(2\pi)^3}\, e^{-i\mathbf{k}\cdot\mathbf{\Delta x}} e^{-\langle[\mathbf{k}\cdot(\delta\mathbf{x}_1 - \delta\mathbf{x}_2)]^2\rangle/2} P\left(k\right)$$

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Lensed Power Spectrum

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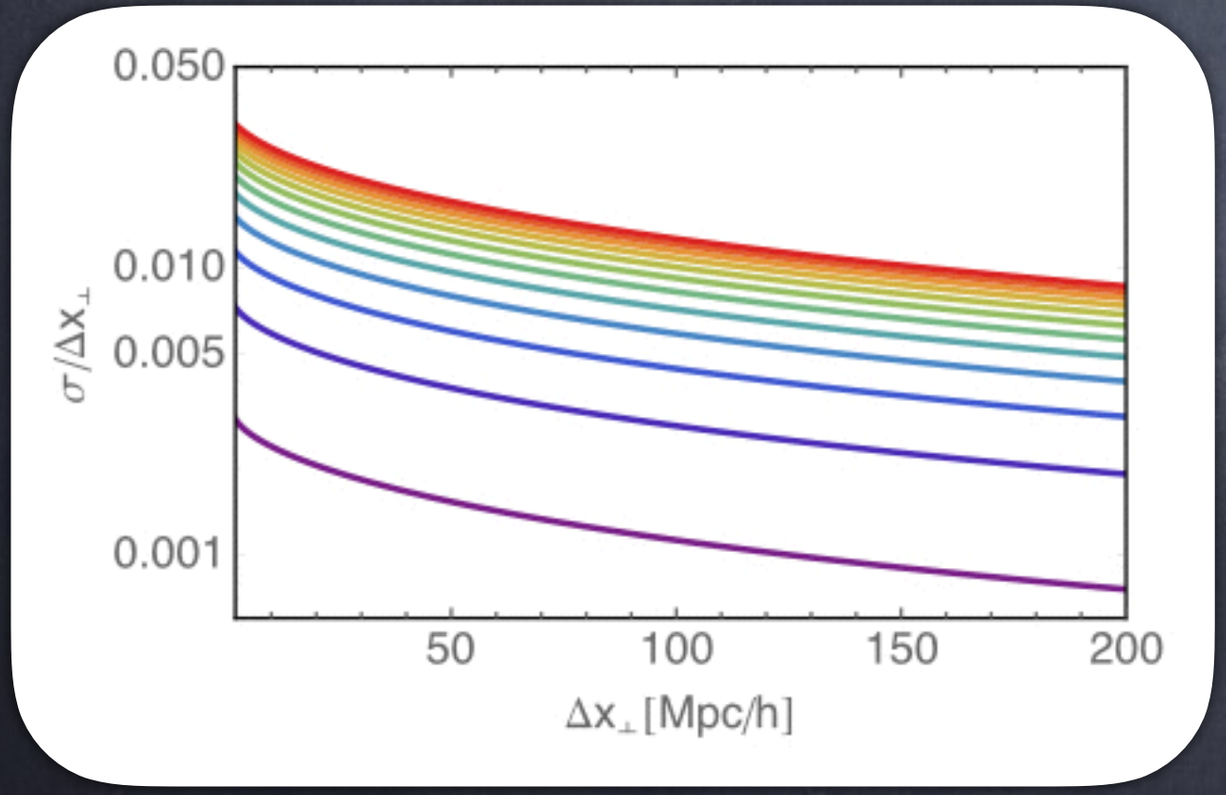
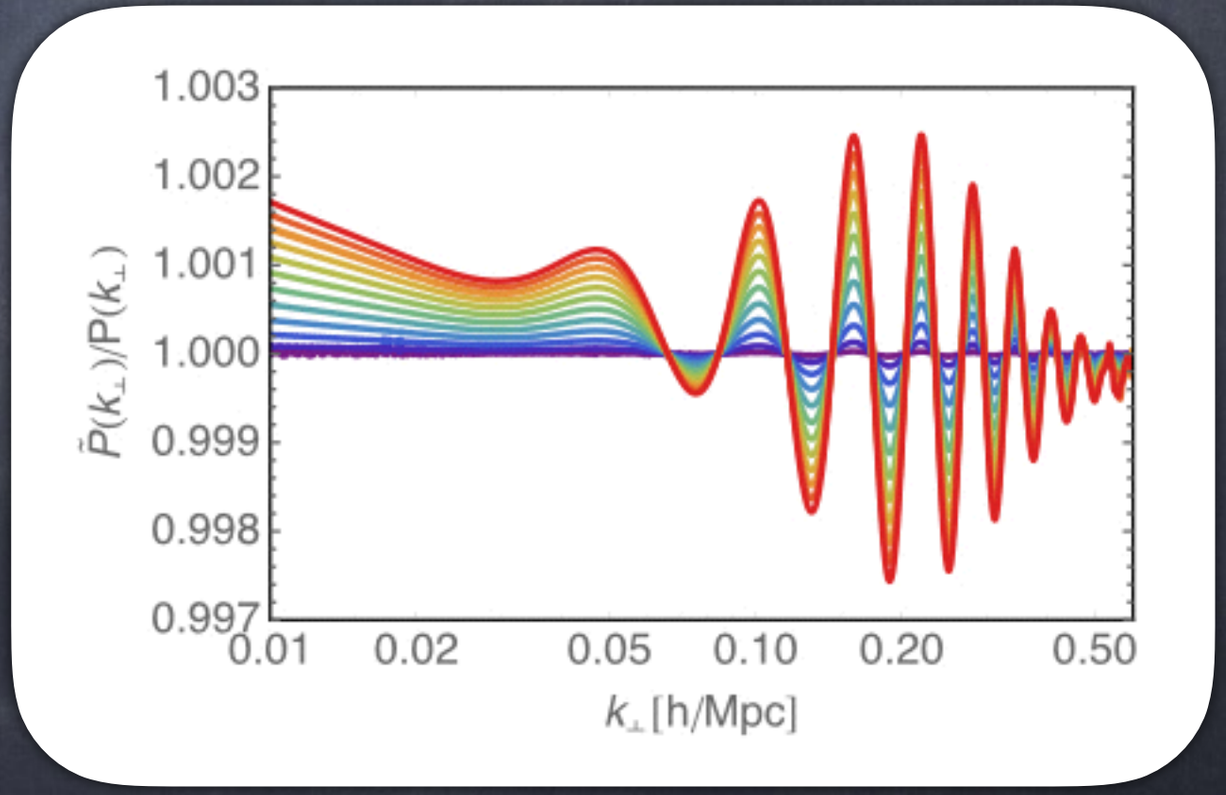
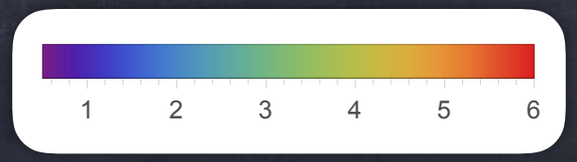
$$- 2\nabla_b\kappa\nabla^b\psi - \frac{1}{2r(z)}\int_0^{r(z)} dr \frac{r(z)-r}{r}\Delta_2\left(\nabla^b\Psi_1\nabla_b\Psi_1\right) - 2\int_0^{r(z)}\frac{dr}{r}\nabla^a\Psi_1\nabla_a\kappa$$

Smoothing effect





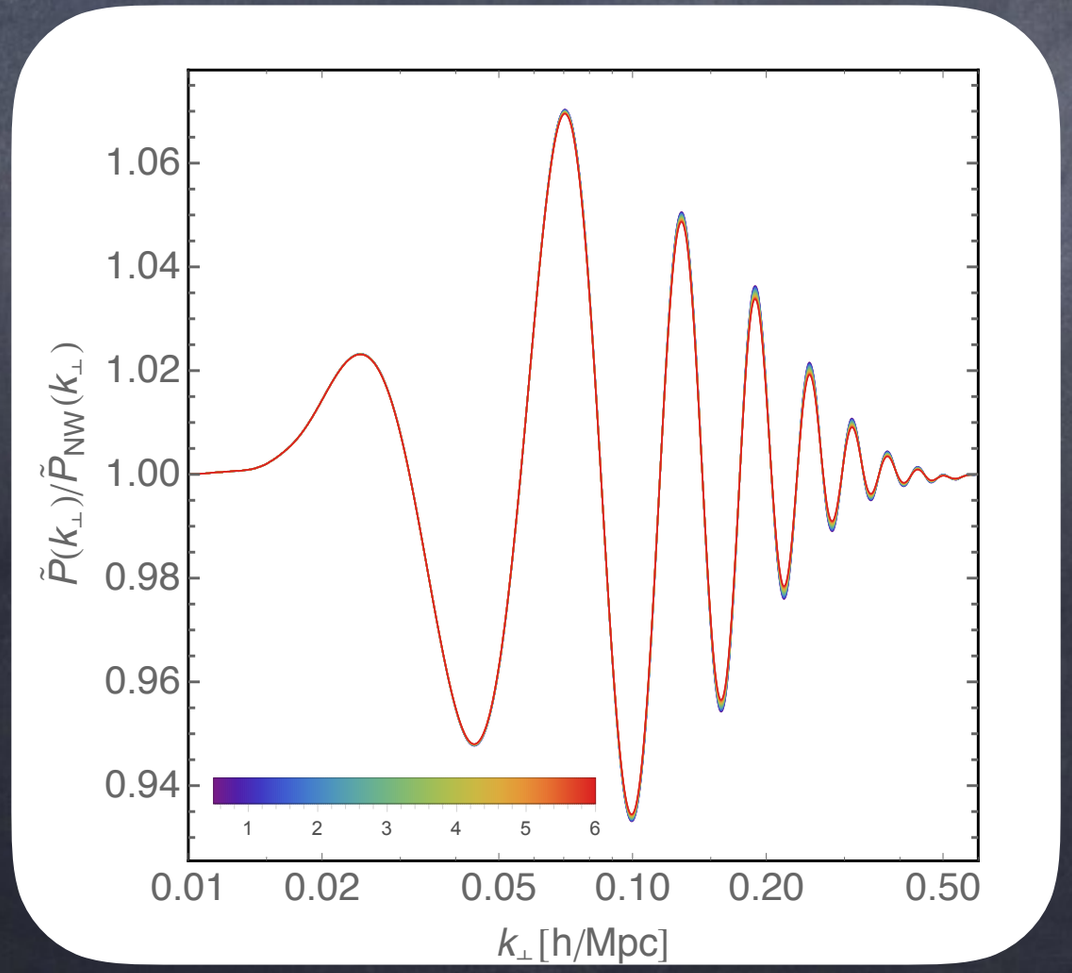


ED [arXiv:1609.09044]

Lensed Power Spectrum

$$\Sigma^{(2)}(\mathbf{n},z) = \delta^{(2)} + \mathcal{H}^{-1}\partial_r^2 v^{(2)} + \mathcal{H}^{-2}\left(\partial_r^2 v\right)^2 + \mathcal{H}^{-2}\partial_r v \partial_r^3 v + \mathcal{H}^{-1}\left(\partial_r v \partial_r \delta + \partial_r^2 v\, \delta\right)$$
$$- 2\kappa^{(2)} - 2\delta\kappa + \nabla_a \delta \nabla^a \psi + \mathcal{H}^{-1}\left[-2(\partial_r^2 v)\kappa + \nabla_a(\partial_r^2 v)\nabla^a\psi\right] + 2\kappa^2$$
$$- 2\nabla_b\kappa\nabla^b\psi - \frac{1}{2r(z)}\int_0^{r(z)} dr \frac{r(z)-r}{r}\Delta_2\left(\nabla^b\Psi_1\nabla_b\Psi_1\right) - 2\int_0^{r(z)}\frac{dr}{r}\nabla^a\Psi_1\nabla_a\kappa$$

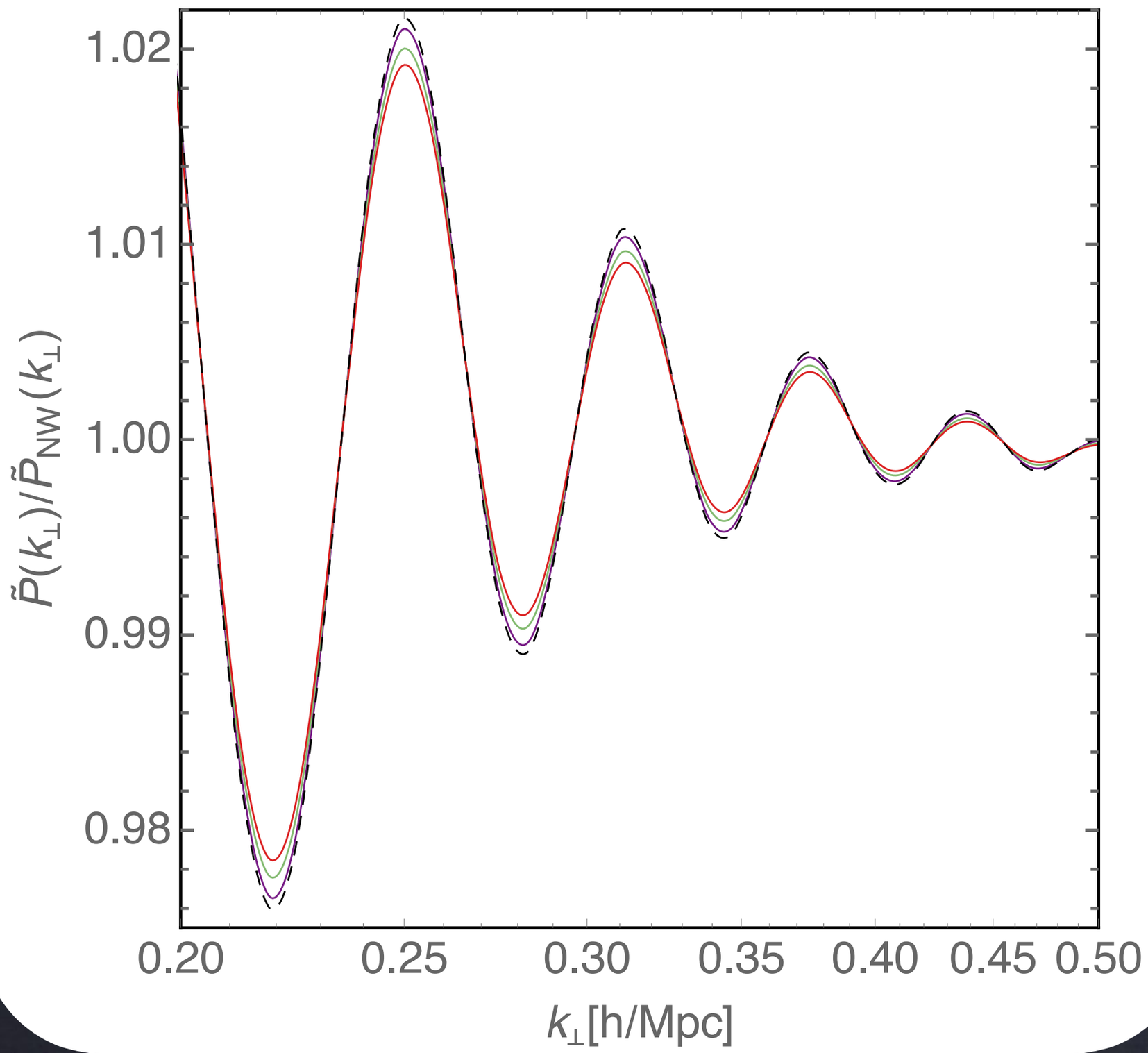
Lensing induced by deflection angle



ED [arXiv:1609.09044]

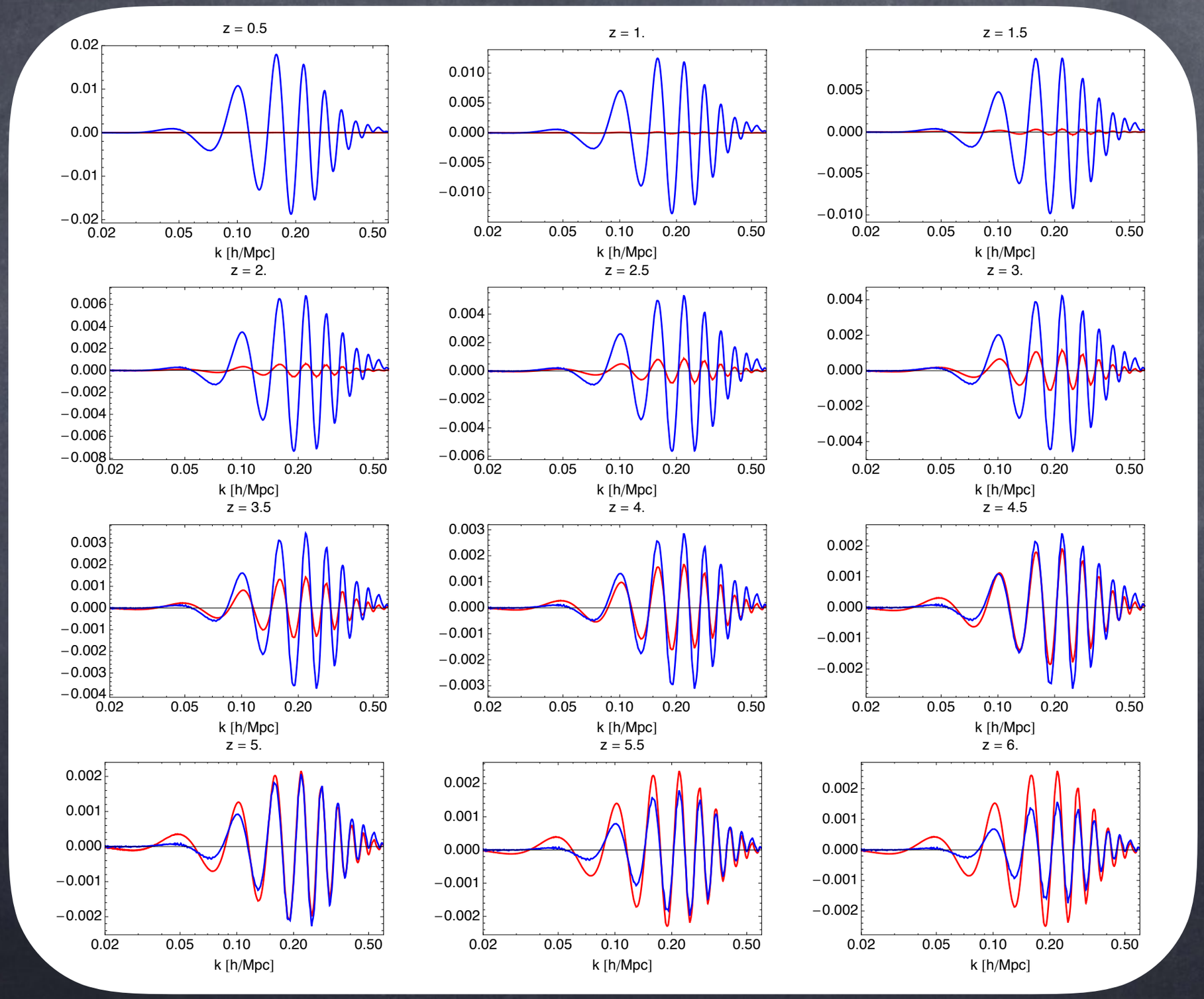
Lensed Power Spectrum

$$\Sigma^{(2)}(\mathbf{n}, z) = \ldots \delta + \partial_r^2 v\, \delta) - \ldots 2\kappa^2 - \ldots \frac{dr}{r} \nabla^a \Psi_1 \nabla_a \kappa$$



ED [arXiv:1609.09044]

Lensed Power Spectrum



ED [arXiv:1609.09044]

Conclusions

- Relativistic corrections
  - encode useful information
  - may bias the parameter estimation, if neglected
- Multi-tracers technique allows to isolate some relativistic correction. Future surveys should be able to measure them
- relativistic number counts can be generalized to non-flat universes
  - neglecting relativistic effects may bias the curvature parameter as well
- Lensing effects are not negligible on the galaxy bispectrum
- Lensing smoothes the BAO wiggles