

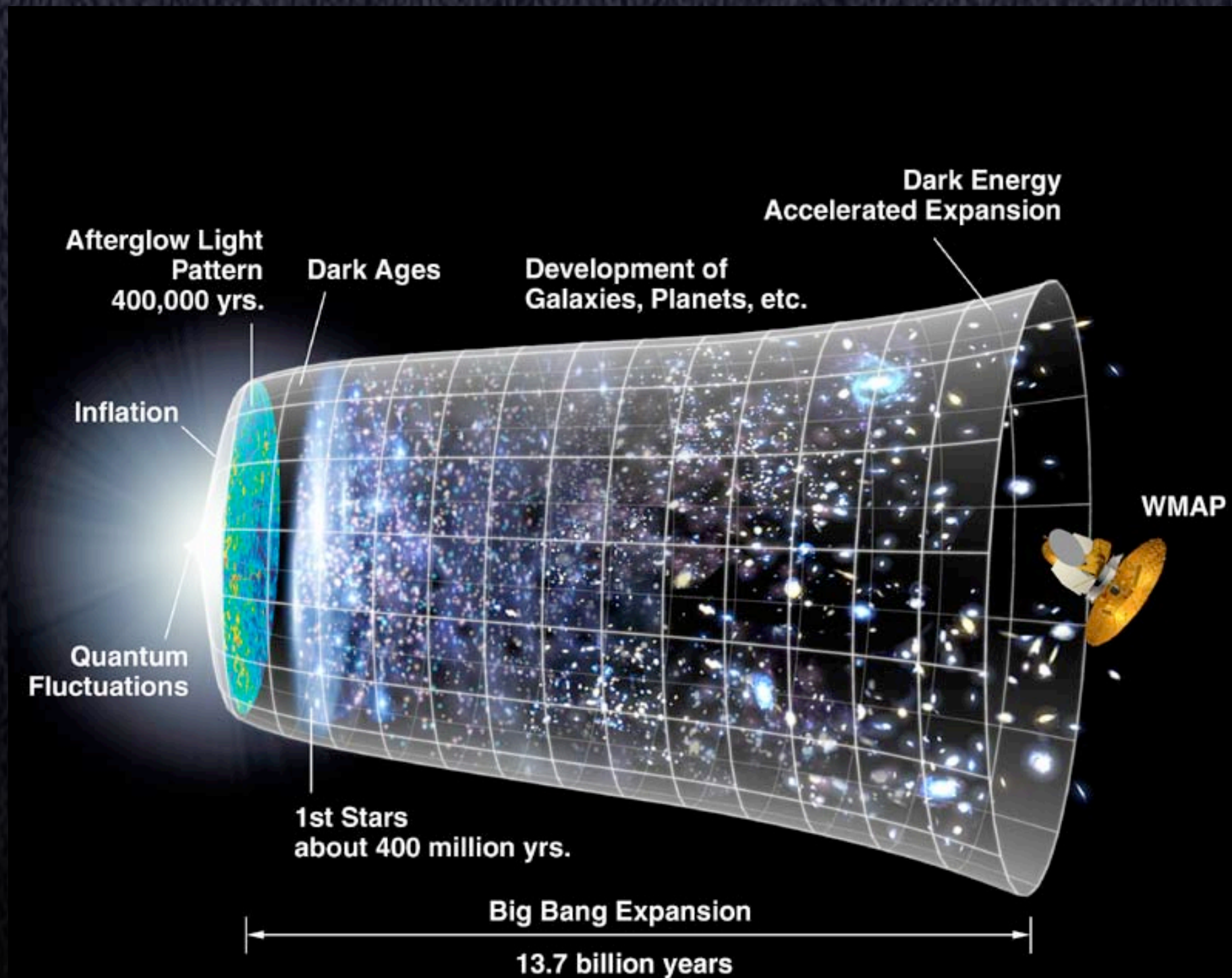
# Probing Cluster Masses with Gravitational Lensing

**Sanghamitra Deb**

**Collaborators: Prof. David M. Goldberg (Drexel University), Prof. Kristian Pedersen (DARK, Copenhagen), Dr. Andrea Morandi (DARK, Copenhagen & Univ. of Tel Aviv.), Dr. Marceau Limousin (LAM Marseille), Dr. Hakon Dahle (Univ. of Oslo), Dr. Signe Riemer-Sørensen (DARK, Copenhagen), Dr. Catherine Heymans (University of Edinburgh, IfA Royal Observatory), Dr. Reiko Nakajima (UC Berkeley), Dr. Rachel Mandelbaum (Princeton University), Prof. Gary Bernstein (University of Pennsylvania)**



# Looking Back ...





# Galaxy Clusters : Formation

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**43 Mpc**

**4.3 Mpc**

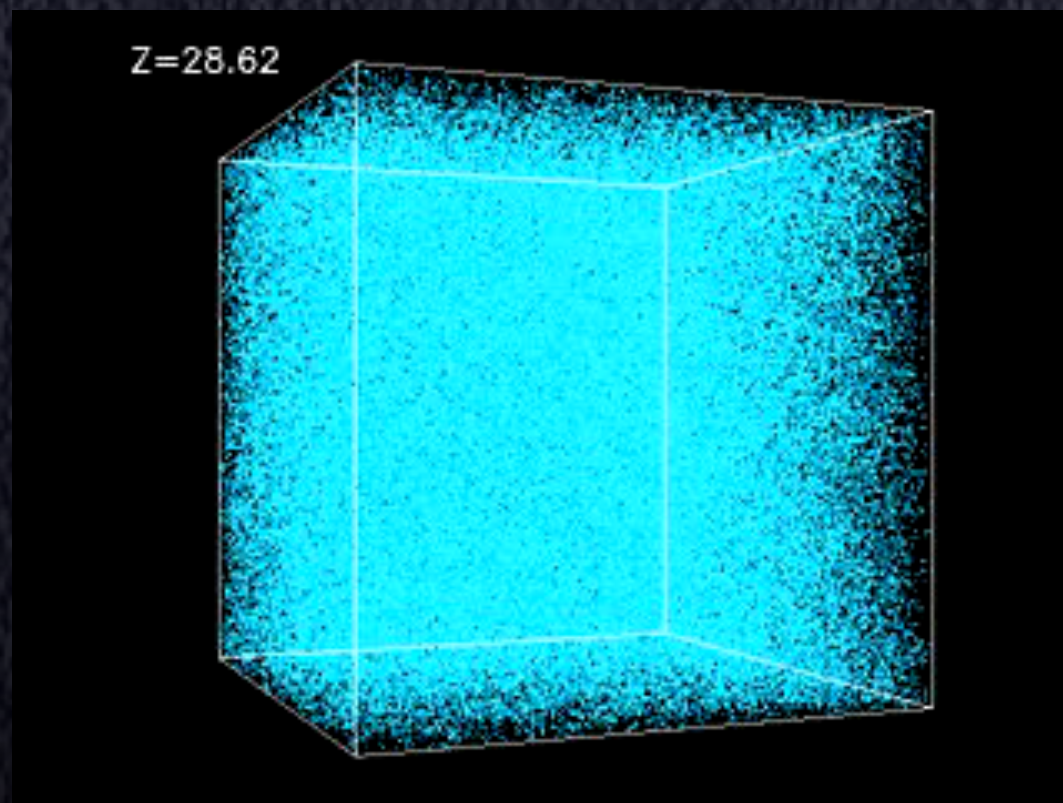
**Andrey Kravtsov**



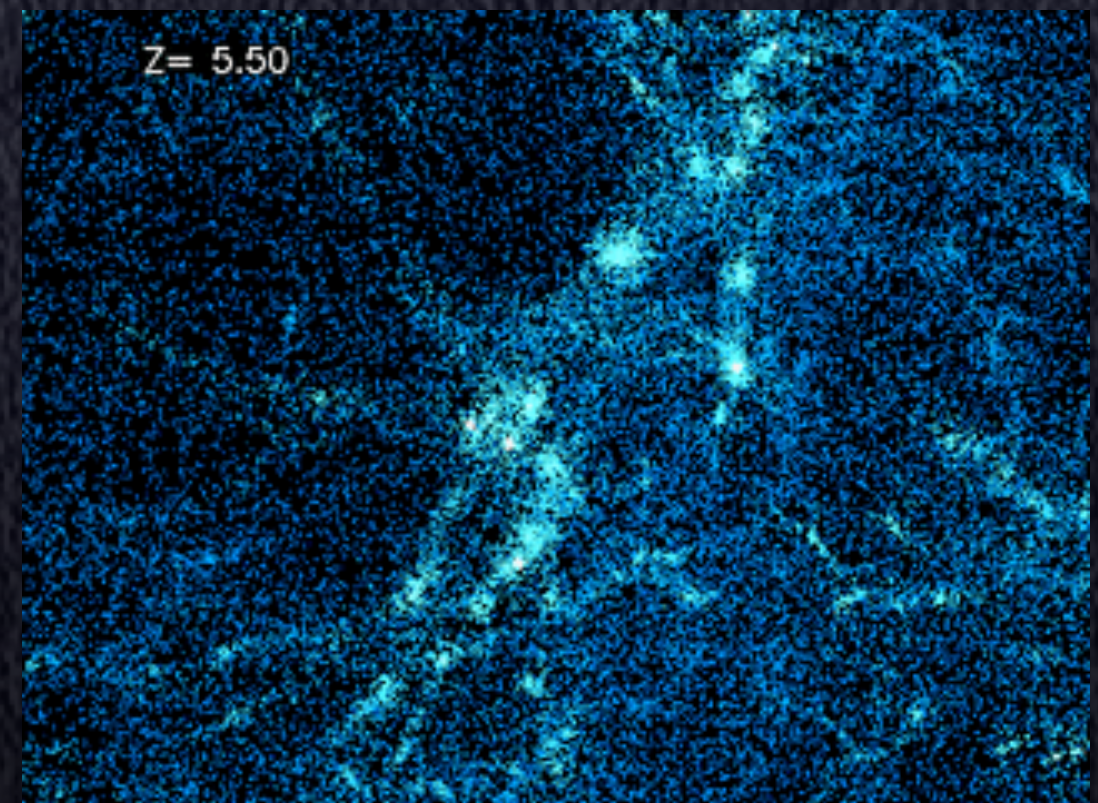
# Galaxy Clusters : Formation

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**43 Mpc**



**4.3 Mpc**

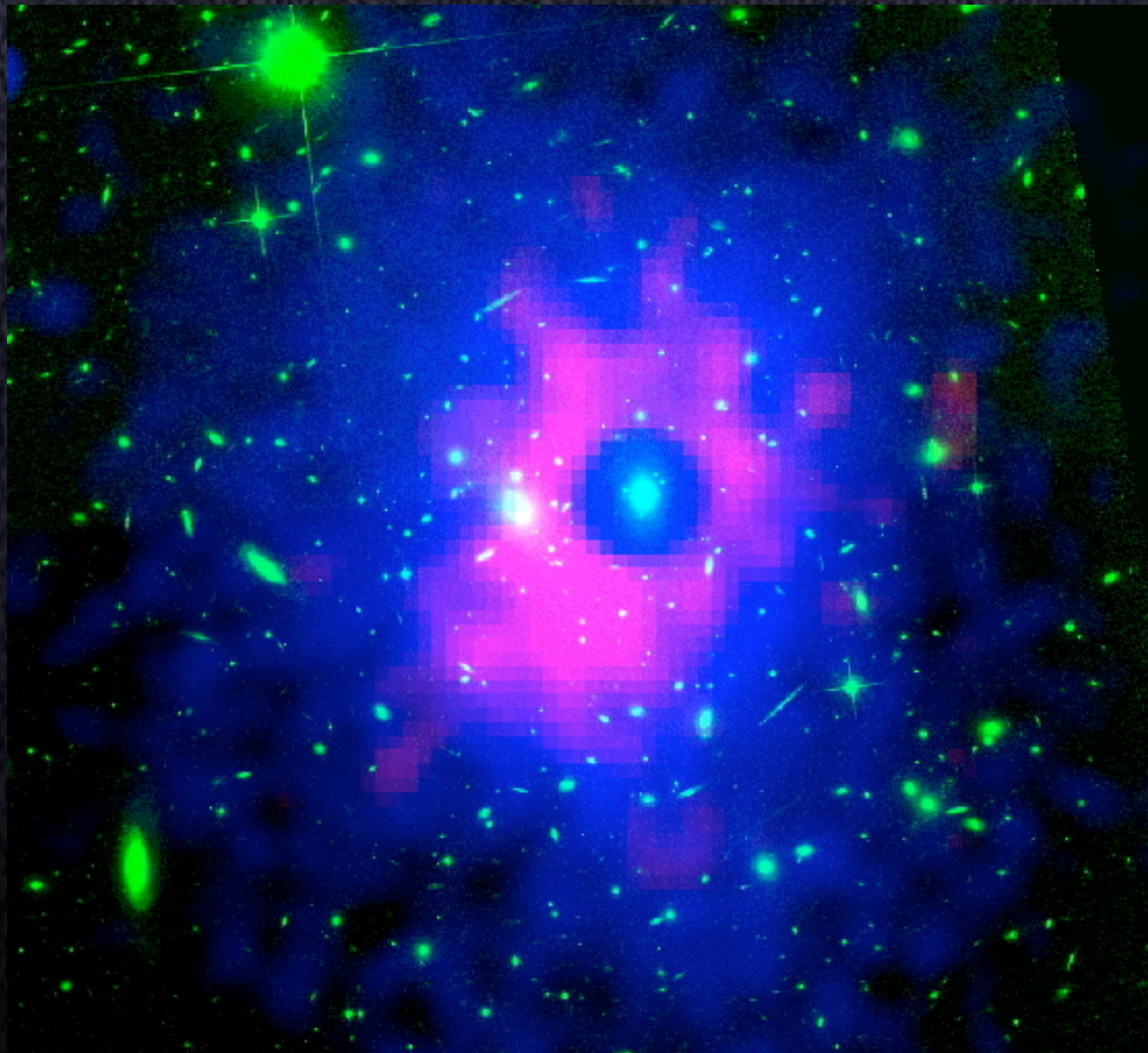


**Andrey Kravtsov**



# Observing Clusters

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**galaxies: HST, optical observations**

**gas: Xray observations**

**gas: Sunyaev Zeldovich Effect**

**Galaxy Cluster RXJ1347-1145**



# What do we want to measure?

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- **Mass**
- **Substructure**
- **Morphology**



# What do we want to measure?

---

- **Mass**
- **Substructure**
- **Morphology**

**Why?**



# What do we want to measure?

---

- **Mass**
- **Substructure**
- **Morphology**

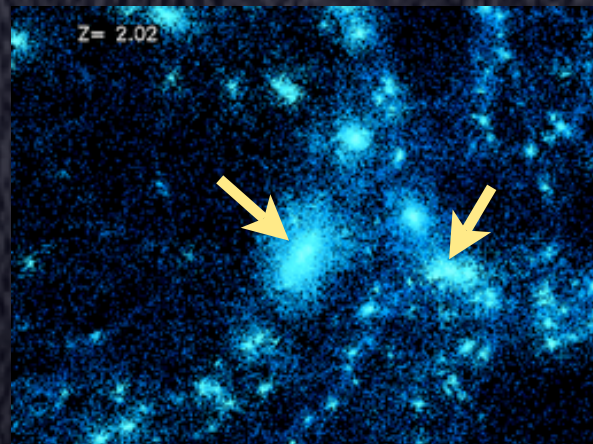
**Why?**

**Test Predictions of  $\Lambda$ CDM cosmology in the local Universe.**

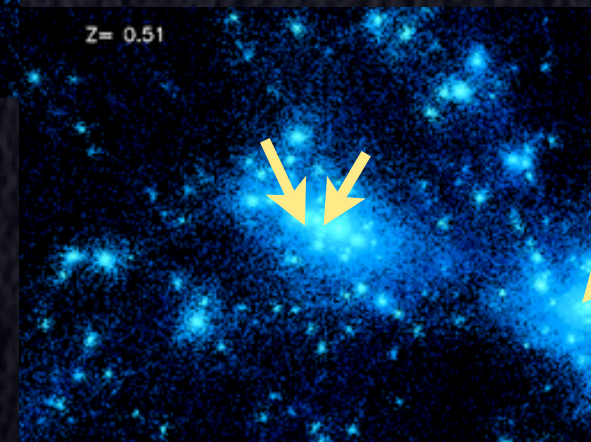
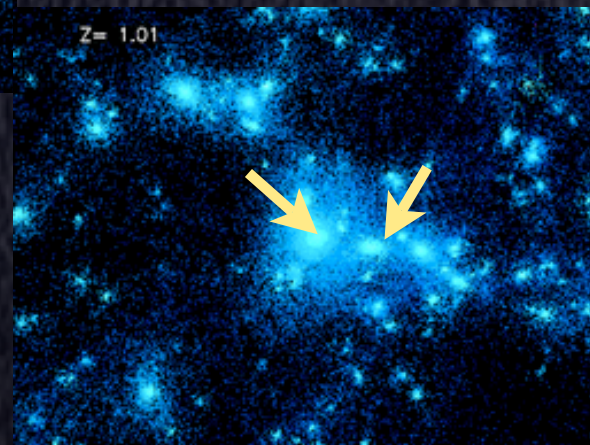
**Compare with the morphologies of gas and light.**



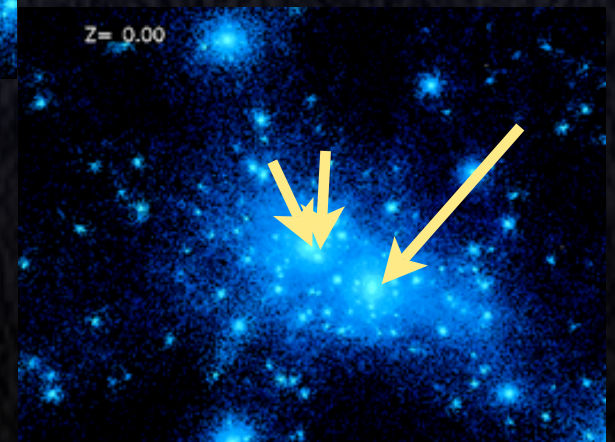
# Substructure: Elliptical Halos



What is the distribution of cluster ellipticity?

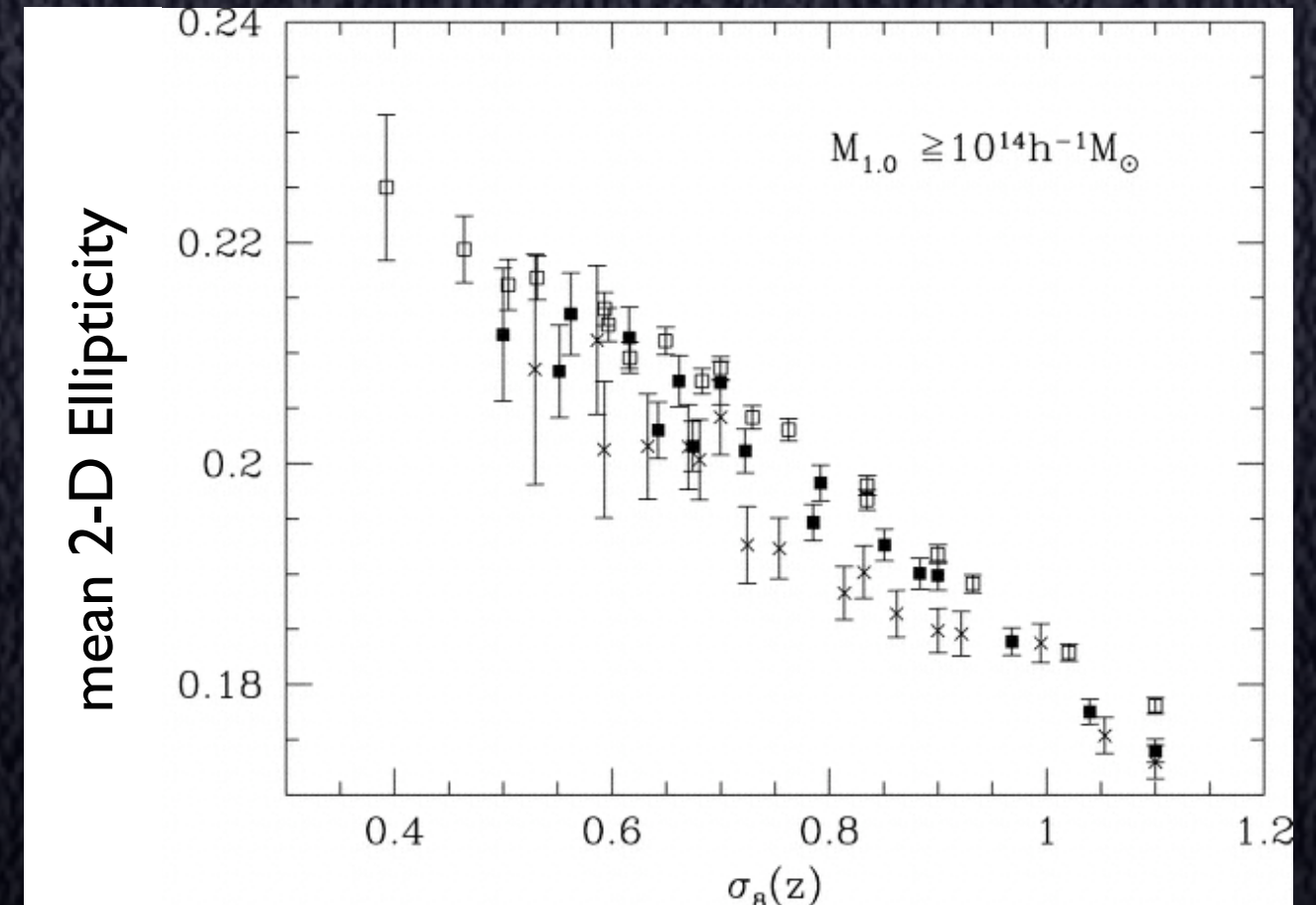
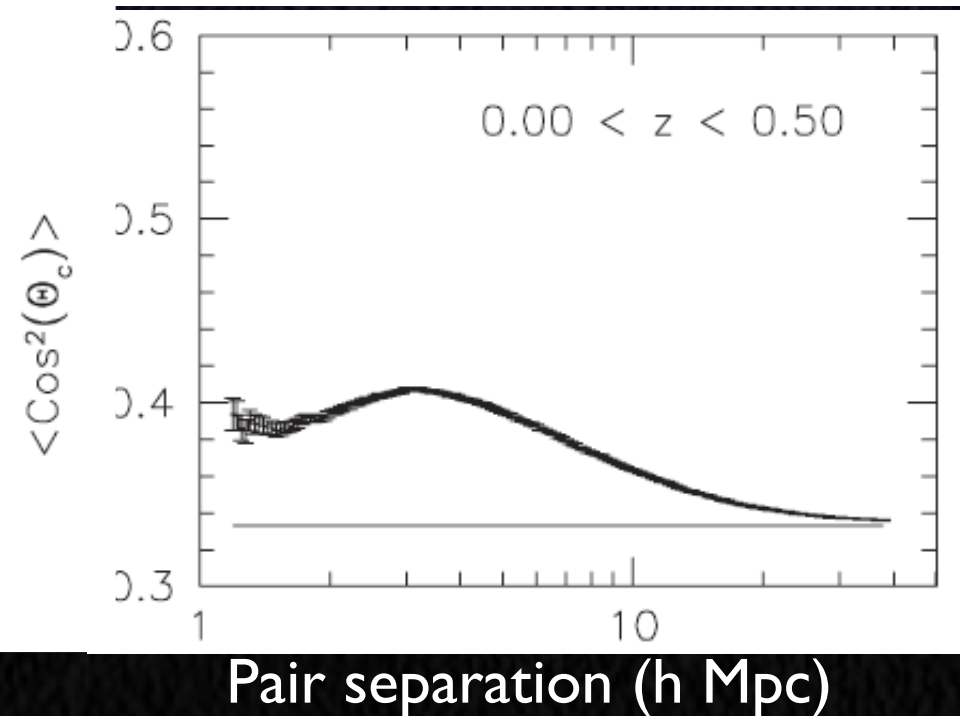
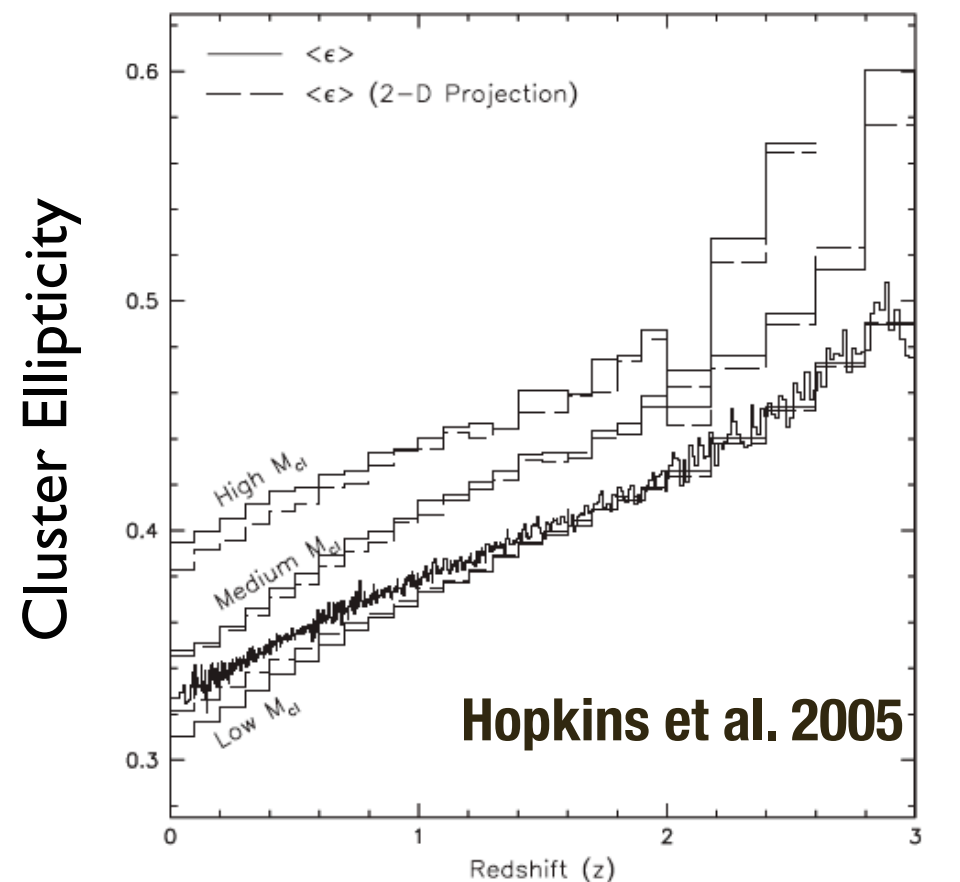


How does the distribution and alignment vary with redshift & mass?





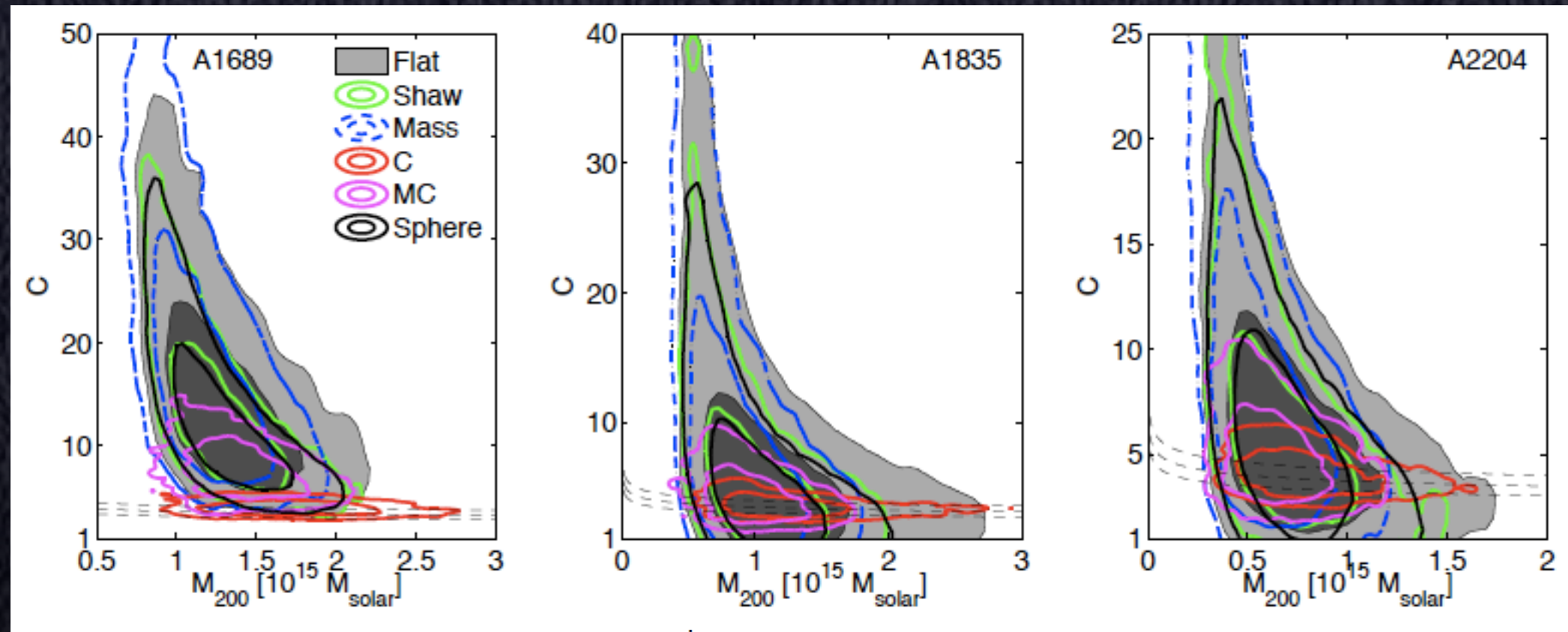
# Prediction from Simulations



**Ho et al. 2006**



# Observations: Lensing



$$M_{200} = (0.67 \pm 0.22) \times 10^{15} h^{-1} M_{\odot}$$

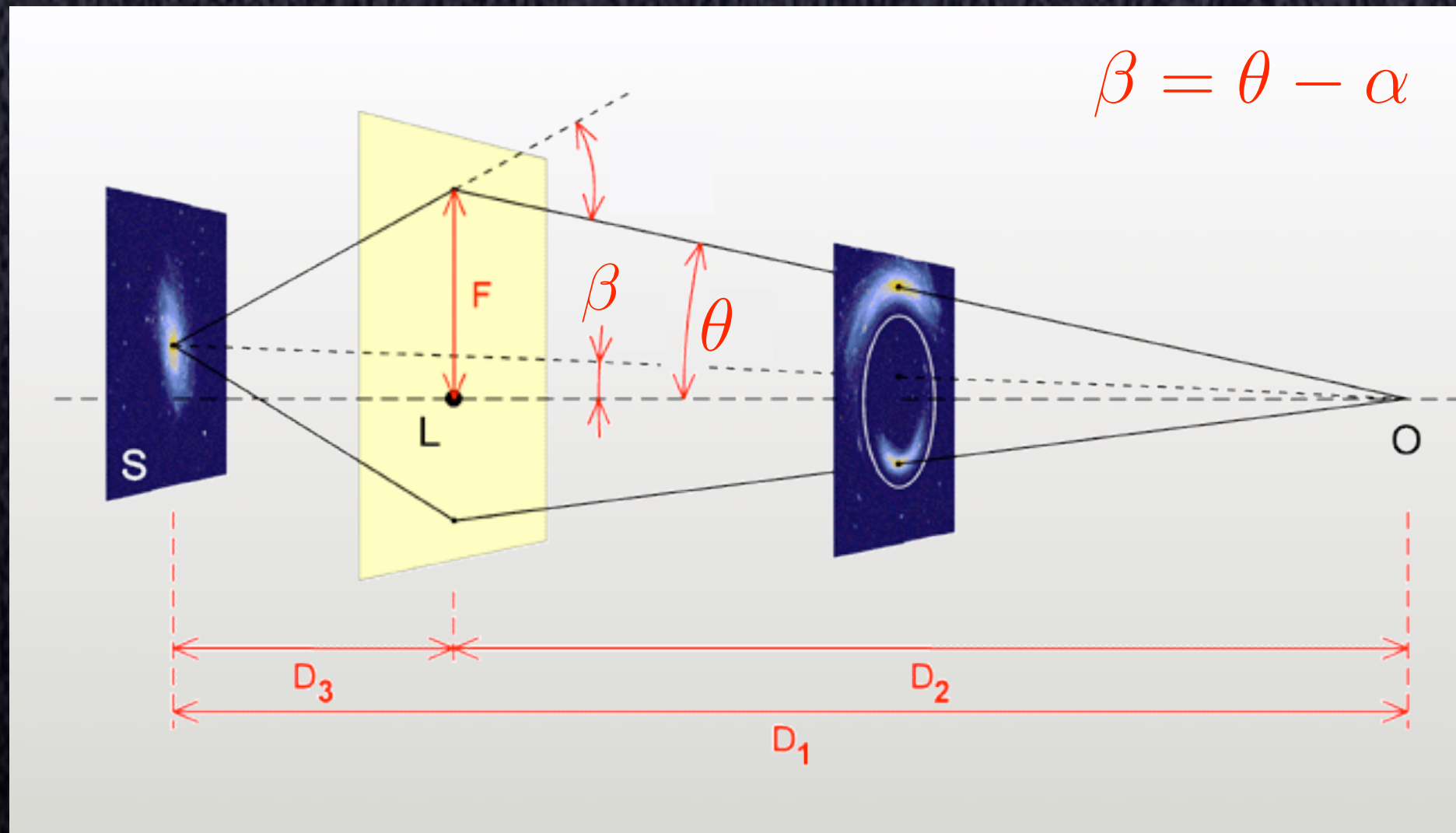
$$M_{200} = 200 \rho_c$$

$$C = \frac{R_{200}}{R_s}$$

Corless & King 2008,2009



# Gravitational Lensing



**Dimension less  
surface mass  
density**

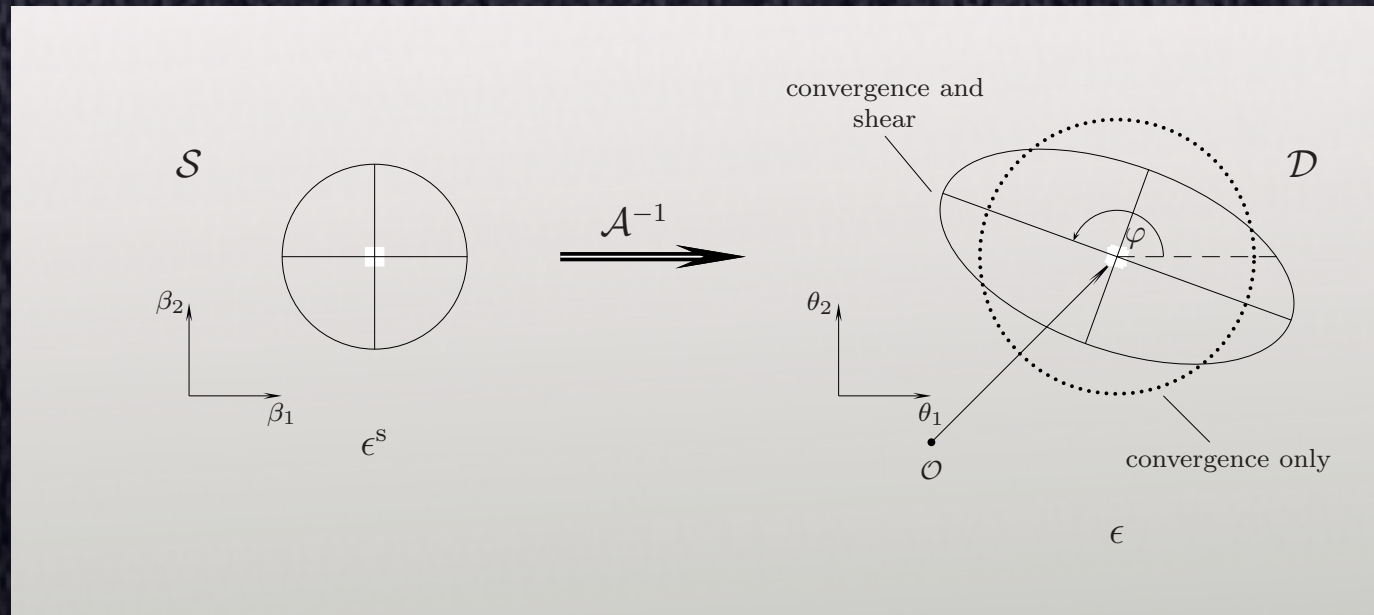
$$\kappa = \frac{\Sigma}{\Sigma_{cr}}$$

**Gravitational Lensing is co-ordinate transformation between the foreground ( $\theta$ ), and background positions( $\beta$ )**



# Shape Distortions

**Critical Curves:  $\det(A)=0$**

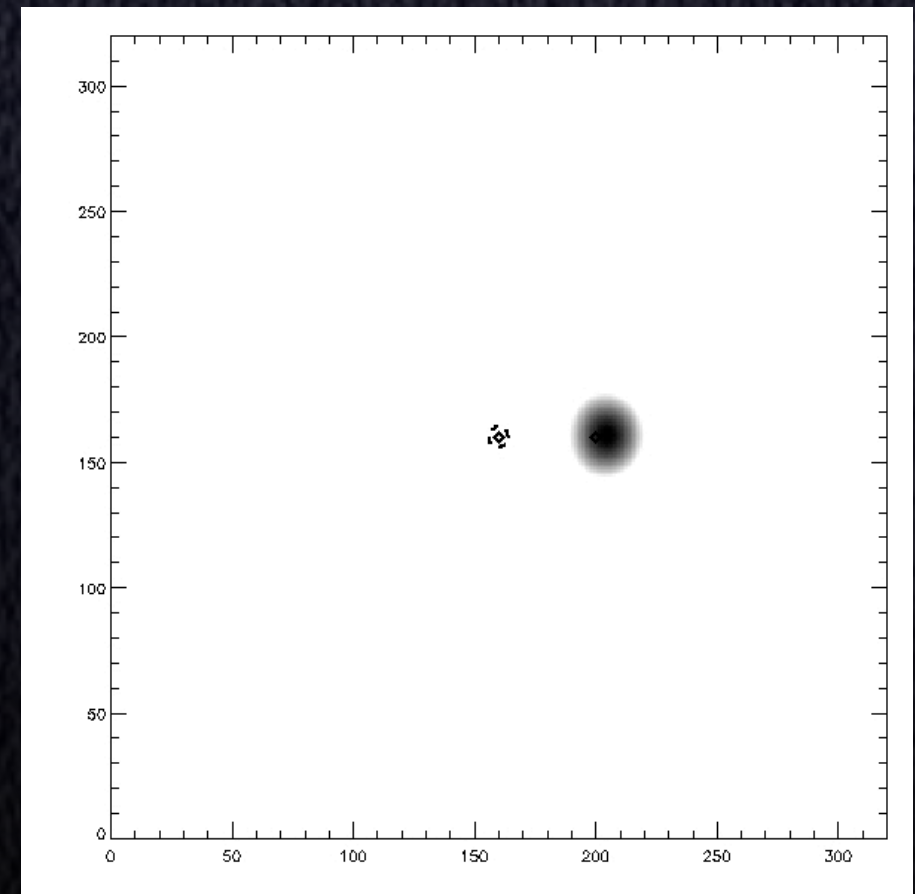


$$\kappa = (\psi_{,11} + \psi_{,22})/2$$

$$\gamma_1 = (\psi_{,11} - \psi_{,22})/2$$

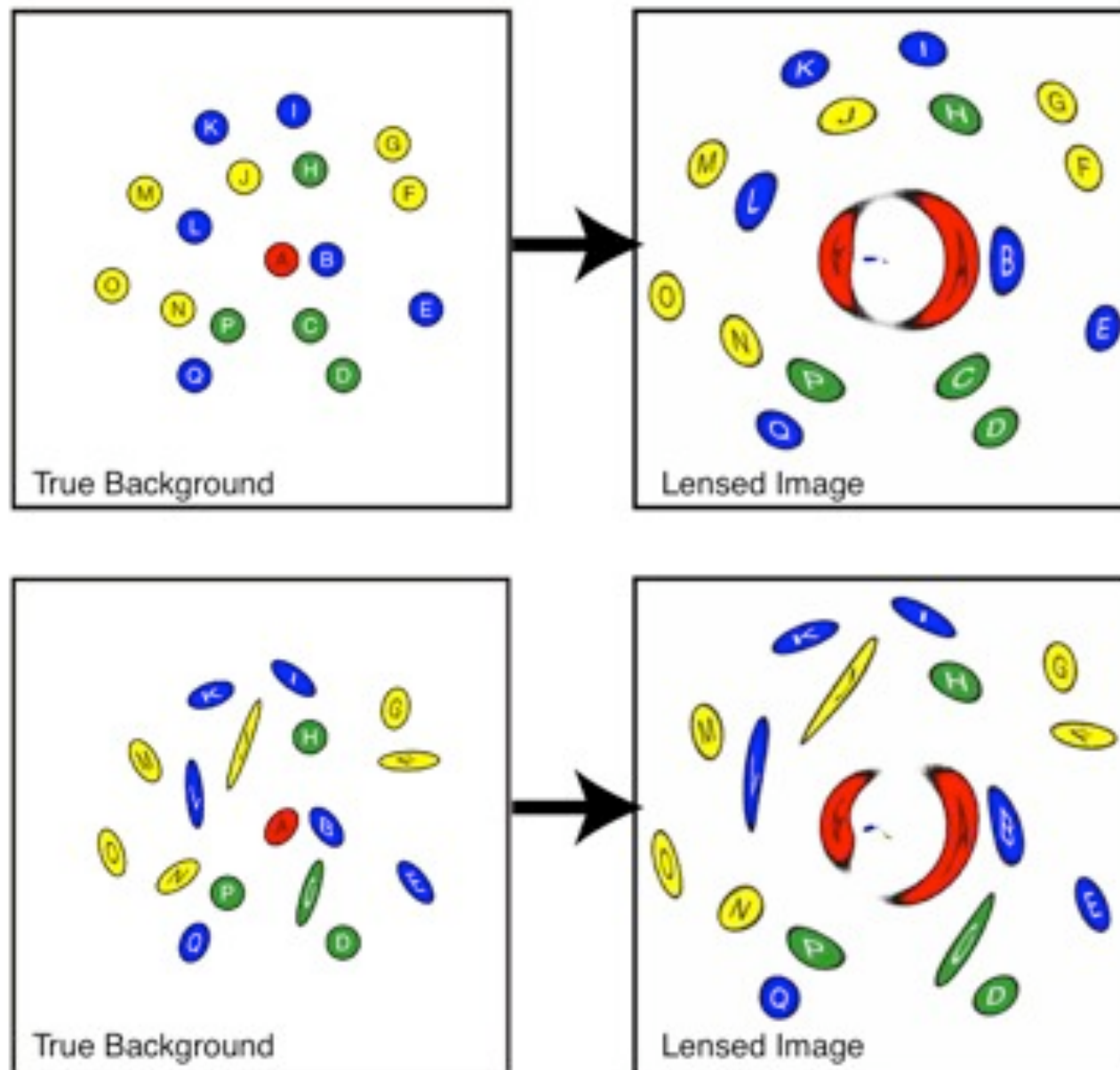
$$\gamma_2 = \psi_{,12}$$

**Distortion Observables are Measured Ellipticities.**  
**For semi-strong regime:  $g = \gamma/(1-\kappa)$**





# Weak Lensing

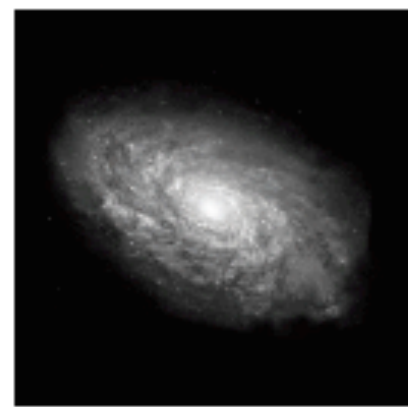


**Weak lensing is a statistical measure of the distortion of background galaxies due to the intervening mass.**

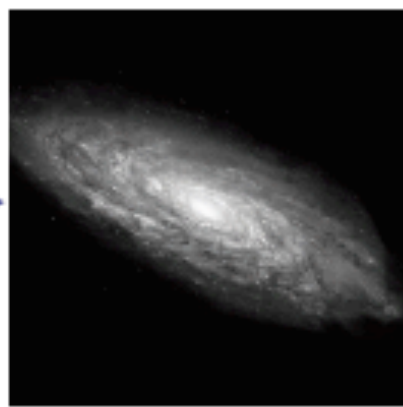
**Williamson et al. 2007.**



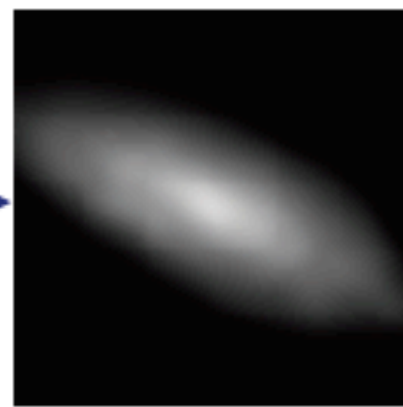
# The shear signal



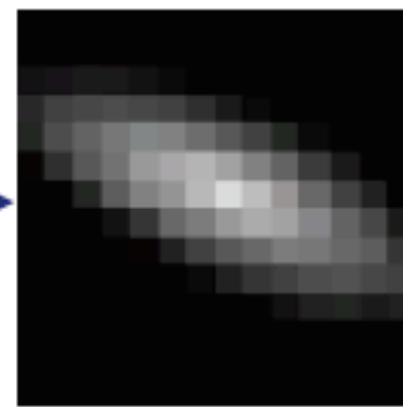
Intrinsic galaxy  
(shape unknown)



Gravitational lensing  
causes a **shear (g)**



Atmosphere and telescope  
cause a convolution



Detectors measure  
a pixelated image

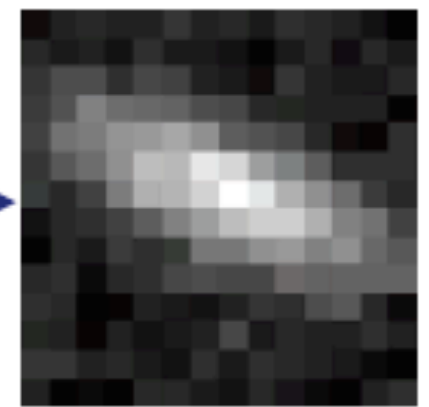
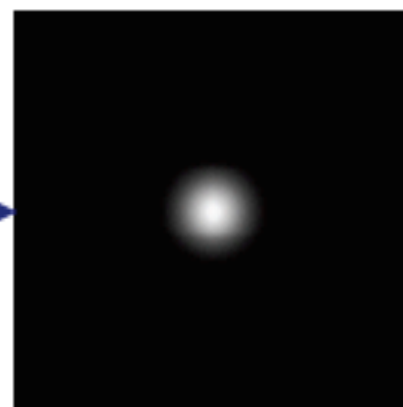


Image also  
contains noise

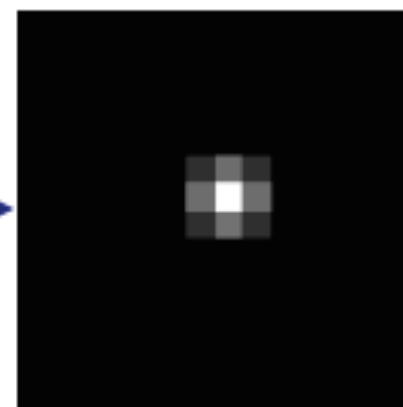
**Stars:** Point sources to star images:



Intrinsic star  
(point source)



Atmosphere and telescope  
cause a convolution



Detectors measure  
a pixelated image

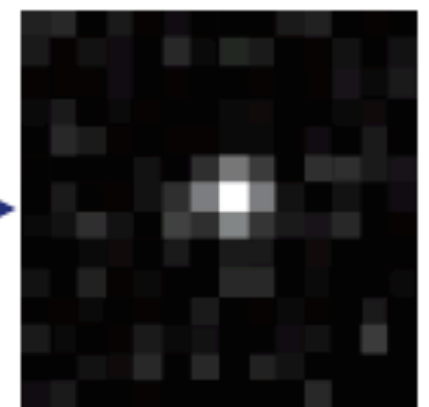


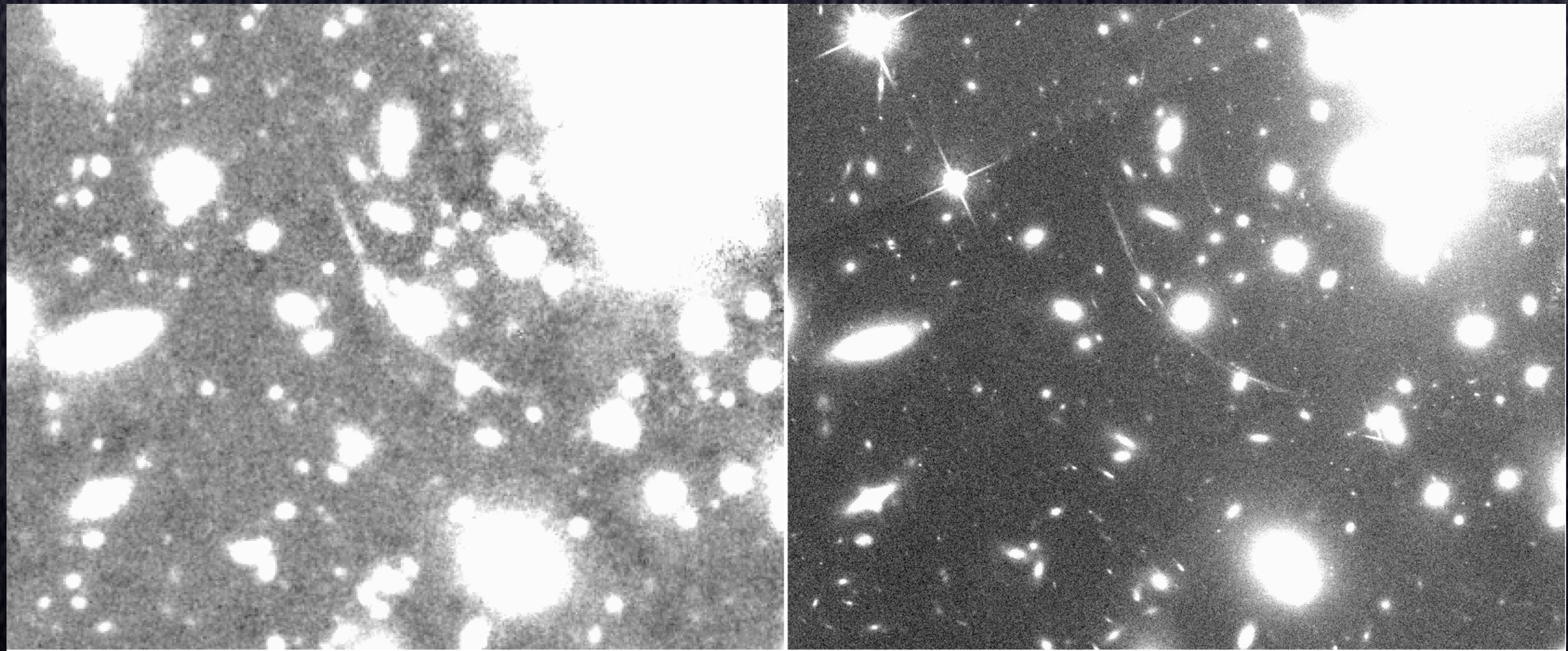
Image also  
contains noise

PSF



# Comparison of Space vs Ground: A1689

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-0.0040

-0.0021

-0.0002

0.0017

0.0035

0.0054

0.0073

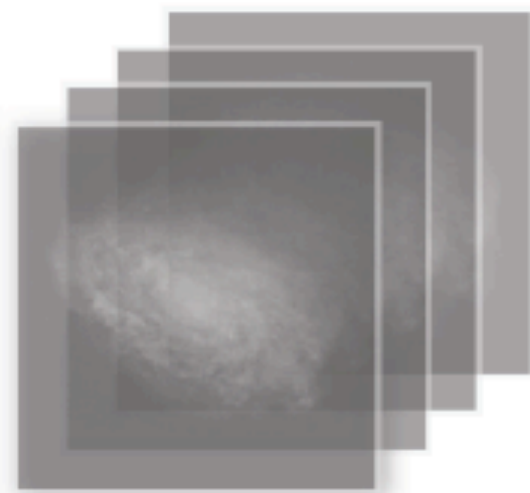
0.0092

0.0110



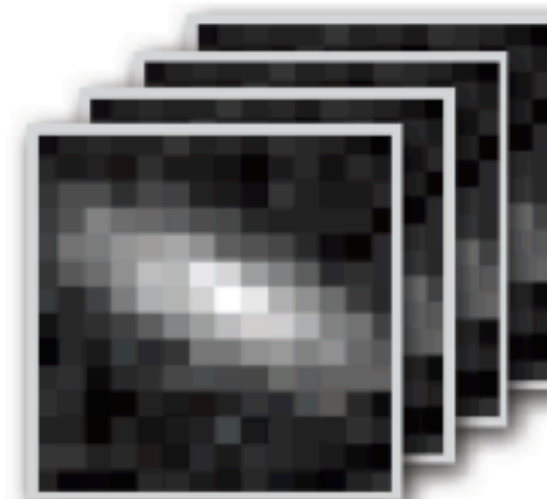
# Recovering the Shear

## The Inverse Problem: Measured images to *shear*



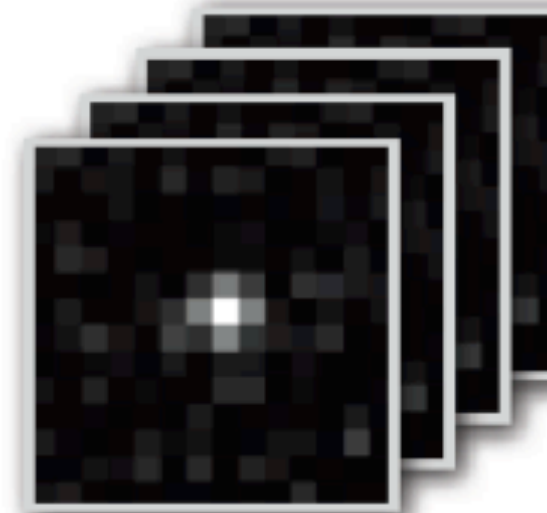
Intrinsic galaxy shapes can be inferred, but are not used beyond shear estimation

**Shear  
Field**



Set of galaxy images.  
Each contains:

- noise
- pixelisation
- convolution
- **shear**
- intrinsic shape



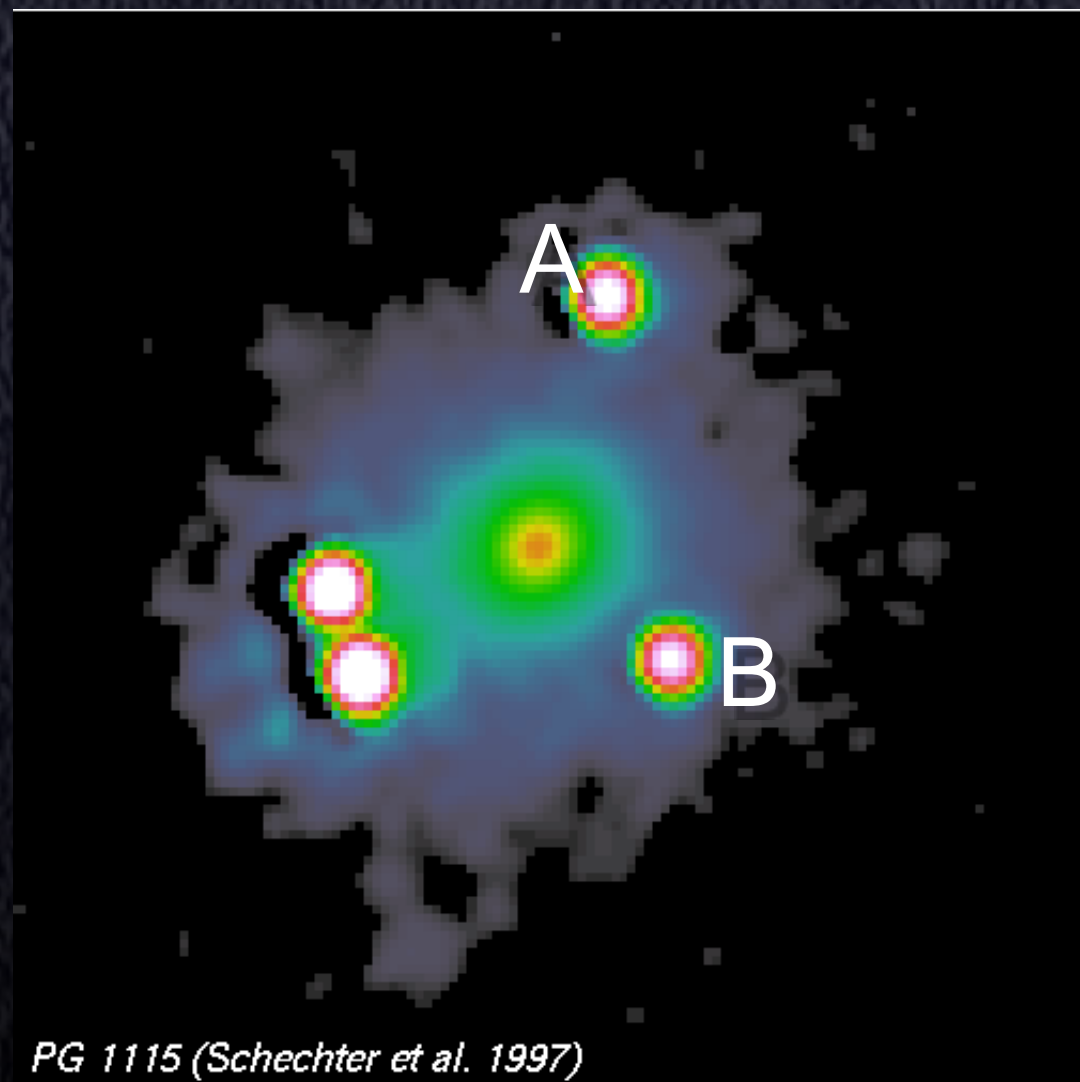
Set of star images.  
Each contains:

- noise
- pixelisation
- convolution



# Strong Lensing

---



$$\alpha_i = \psi_i$$

$$\alpha_A - \alpha_B = \theta_A - \theta_B$$

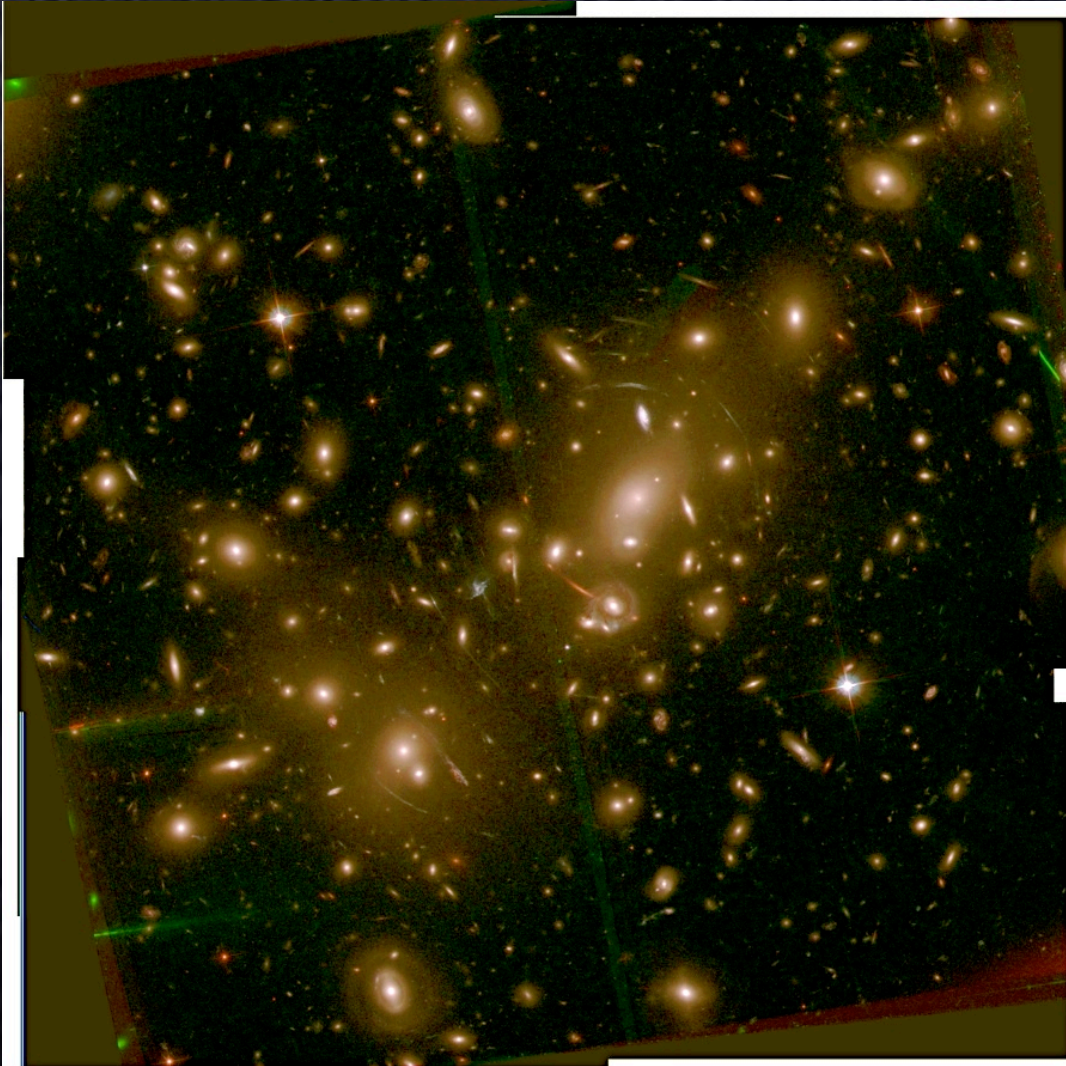


# Parametric technique

---

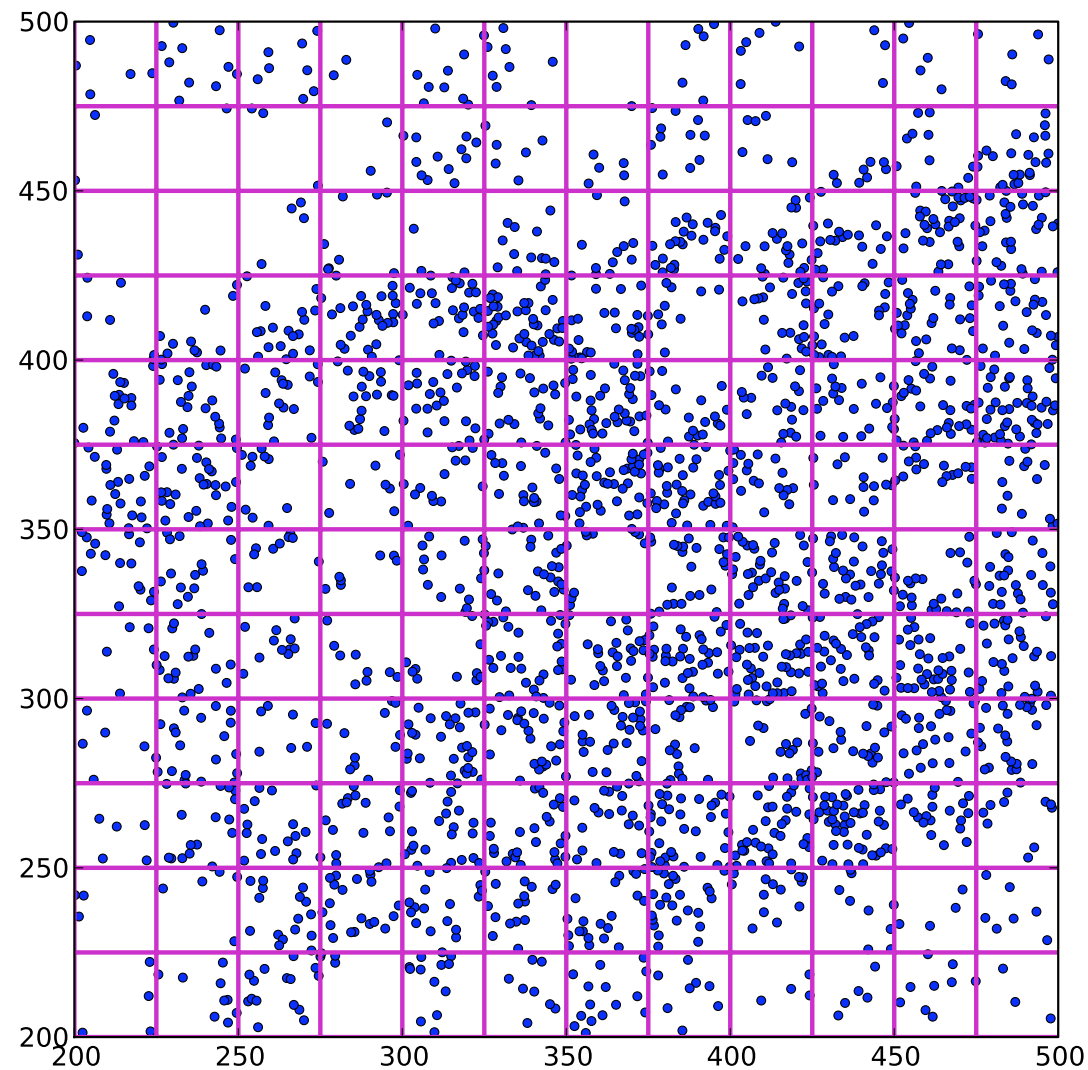
**Assumption: Light traces mass**

- **Place galaxy sized halos at the location of Cluster Members.**
- **Have one or more dark matter halos with free parameters that are fit from data.**





# Grid Based Lensing

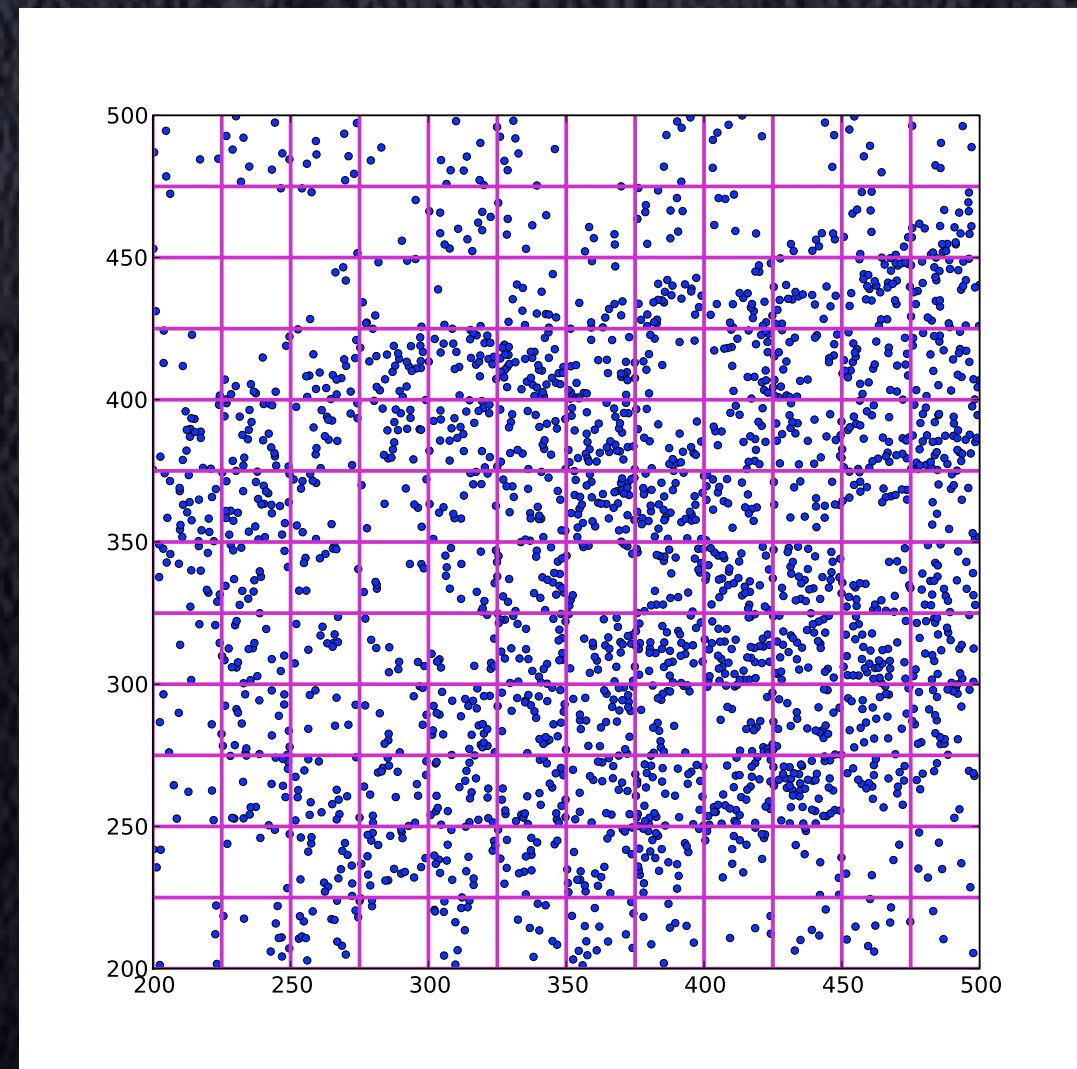
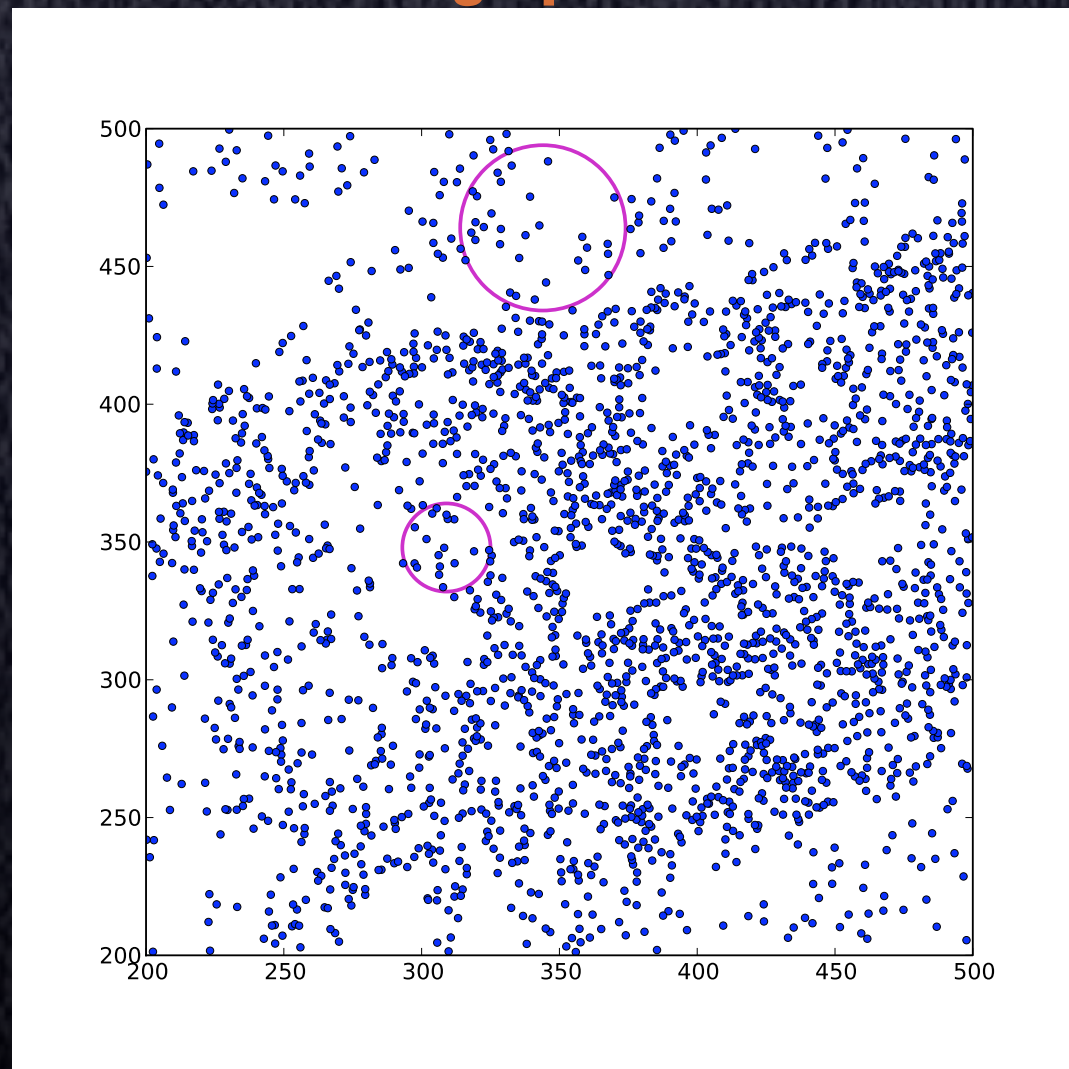


$$\langle \epsilon \rangle = \gamma \xrightarrow{\text{Convolution}} \kappa$$



# Particle Based Lensing

Particles-> lensed  
image positions

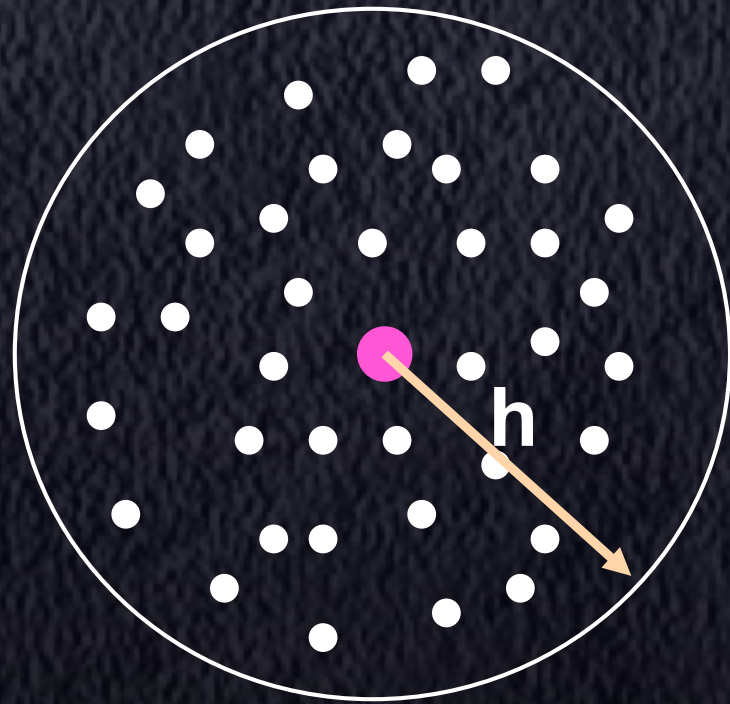


- Variable Resolution with the same complexity as finite differencing on a regular grid.
- No empty grid cells.



# How does it work?

## Taylor Expansion



$$\psi(\boldsymbol{\theta}) = \psi_n + \theta_j \psi_{n,j} + \frac{1}{2} \theta_j \theta_k \psi_{n,jk} + \dots$$

$$\psi_{n,j} = D_{nm}^{(j)} \psi_m$$

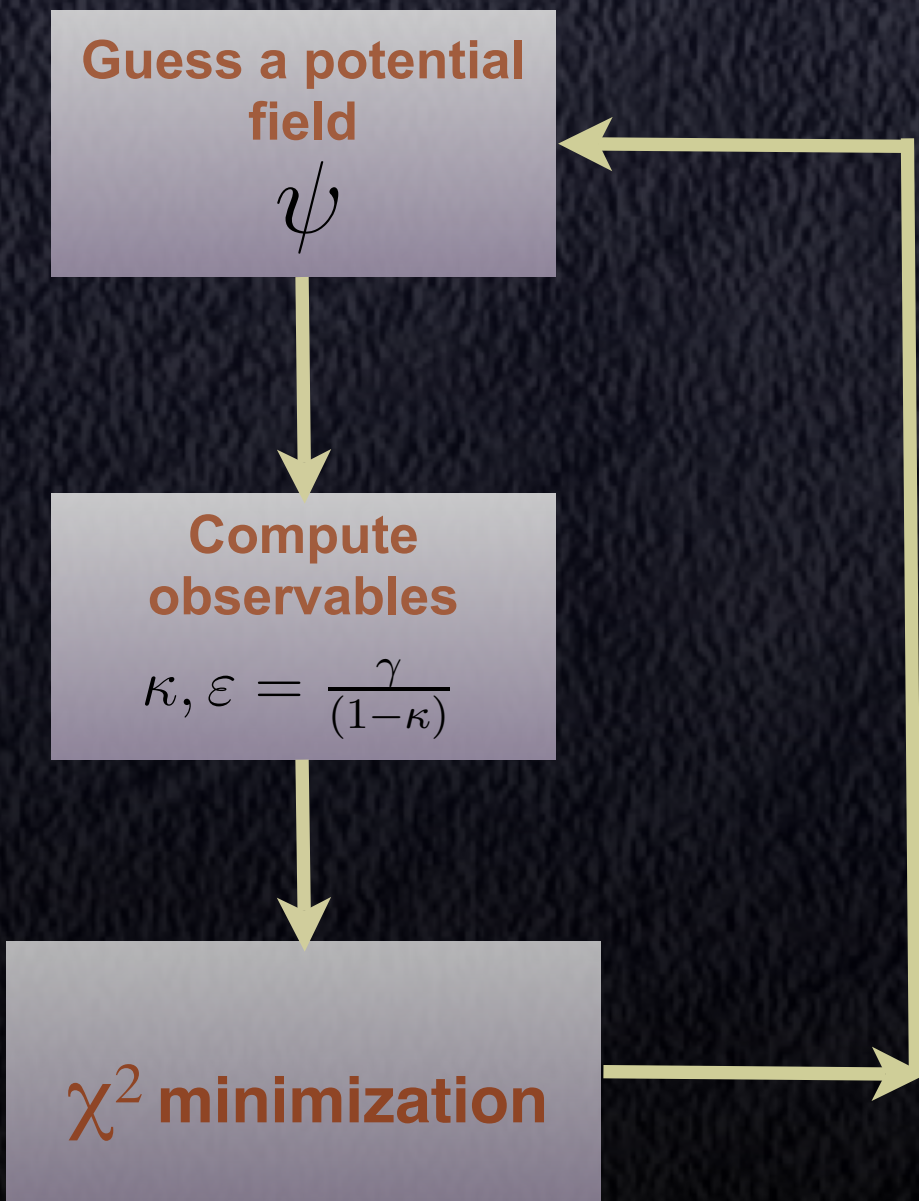
$$\psi_{n,jk} = D_{nm}^{(jk)} \psi_m$$

**Similar to Finite Differencing  
on uniform grids**



# Reconstruction Procedure

Keep Iterating ...



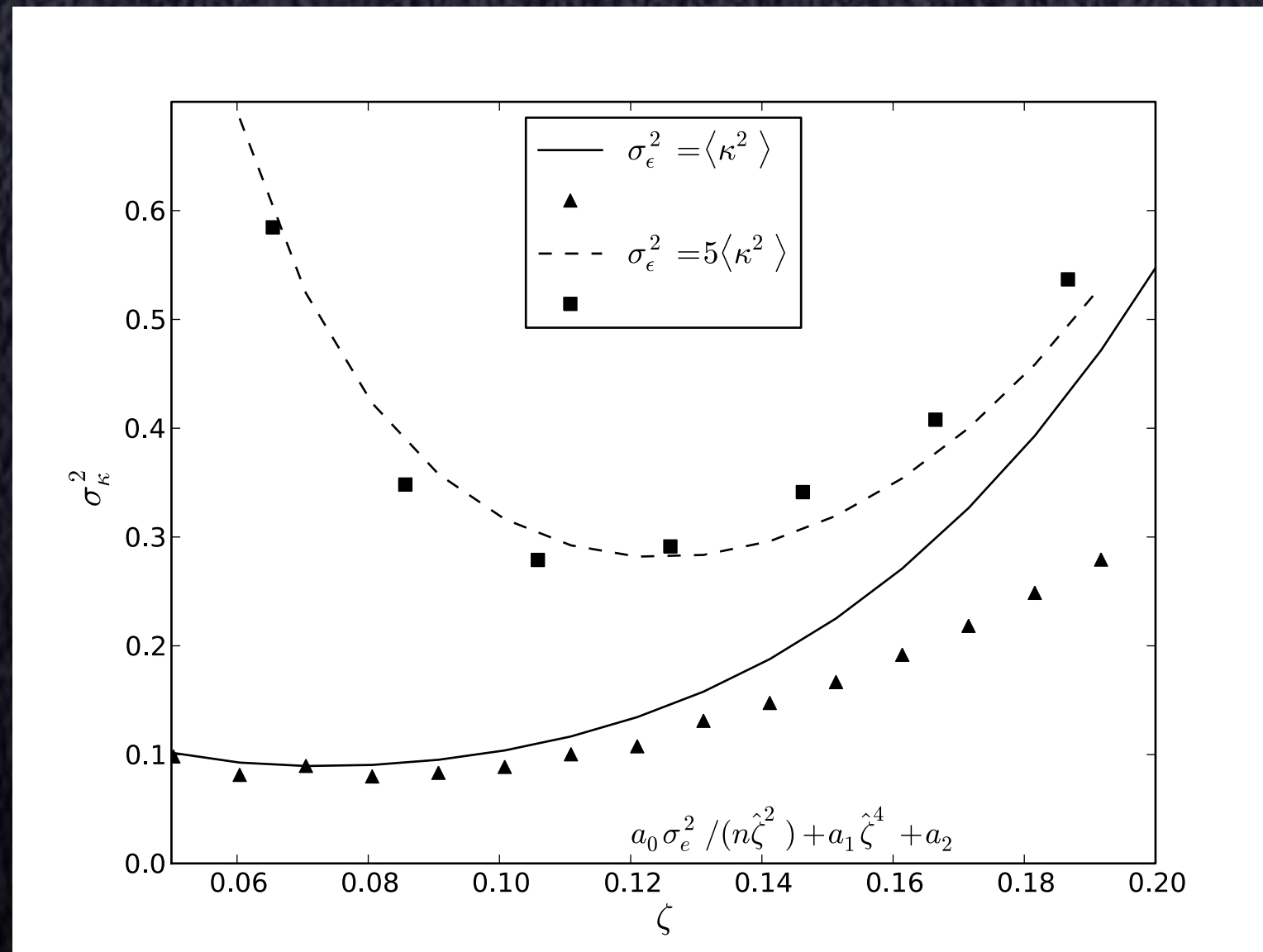
$$\chi_{\text{weak}}^2 = \sum_i \frac{\left[ \varepsilon_i - \frac{\gamma_i}{(1-\kappa_i)} \right]^2}{\sigma_i^2}$$

In case of weak lensing, a  $\chi_{\text{w}}^2$  like this will fit best to noisy data

Smoothing the ellipticity field before minimization and using the full covariance matrix in the minimization



# Choice of smoothing scale

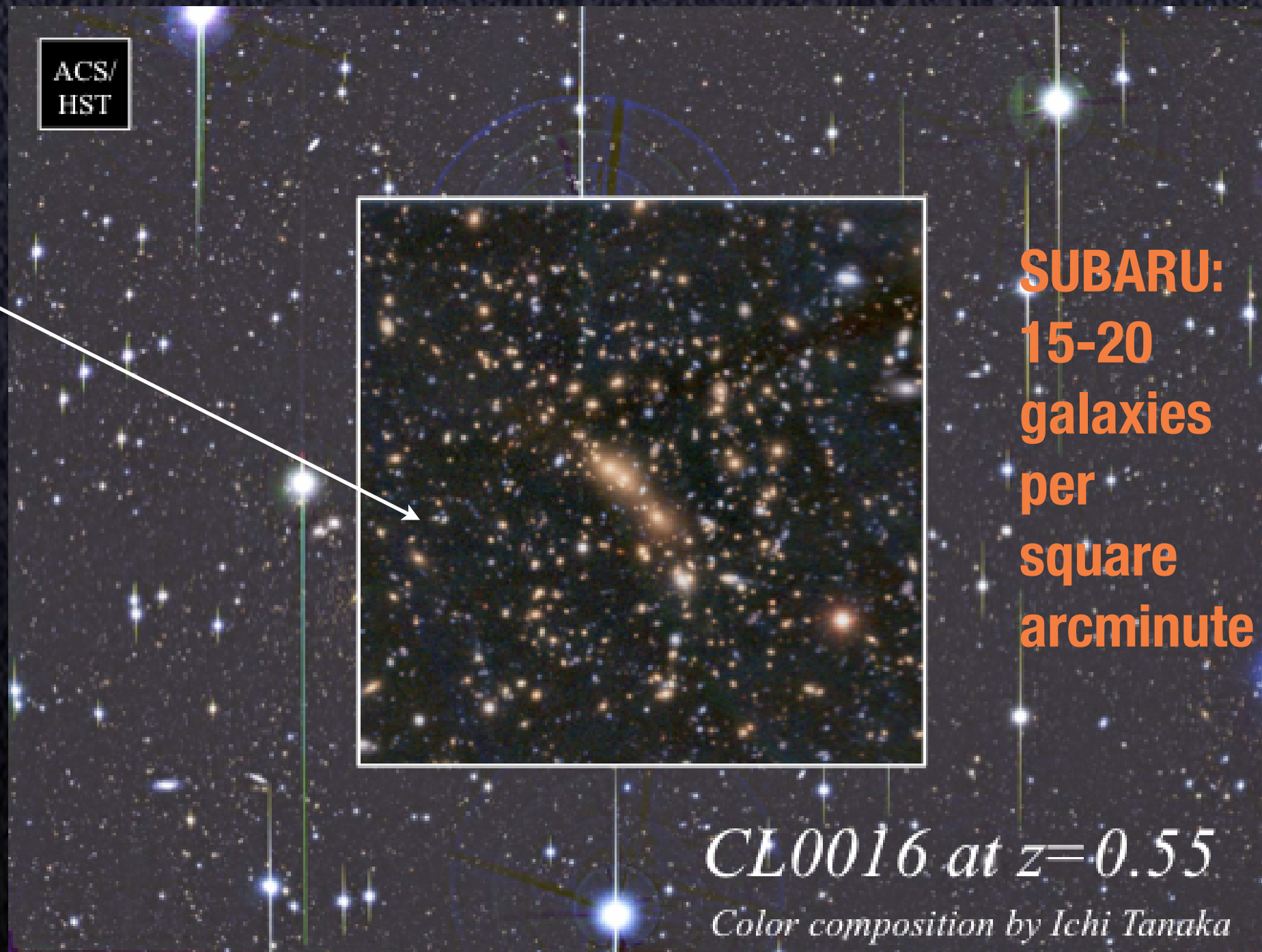


$$\sigma_\kappa^2 = A_0 \left( \frac{\zeta}{\lambda} \right)^4 + B_0 \frac{\sigma_\epsilon^2 / \langle \kappa^2 \rangle}{(\zeta)^2 n} + C_0$$



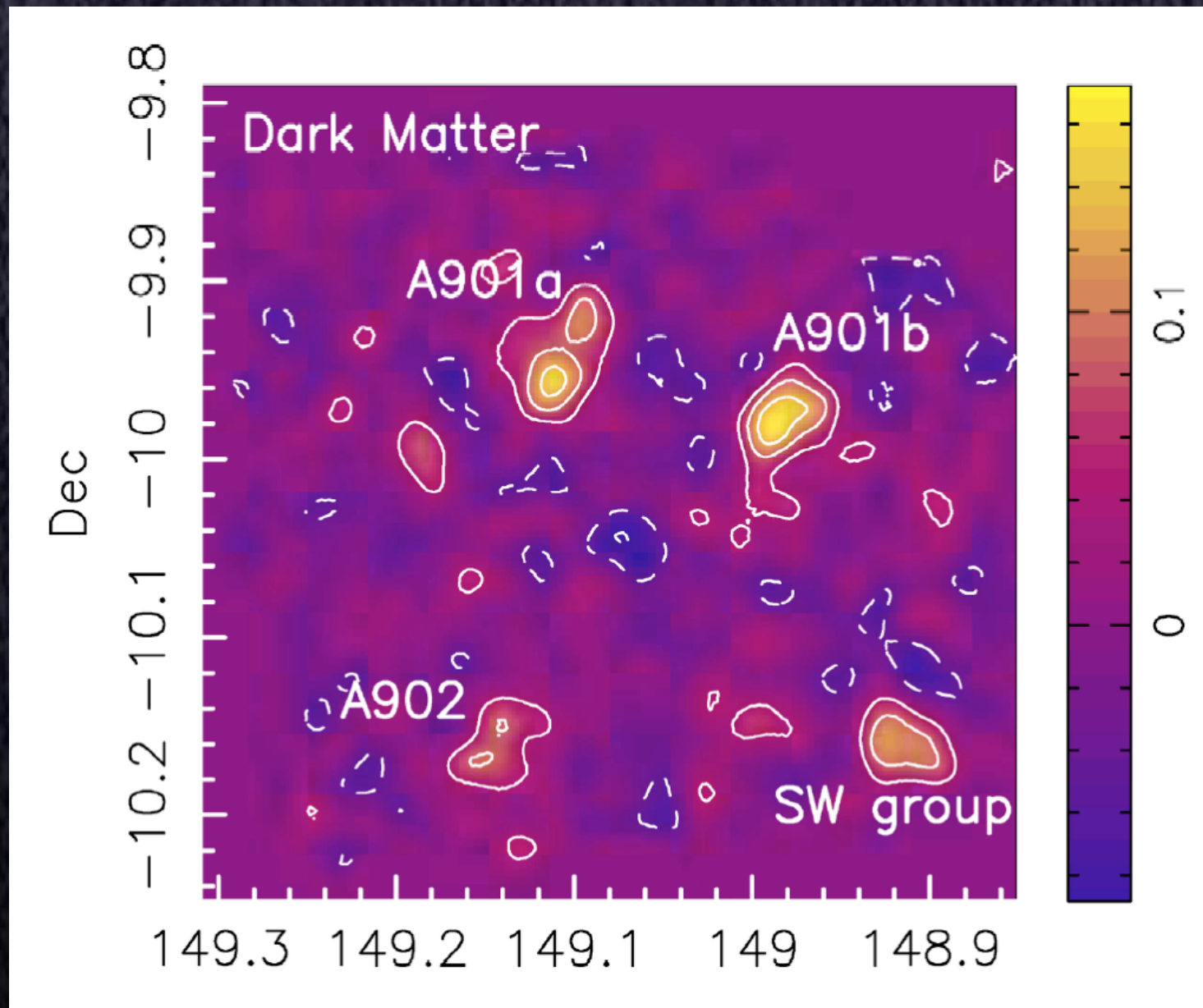
# Heterogeneous Datasets

**HST:**  
50-60  
galaxies  
per  
square  
arcminute





# Abell 901/902



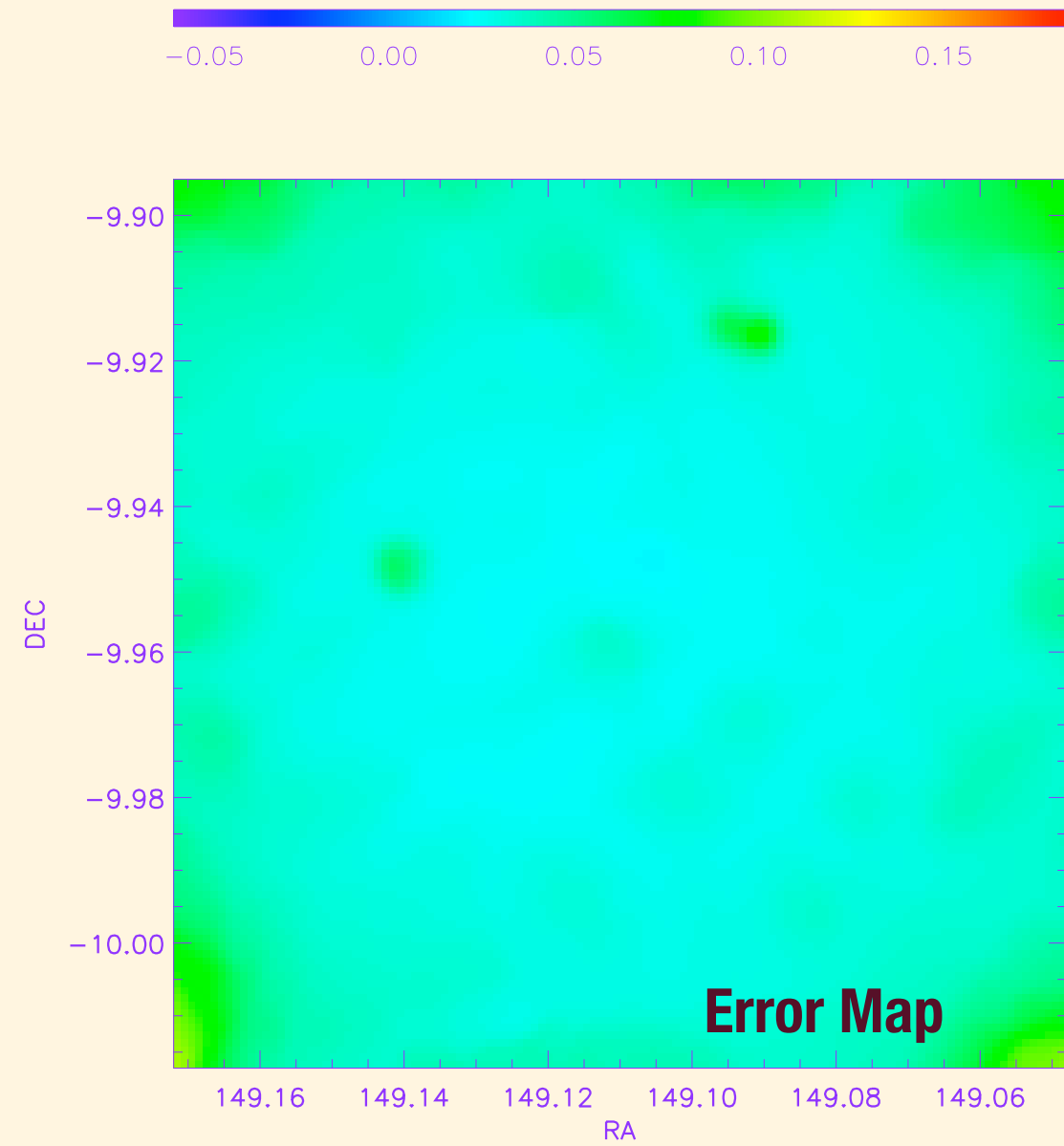
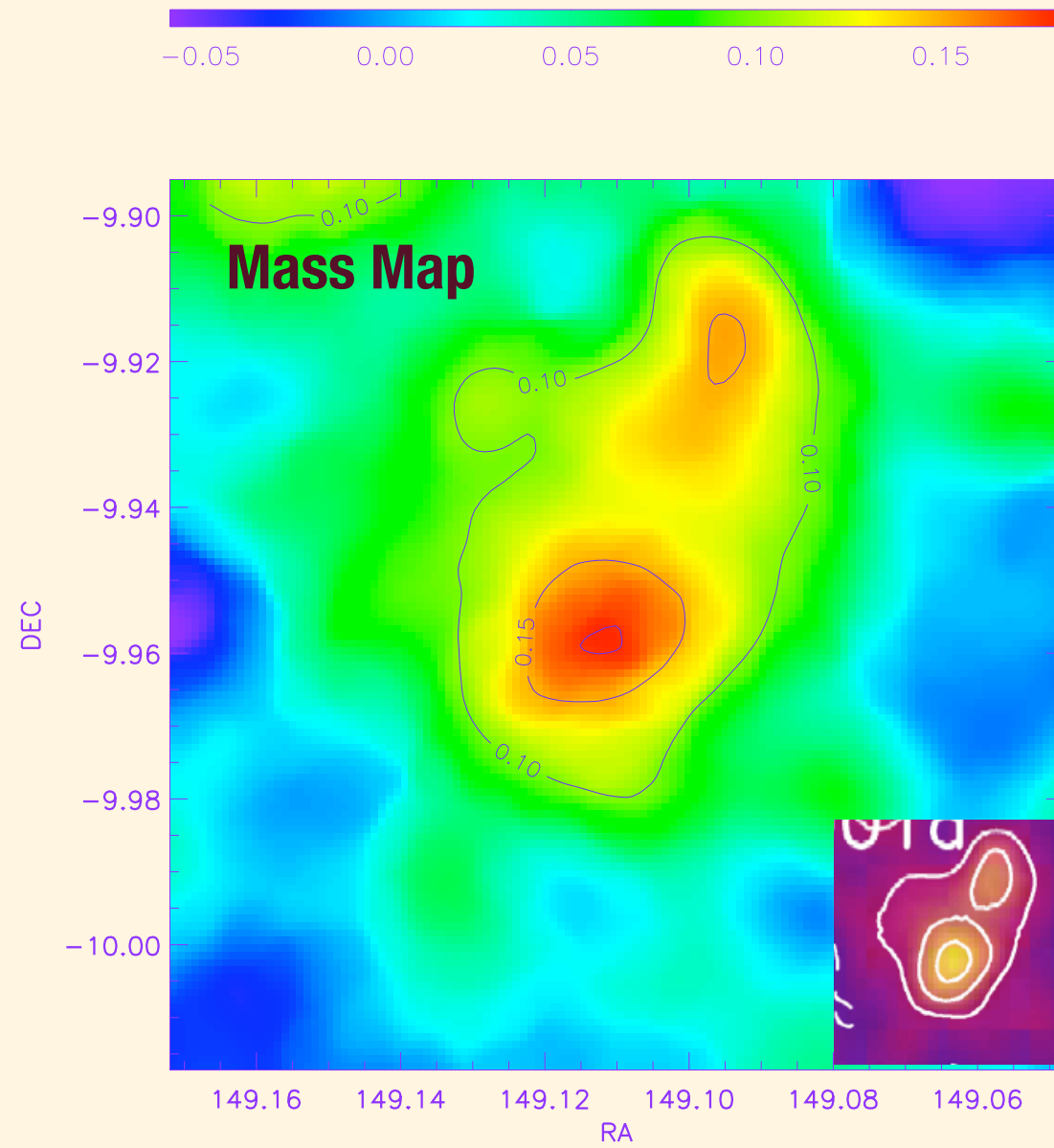
0.5 degree<sup>2</sup> fov  
STAGES HST  
survey  
60,000 background  
images.

Heymans et. al. 2008.



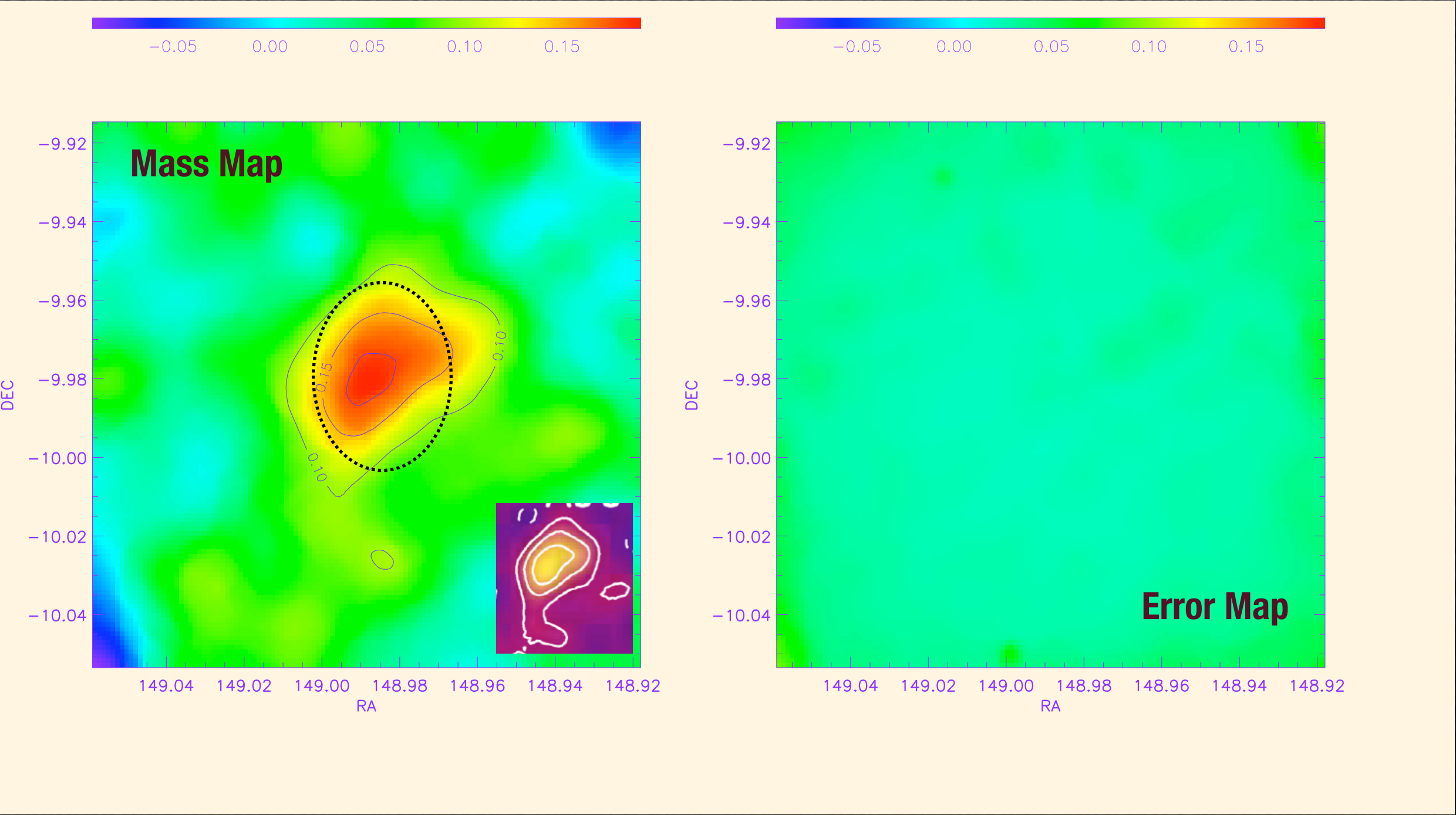
# A901a

From Deb et al. 2009





# A901b



From Deb et al. 2009

Axis Ratio

Position angle

Axis Ratio

Position angle

Non-

Parametric:

$$0.37^{+0.1}_{-0.1}$$

$$91.4^{+8.2}_{-8.2}$$

Parametric:

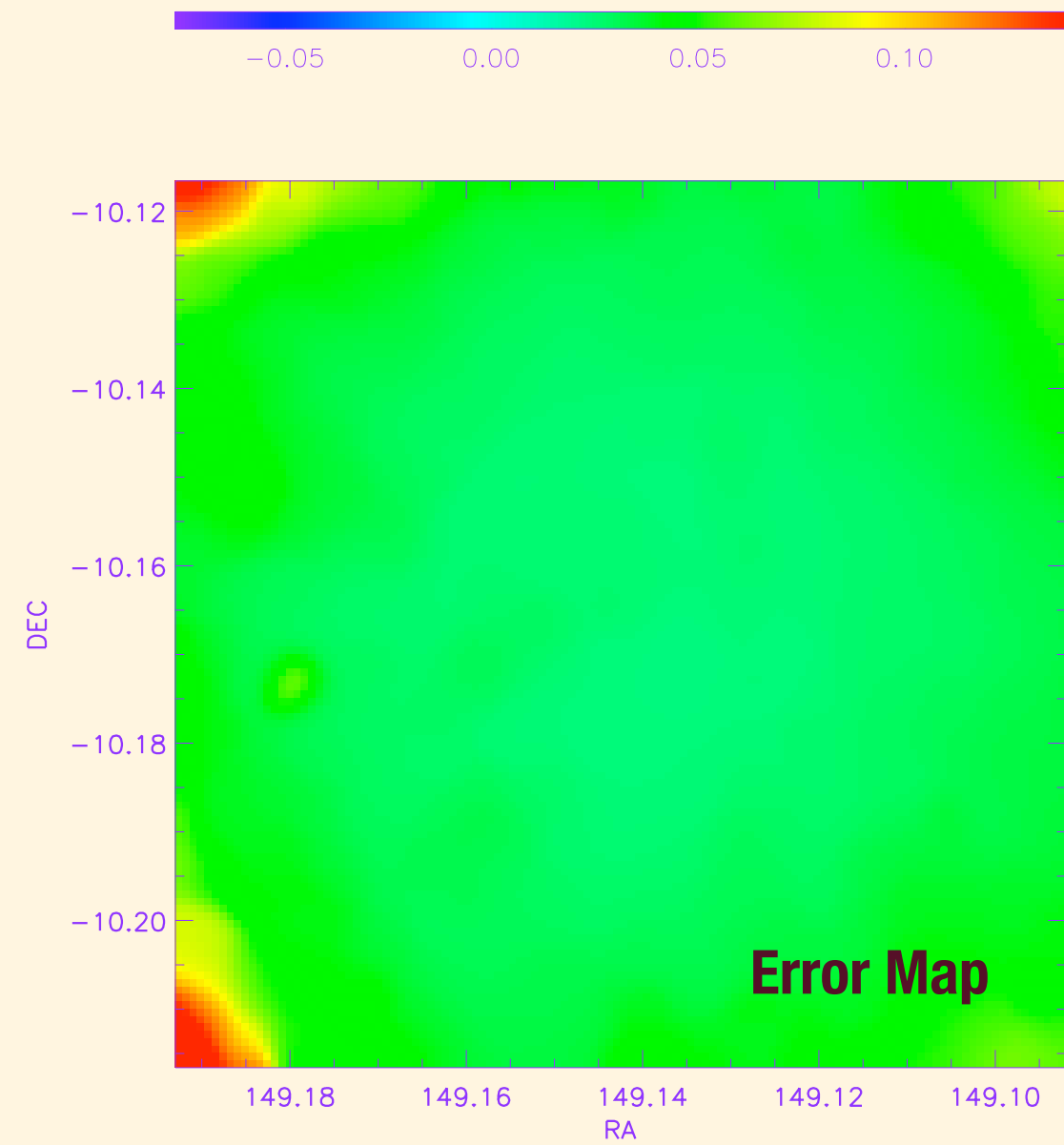
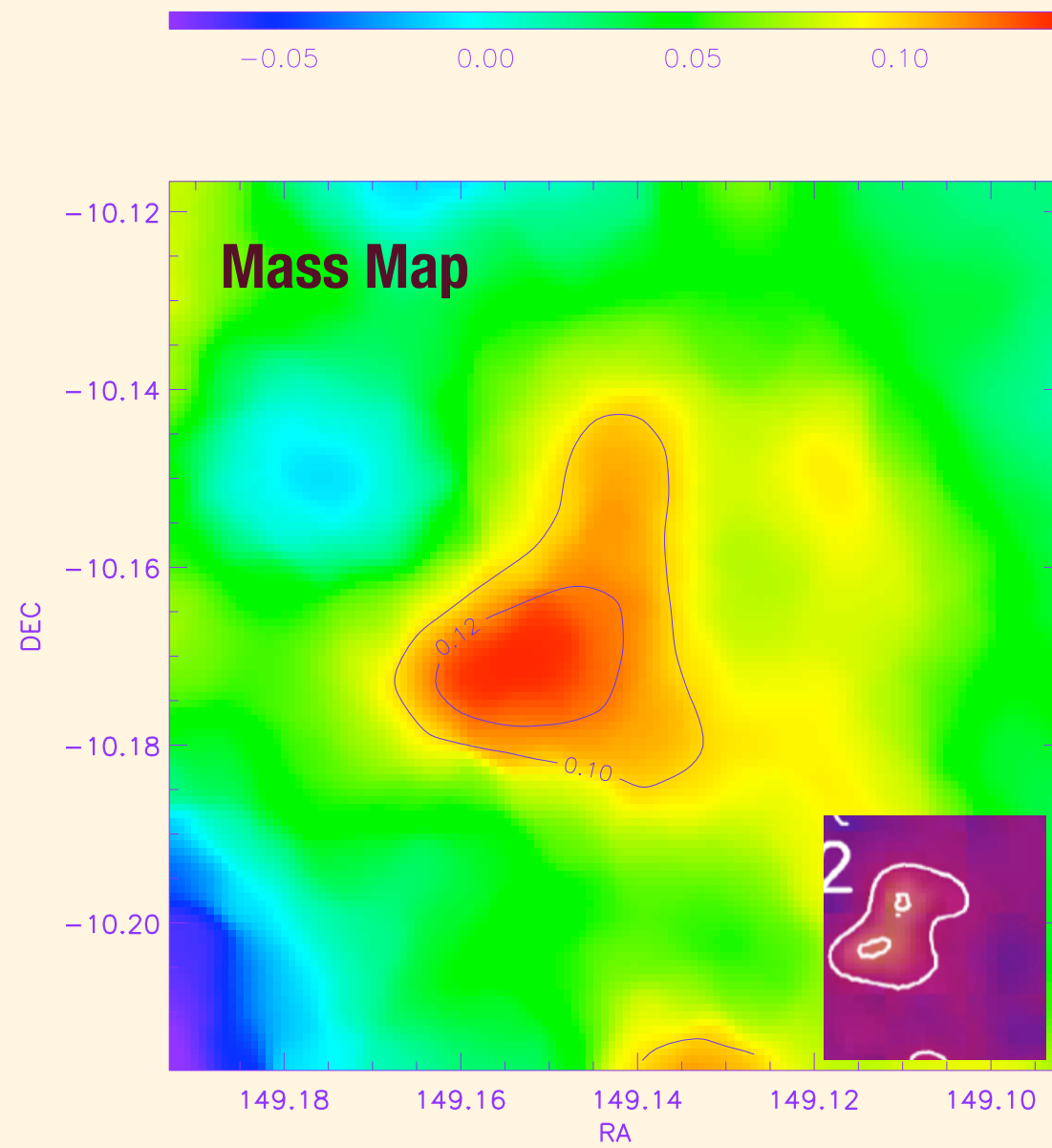
$$0.437^{+0.1}_{-0.087}$$

$$90.0^{+2.25}_{-2.25}$$



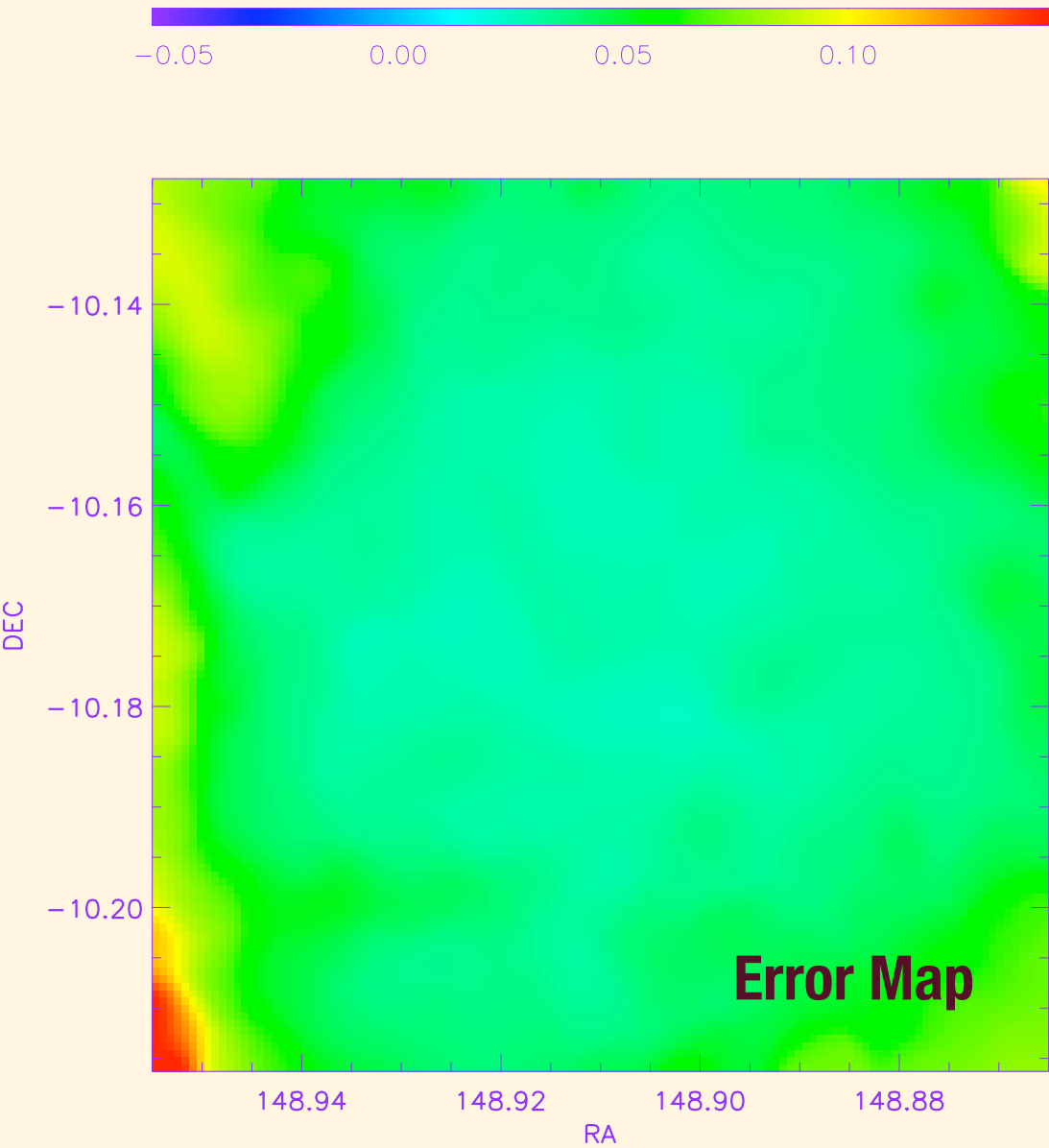
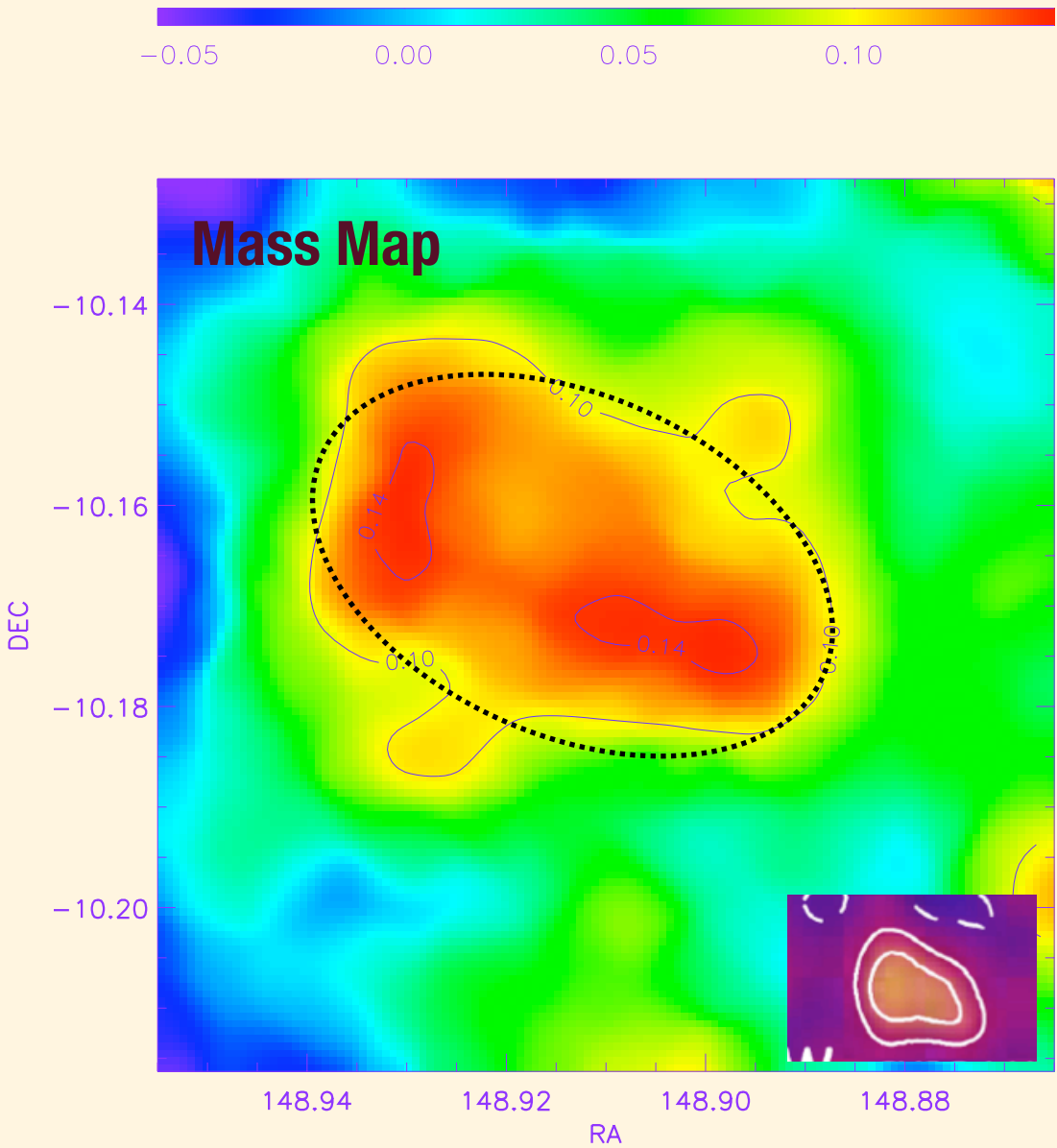
# A902

From Deb et al. 2009





# Southwest Group



From Deb et al. 2009

Axis Ratio

Position angle

Axis Ratio

Position angle

Non-

Parametric:

$0.54^{+0.08}_{-0.09}$

$120.0^{+4.8}_{-4.8}$

Parametric:

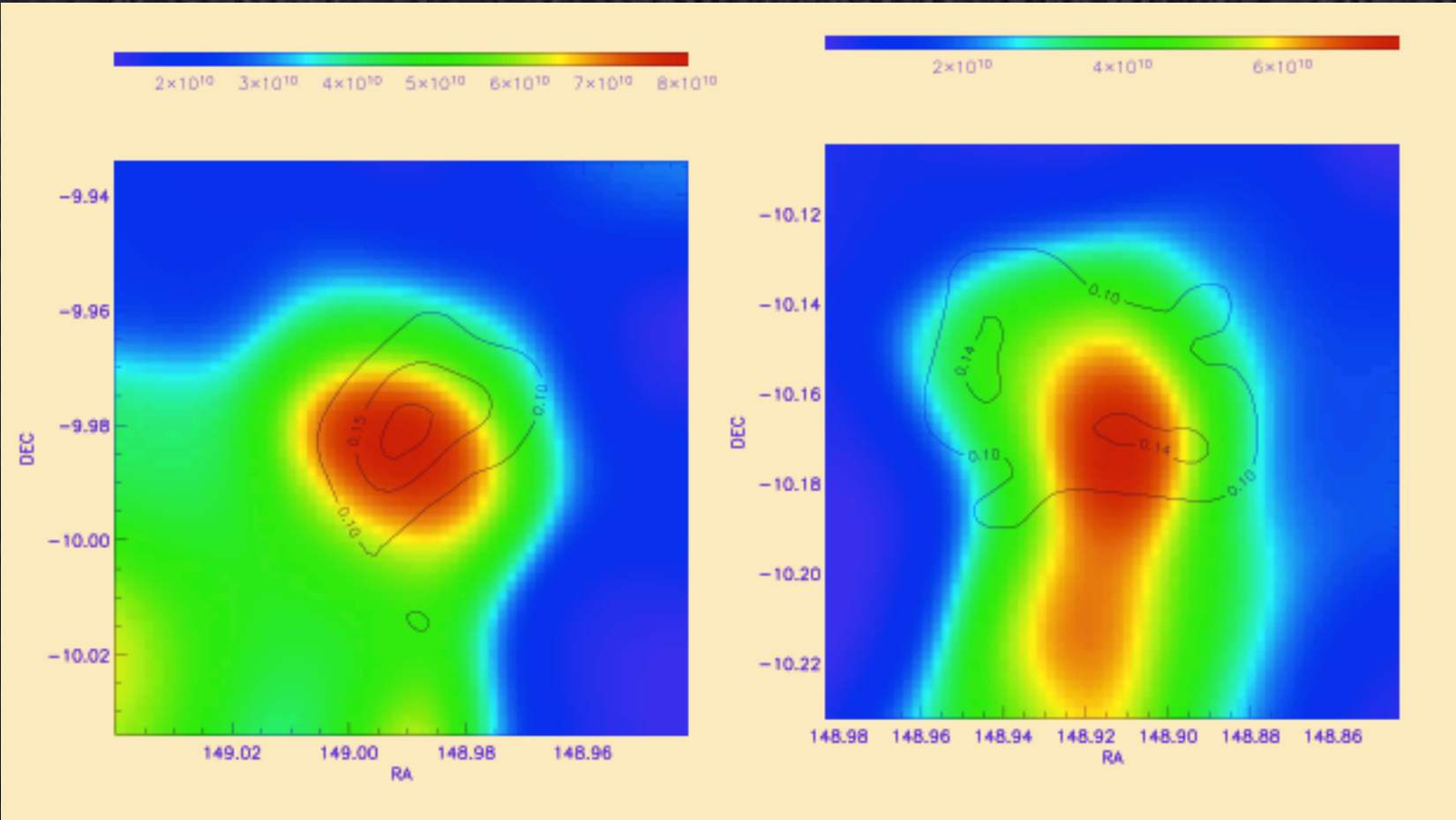
$0.42^{+0.18}_{-0.12}$

$180.0^{+7.73}_{-5.15}$



# Dark Matter vs Light

From Deb et al. 2009



Peak

$f$

$\alpha$ (degrees)

Dark Matter: Non-parametric

A901b	—	$0.37^{+0.1}_{-0.1}$	$91.4^{+8.2}_{-8.2}$
SW group	—	$0.54^{+0.08}_{-0.09}$	$120.0^{+4.8}_{-4.8}$

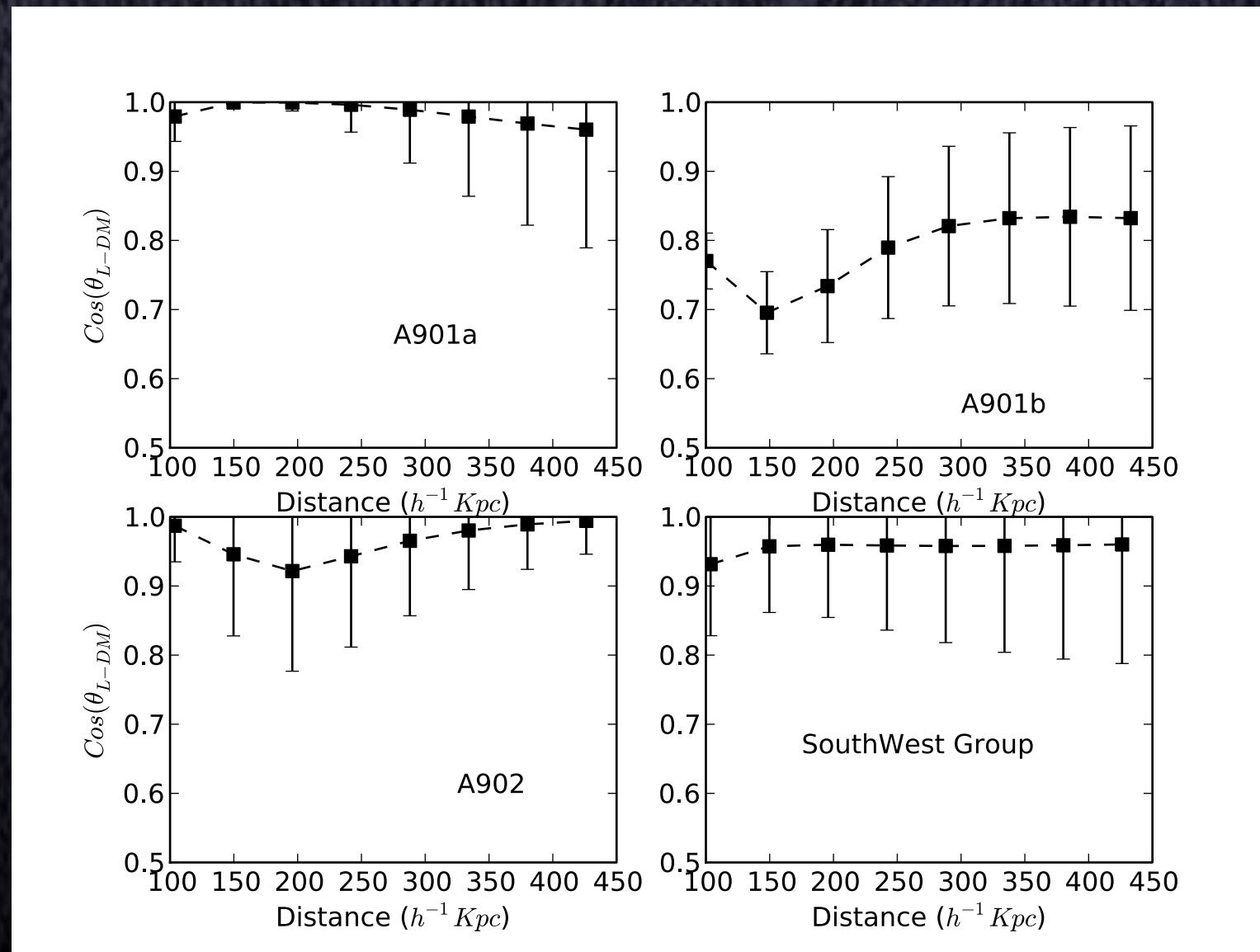
Light Distribution

A901b	—	$0.58^{+0.1}_{-0.09}$	$121.5^{+12.0}_{-12.0}$
SW group	—	$0.69^{+0.06}_{-0.05}$	$100.0^{+10.0}_{-10.0}$



# Alignment between Light and Dark Matter

From Deb et al. 2009



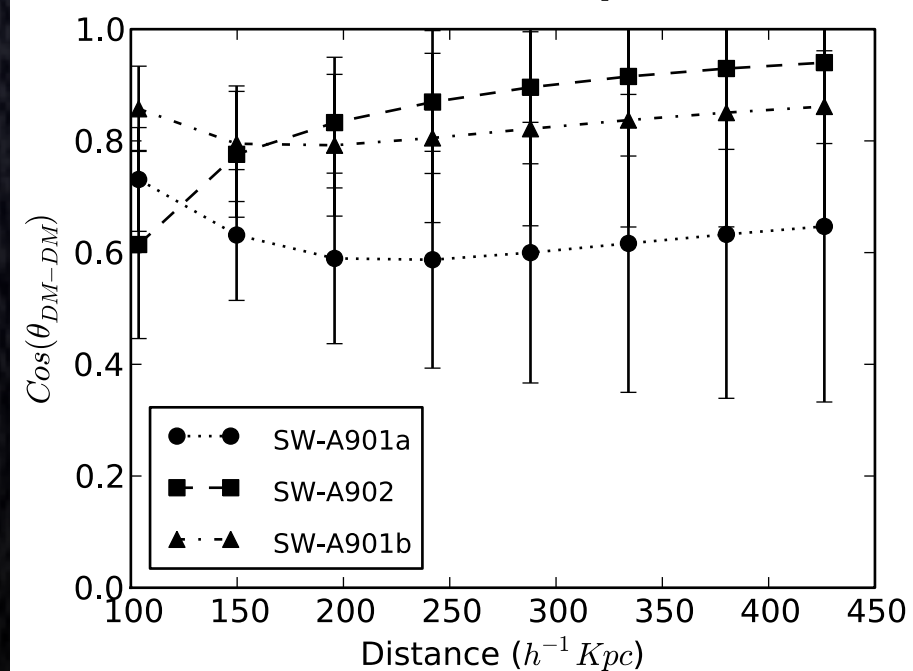
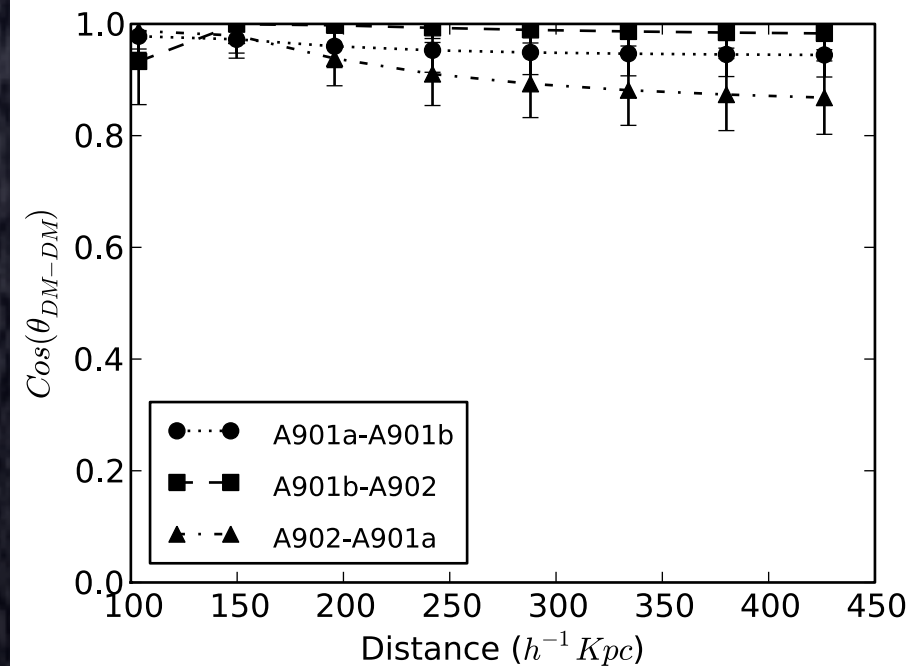
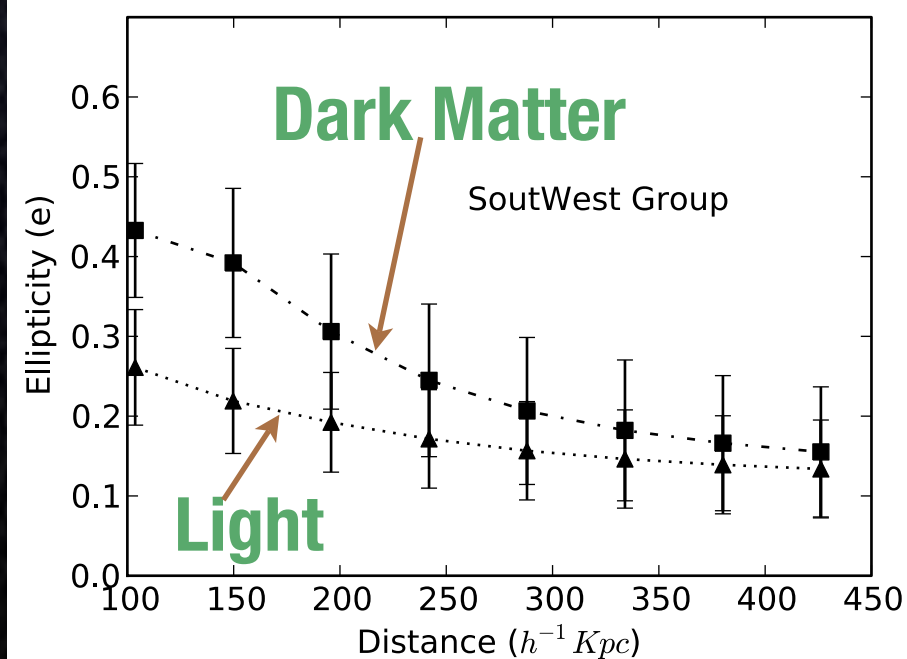
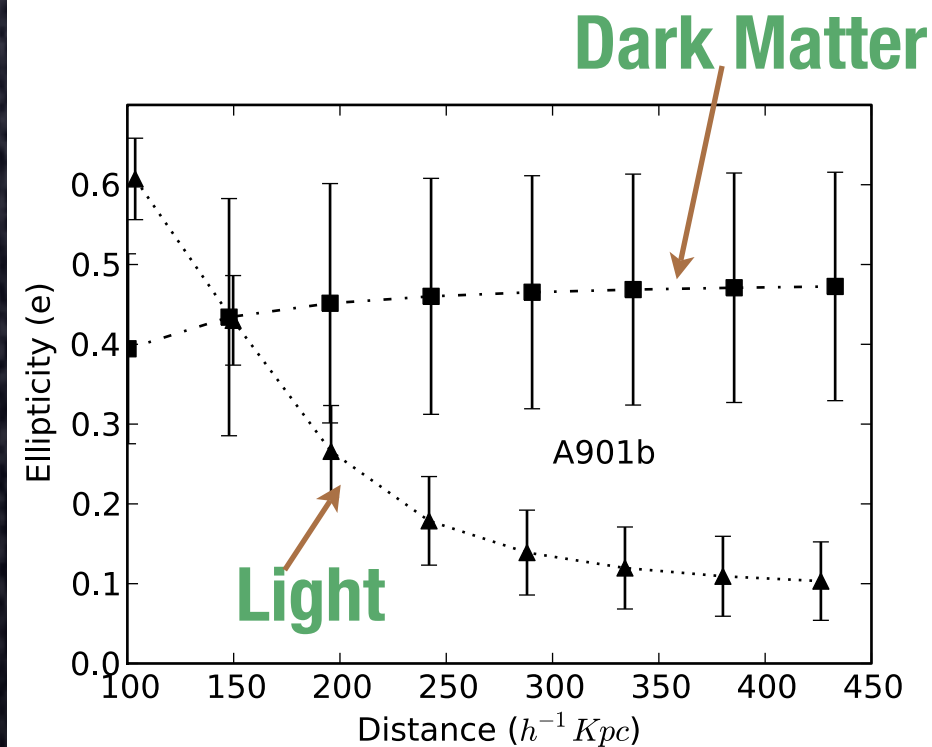


# Radial dependence

## Ellipticity

From Deb et al. 2009

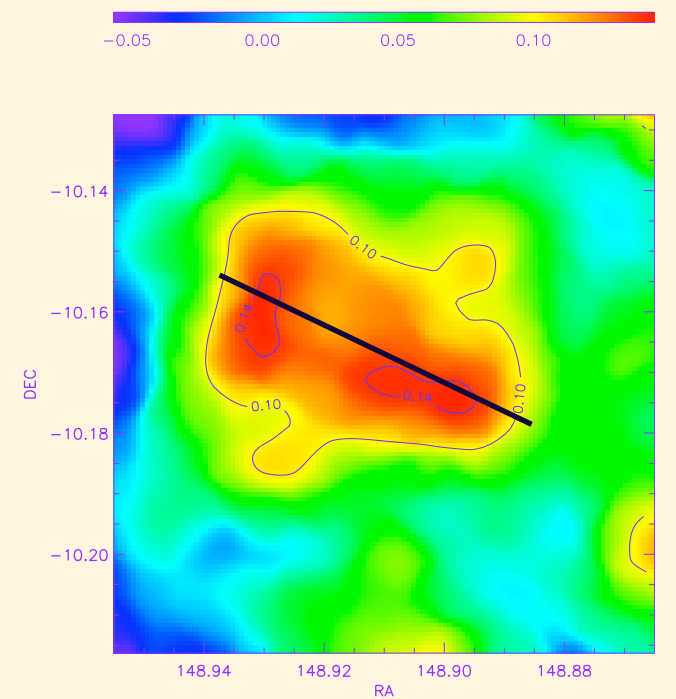
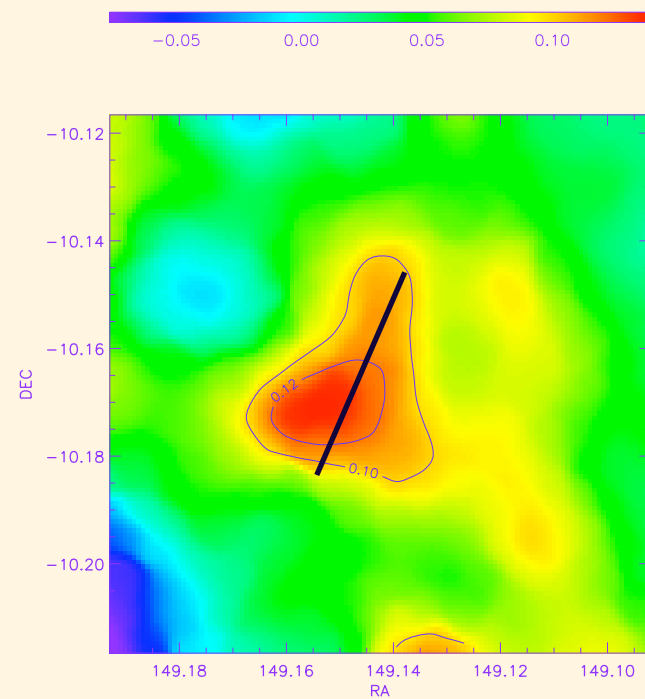
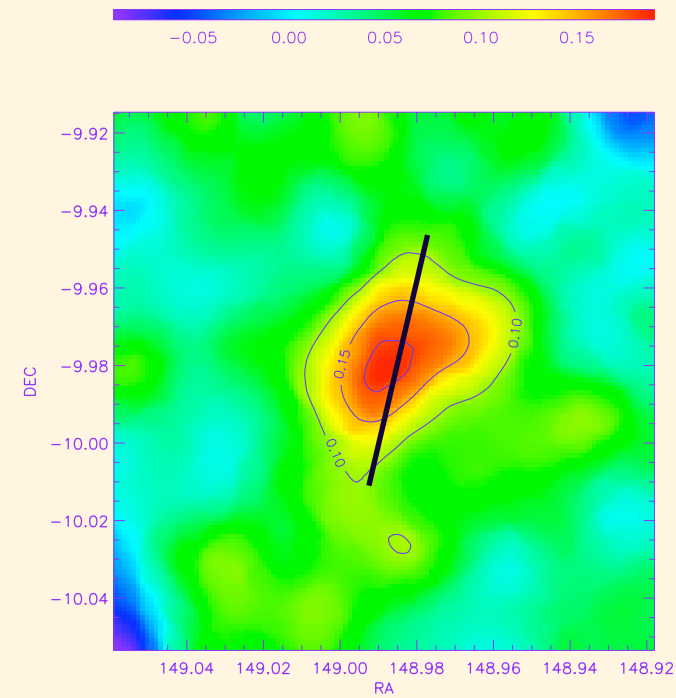
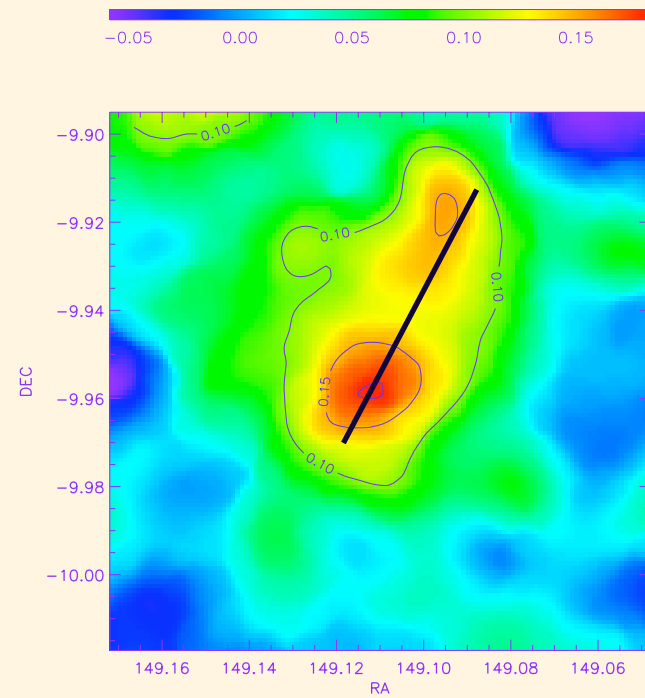
## Alignment





# Alignment

From Deb et al. 2009





# Reconstruction Methods

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## Parametric:

### Strong Lensing

GRAVLENS - Keeton et al., 2001

LENSTOOL - Jullo et al., 2007

### Galaxy-Galaxy

### Lensing

Natarajan et al. 2005

## Hybrid

**LENSTOOL -  
Jullo et al., 2009**

## Non-Parametric

### Strong Lensing

Jullo & Kneib 2009,

LENSPERFECT -Coe et al, 2008,2010 (Mesh Free)

### Weak Lensing

Kaiser 1995, Seitz & Schneider 1995-2001

### Strong+Weak Lensing

### Finite Differencing Based

Bradac et al. 2004,

(ADAPTIVE) - Diego et al. , Cacciato et al.

Merten et al., Saha et. al. Pixelens

and more ....

### Mesh-Free Technique

### Particle Based Lensing

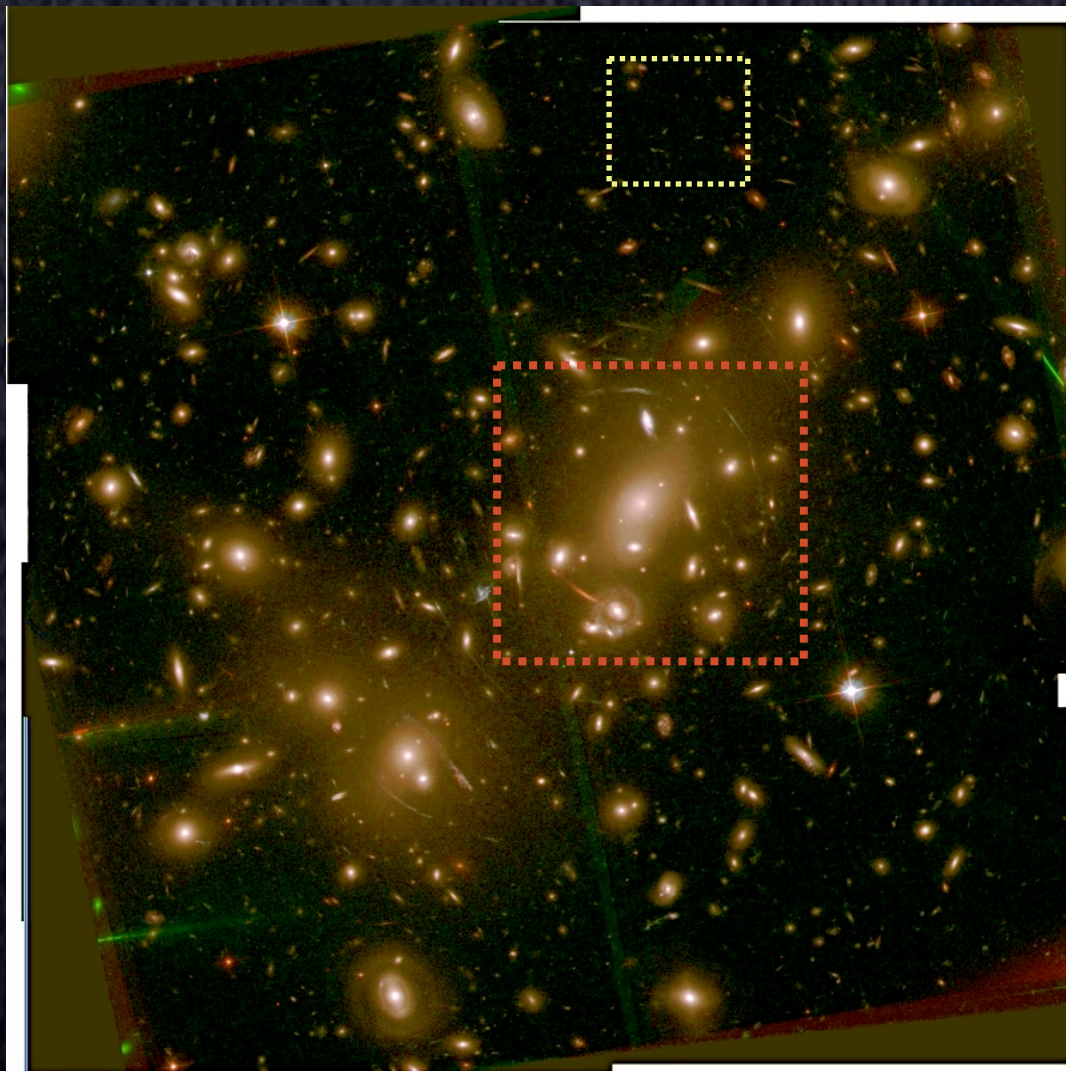
### (PBL)

Deb et al. ,2008, 2009



# Strong+Weak Lensing: Challenges

---

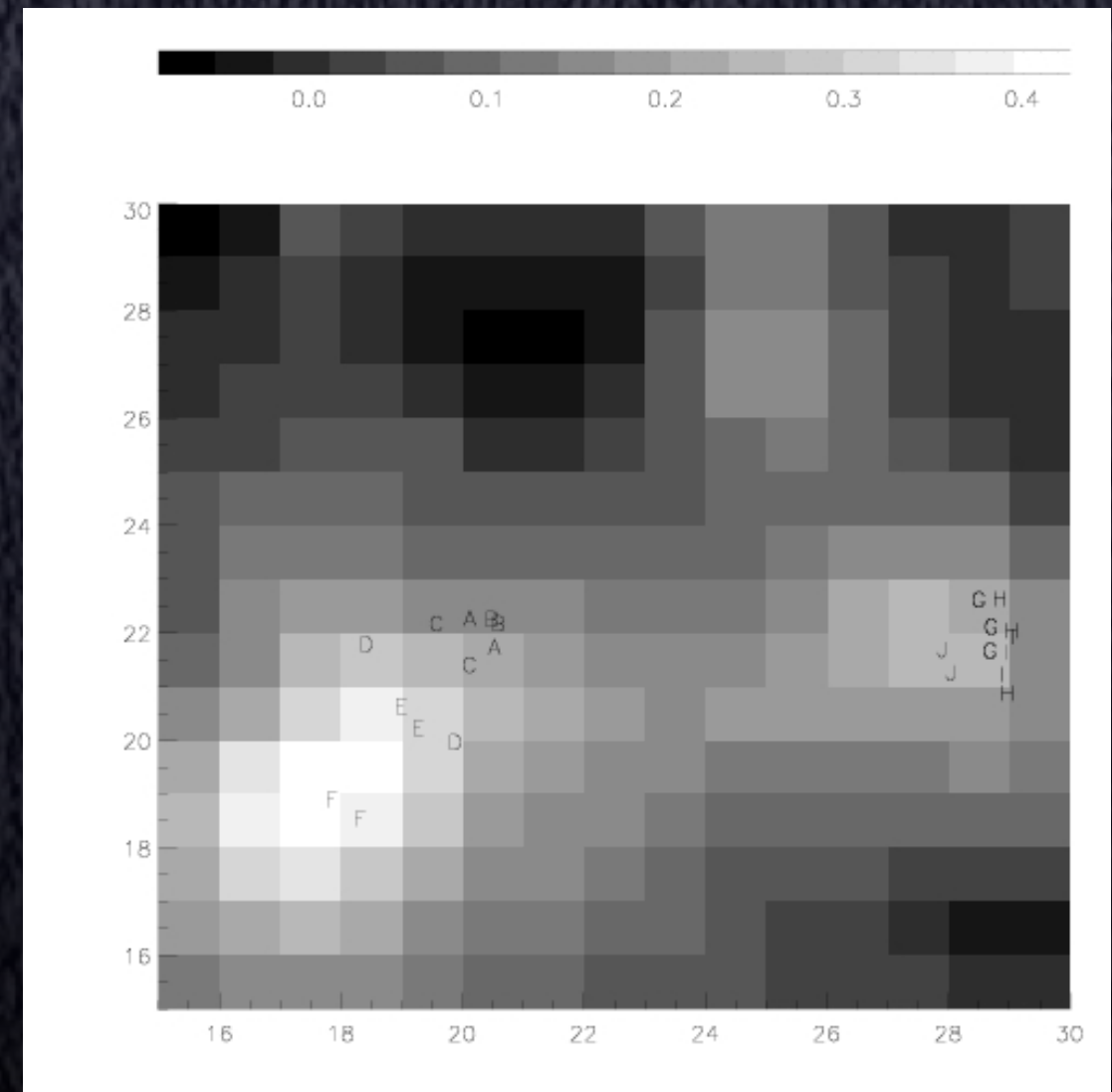
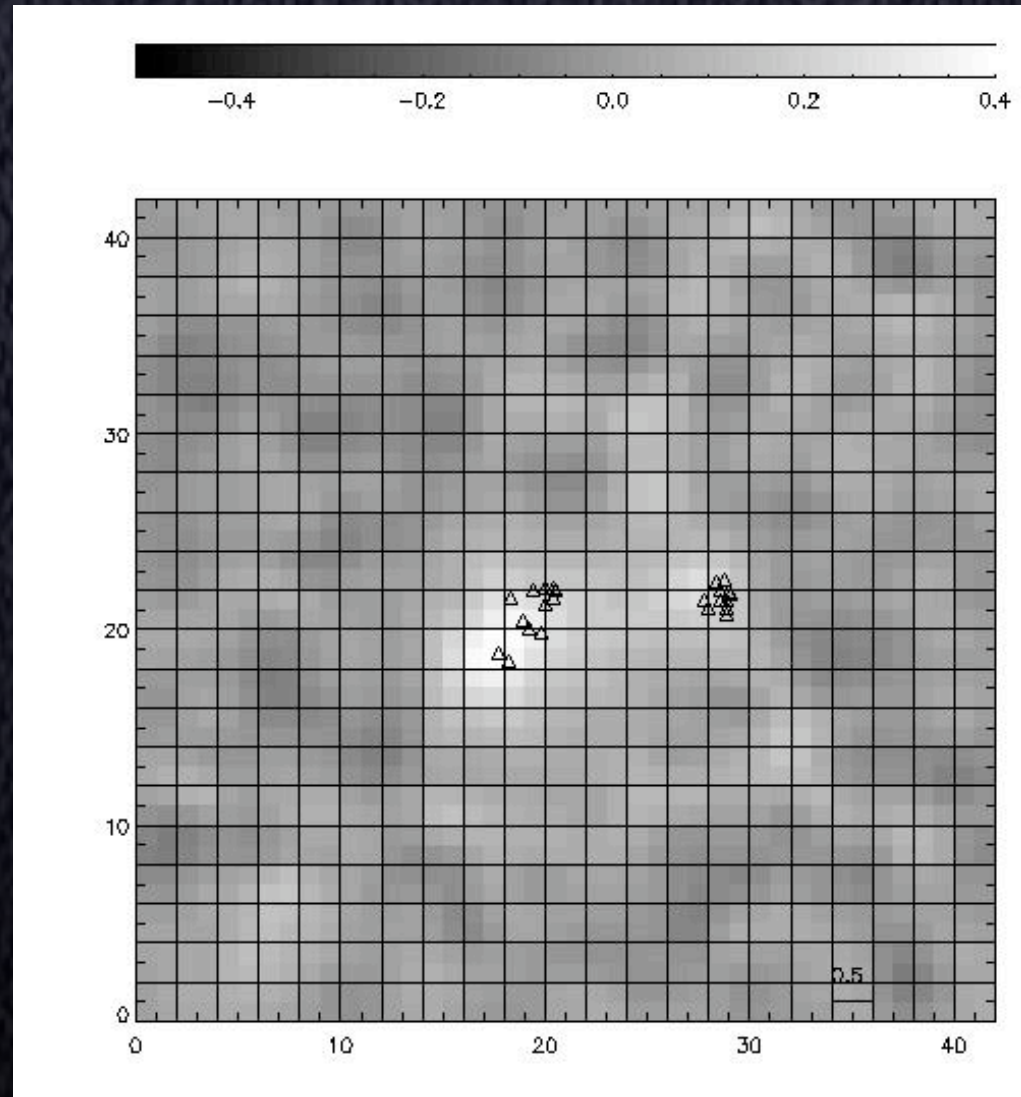


HST/ACS image of Abell 2218 (Sánchez et al. 2006)



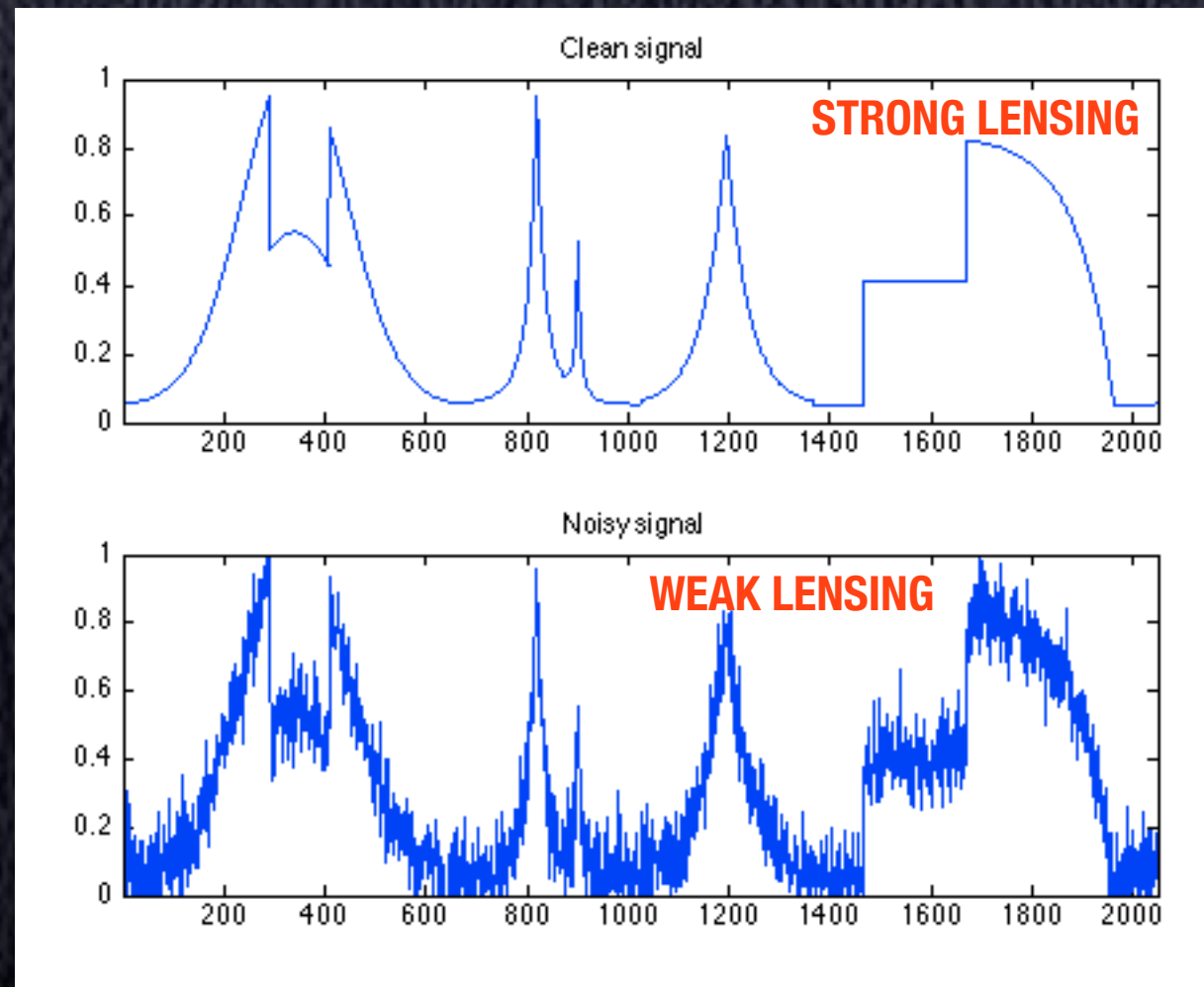


# Difference of Scales: Bullet Cluster





# Strong+Weak Lensing



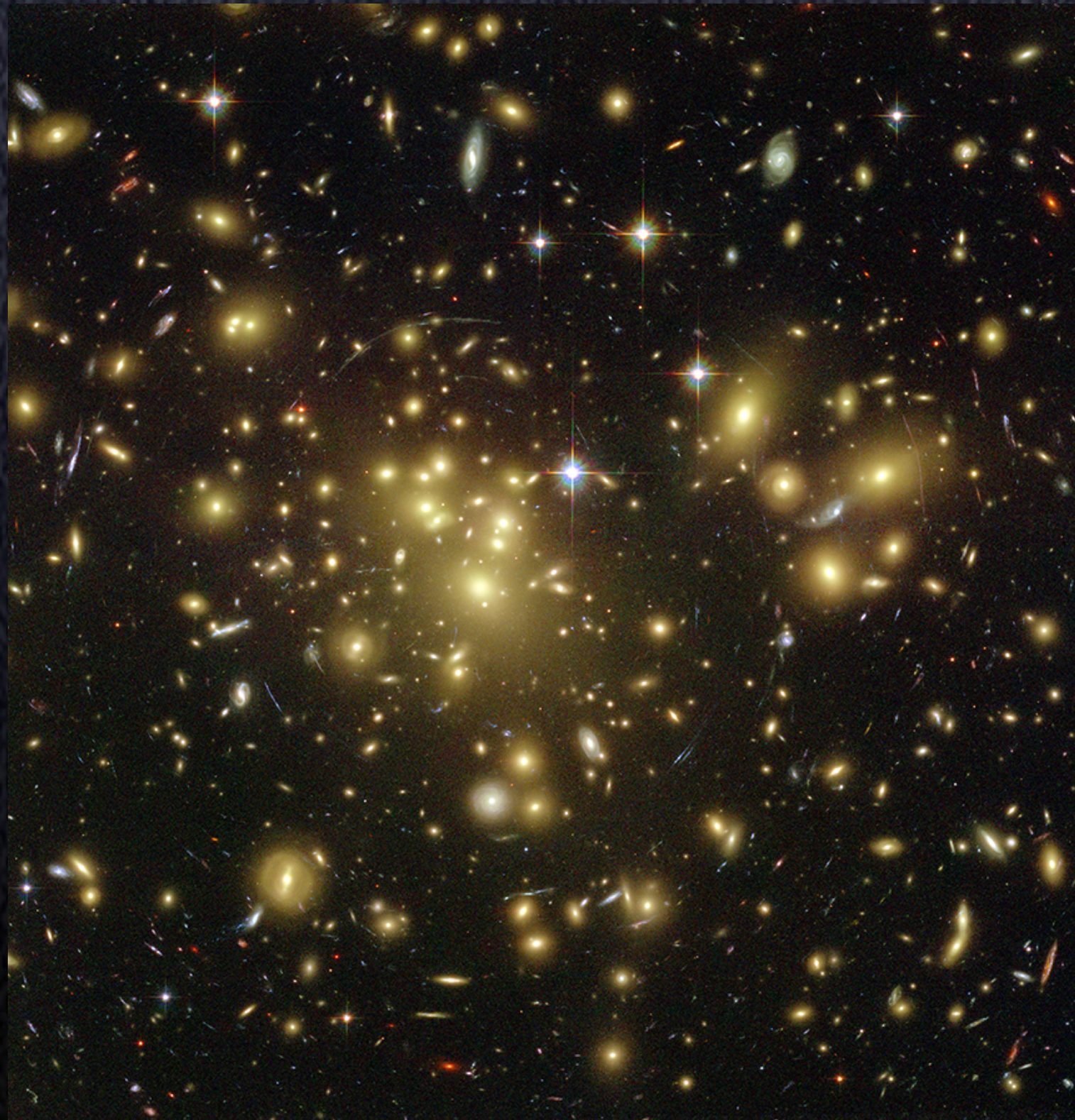
$$\chi^2 =$$

$$\sum_{mn} \left( \varepsilon_m^i - \frac{\gamma_m^i(\psi_m)}{1 - \kappa_m(\psi_m)} \right) C_{mn}^{-1} \left( \varepsilon_n^i - \frac{\gamma_n^i(\psi_n)}{1 - \kappa_n(\psi_n)} \right) + \sum_{i, \text{pairs}} \frac{((\boldsymbol{\alpha}^A(\{\psi\}) - \boldsymbol{\alpha}^B(\{\psi\}))Z(z_i) - (\boldsymbol{\theta}^A - \boldsymbol{\theta}^B))^2}{\sigma_i^2}$$



# A1689

---

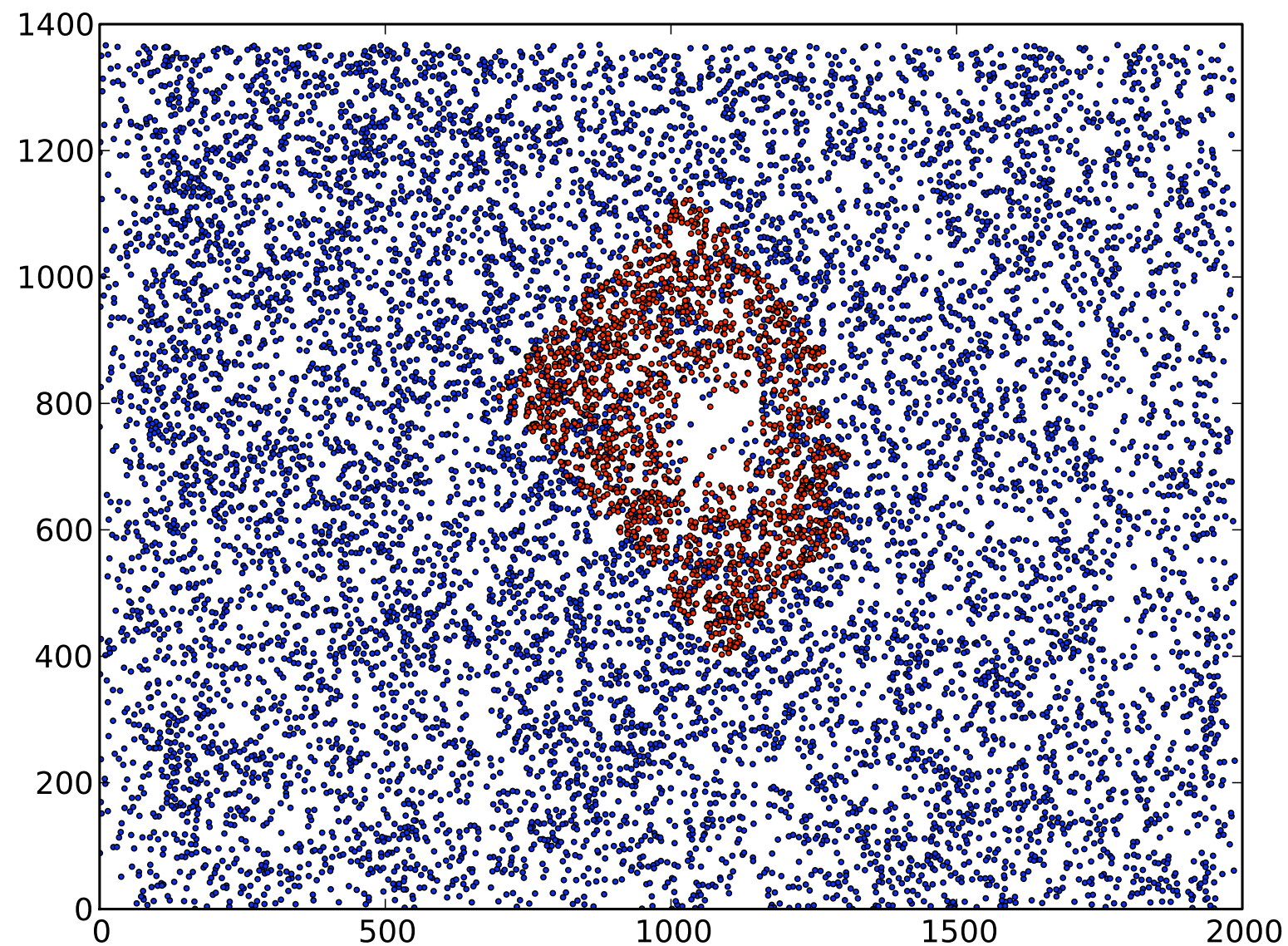


<http://chandra.harvard.edu/photo/2008/a1689/>



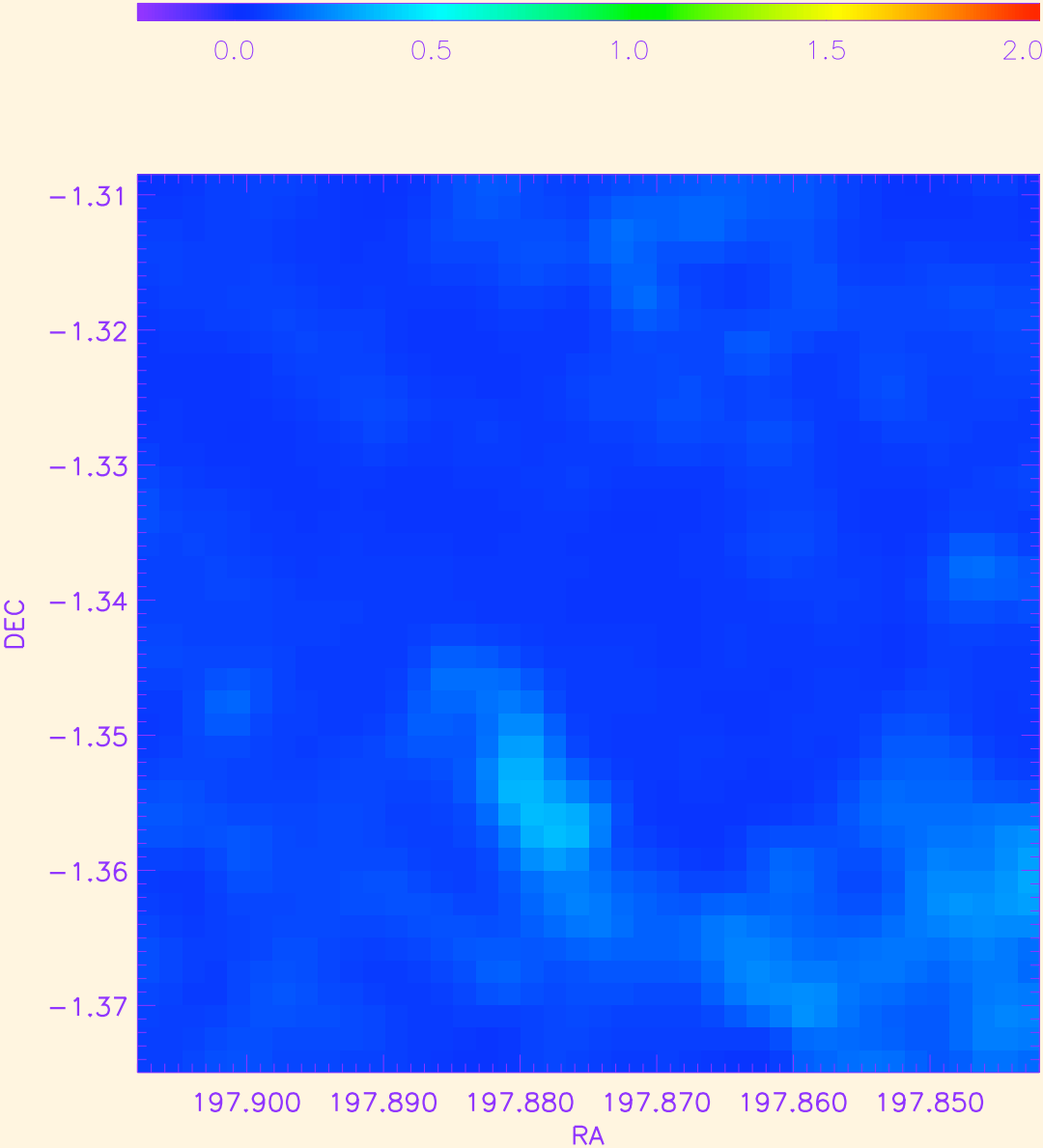
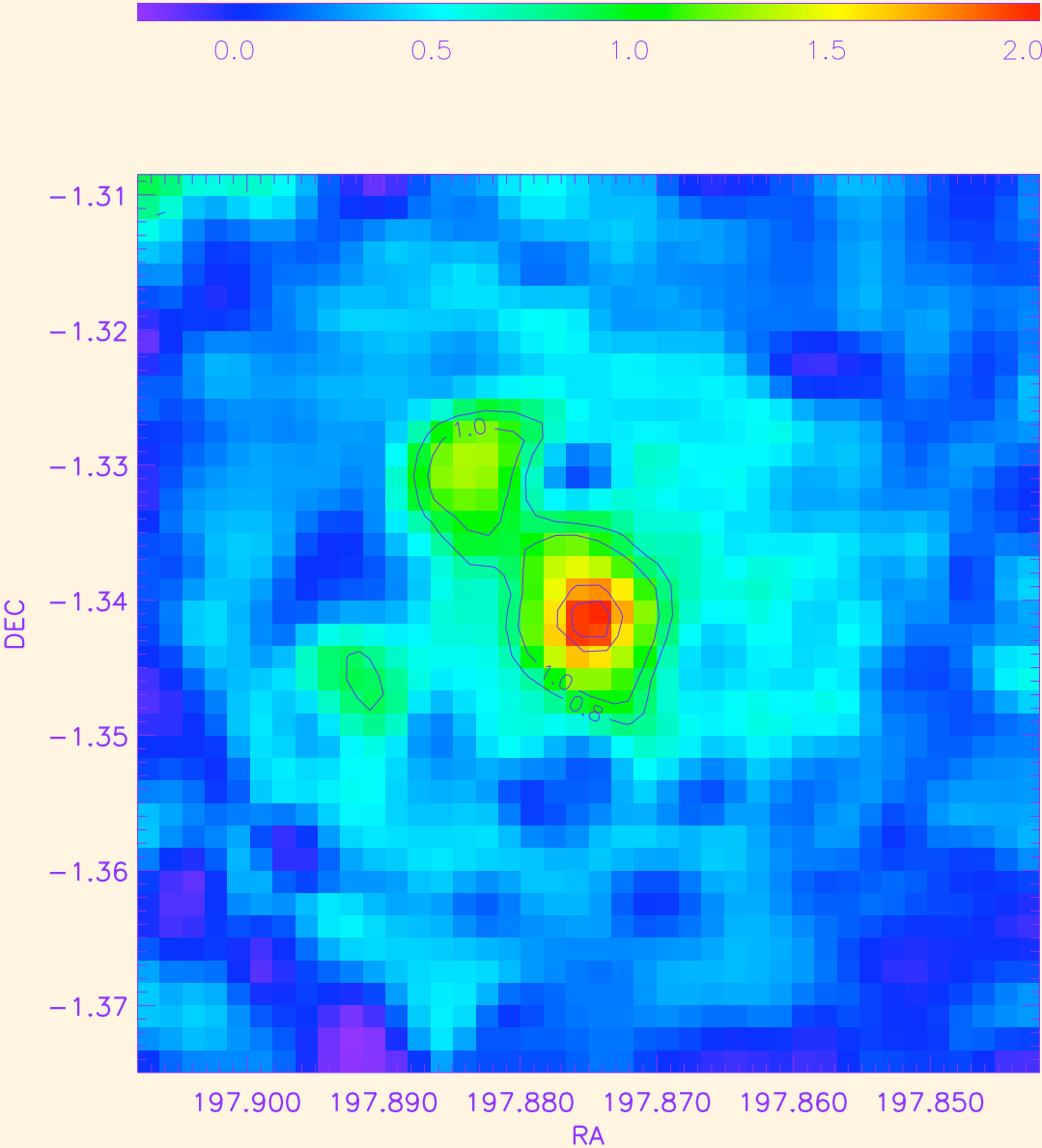
# A1689: ACS+SUBARU

---



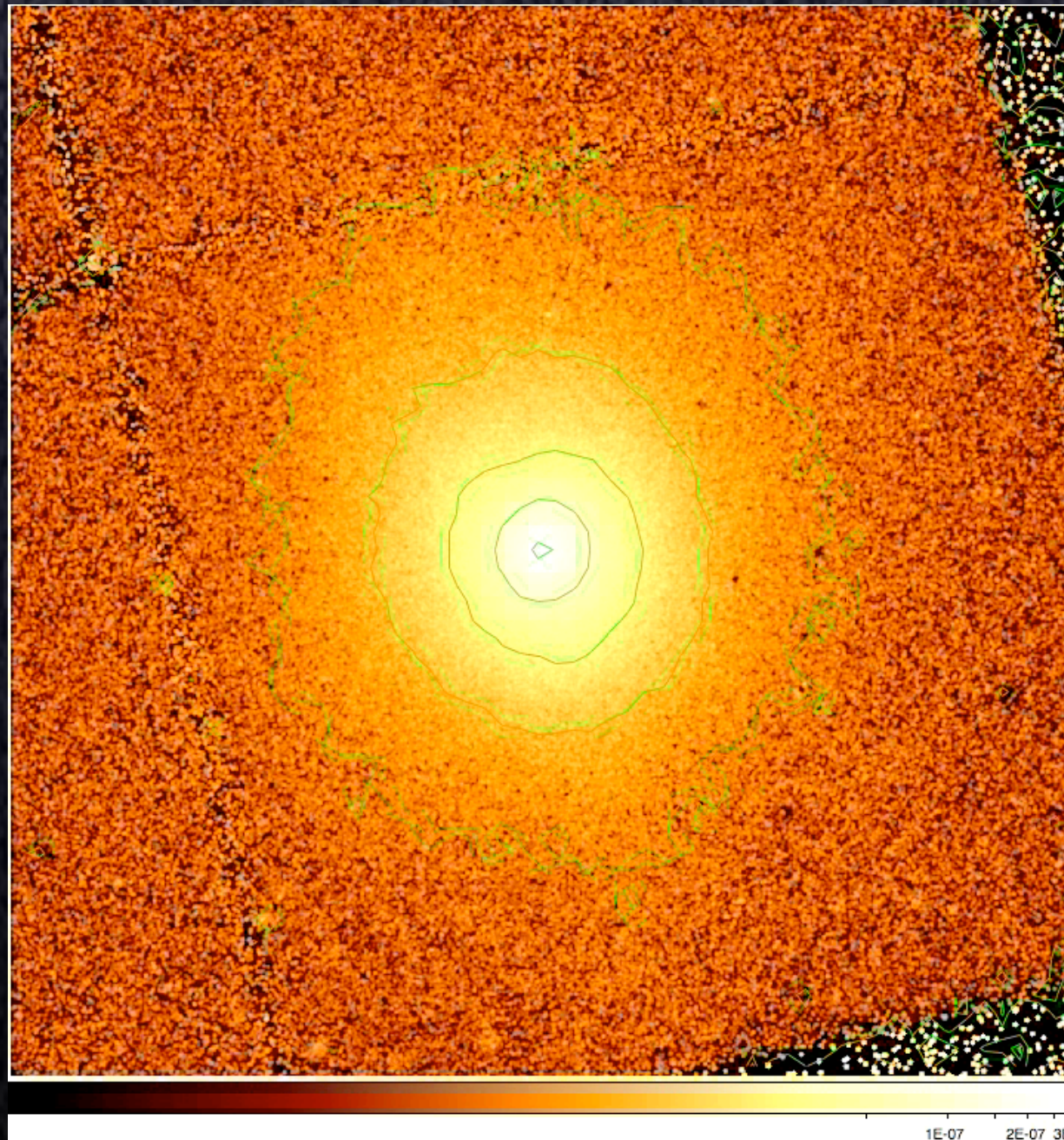


# Mass Map

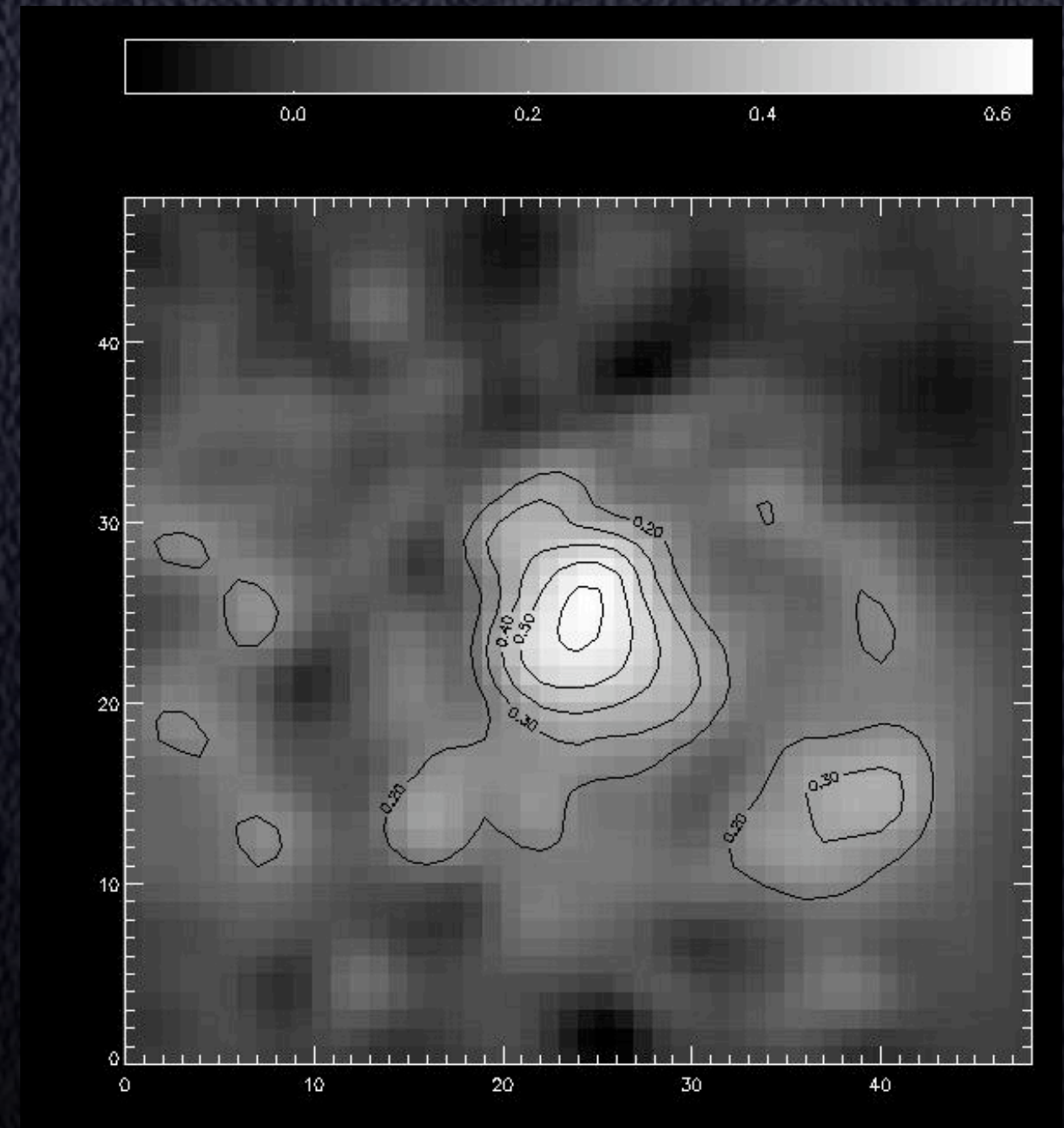




# X-ray vs Weak Lensing

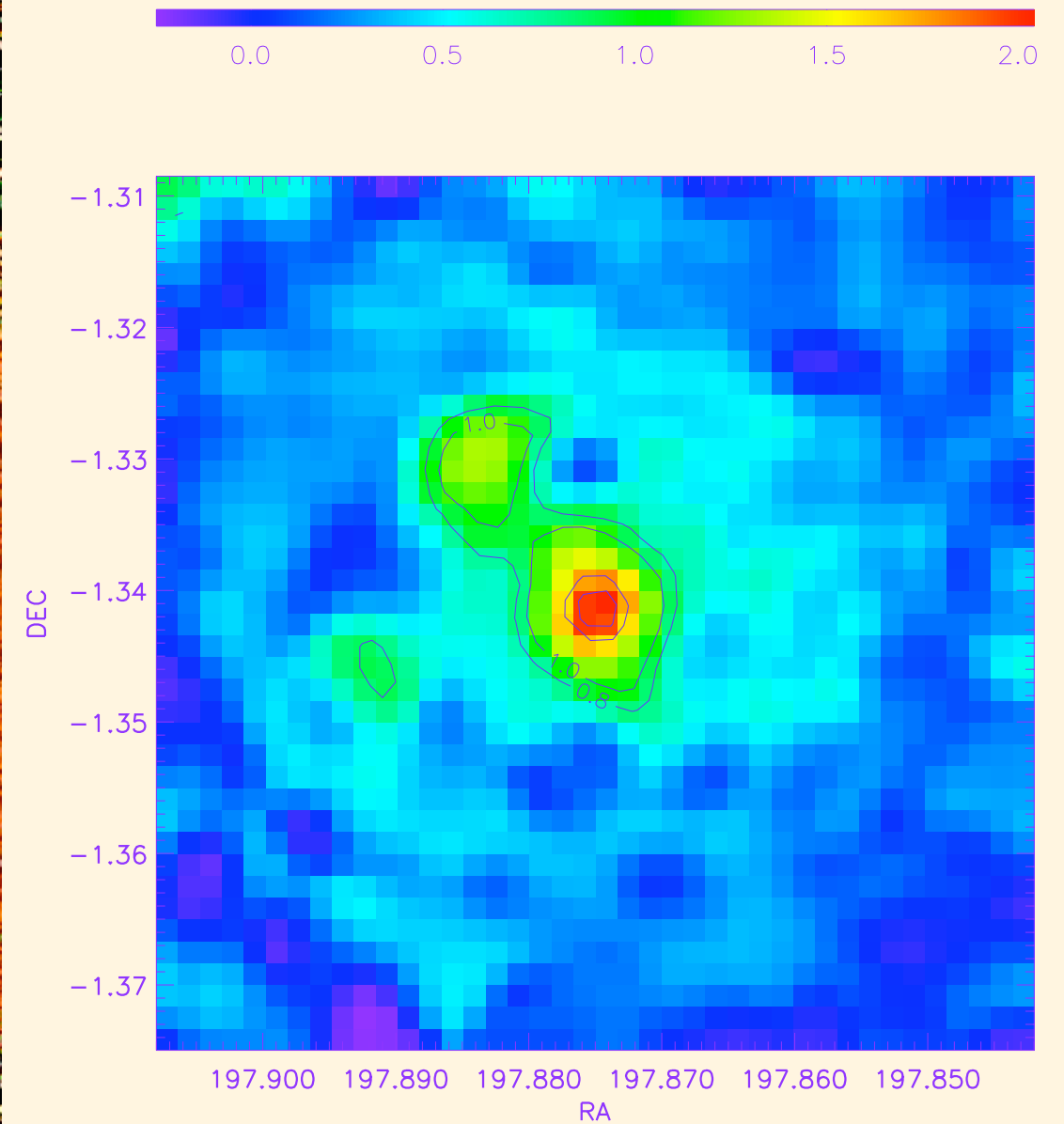
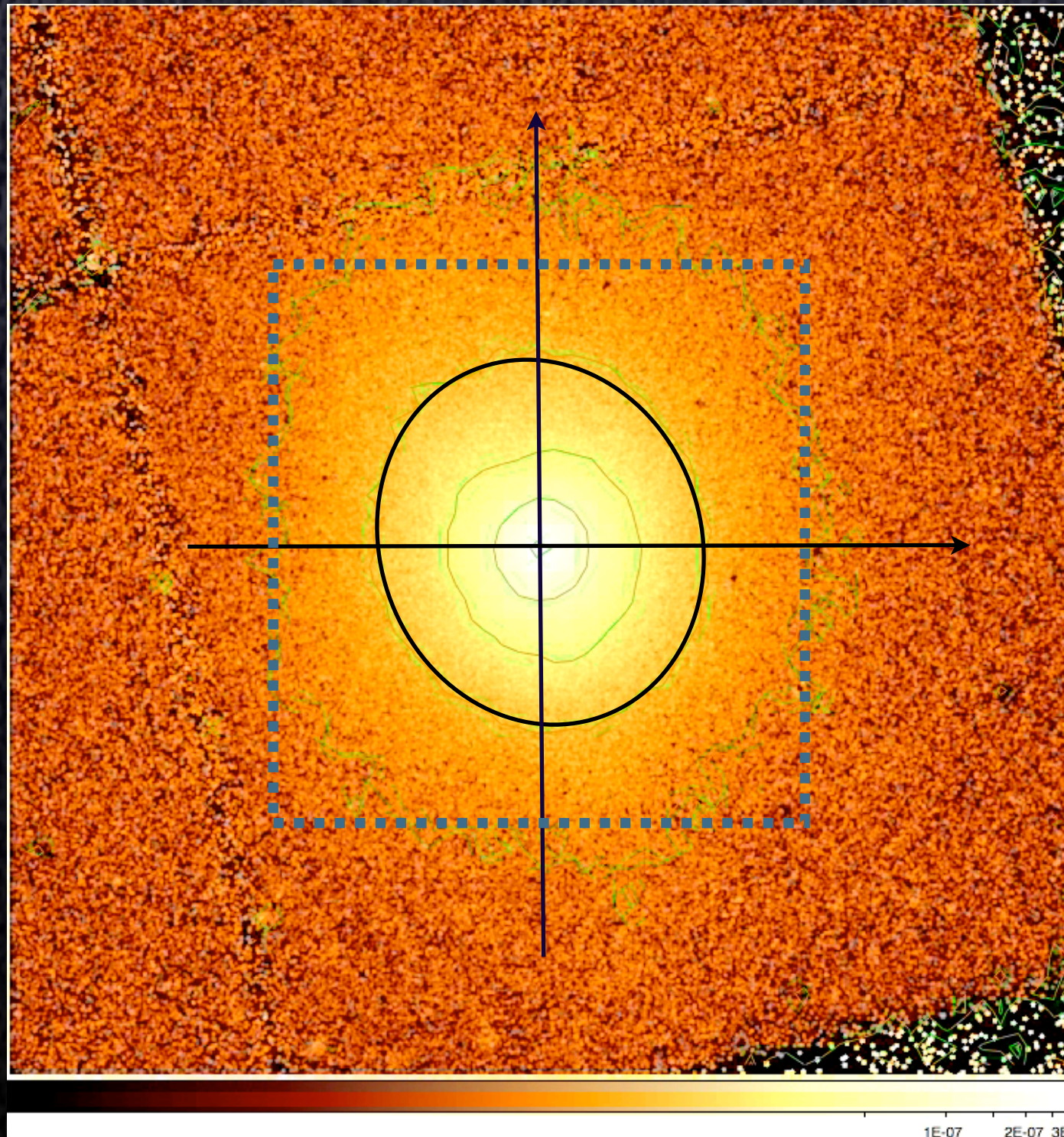


Sorensen et al. 2009



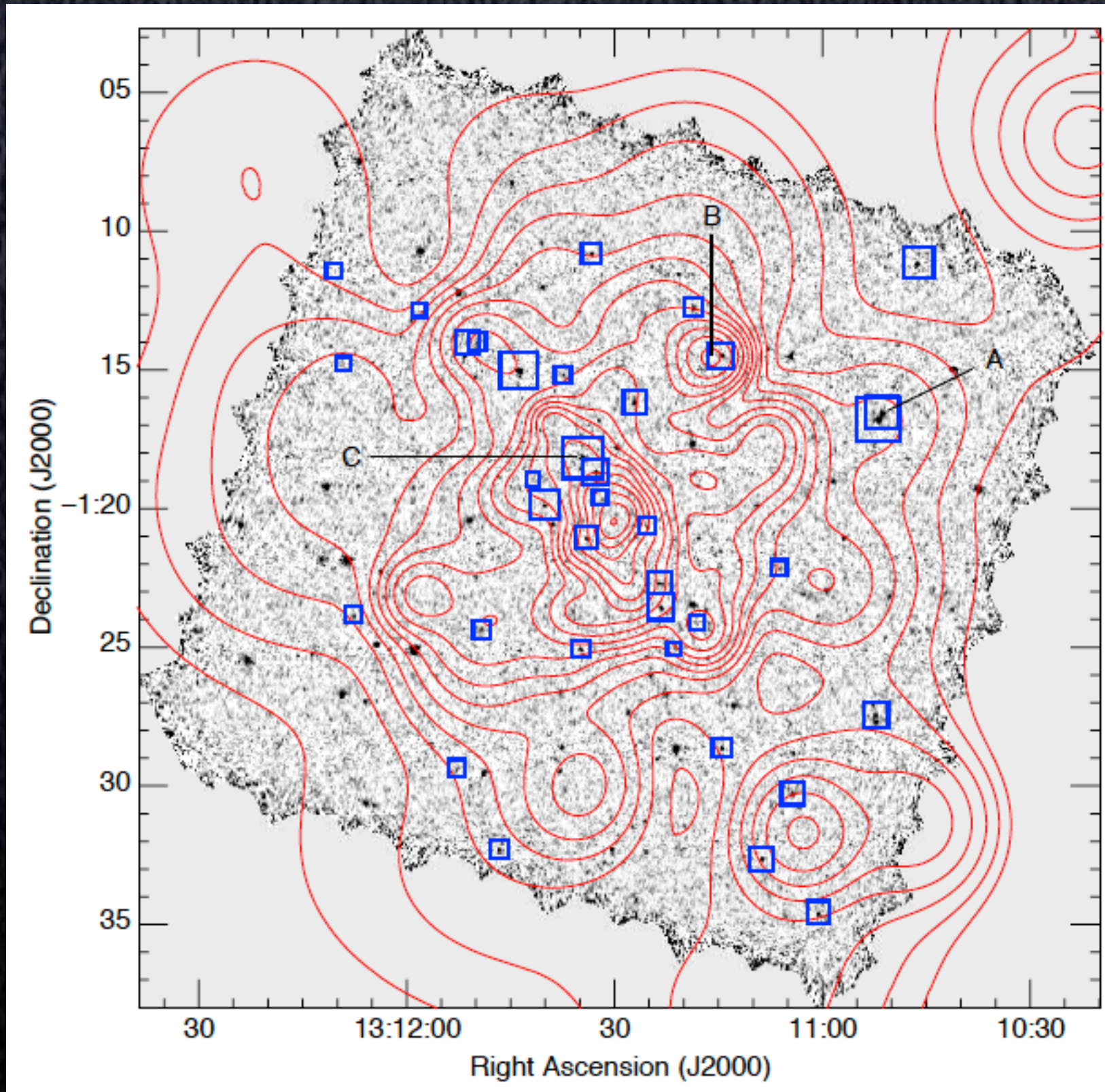


# X-ray vs S+W Lensing





# Recent Observations



**Herschel  
Observations reveal  
an excess of 100  
micron-selected  
galaxies extending  
~6 Mpc in length  
along an axis that  
runs NE-SW through  
the cluster centre**

Haines et al.  
arXiv:1005.3811



# Power Ratios

**Moments of the mass distribution characterize the morphology and substructure in dark matter distribution.**

$$a_m(r) = \int_{r' < r} \Sigma(\vec{r}') (r')^m \cos(m\phi') d^2\vec{r}',$$

$$b_m(r) = \int_{r' < r} \Sigma(\vec{r}') (r')^m \sin(m\phi') d^2\vec{r}'.$$

$$P_0 = [a_0 \ln(R)]^2,$$

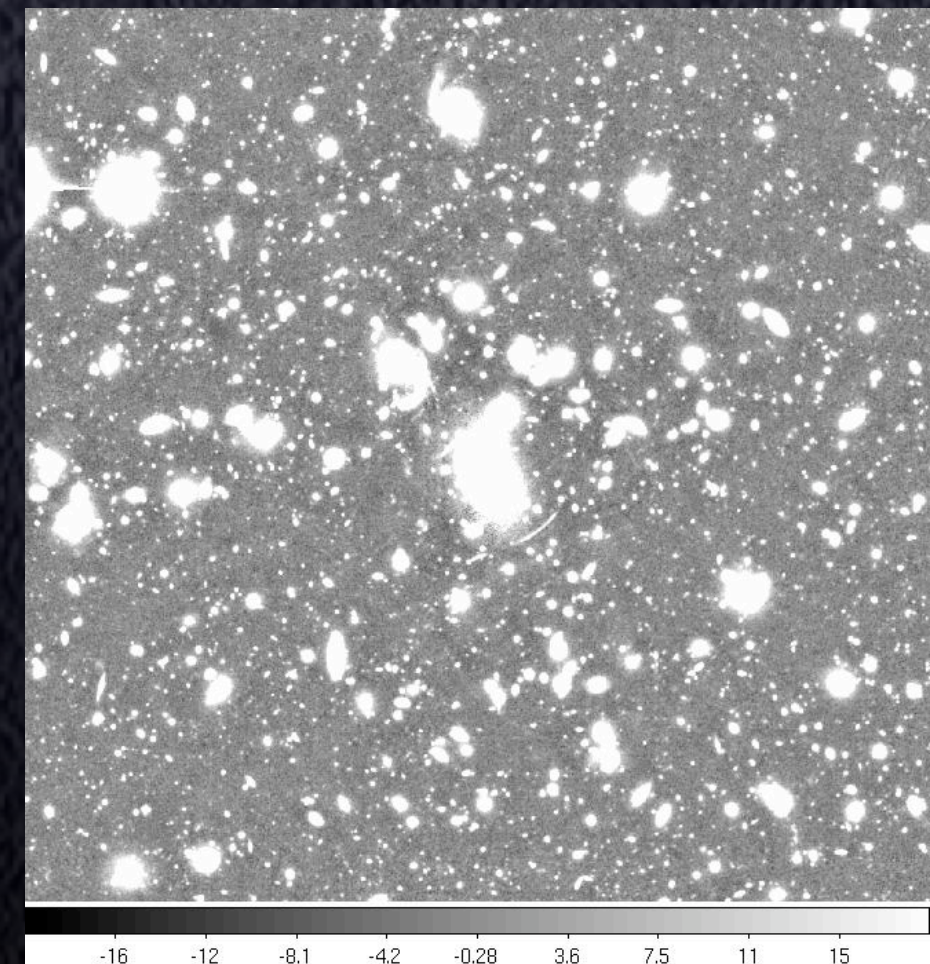
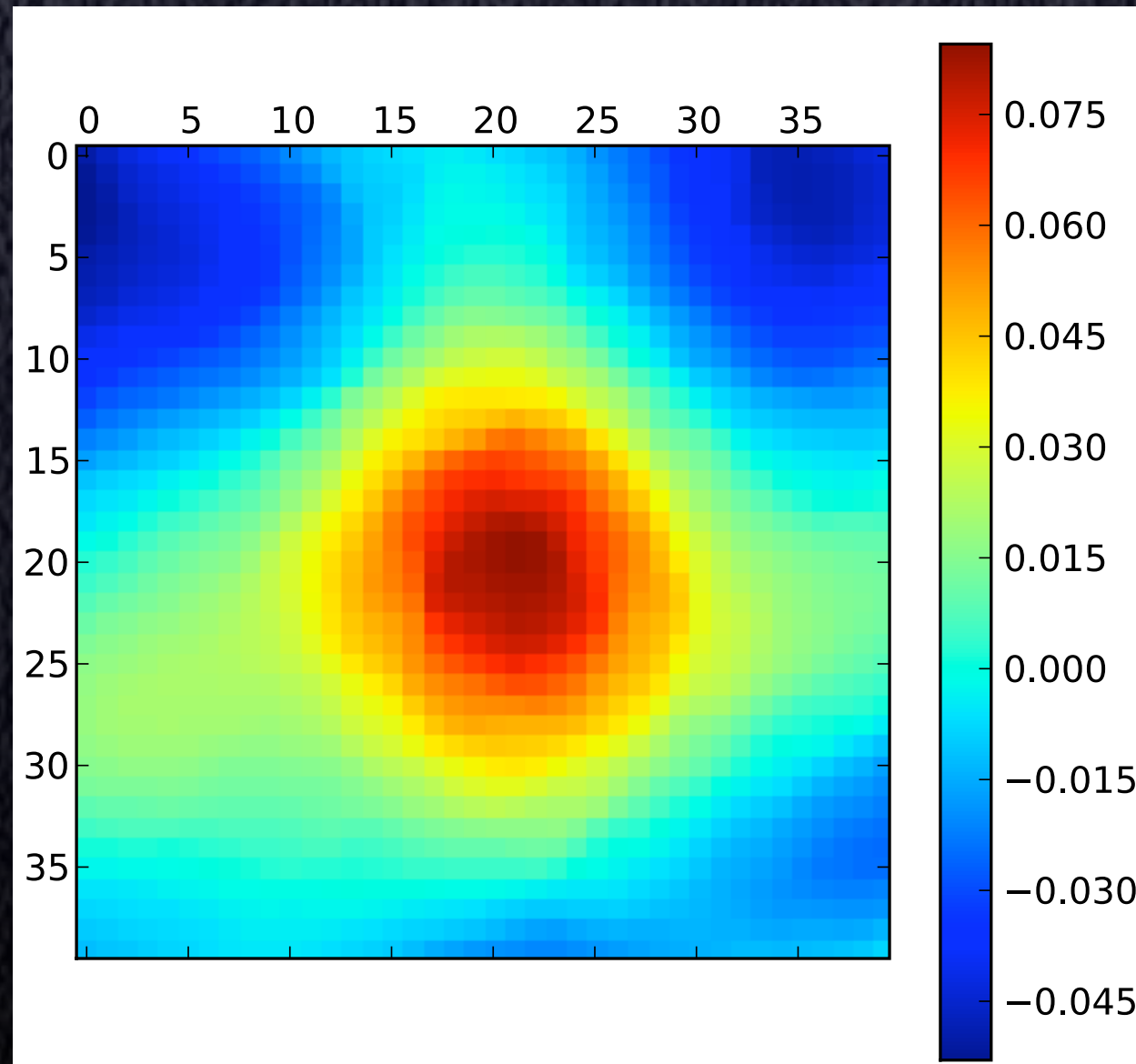
$$P_m = \frac{1}{2m^2 r^{2m}} (a_m^2 + b_m^2).$$

Power	X-ray	Lensing
$P_2/P_0$	$(6.68 \pm 0.27) \times 10^{-06}$	$(1.6 \pm 0.25) \times 10^{-5}$
$P_3/P_0$	$(3.71 \pm 1.12) \times 10^{-07}$	$(0.9 \pm 0.14) \times 10^{-5}$
$P_4/P_0$	$(6.42 \pm 2.65) \times 10^{-08}$	$(8.6 \pm 0.3) \times 10^{-5}$



# Current Research

## A2219: Optical vs Lensing mass reconstruction

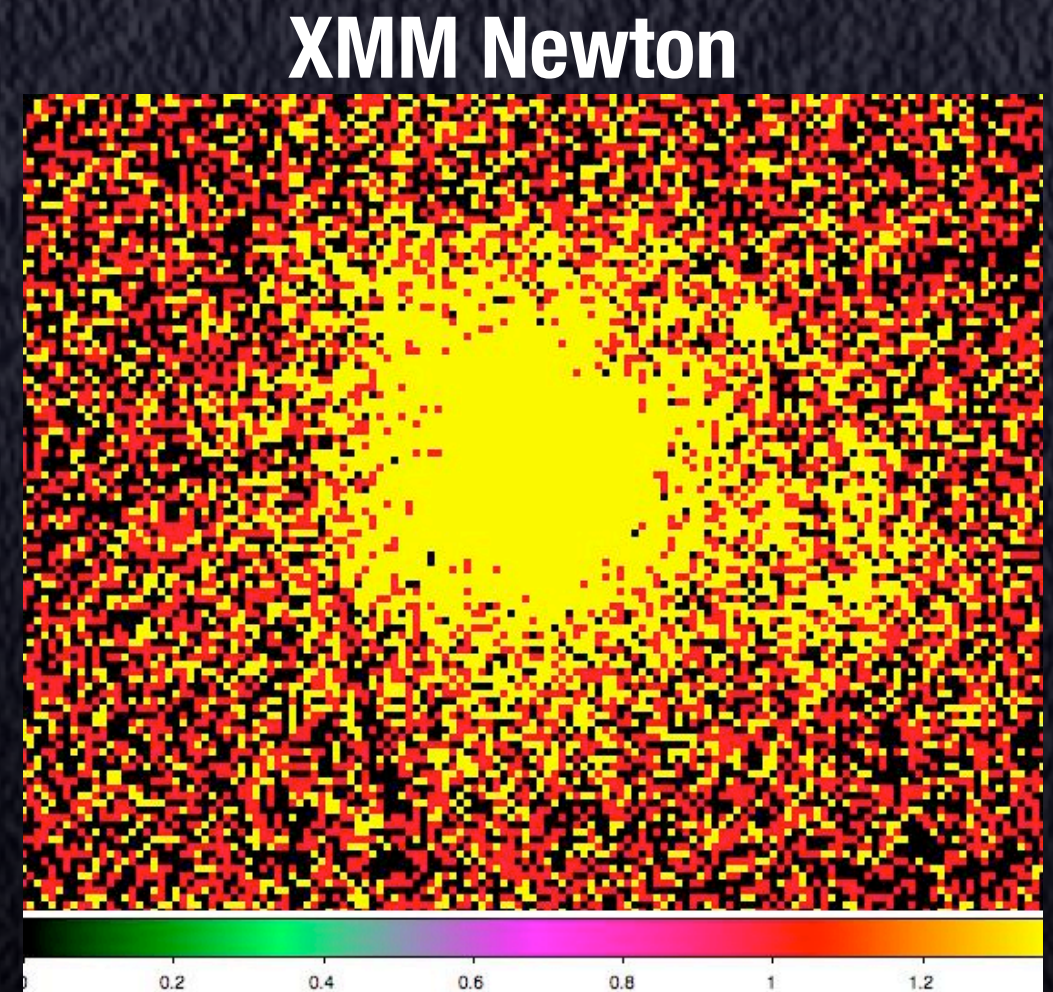
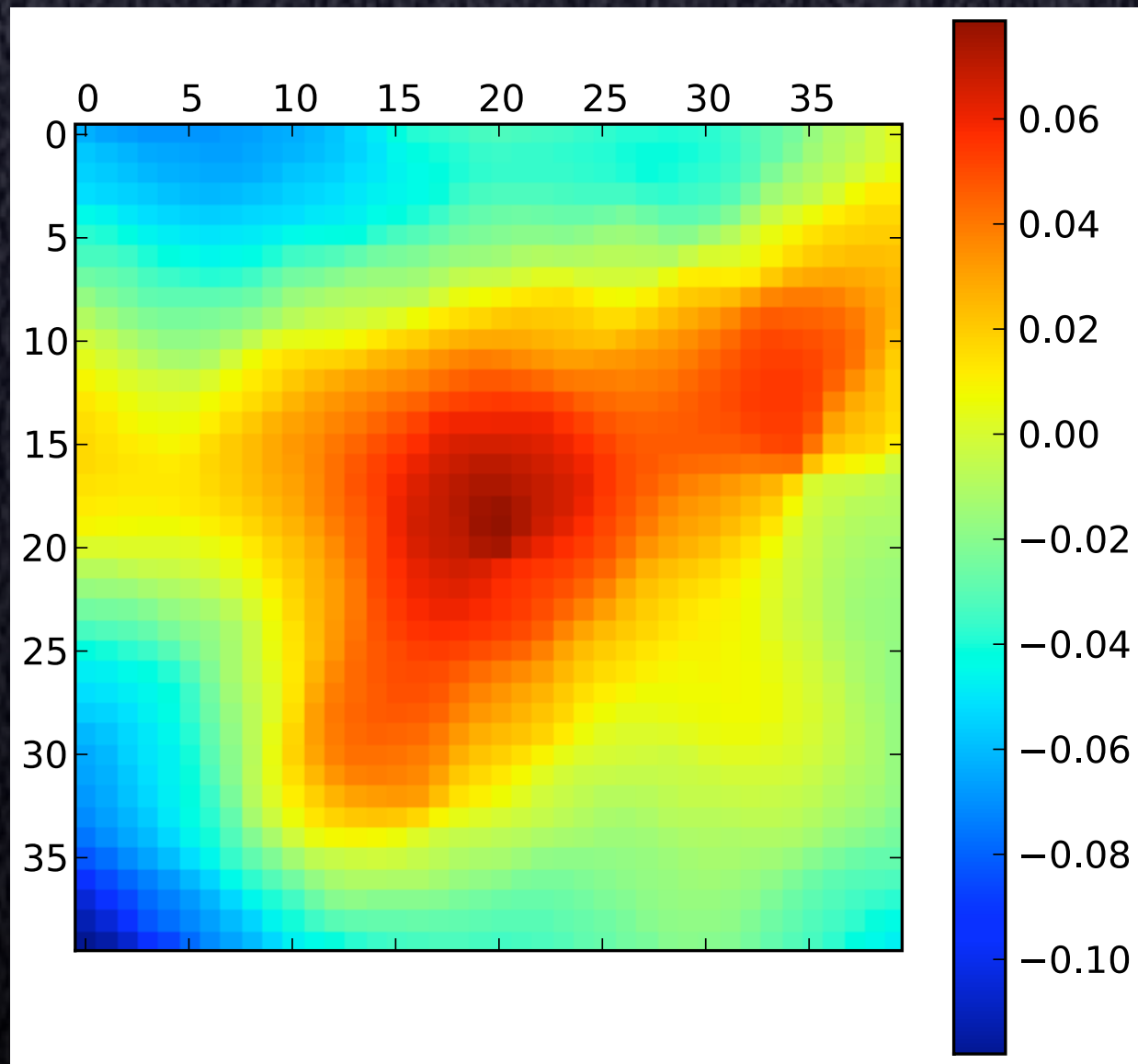


**PRELIMINARY**



# Current Research

## A2261: X-ray vs Lensing mass reconstruction

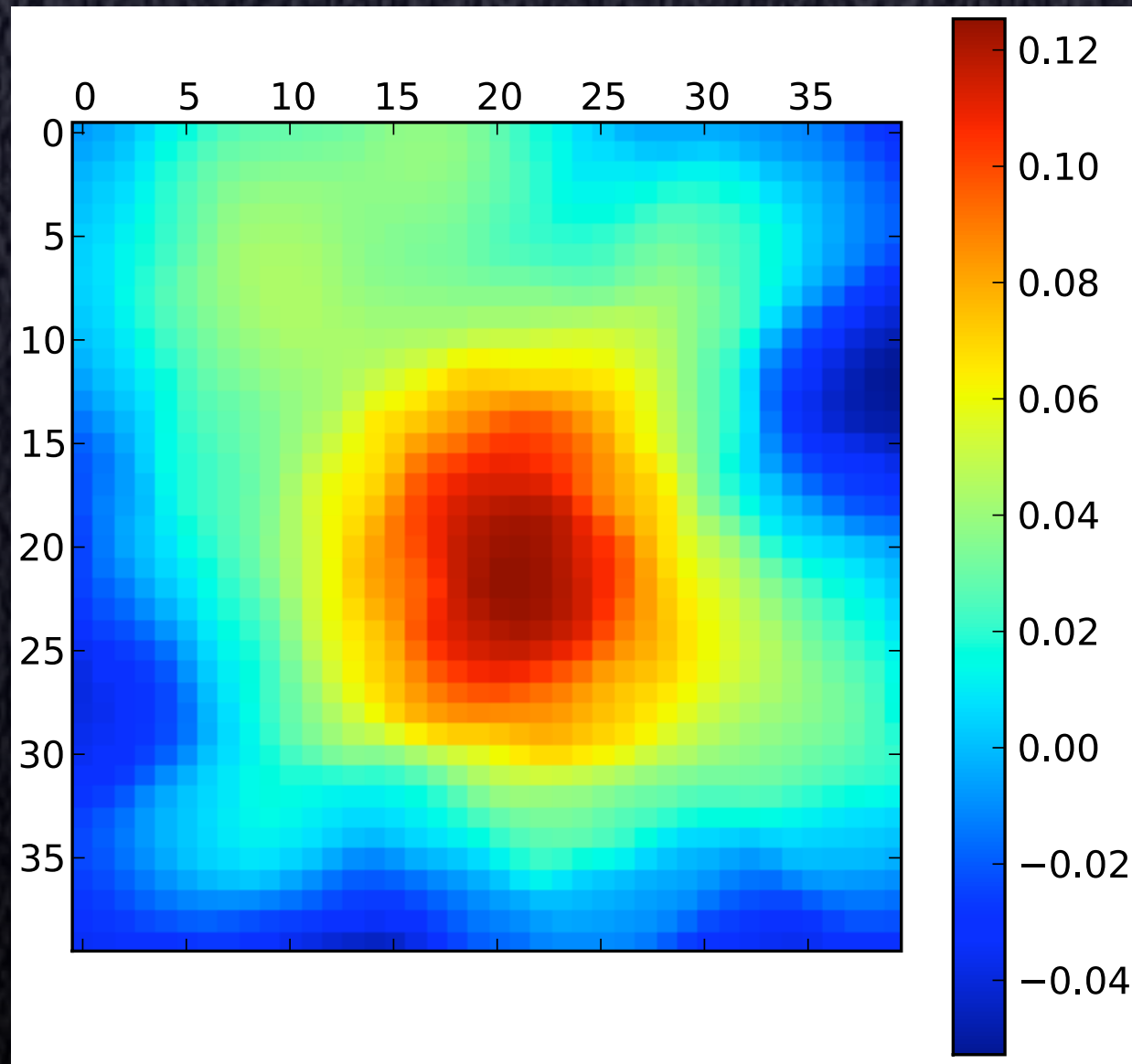


**PRELIMINARY**



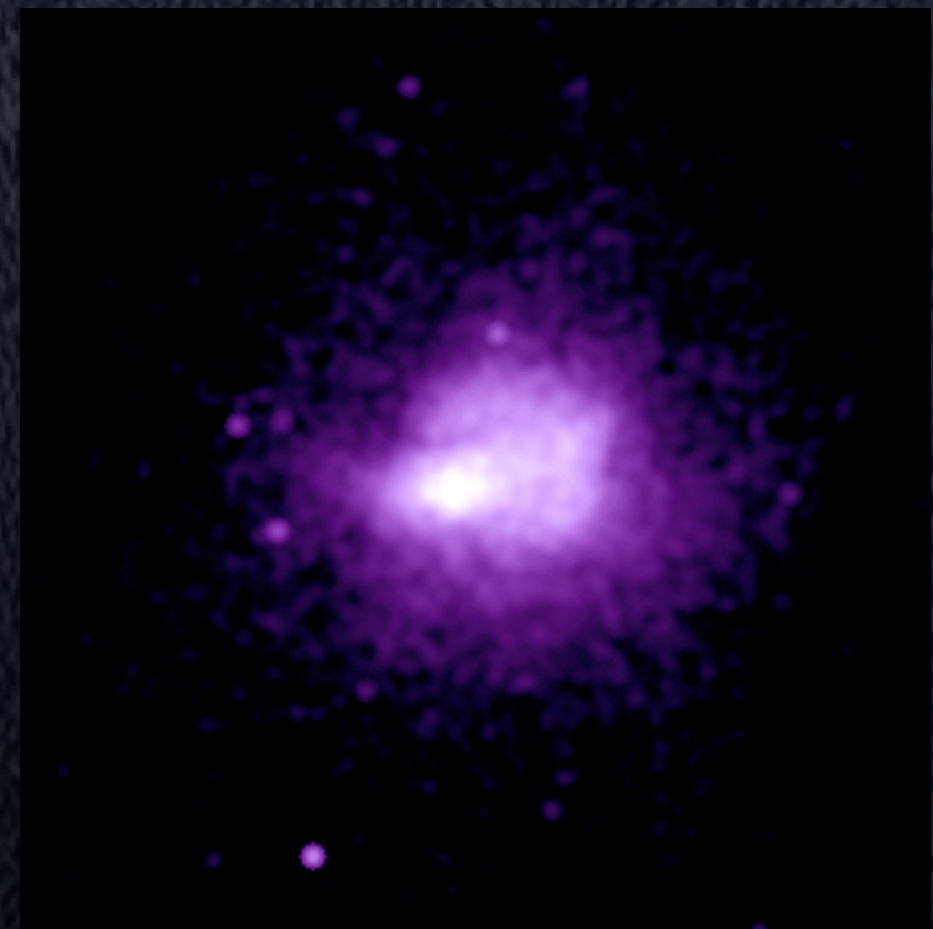
# Current Research

## A1914: X-ray vs Lensing mass reconstruction



**PRELIMINARY**

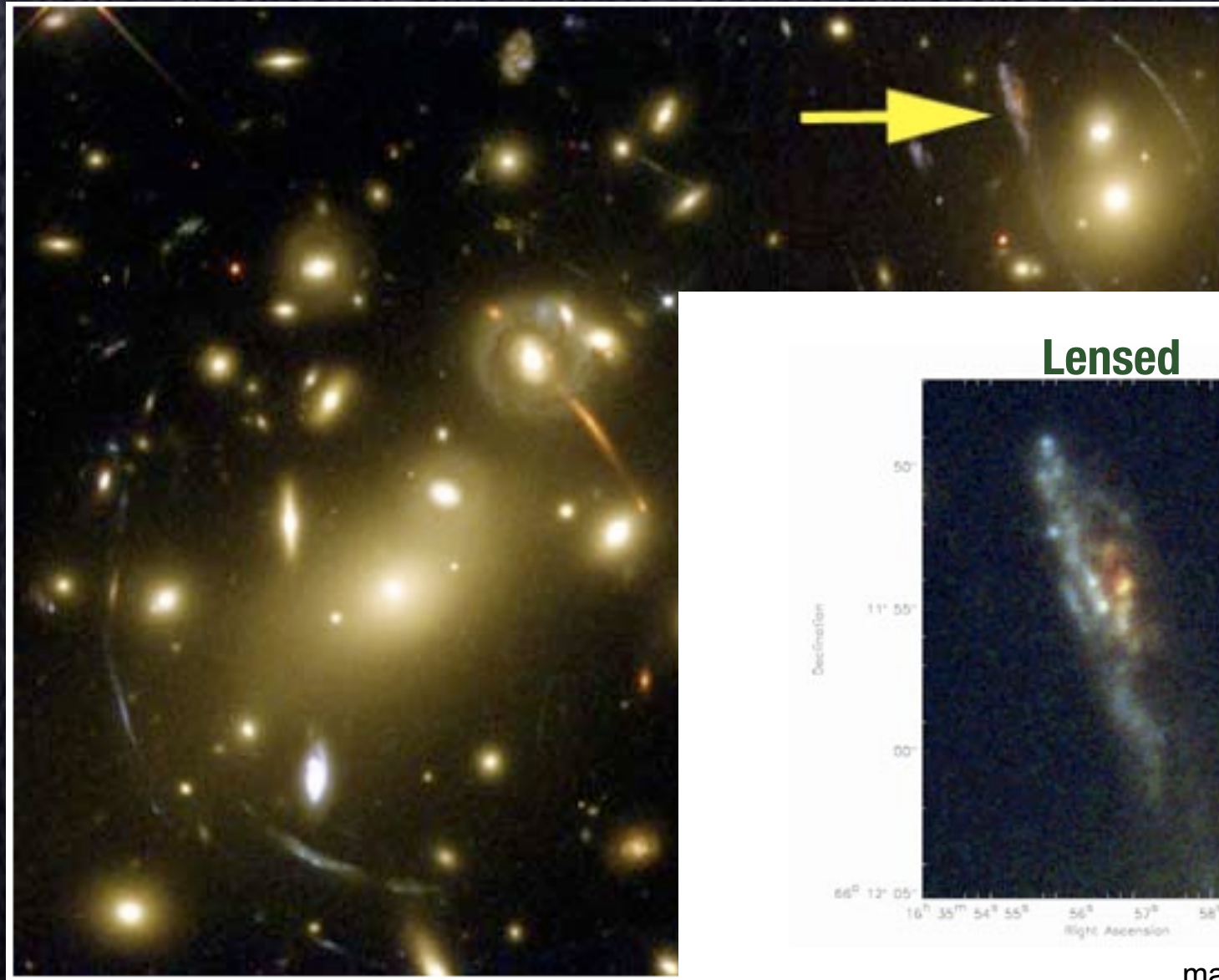
## Chandra data



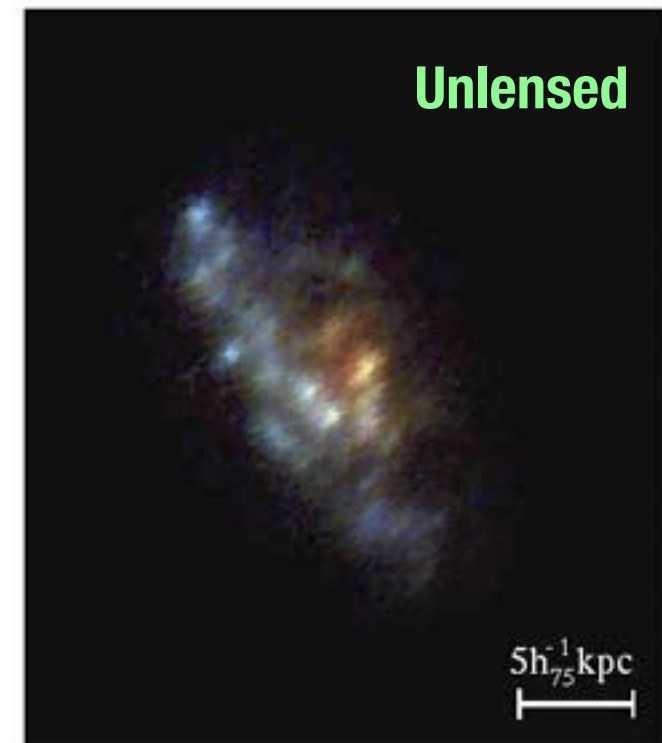
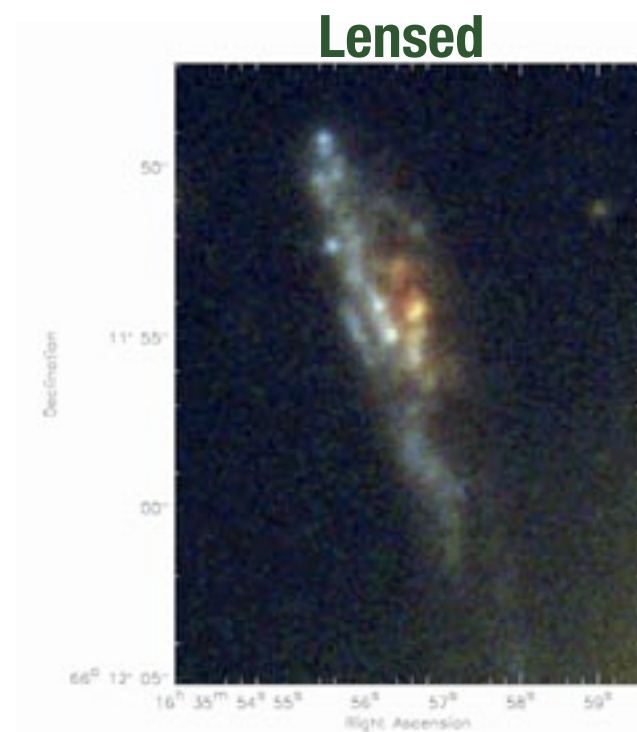
<http://chandra.harvard.edu/photo/2006/clusters/>



# Application: Studying high redshift galaxies



Swinbank et al. 2006

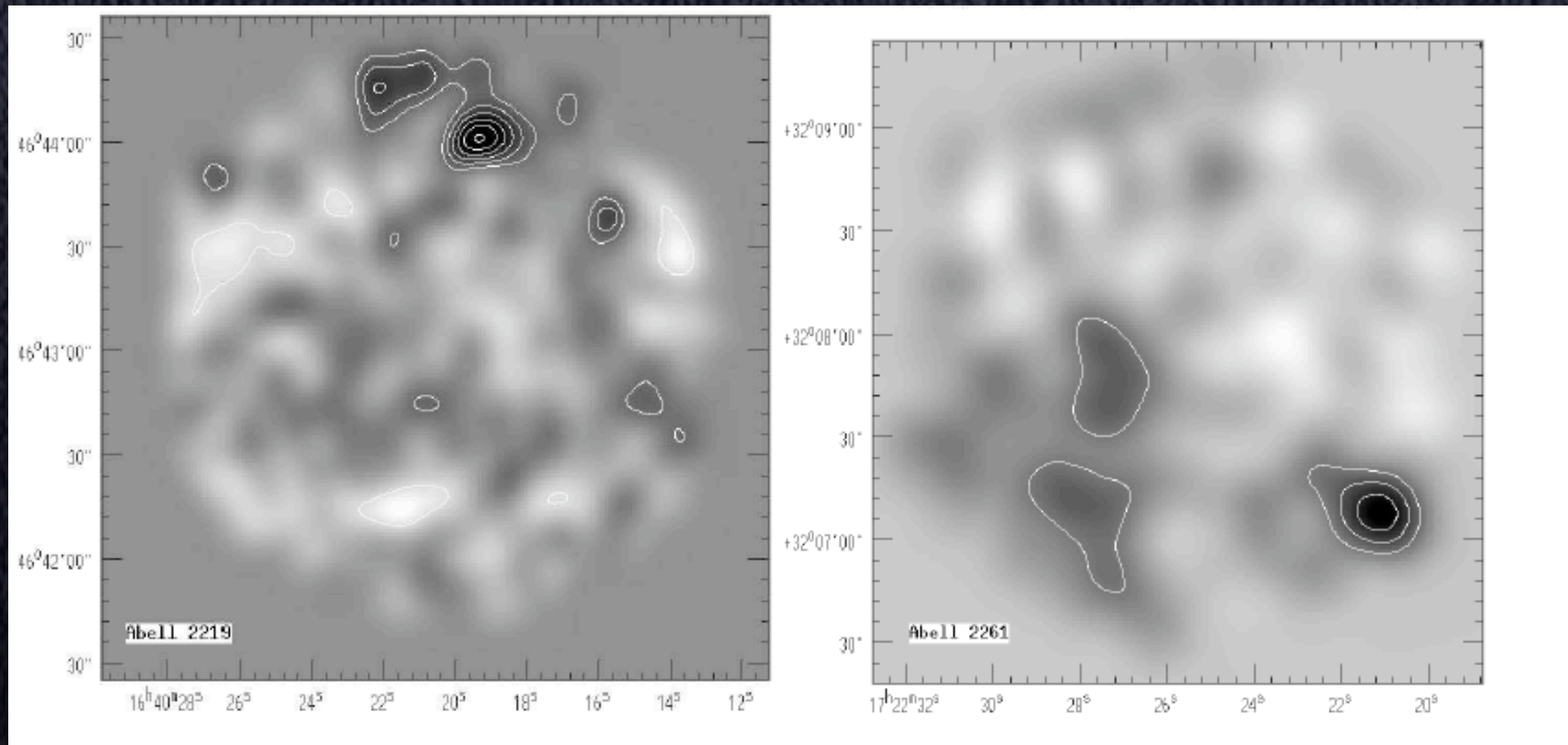


magnification:  $4.92 \pm 0.15$

$z=1.034$   $\xrightarrow{\text{Lensed}}$   $z=0.1$



# Sumilimeter Imaging



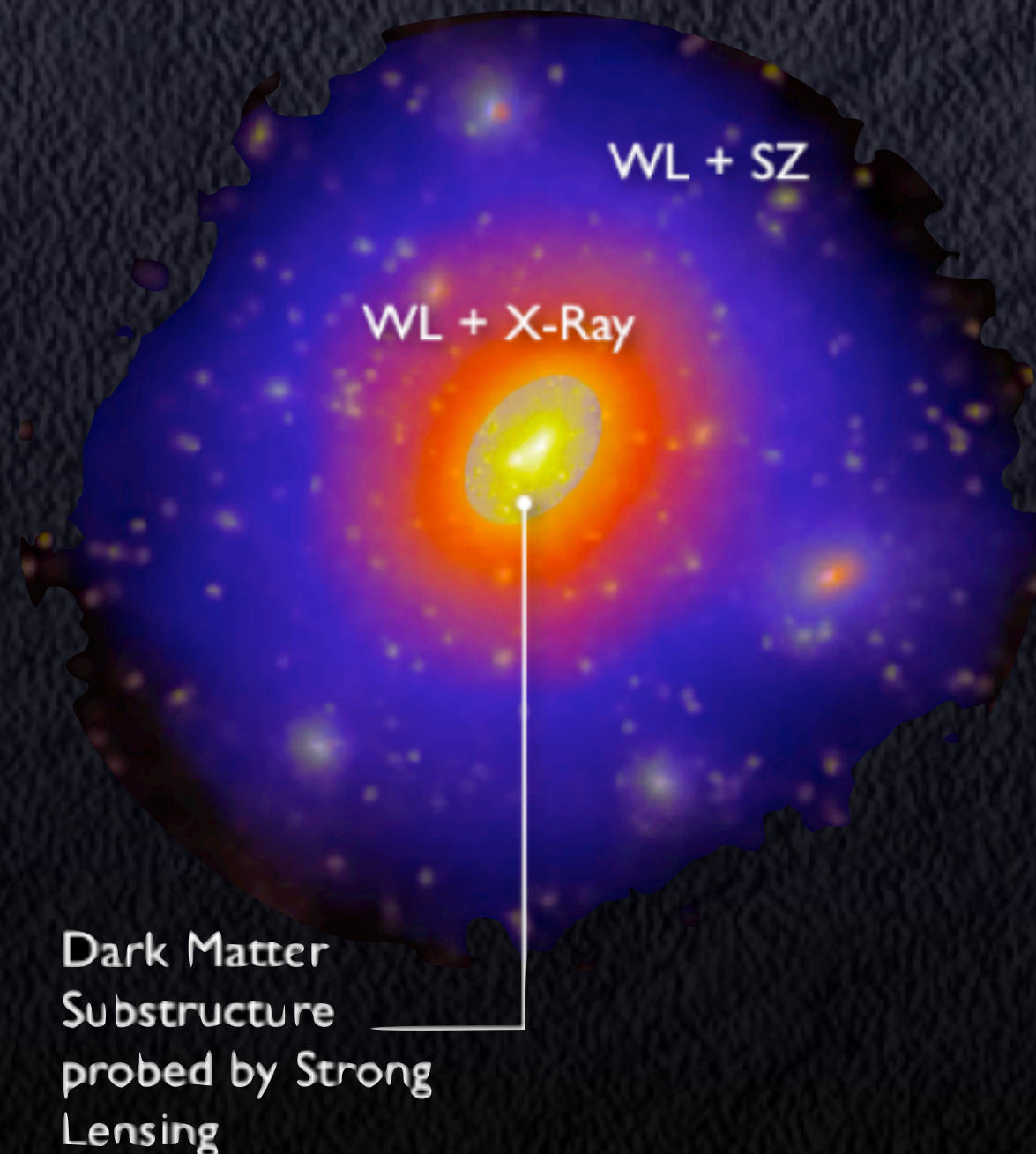
**Cluster field images at 850 microns**

Chapman et. al. 2000



# Multiwavelength analysis

**Multiwavelength shape analysis of galaxy clusters for the current sample of 20 clusters.**



**X-ray data: Chandra & XMM archival data**

**SZ data: Future plans of writing CARMA proposals with Dr. Morandi for some of these clusters.**

**Clusters are being discovered as we speak with SZ experiments like ACT/SPT. Lensing observations of these clusters will also increase this data set.**



# Summary

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## METHOD

Developed a non-parametric mass reconstruction technique “Particle Based Lensing” (PBL).

PBL is applied to compute mass maps of variable resolution and signal-to-noise.

## RESULTS

- \* The **ellipticity** for the light distribution is smaller than the ellipticity of the dark matter distribution for A901b and the Southwest Group.
- \* A901a, A901b and A902 have **strong alignment** whereas the Southwest group is not aligned with the rest of the peaks.
- \* The **gas** distribution of A1689 **is smoother** than the **dark matter** distribution.

## Future Research

Mutiwavelength analysis for a sample of 20 Supermassive Clusters.



# Smoothing Ellipticities: Error Covariance

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$$\epsilon_{sm,p}^i = Q_{pq} \epsilon_q^i \longrightarrow C_{pq}^{\text{MIN}} = Q_{kp} Q_{kq} \sigma_q^2$$



# Smoothing Ellipticities: Error Covariance

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$$\epsilon_{sm,p}^i = Q_{pq} \epsilon_q^i$$



$$C_{pq}^{\text{MIN}} = Q_{kp} Q_{kq} \sigma_q^2$$

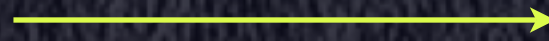


**singular**



# Smoothing Ellipticities: Error Covariance

$$\epsilon_{sm,p}^i = Q_{pq} \epsilon_q^i$$

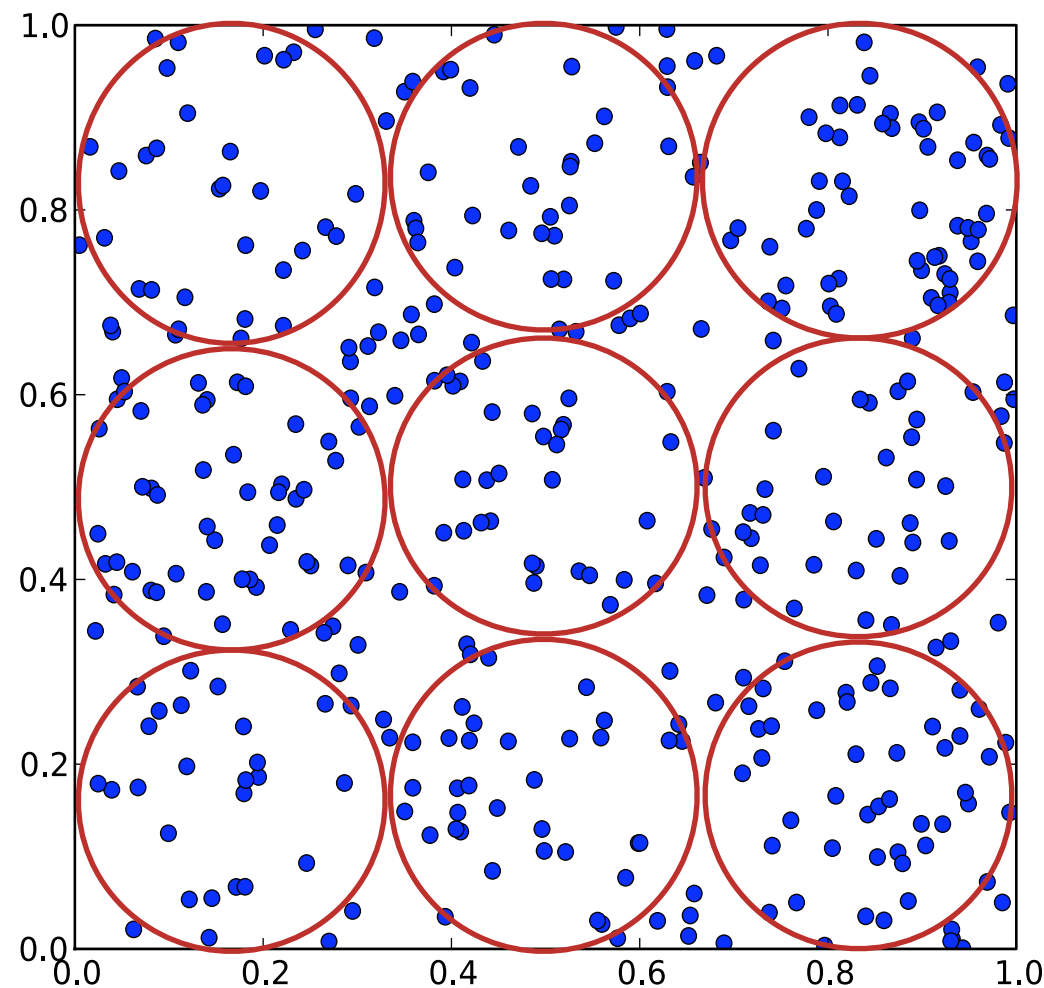


$$C_{pq}^{\text{MIN}} = Q_{kp} Q_{kq} \sigma_q^2$$



singular

**Number of particles  
(background images)  
larger than the number of  
resolution units.**





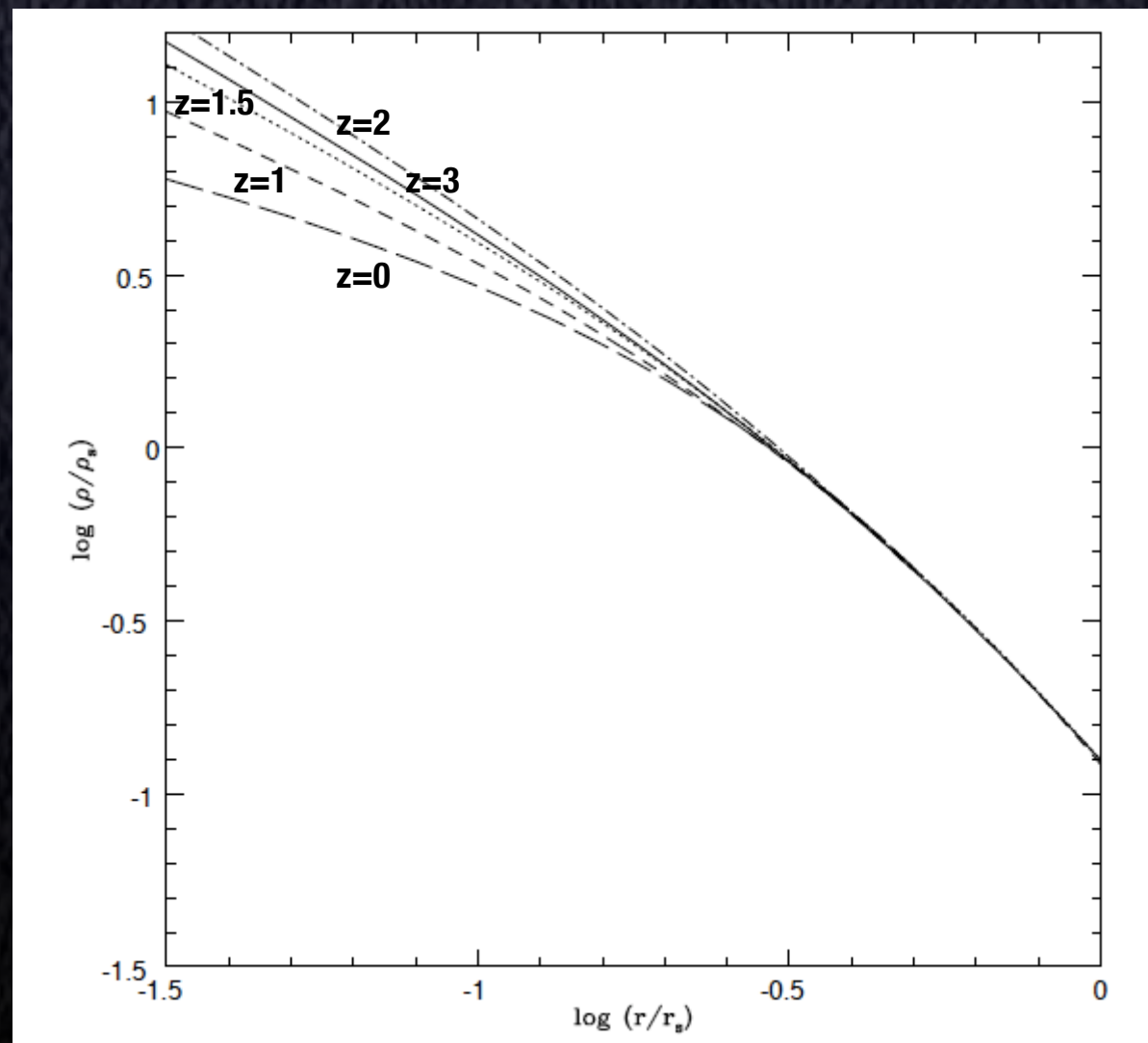
# Ellipticity Observations: Triaxial NFW

$$\rho(R) = \frac{\delta_c \rho_c(z)}{\frac{R}{R_s} (1 + R/R_s)^2}$$

$\rho_c(z)$  :critical density of the Universe

$\delta_c$  :characteristic overdensity of the halo

$R_s$  : scale radius



$$R^2 = \frac{X^2}{a^2} + \frac{Y^2}{b^2} + \frac{Z^2}{c^2}$$