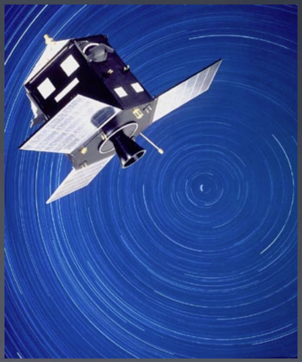
Detecting a Gravitational Wave Background with Astrometry

L. Book (Caltech) and É.É. Flanagan (Cornell),
PRD 83, 024024; arXiv:1009.419

Summary

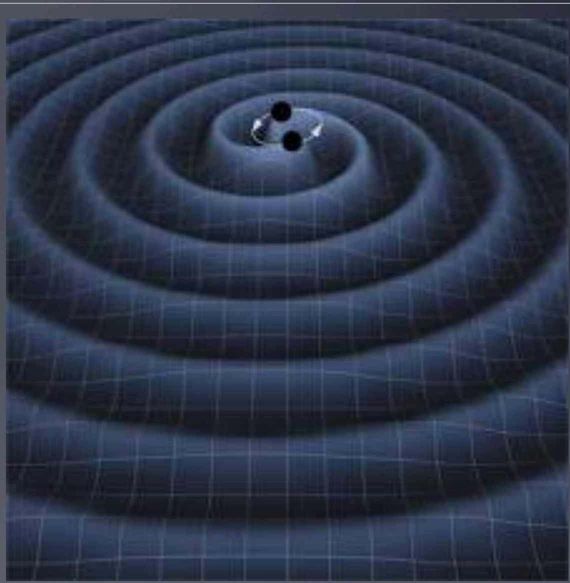
- Gravitational Wave Background (GWB) sources, properties, constraints
- Astrometry: Present and Near future
- History of the idea
- Deflection Results
- Signal to Noise



Gravitational Wave Background Sources

- Astrophysical Sources:

  compact object inspiral, merger or disruption and ringdown; stellar collapse; spinning neutron stars



- Early Universe Sources:

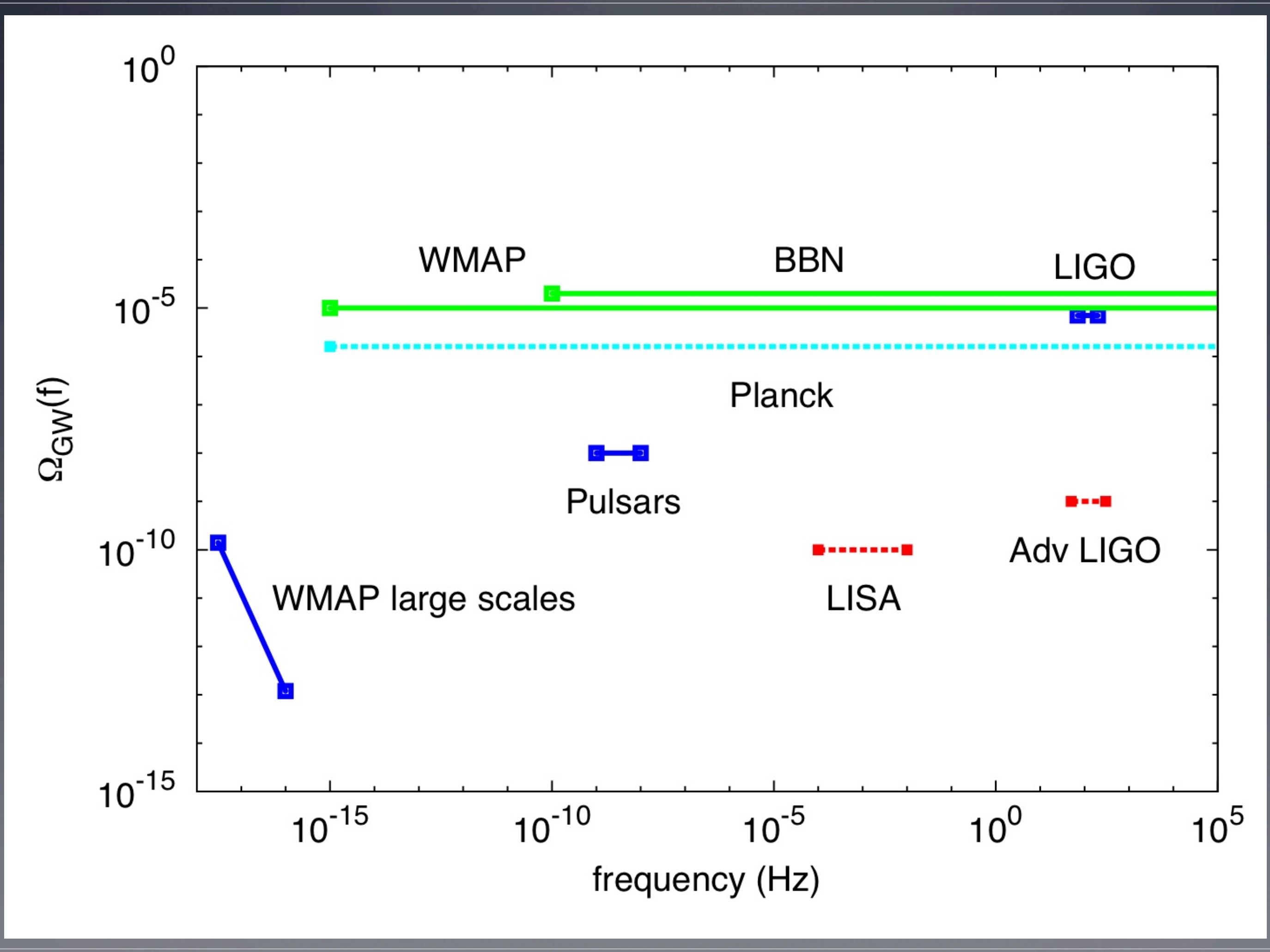
  ﬂuctuations during inﬂation, bubble collisions at a ﬁrst-order phase transition, cosmic string vibrations, etc.

Gravitational Wave Background Basics

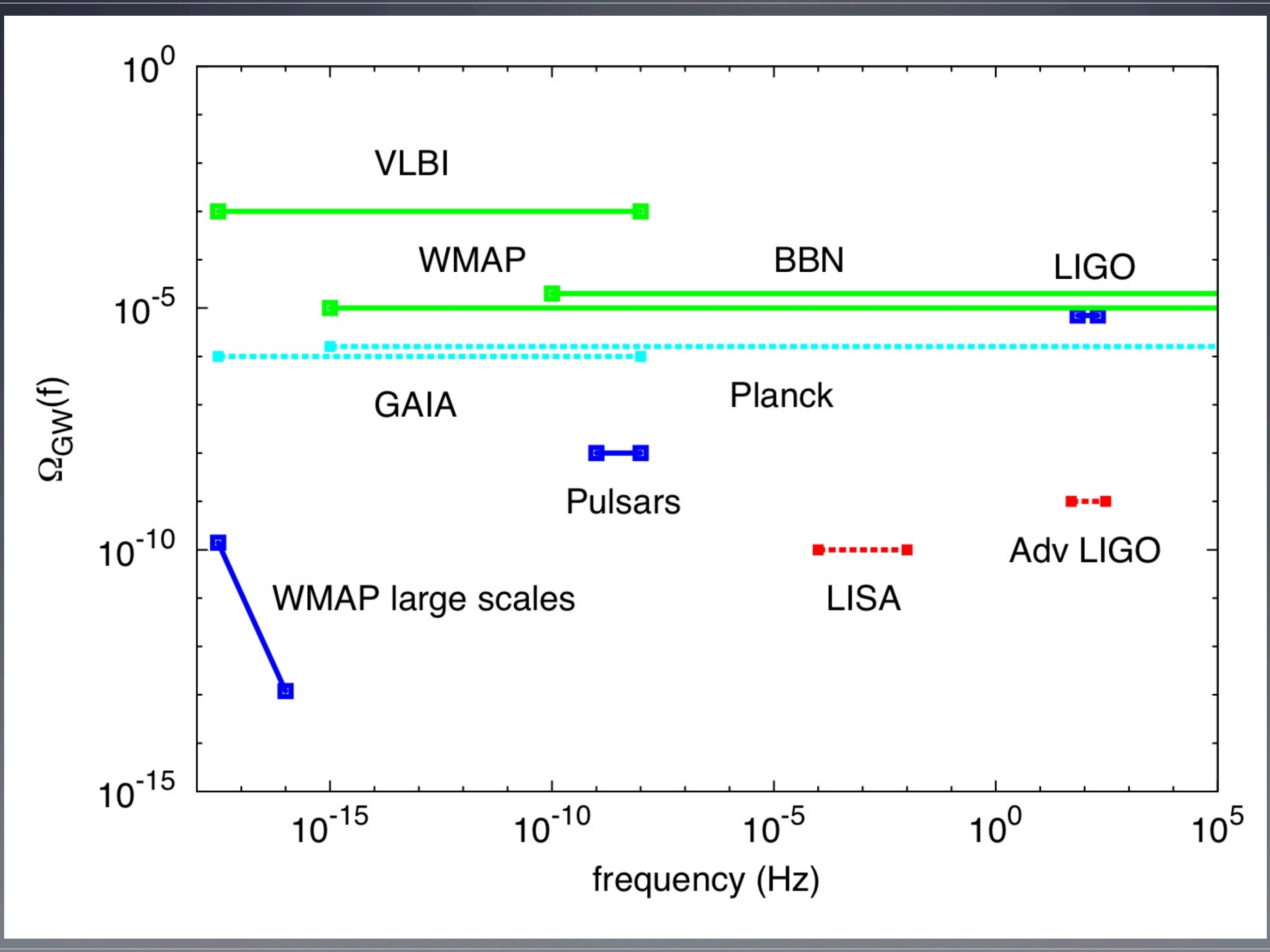
- Assume:
  - statistically homogeneous and isotropic
  - static
  - Gaussian
- Characterized by

$$\Omega_{\rm gw}(f) \equiv \frac{1}{\rho_{\rm c}} \frac{d\rho_{\rm gw}}{d \ln f} \sim f^2 H_0^2 h_{\rm rms}(f)^2$$

Gravitational Wave Background Constraints



Gravitational Wave Background Constraints



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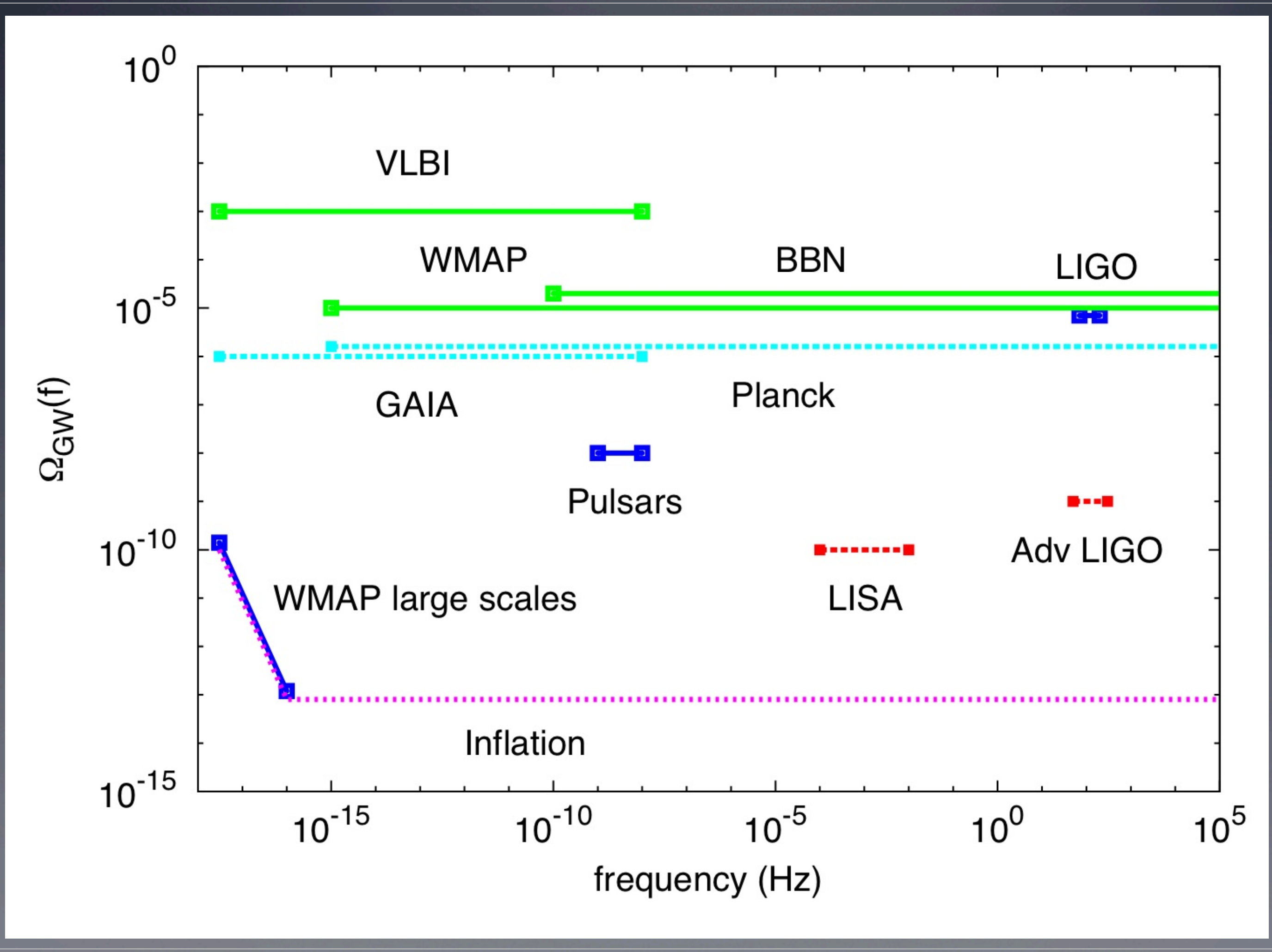

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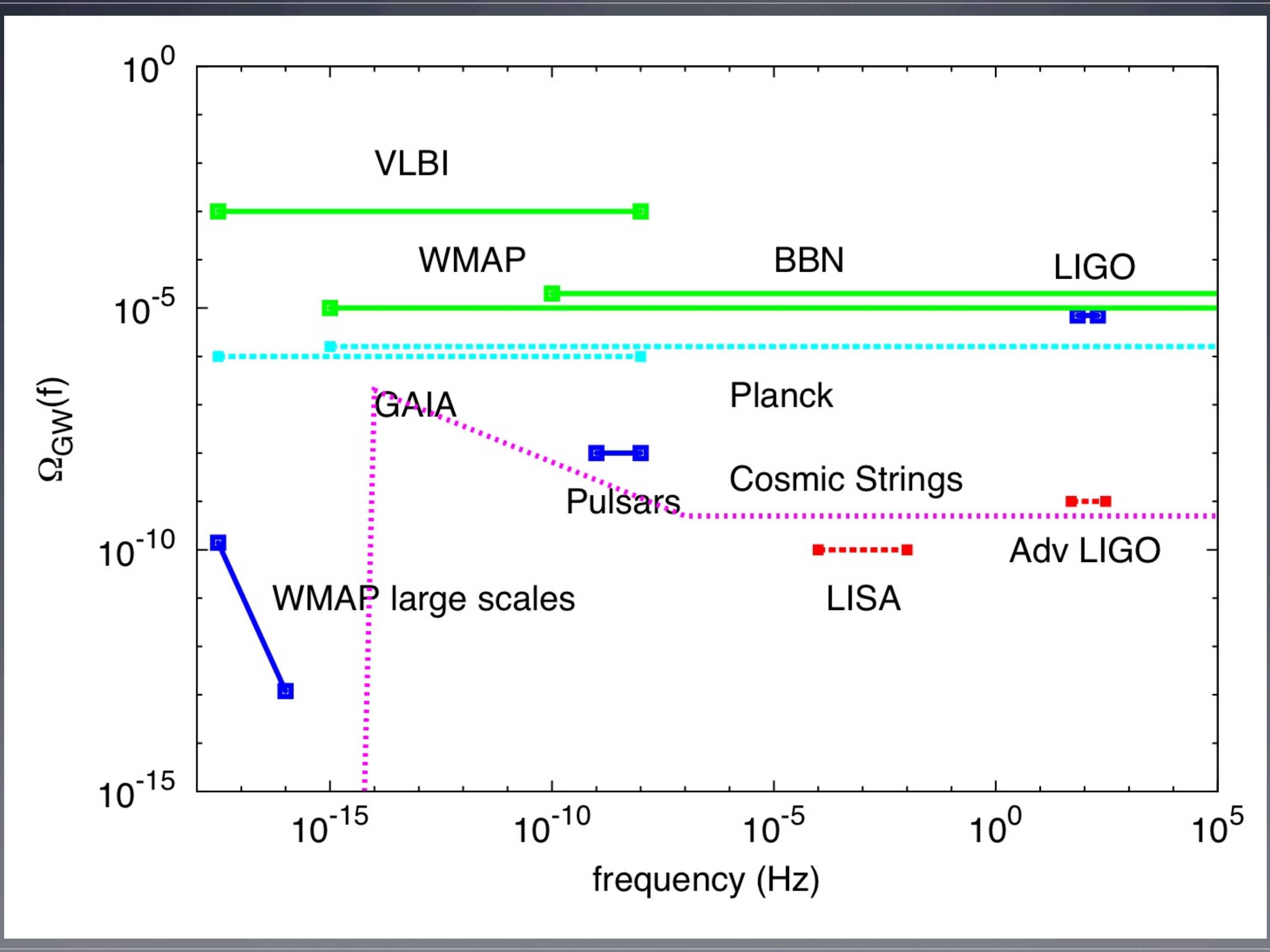

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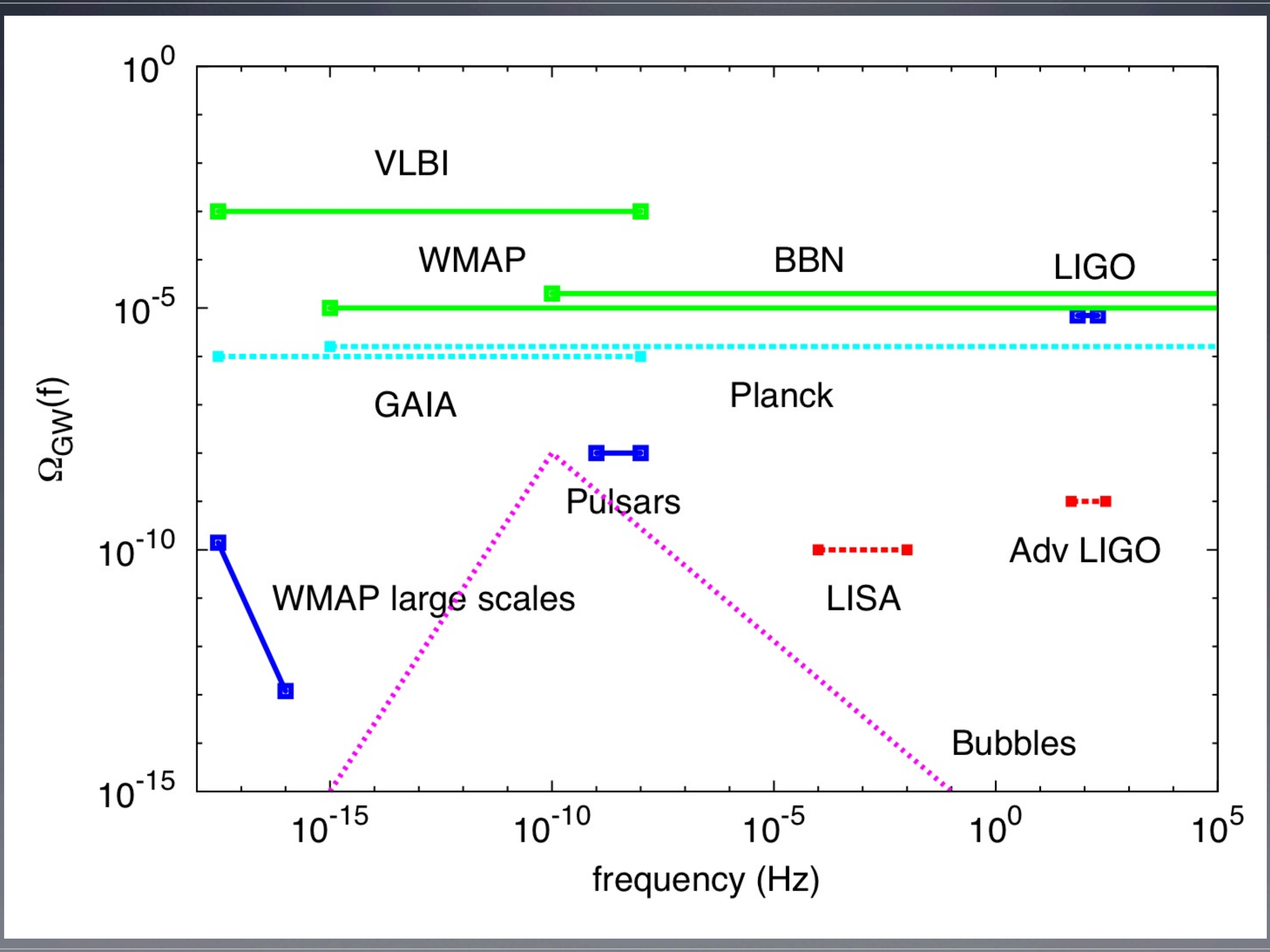

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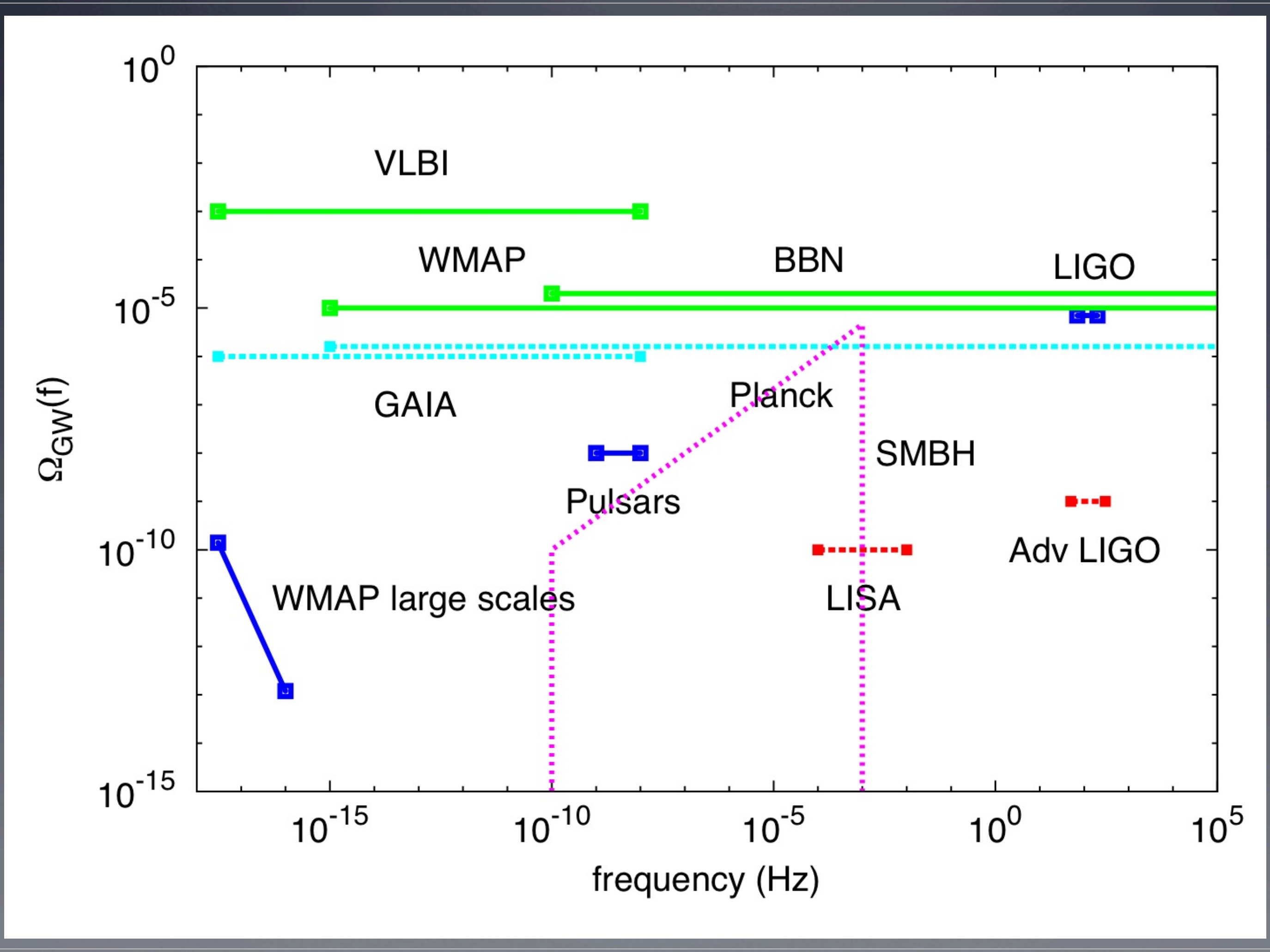


High Precision Astrometry: Current State of the Art

Radio Interferometry

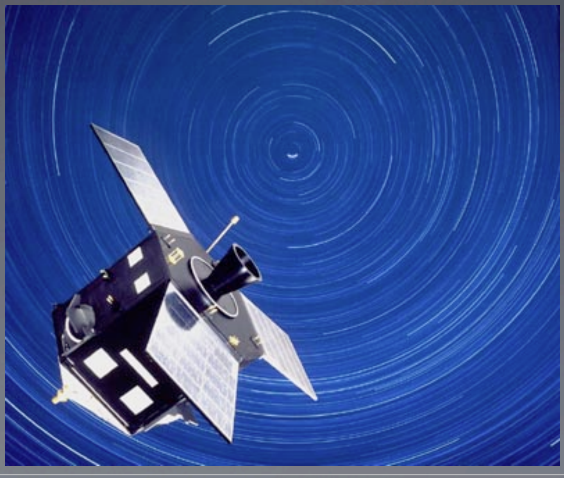
Current $N \sim 10^3$ quasars
$\delta\theta \sim 10\ \mu\mathrm{as}$

VLBA + EVN

Optical Satellites

HIPPARCOS (1989-93)

$N \sim 10^6$ stars
$N \sim 0$ quasars
$\delta\theta \sim$ few mas



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High Precision Astrometry: Near Future

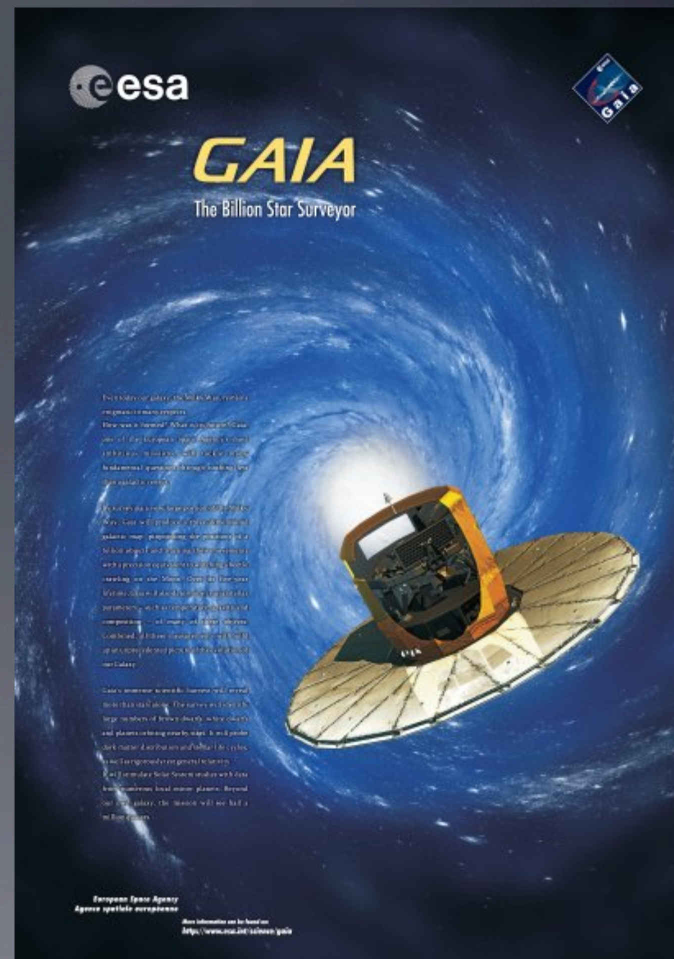
Optical Satellites

GAIA (2013—2018)

$$N \sim 10^9 \text{ stars}$$
$$N \sim 10^6 \text{ quasars}$$
$$\delta\theta \sim 10\ \mu\text{as}$$

Each source will be measured ~ 70 times giving proper motions

$$\delta\omega \sim 10\ \mu\text{as yr}^{-1}$$



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High Precision Astrometry: Near Future

Radio Interferometry
SKA (2016)

$$N \sim 10^7 \text{ stars}$$
$$N \sim 10^6 \text{ quasars}$$
$$\delta\theta \sim 10 \, \mu\text{as}$$

If sources are measured once a year, comparable angular velocities $\delta\omega \sim 10 \, \mu\text{as yr}^{-1}$

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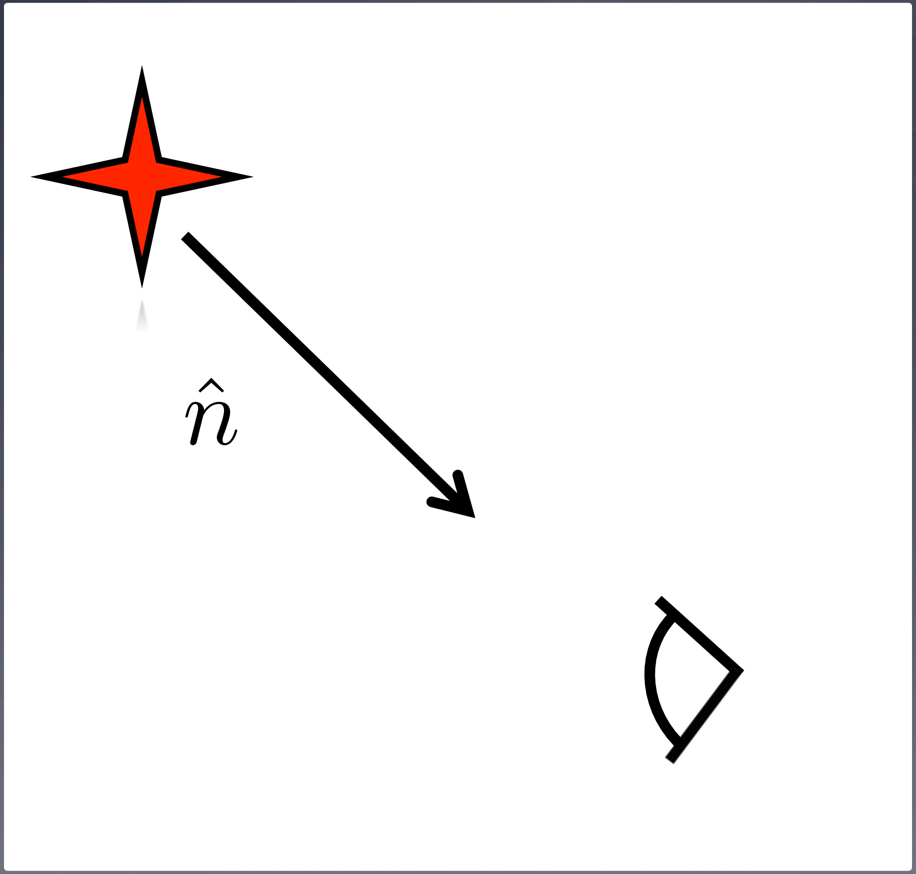
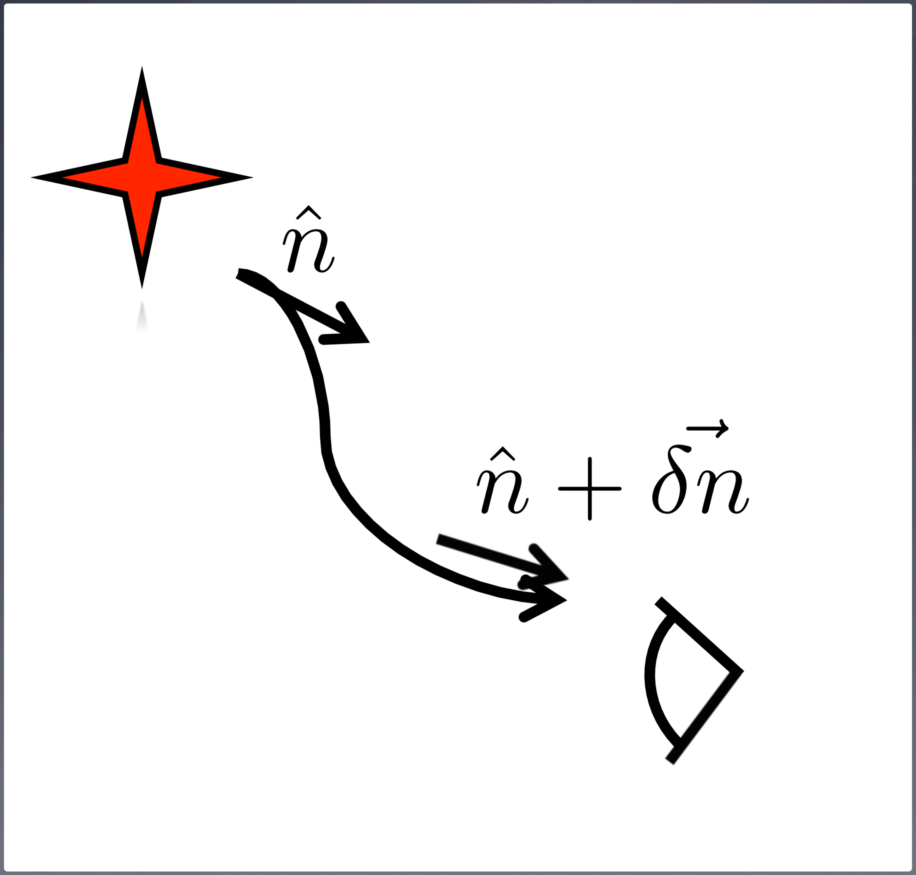
How to Detect GWs Using Astrometry



How to Detect GWs Using Astrometry



Astrometry and Gravitational Waves (History)

- GWs cause deflection $\delta\theta \sim h$
- Initial idea (Fakir) $\delta\theta \sim h \sim 1/b$
- Does not work (Damour, Kopekin)

$$\delta\theta \sim \int h \sim 1/b^3$$

- Second Idea: search for GWB (Braginsky)

GWB causes apparent angular velocities correlated on the sky

Distant sources have small proper motions

Search in data for expected statistical deflection pattern

Order of Magnitude Estimates

- Signal $\delta\theta_{\rm rms}(f) \sim h_{\rm rms}(f) \sim \frac{H_0}{f}\sqrt{\Omega_{\rm gw}(f)}$

$$\delta\omega_{\rm rms}(f) \sim f\,\delta\theta_{\rm rms}(f) \sim H_0\sqrt{\Omega_{\rm gw}(f)}$$

$$\sim 10^{-2}\mu{\rm as\ yr}^{-1}\sqrt{\Omega_{\rm gw}/10^{-6}}$$

- Monitor N sources with accuracy $\Delta\theta$ for a time T

$$\omega > \Delta\theta/(T\sqrt{N})$$

$$\Omega_{\rm gw}(f) \geq \frac{\Delta\theta^2}{NT^2H_0^2} \sim 10^{-6}\left(\frac{\delta\theta}{10\mu{\rm as}}\right)^2\left(\frac{N}{10^6}\right)^{-1}$$

- Limit applies to $\displaystyle\int_{\ln H_0}^{\ln(1/T)} d\ln f\,\Omega_{\rm gw}(f)$

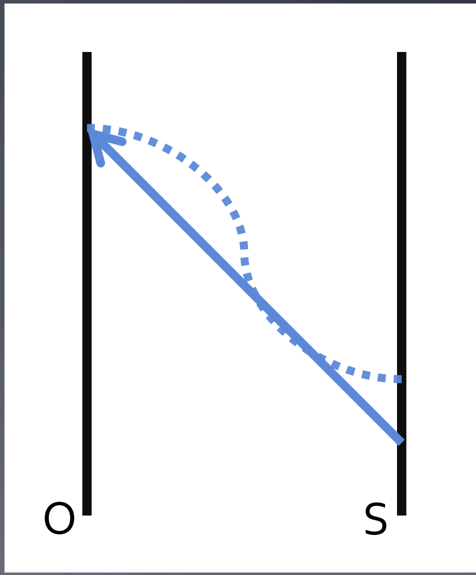
Angular Deflection Calculation

Metric: $$ds^2 = a(\eta)^2[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$$

Find the perturbed geodesic which

1. Intersects detection event
2. Is null
3. Intersects source worldline
4. Emitted with expected f

$$\vec{k}_O = \frac{\omega_O}{1+\delta z}\{\vec{u}_O + [n^i + \delta n^i(\mathbf{n}, t)]\vec{e}_{\hat{i}}\}$$



Result $$\vec{x}(\lambda) = (\eta_0 + \lambda, -n^i\,\lambda)$$

$$\delta n^i = \frac{1}{2}(\delta^{ik} - n^i n^k)\, n^j \left\{-h_{jk}(0) + \frac{1}{\lambda_s}\int_0^{\lambda_s} d\lambda \left[h_{jk}(\lambda) + \frac{1}{2}(\lambda_s - \lambda)\, n^l h_{jl,k}(\lambda)\right]\right\}$$

Deflection Result

- Assuming a plane wave travelling in direction $\hat{p}$, distant sources $D \gg \lambda$ and subhorizon modes $\lambda \ll L_H$

$$\delta n^i(\tau, \mathbf{n}) = \frac{n^i + p^i}{2(1 + \mathbf{p} \cdot \mathbf{n})} h_{jk}(0) n_j n_k - \frac{1}{2} h_{ij}(0) n_j$$

- Spatial Correlation Function for Stochastic Background

$$\langle \delta n_i(\mathbf{n}, t)\, \delta n_j(\mathbf{n}', t') \rangle = \int_0^\infty df\, e^{-2\pi\, \mathrm{i}\ f(t-t')} \frac{H_0^2\, \Omega_{\mathrm{gw}}(f)}{16\, \pi^2 f^3} H_{ij}(\mathbf{n}, \mathbf{n}')$$

where $H_{ij} = \alpha(\Theta)[A_i A_j - B_i C_j]$ $\qquad \cos(\Theta) = \mathbf{n} \cdot \mathbf{n}'$

$$\mathbf{A} = \mathbf{n} \times \mathbf{n}' \quad \mathbf{B} = \mathbf{n} \times \mathbf{A} \quad \mathbf{C} = -\mathbf{n}' \times \mathbf{A}$$

$$\alpha(\Theta) = \frac{7 \cos \Theta - 5}{2 \sin^2 \Theta} - \frac{48 \sin^6(\Theta/2)}{\sin^4 \Theta} \ln[\sin(\Theta/2)]$$

Angular deflection power spectrum

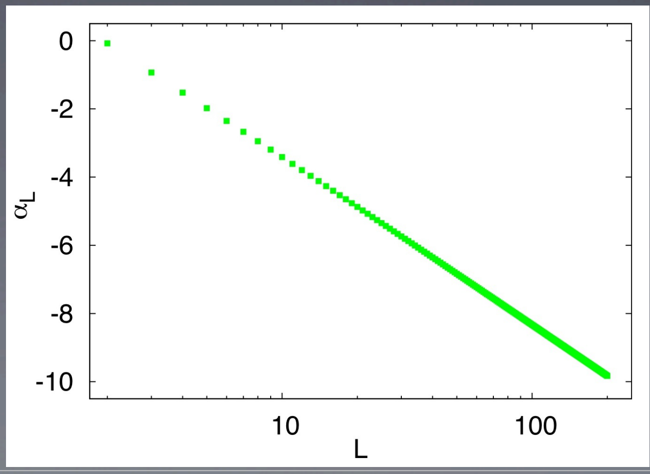
Expand in vector spherical harmonics (E and B)

$$\langle \delta n_{Qlm}(t)\, \delta n_{Q'l'm'}(t')^* \rangle = \delta_{QQ'} \delta_{ll'} \delta_{mm'} \int_0^\infty df \cos[2\pi f (t-t')] S_{Ql}(f)$$

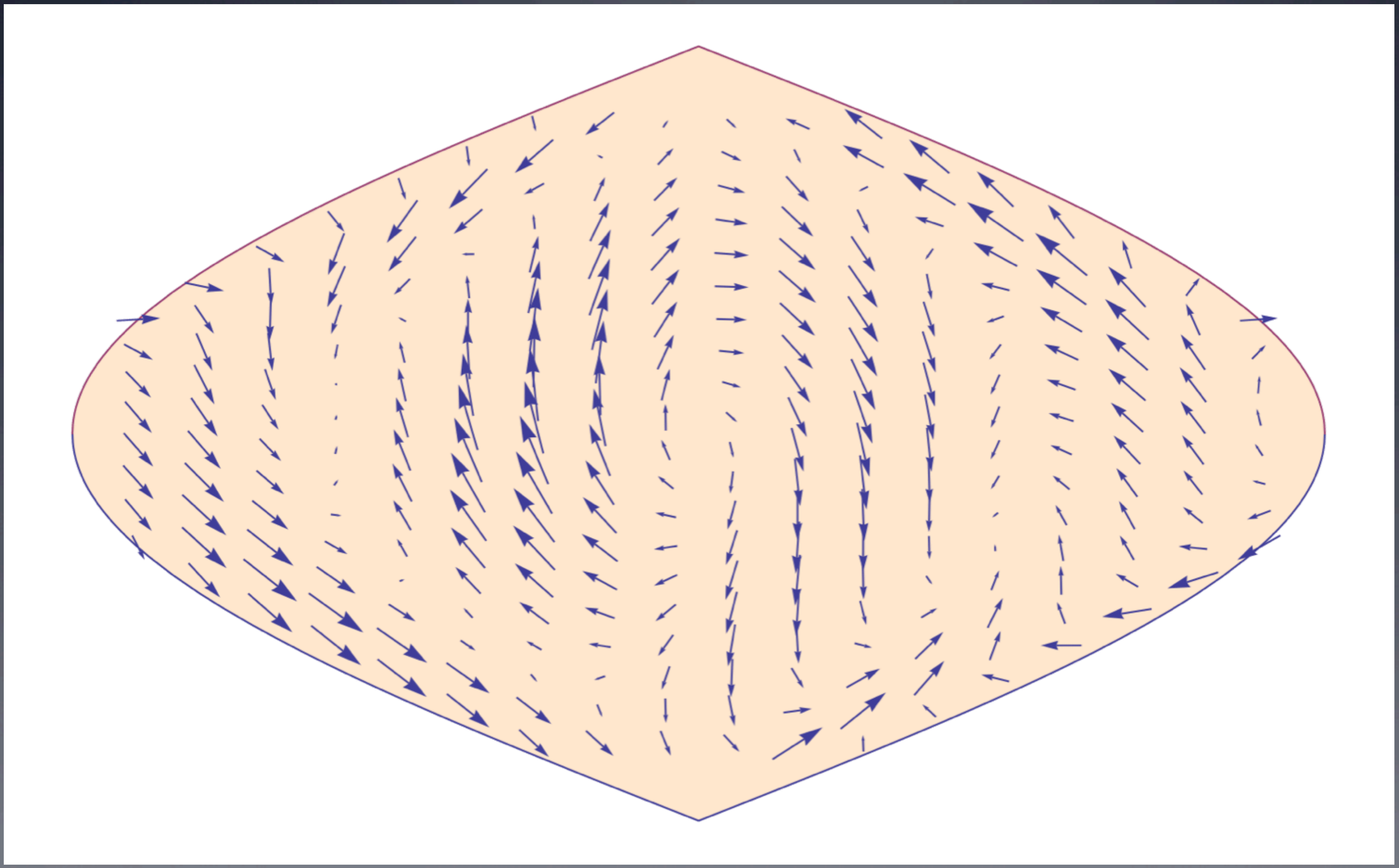
Specialized to GWB that is Gaussian, stationary, isotropic and unpolarized

$$S_{Ql}(f) = \frac{H_0^2\, \Omega_{\rm gw}(f)}{2\pi f^3} \frac{\alpha_l}{2l+1}$$

$$\alpha_l = \left(\frac{5}{6}, \frac{7}{60}, \frac{3}{100}, \frac{11}{1050}, \ldots \right)$$



Typical Deflection Pattern

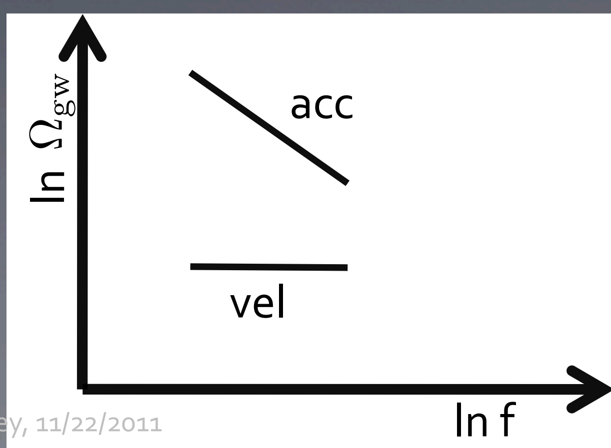


Prediction for Sensitivity of Astrometry

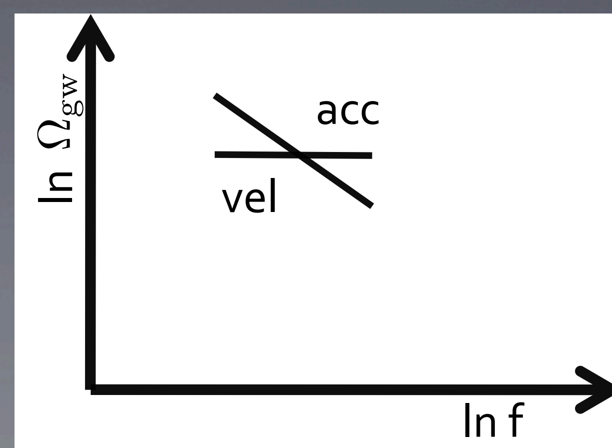
$$\frac{S^2}{\mathcal{N}^2} = \sum_{l\geq 2} \frac{N^2 H_0^4 \alpha_l^2}{(2l+1)} \left\{ \frac{[\int d\ln f \; \Omega_{\rm gw}(f)]^2}{\sigma_\omega^4} + \frac{[\int d\ln f \; (2\pi f)^2 \Omega_{\rm gw}(f)]^2}{\sigma_\alpha^4} + \dots \right\}$$

Parameters:	**Angular Velocity**	**Angular Acceleration**
Measurement Error:	$\sigma_\omega \sim 10\ \mu\text{as yr}^{-1}$	$\sigma_\alpha \sim 10\ \mu\text{as yr}^{-2}$
Stellar Motion:	$\sigma_\omega \sim 10^4\ \mu\text{as yr}^{-1}$	$\sigma_\alpha \sim 10^{-3}\ \mu\text{as yr}^{-2}$
Quasar Motion:	$\sigma_\omega \sim 10\ \mu\text{as yr}^{-1}$	$\sigma_\alpha \sim$ negligible

Quasars today
Measurement error dominates
Velocity data gives best constraint



Stars today, quasars in the future?
Proper motion dominates
Acceleration data also useful



Conclusions

- Astrometry will provide an interesting constraint on the GWB, at the level of $\Omega_{\mathrm{gw}} \sim 10^{-6}$, using future optical (GAIA) and radio (SKA) observations
- The limit is scale invariant at frequencies low compared to the observation time
- For GAIA, using the acceleration data from the $\sim 10^{9}$ stars may be competitive with the velocity data from the $\sim 10^{6}$ quasars at certain frequencies

Future Work

Characterize Errors and their effect on the GWB constraint:

- Foregrounds from SMBH binaries
- Foreground from scalar modes
- Errors from local universe modeling
- Instrumental errors

Astrometric Effects of a Stochastic Gravitational Wave Background

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L.G. Book and É.É. Flanagan (Cornell), 2011, in prep

Prediction for Sensitivity of Astrometry

Generalizing analysis to include time dependence of fluctuations gives

$$\frac{S^2}{\mathcal{N}^2} = \sum_{l\geq 2} \frac{N^2 H_0^4 \alpha_l^2}{(2l+1)\,\sigma_\omega^4} \left\{ \left[\int^{\ln(1/T)} d\ln f\, \Omega_{\rm gw}(f) \right]^2 + \frac{9}{2\,\pi^4} \int_{\ln(1/T)} d\ln f\, \frac{\Omega_{\rm gw}(f)^2}{f^5\, T^5} \right\}$$