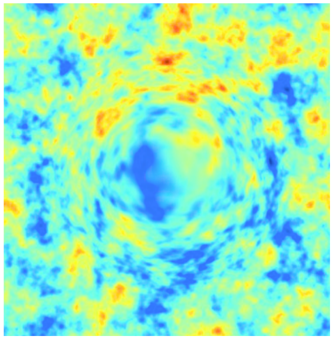
A bias to CMB lensing measurements

from the bispectrum of large-scale structure



Vanessa Böhm

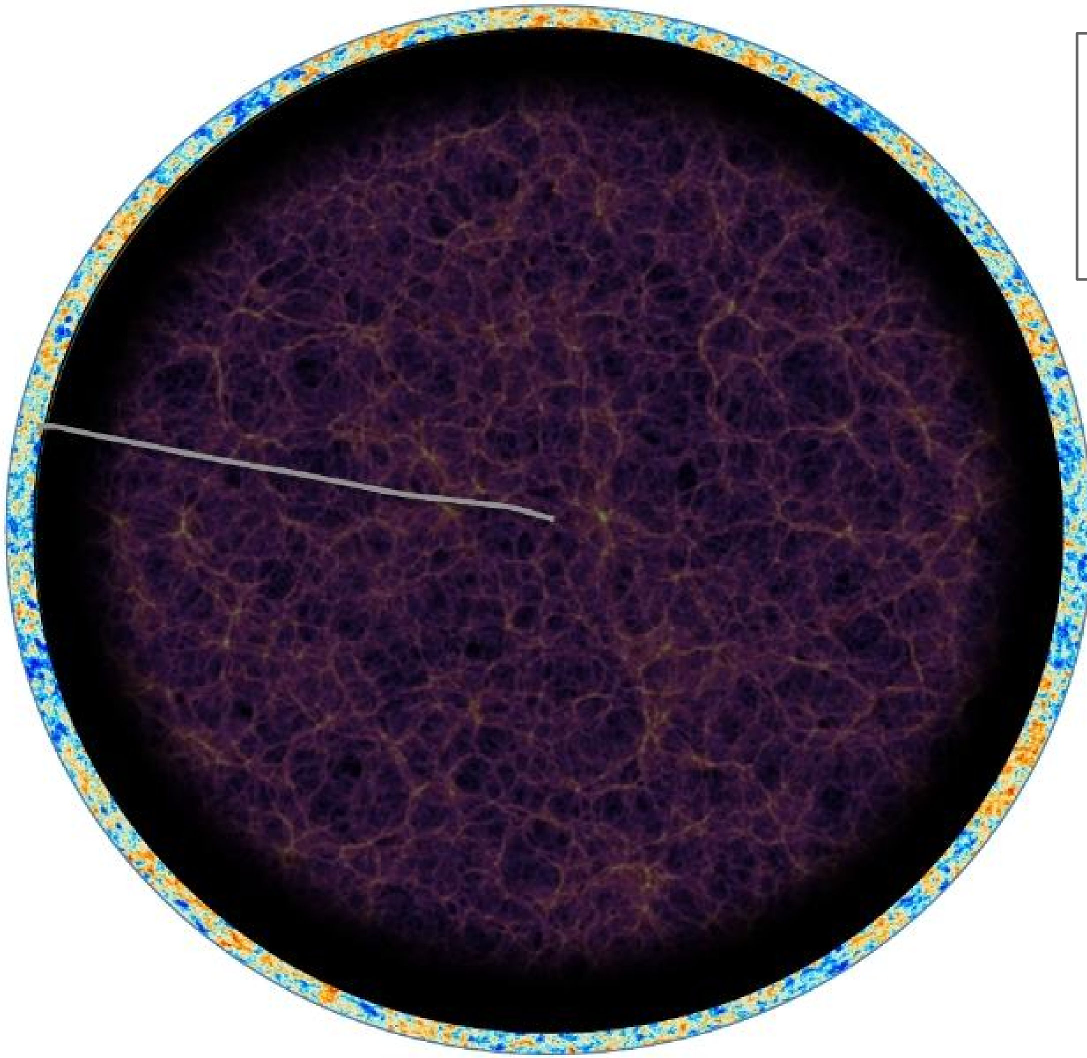
Max Planck Institut für Astrophysik

Garching

VB, Marcel Schmittfull, Blake Sherwin

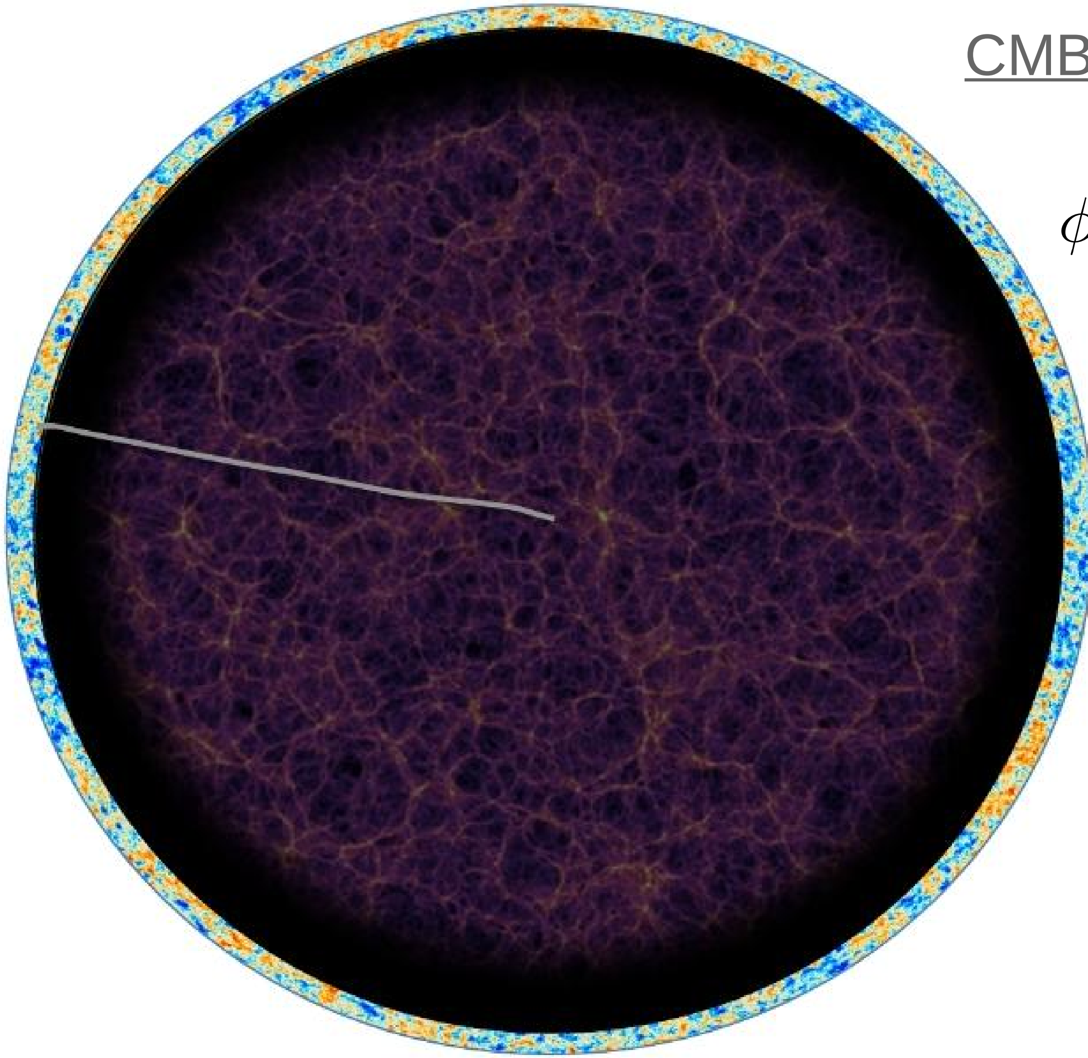
(*Phys. Rev. D 94, 043519; arXiv:1605.01392*)

Lensing of the cosmic microwave background



$$\tilde{T}(\mathbf{x}) = T\left[\mathbf{x} + \boldsymbol{\alpha}(\mathbf{x})\right]$$
$$= T\left[\mathbf{x} + \nabla\phi(\mathbf{x})\right]$$

Lensing of the cosmic microwave background



CMB lensing potential

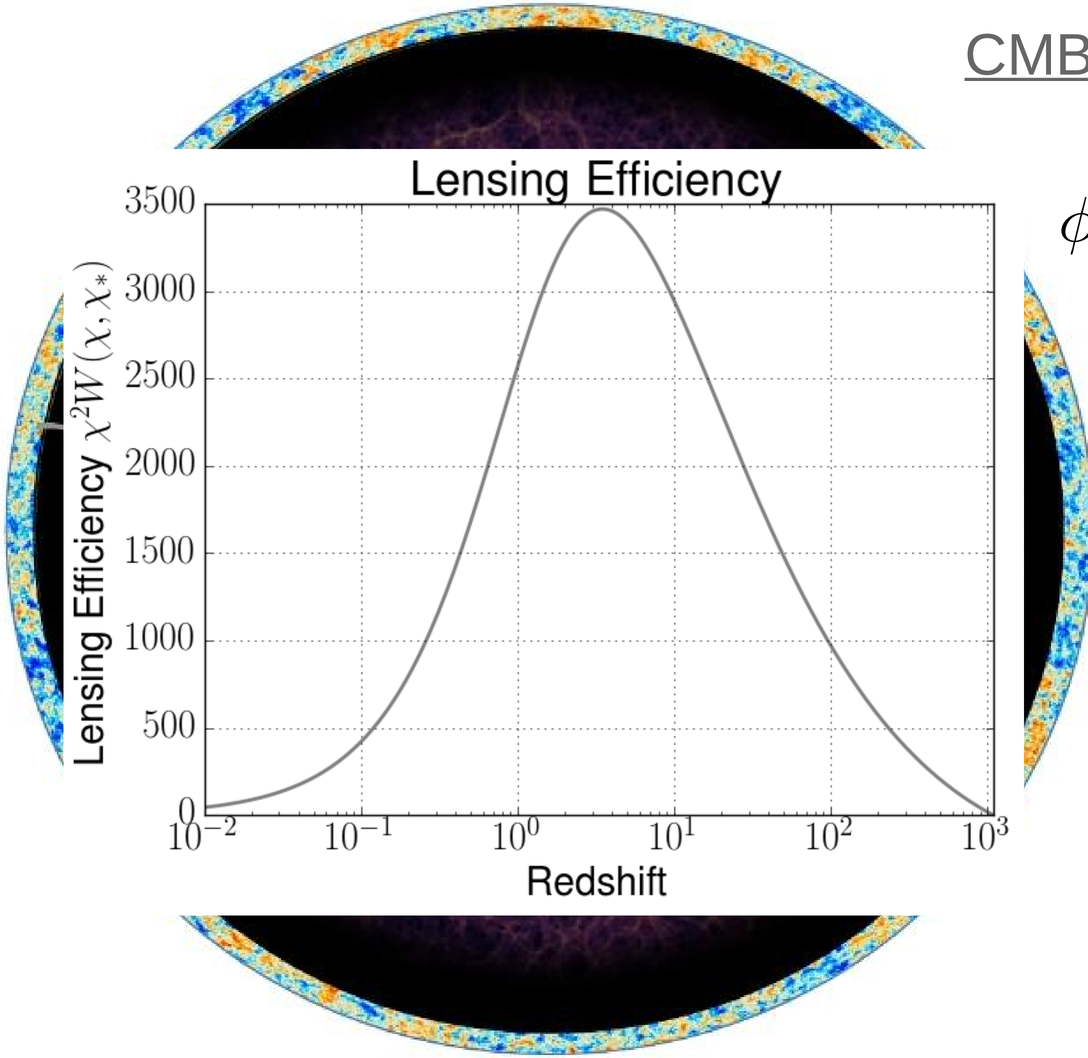
$$\phi(\mathbf{x}) = -2\int_0^{\chi_*} \mathrm{d}\chi\, W(\chi)\, \psi\left(\mathbf{x},\chi\right)$$

$$W(\chi) = \frac{f_k(\chi_* - \chi)}{f_k(\chi_*) f_k(\chi)}$$

Power spectrum

$$C_L^{\phi\phi} \leftarrow P_\delta(L/\chi;\chi)$$

Lensing of the cosmic microwave background



CMB lensing potential

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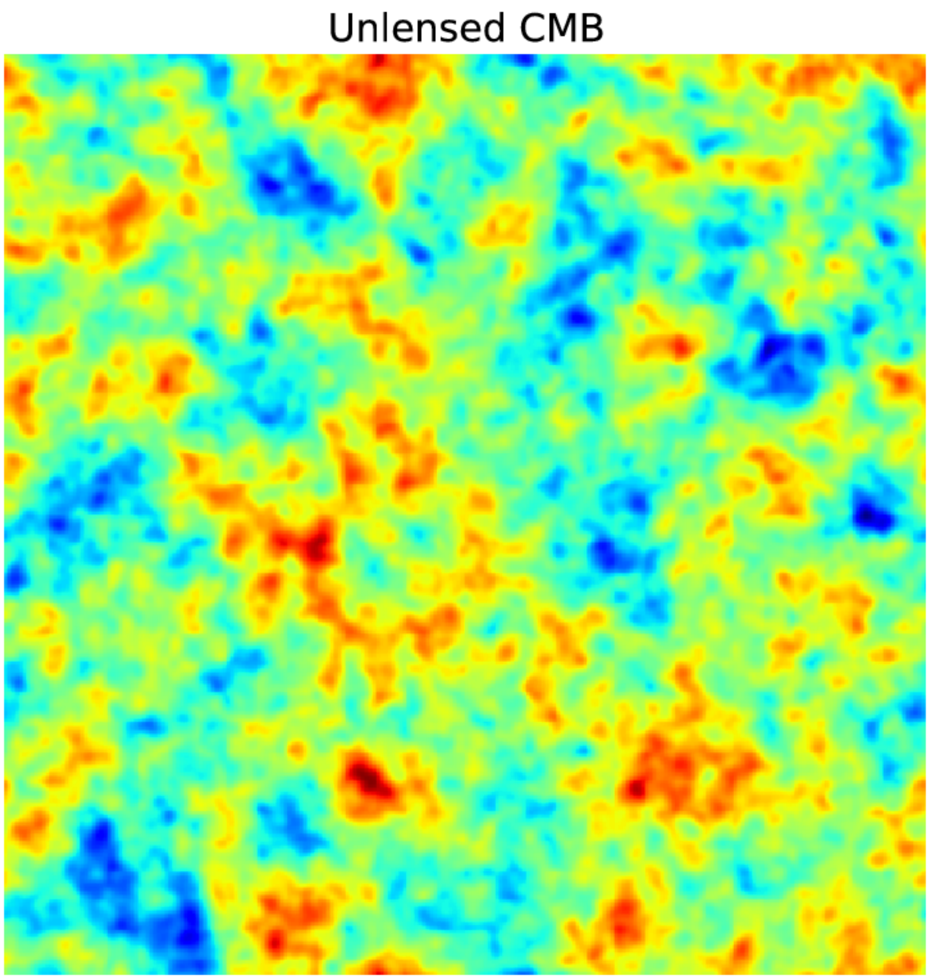
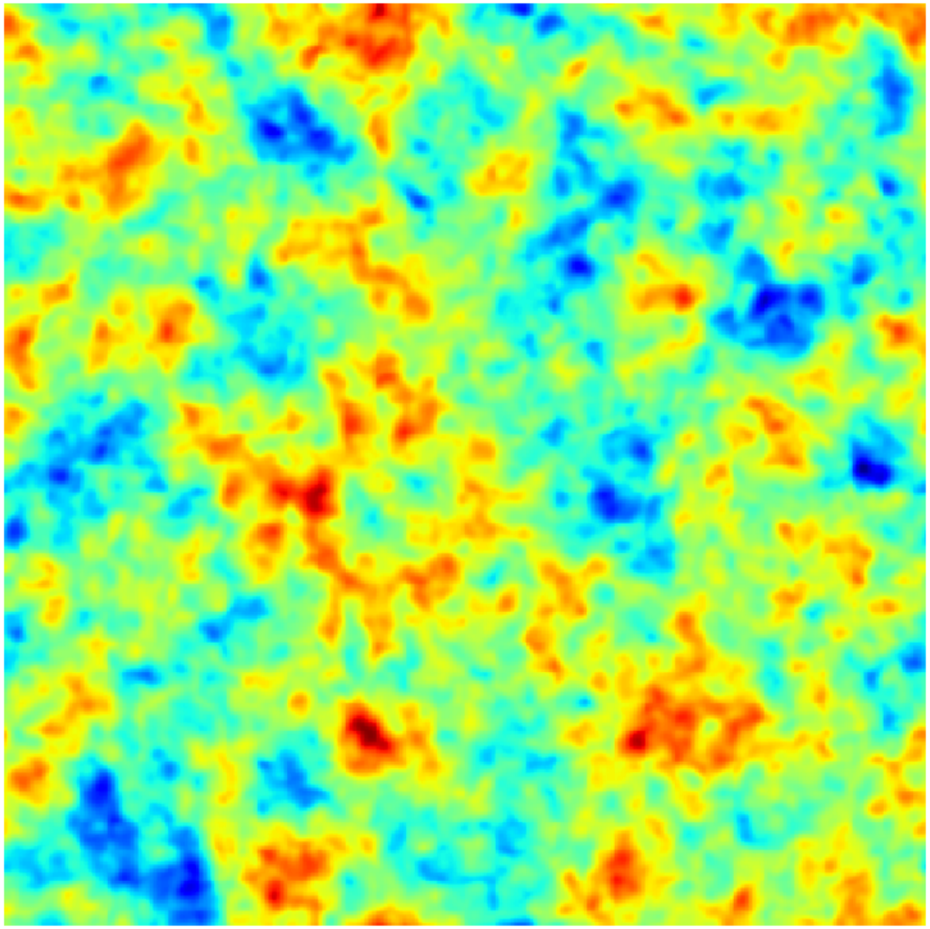
Lensing of the cosmic microwave background

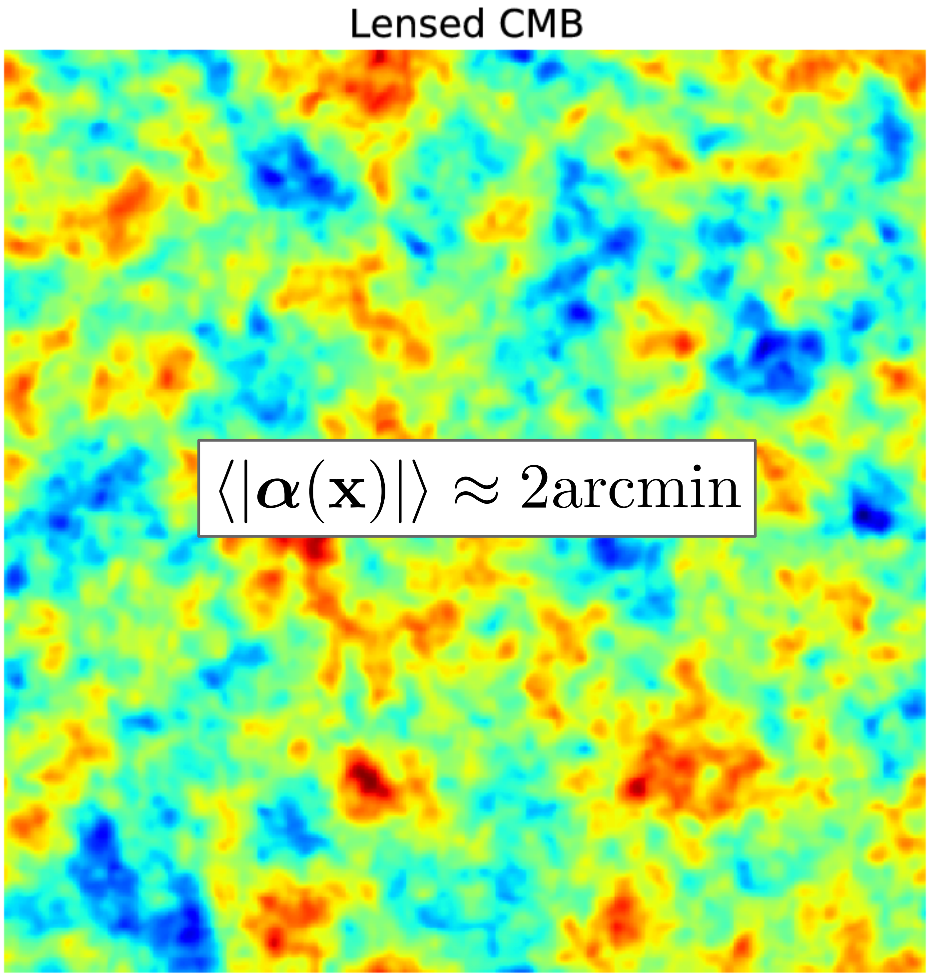


Lensing of the cosmic microwave background

Lensed CMB



Lensing of the cosmic microwave background



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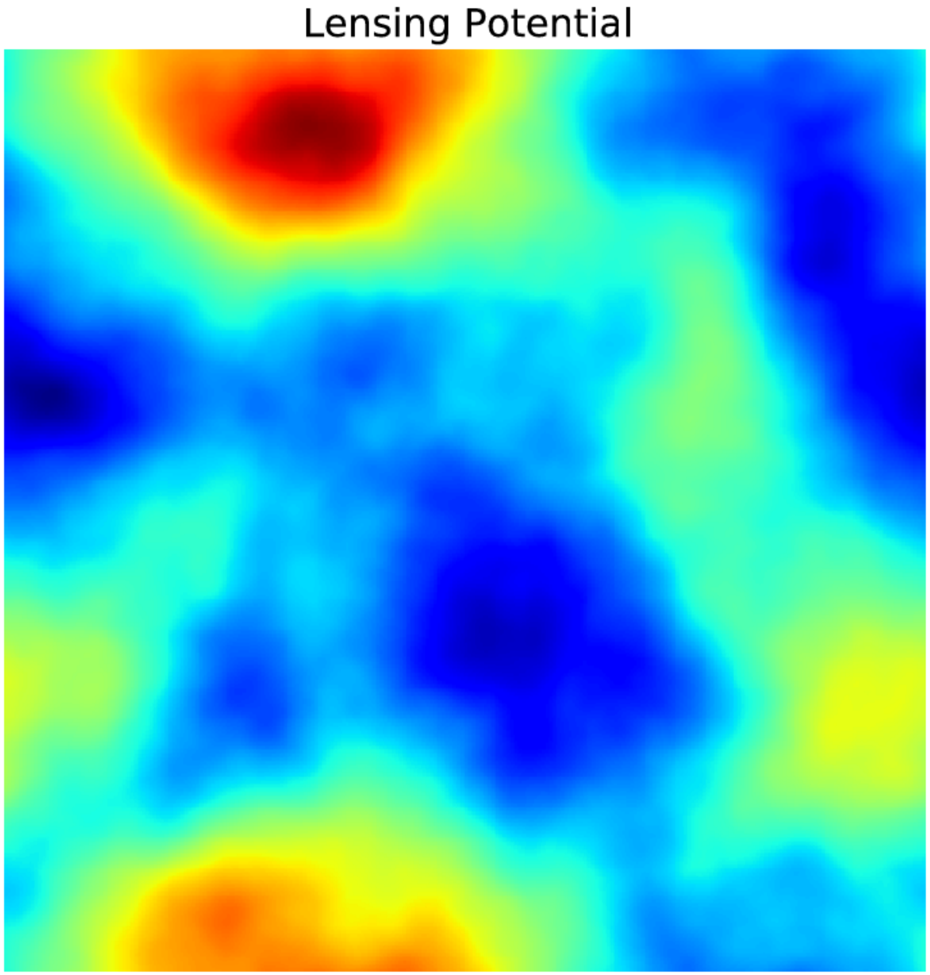
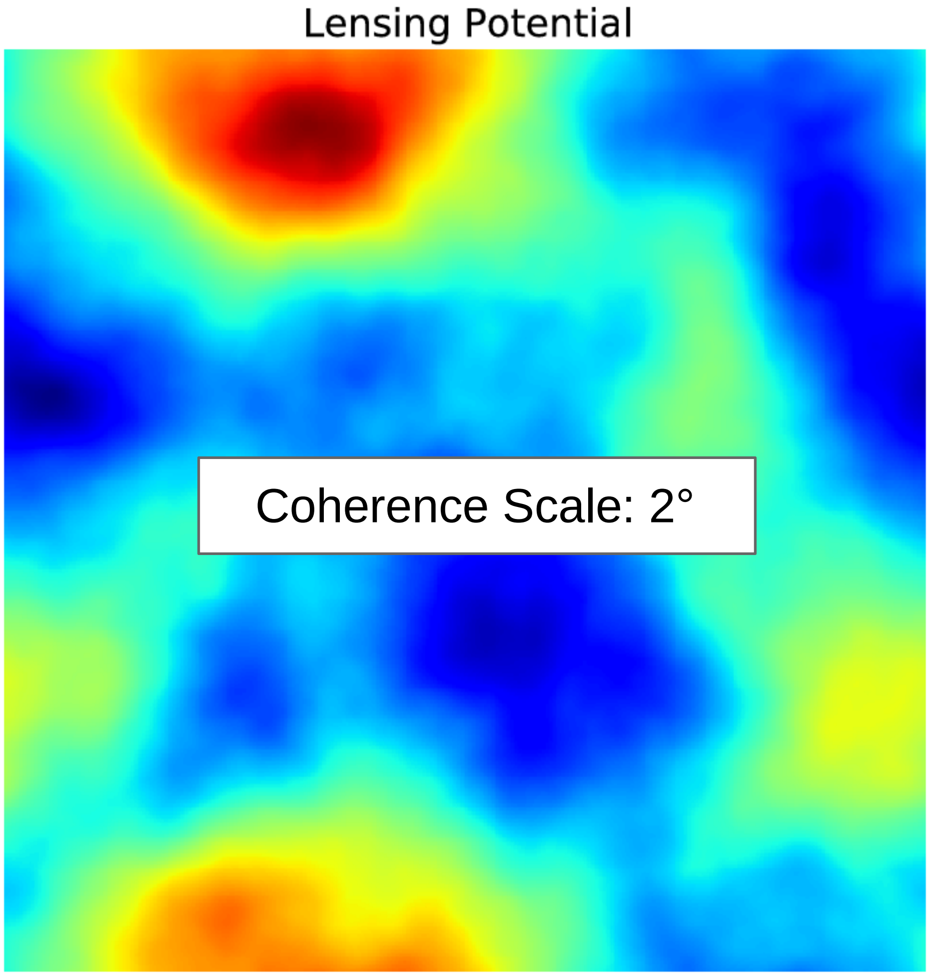


Lensing of the cosmic microwave background

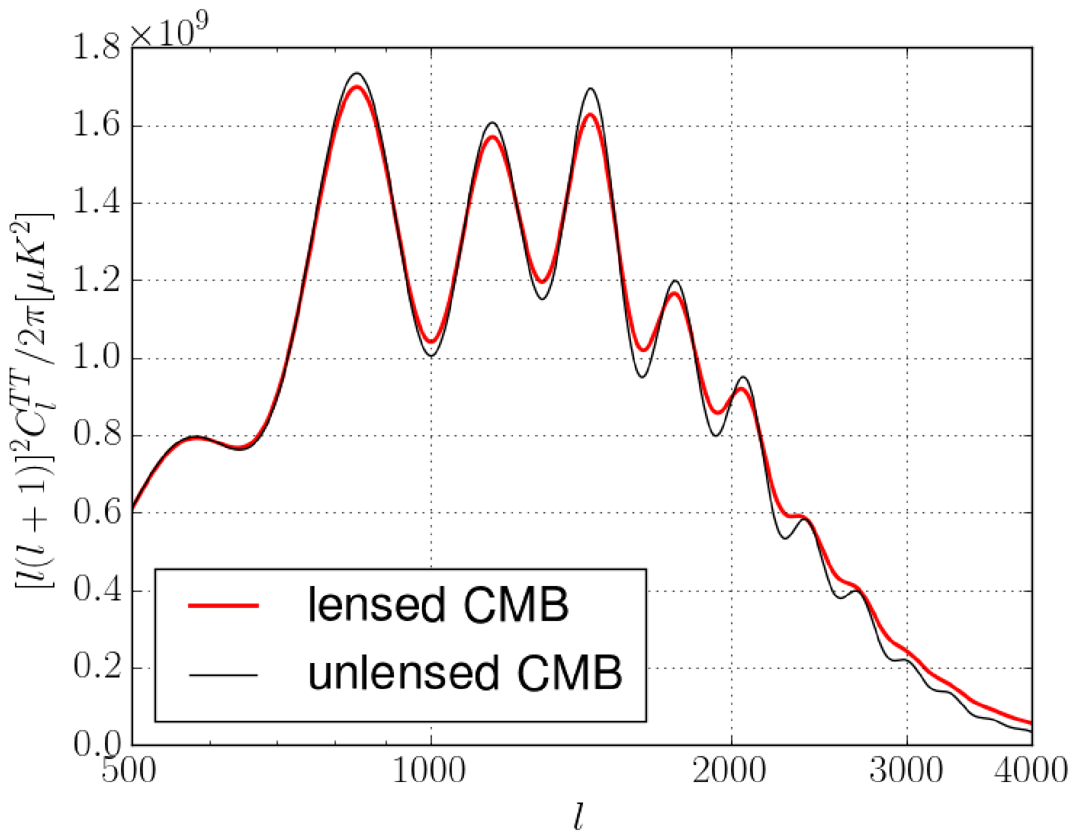


Lensing of the cosmic microwave background

- **Fixed lens**
  Two-point correlator in Fourier space → non-diagonal

- **Average over lens realizations**
  - CMB power spectrum →
  - temperature trispectrum → non-zero

T → non-Gaussian



CMB lensing reconstruction

Quadratic estimator (Hu 2001)

$$\hat{\phi}(\mathbf{L}) = A_L \int_{\mathbf{l}} g(\mathbf{l}, \mathbf{L}) \tilde{T}_{\rm expt}(\mathbf{l}) \tilde{T}^*_{\rm expt}(\mathbf{l} - \mathbf{L})$$

Power spectrum estimate ← mean square of the estimator

$$C^{\hat{\phi}\hat{\phi}}(\mathbf{L}) \leftarrow \langle \hat{\phi}(\mathbf{L}) \hat{\phi}^*(\mathbf{L}') \rangle_{(\phi,T)}$$

$$\langle \hat{\phi}(\mathbf{L}) \hat{\phi}^*(\mathbf{L}') \rangle_{(\phi,T)} \propto \langle \, \tilde{T}_{\rm expt}(\mathbf{l}_1) \, \tilde{T}_{\rm expt}(\mathbf{L} - \mathbf{l}_1) \, \tilde{T}_{\rm expt}(-\mathbf{l}_2) \, \tilde{T}_{\rm expt}(\mathbf{l}_2 - \mathbf{L}') \, \rangle$$

Status of CMB lensing

- **Planck**

$$\sum m_\nu < 145\,\mathrm{meV}\;(68\%)$$

Planck TT+low P+lensing+BAO

- **Ground-based (Stage-II)**
  - ACTPol
  - SPTPol
  - Polarbear

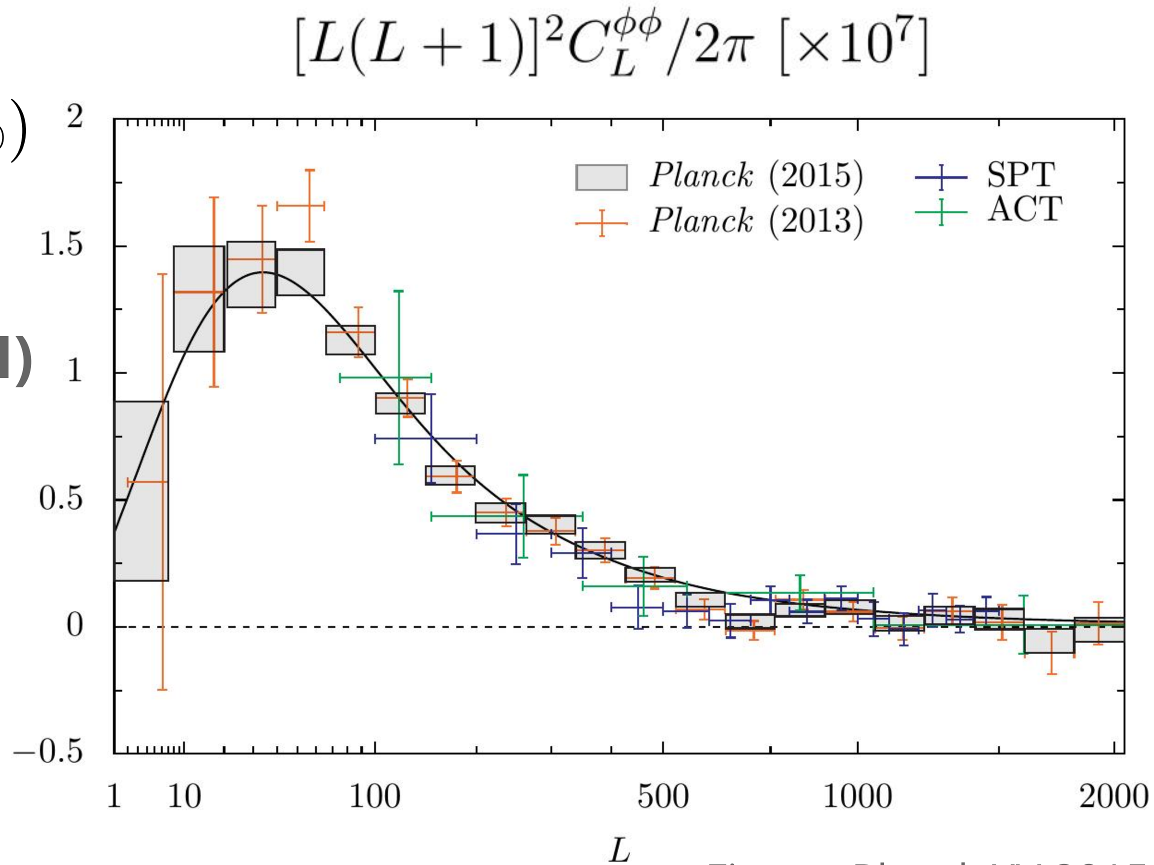


Figure: Planck XV 2015

Rapid improvement in precision

- Ground-based experiments
  - **Stage-III**

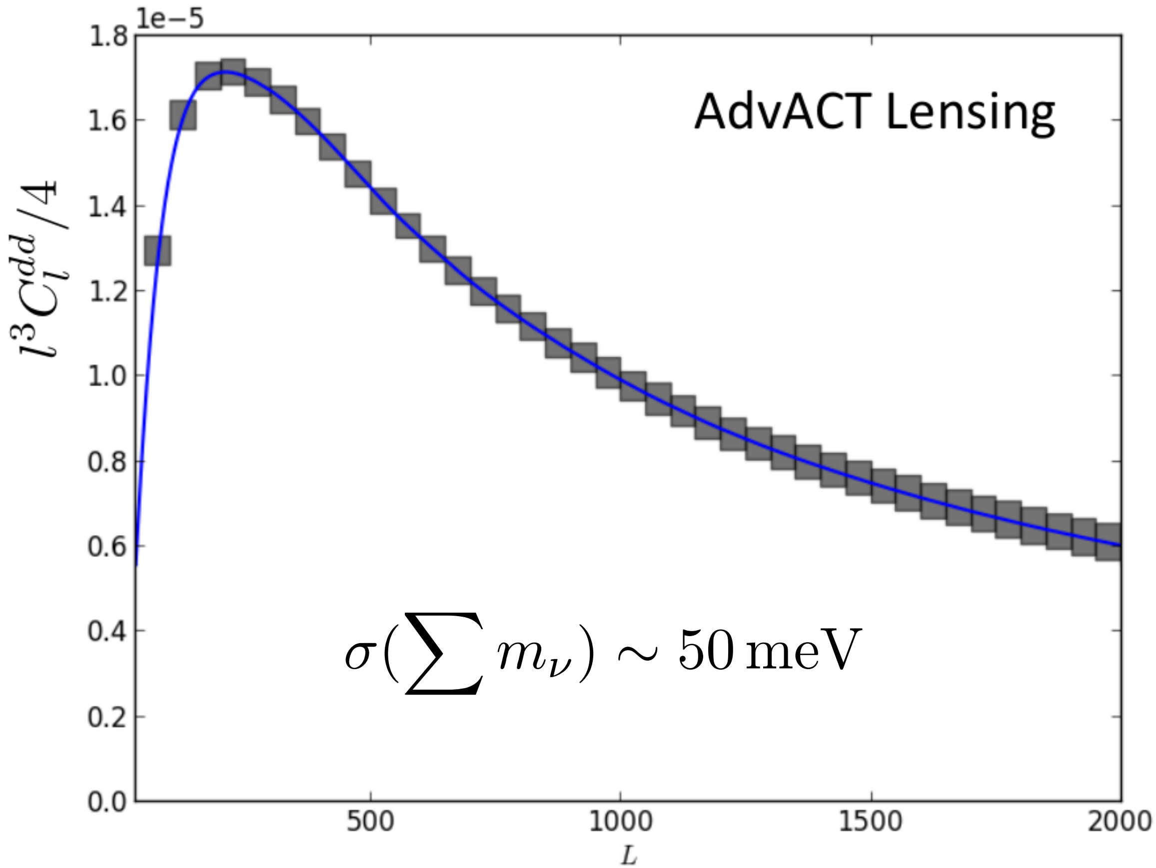
    Advanced ACT

    SPT3G

    Polarbear2
  - **Stage-IV**

    CMB Stage 4

    $\sigma(\sum m_\nu) \sim 15\,\mathrm{meV}$



- Increasing precision requires increasingly accurate theoretical modeling

CMB lensing reconstruction

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Known biases

Lensed temperature perturbation series

$$\tilde{T}(\mathbf{l}) = T(\mathbf{l}) + \delta T(\mathbf{l}) + \delta^2 T(\mathbf{l}) + \mathcal{O}(\phi^3)$$

$$\left\langle \tilde{T}\tilde{T}\tilde{T}'\tilde{T}' \right\rangle = \langle TTT'T' \rangle + \langle \delta T \delta T T' T' \rangle + \left\langle \delta^2 T T T' T' \right\rangle + \ldots + O(\phi^4)$$

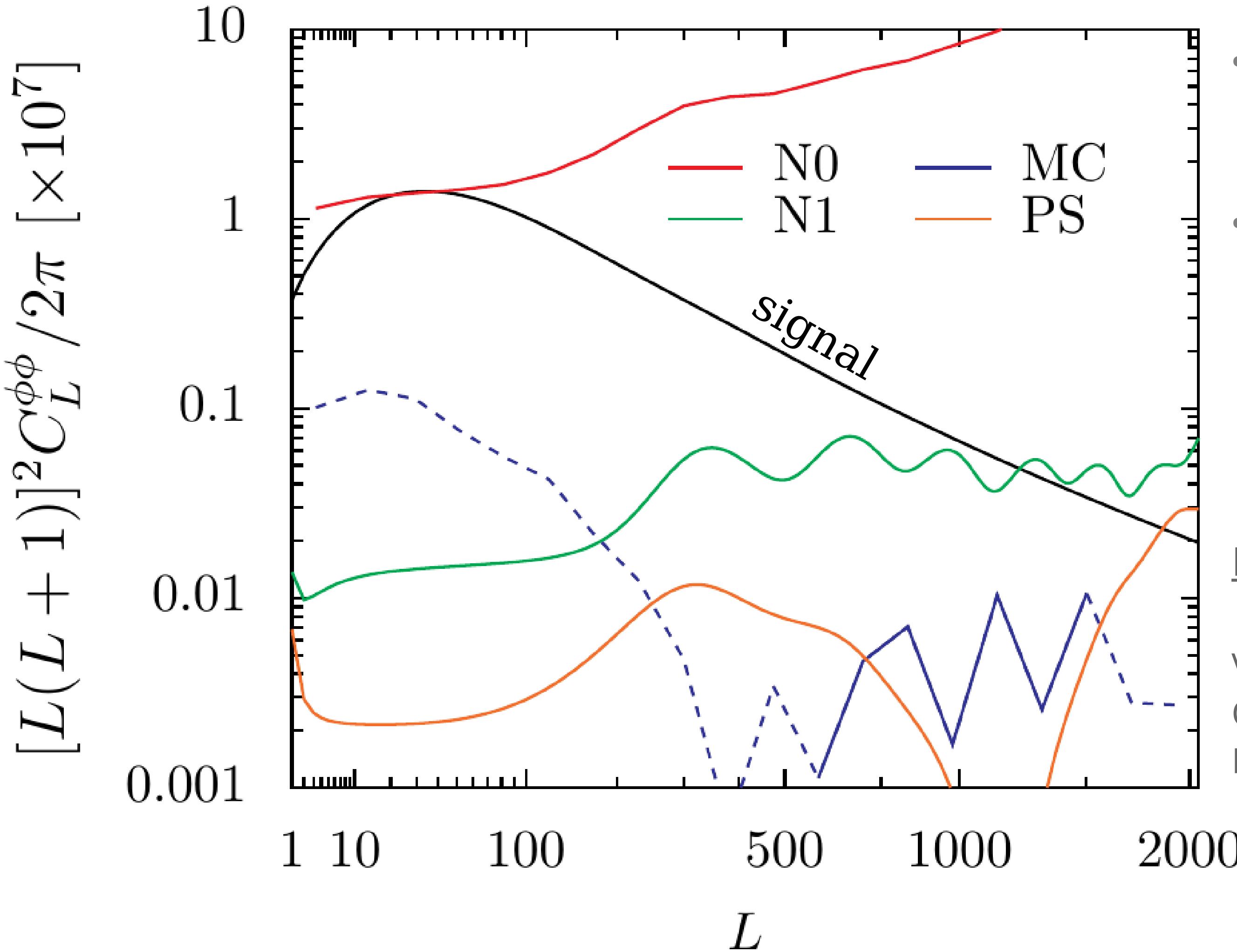
$N^{(0)}$ $\quad C_L^{\phi\phi},\ N^{(1)}$ $\quad N^{(0)}$

Different couplings give rise to different bias terms

$$\langle \hat{C}_L^{\hat{\phi}\hat{\phi}} \rangle = N_L^{(0)} + C_L^{\phi\phi} + N_L^{(1)} \qquad (\text{Gaussian } \phi)$$

Kesden et al. 2003, Hanson et al. 2011, Lewis et al. 2011

Known biases

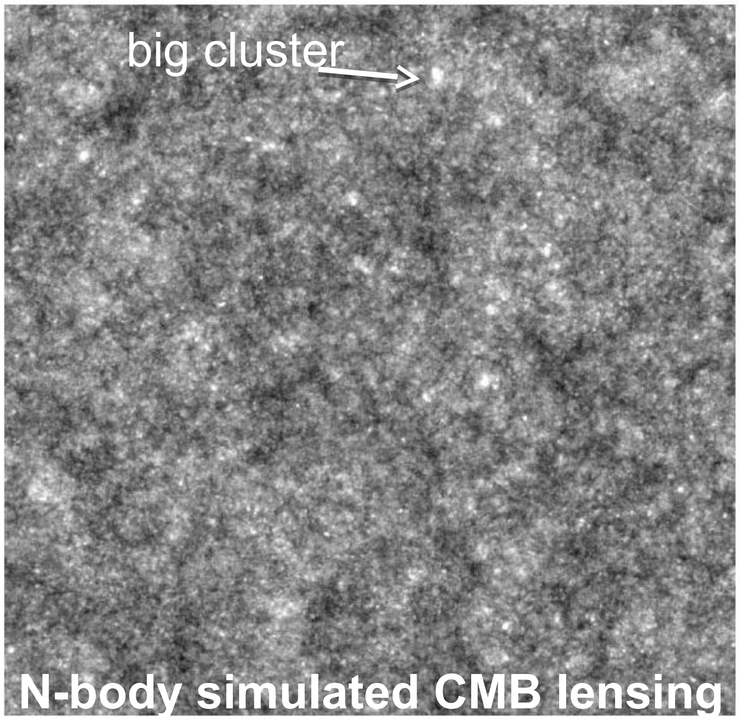


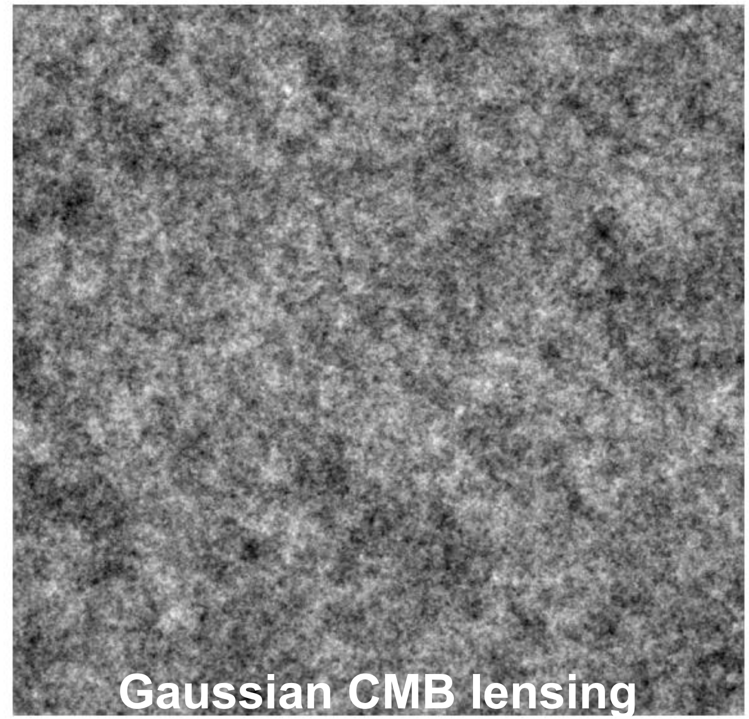
- **PS**
  foreground-sourced biases
- **MC**
  simulation-based corrections
  e.g. leakage from masking

Figure: Planck XV 2015

Van Engelen et al. (2014)
Osborne et al. (2014)
Namikawa et al. (2013)

Lensing potential
validity of Gaussian approximation





- 1-point PDF, peak count distribution → adds skewness
  Liu, Hill, Sherwin, Petri, VB, Haiman 2016
- Non-negligible bispectrum
  VB, Schmittfull, Sherwin 2016, Sherwin,VB, Liu, Hill in prep.

Lensing potential bispectrum

- Lensing potential is sourced by large-scale matter distribution
- Non-linear structure formation $\rightarrow$ matter distribution becomes non-Gaussian and acquires bispectrum

$$B_\phi(\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}_3) \leftarrow B_\delta(l_1/\chi, l_2/\chi, l_3/\chi)$$

Lensing potential bispectrum

- Lensing potential is sourced by large-scale matter distribution
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$$B_\phi(\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}_3) \leftarrow B_\delta(l_1/\chi, l_2/\chi, l_3/\chi)$$

- Does the lensing potential become non-Gaussian, too? *Many lenses along LOS should 'gaussianize' the lensing potential...*
- Namikawa 2016: bispectrum as signal
  $\rightarrow$ significant detection with a Stage-IV experiment

New 4-point bias

$$\left\langle \tilde{T}\tilde{T}\tilde{T}'\tilde{T}' \right\rangle = O(\phi^0) + O(\phi^2) + O(\phi^3) + O(\phi^4) + \ldots$$

contributions involving the lensing potential bispectrum

$$\underbrace{\langle \delta T \delta T\, \delta T' T' \rangle}_{\text{Type A}}, \underbrace{\langle \delta^2 T \delta T\, T' T' \rangle}_{\text{Type B}}, \underbrace{\langle \delta^2 T T\, \delta T' T' \rangle}_{\text{Type C}}, \underbrace{\langle \delta^3 T T\, T' T' \rangle}_{\text{Type D}}$$

VB, Schmittfull, Sherwin 2016

New 4-point bias

$$\underbrace{\langle \delta T \delta T\, \delta T' T' \rangle}_{\text{Type A}}, \underbrace{\langle \delta^2 T \delta T\, T' T' \rangle}_{\text{Type B}}, \underbrace{\langle \delta^2 T T\, \delta T' T' \rangle}_{\text{Type C}}, \underbrace{\langle \delta^3 T T\, T' T' \rangle}_{\text{Type D}}$$

More or less coupled 6D integrals over lensing bispectrum and temperature power spectra

New 4-point bias

$$\underbrace{\langle \delta T \delta T \, \delta T' T' \rangle}_{\text{Type A}}, \underbrace{\langle \delta^2 T \delta T \, T' T' \rangle}_{\text{Type B}}, \underbrace{\langle \delta^2 T T \, \delta T' T' \rangle}_{\text{Type C}}, \underbrace{\langle \delta^3 T T \, T' T' \rangle}_{\text{Type D}}$$

$$\langle T_{,i} \phi_{,i} T_{,j} \phi_{,j} \, T'_{,k} \phi'_{,k} T' \rangle$$

$$\langle T_{,ij} \phi_{,i} \phi_{,j} T \, T'_{,k} \phi'_{,k} T' \rangle$$

New 4-point bias

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$$\langle T_{,i} \phi_{,i} T_{,j} \phi_{,j}\, T'_{,k} \phi'_{,k} T' \rangle$$

$$\langle T_{,ij} \phi_{,i} \phi_{,j} T\, T'_{,k} \phi'_{,k} T' \rangle$$

$$-4 A_L^2 S_L \int_{\mathbf{l}_1, \mathbf{l}} g(\mathbf{l}_1, \mathbf{L}) [(\mathbf{l}_1 - \mathbf{l}) \cdot \mathbf{l}][(\mathbf{l}_1 - \mathbf{l}) \cdot (\mathbf{L} - \mathbf{l})] C^{TT}_{|\mathbf{l}_1 - \mathbf{l}|} B_\phi(\mathbf{l}, \mathbf{L} - \mathbf{l}, -\mathbf{L})$$

$$S_L = \int_{\mathbf{l}_2} g(\mathbf{l}_2, \mathbf{L}) (\mathbf{l}_2 \cdot \mathbf{L}) C^{TT}_{l_2}$$

New 4-point bias

$$\underbrace{\langle \delta T \delta T\, \delta T' T' \rangle}_{\text{Type A}},\ \underbrace{\langle \delta^2 T \delta T\, T' T' \rangle}_{\text{Type B}},\ \underbrace{\langle \delta^2 T T\, \delta T' T' \rangle}_{\text{Type C}},\ \underbrace{\langle \delta^3 T T\, T' T' \rangle}_{\text{Type D}}$$

$$\langle T_{,i} \phi_{,i} T_{,j} \phi_{,j}\, T'_{,k} \phi'_{,k} T' \rangle$$

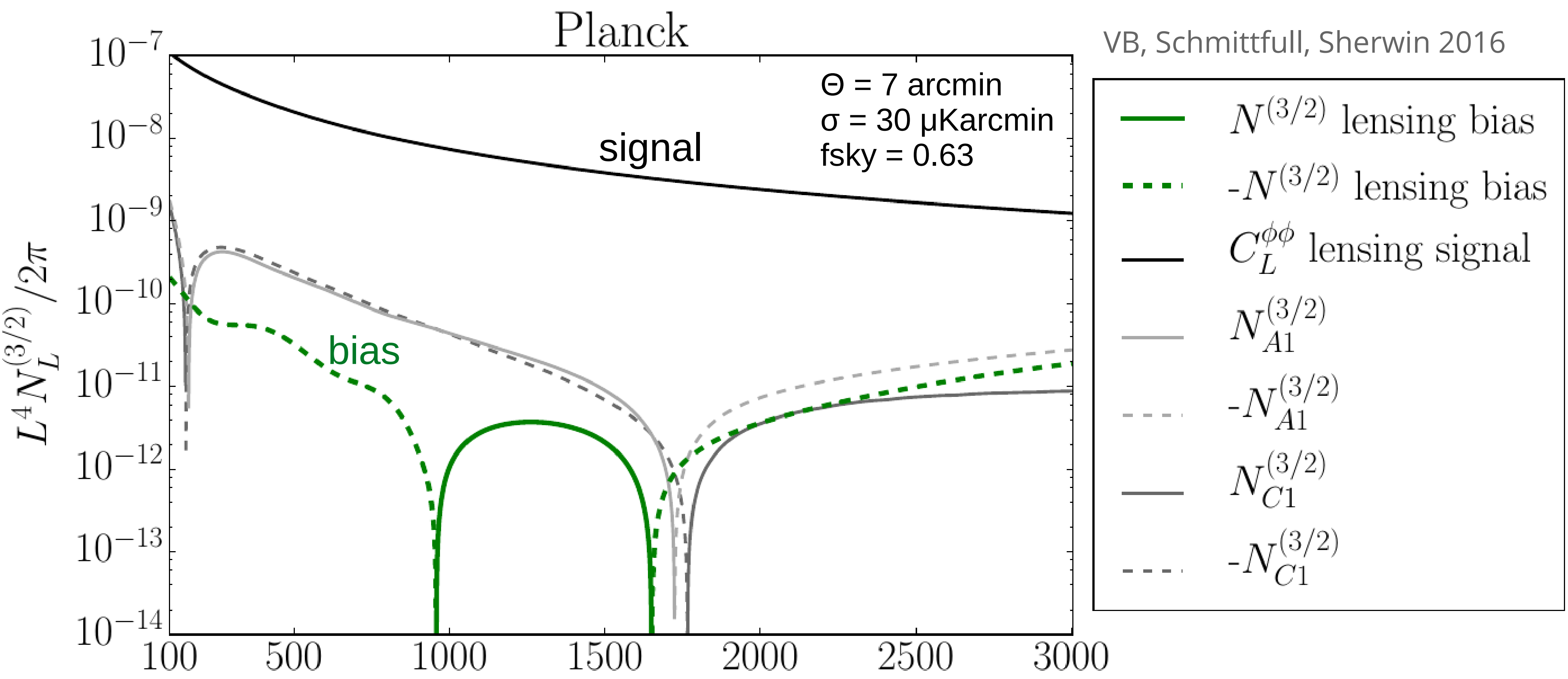
$$\langle T_{,ij} \phi_{,i} \phi_{,j} T\, T'_{,k} \phi'_{,k} T' \rangle$$

$$\langle \hat{C}_L^{\hat{\phi}\hat{\phi}} \rangle = N_L^{(0)} + C_L^{\phi\phi} + N_L^{(1)} + N_L^{(3/2)}$$

Results

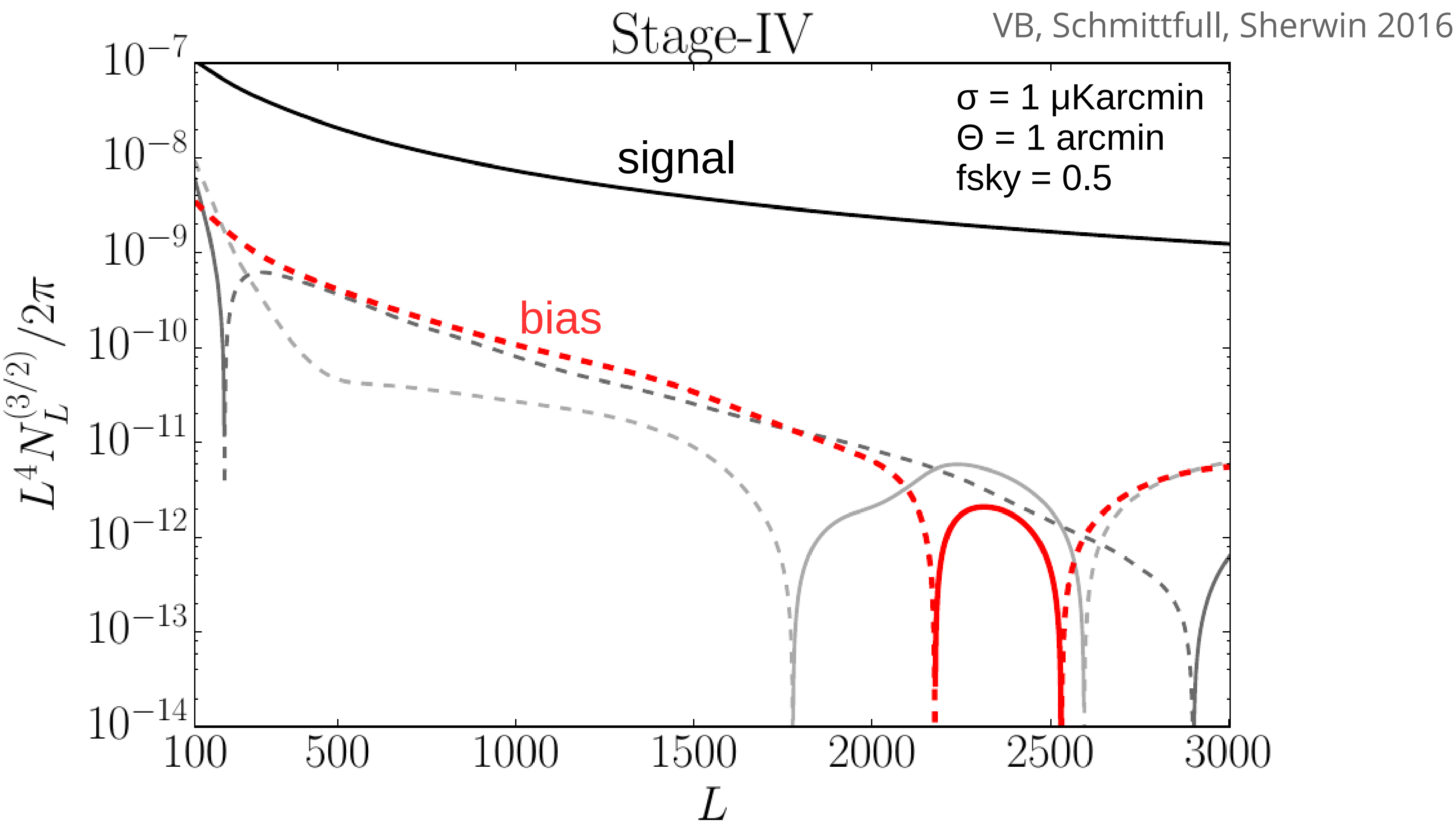
Temperature TT,TT

VB, Schmittfull, Sherwin 2016



Results

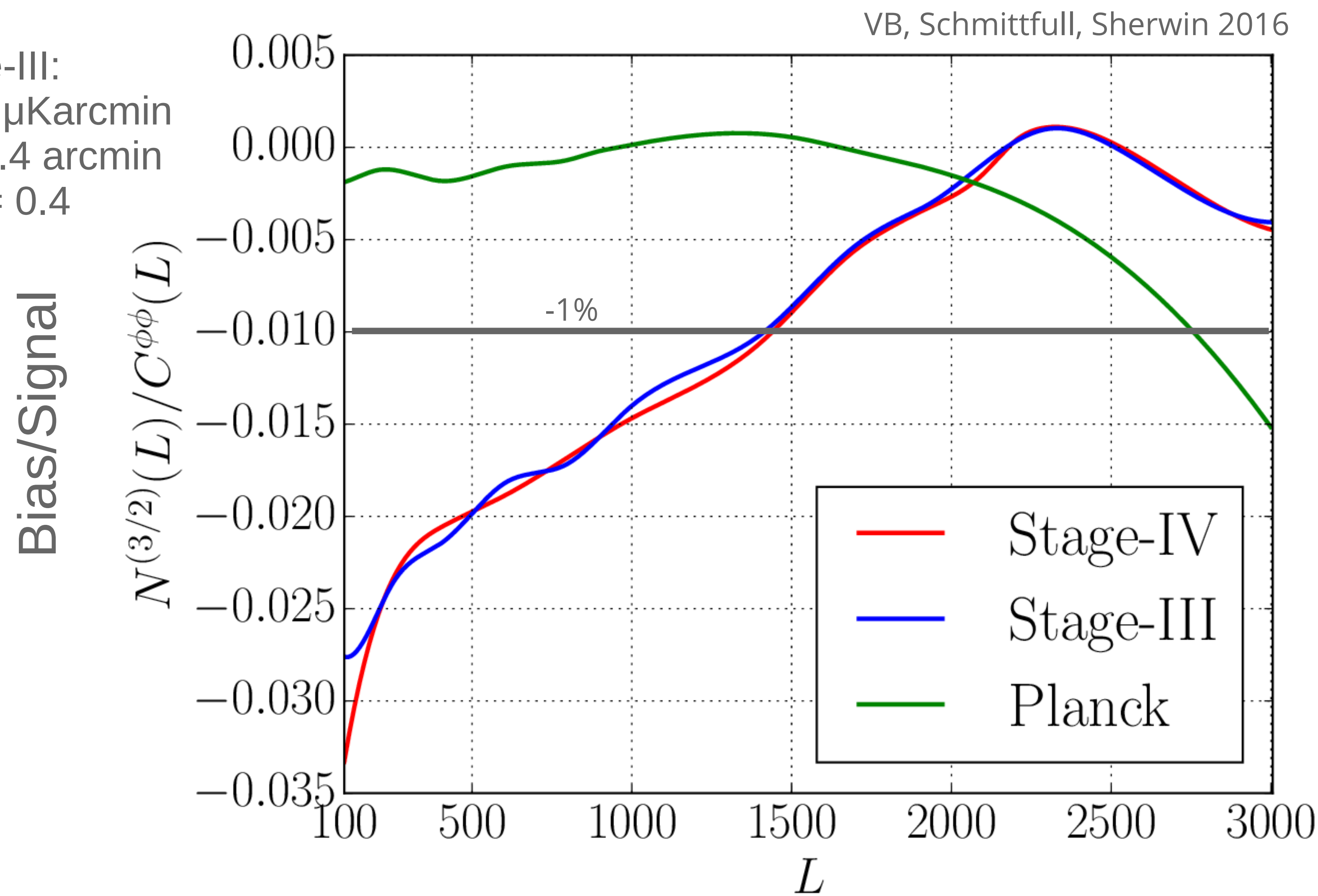
Temperature TT,TT



Results

Temperature TT,TT

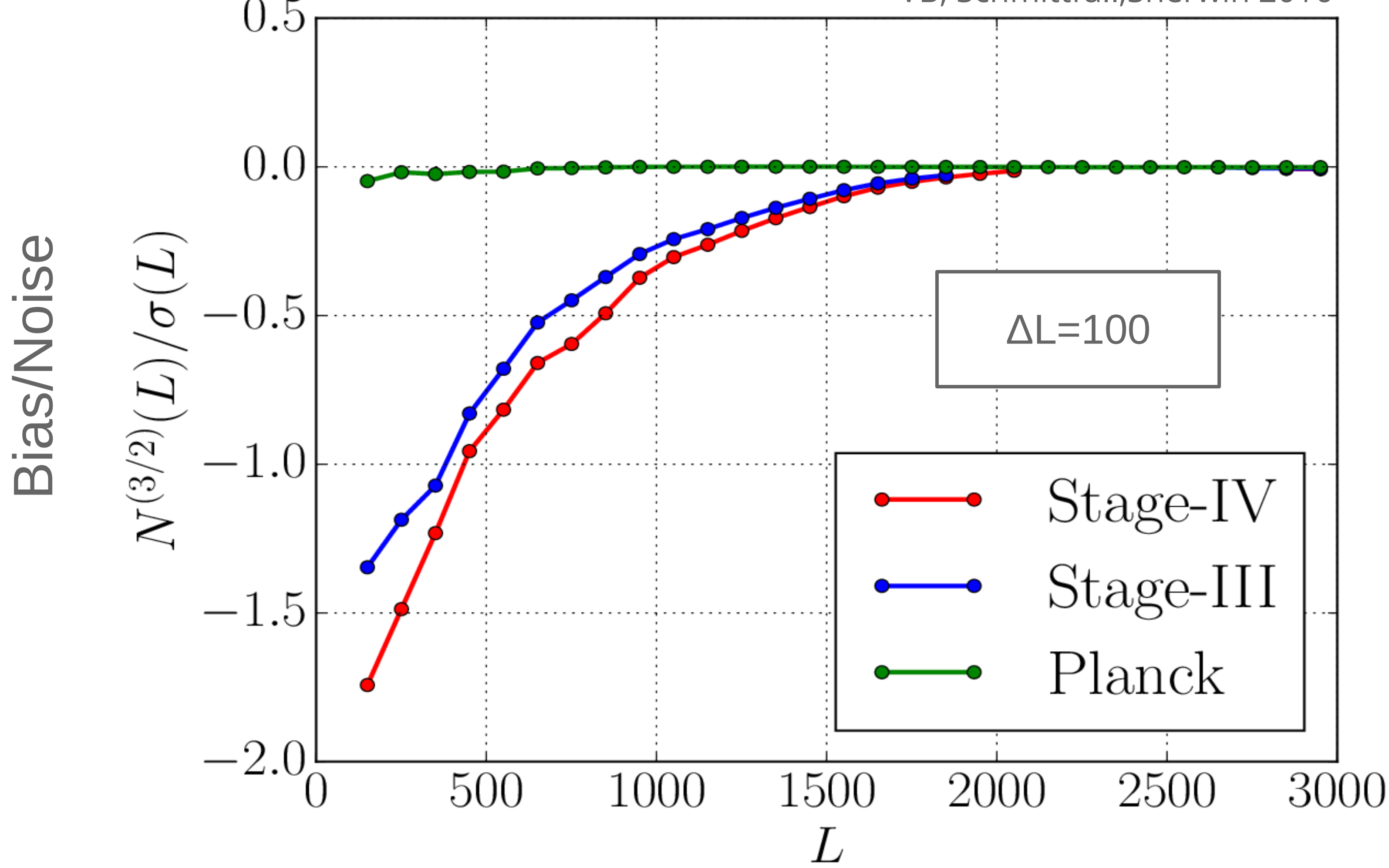
Stage-III:
σ = 6 μKarcmin
Θ = 1.4 arcmin
fsky = 0.4



Results

Temperature TT,TT

VB, Schmittfull,Sherwin 2016



Lensing Bispectrum Upgrades

Semi-analytic Fit

So far

Eulerian perturbation theory at leading-order (tree-level)

$$B_\delta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3; \eta) = 2\, F_2(\mathbf{k}_1, \mathbf{k}_2) P_\delta(k_1, \eta) P_\delta(k_2, \eta) + (1 \leftrightarrow 3) + (2 \leftrightarrow 3)$$

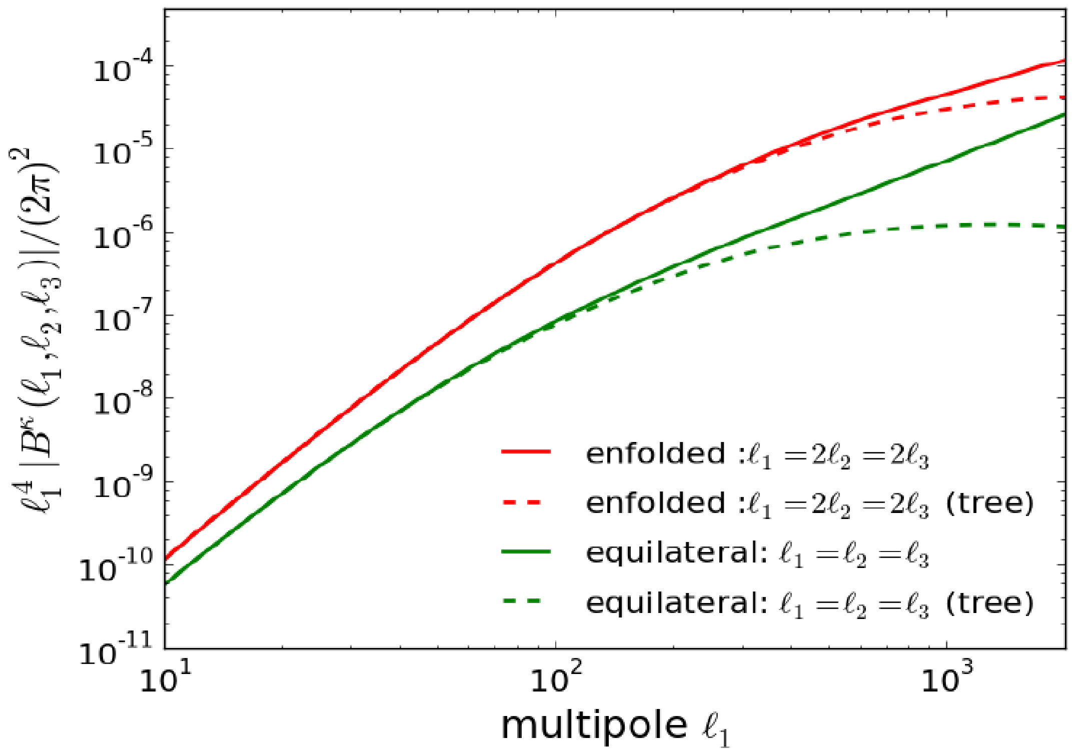
Upgrade

Fitting formula

$$F_2(\mathbf{k}_i, \mathbf{k}_j) \to F_2^{\mathrm{eff}}(\mathbf{k}_i, \mathbf{k}_j, n_i, n_j)$$

Scoccimarro&Couchman 2001,
Gil-Marin et al. 2012

Figure: Namikawa 2016



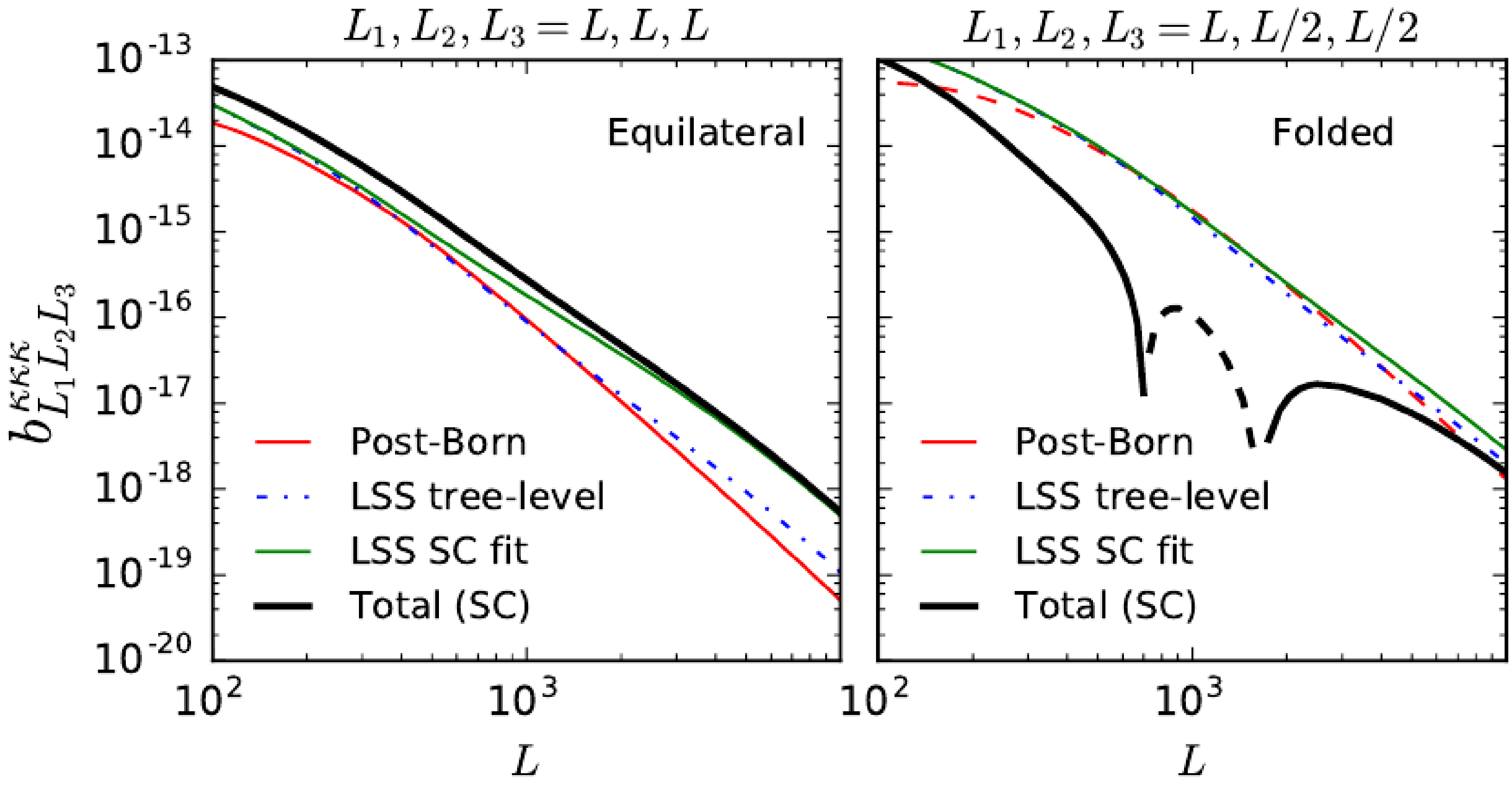
Lensing Bispectrum Upgrades

Post-Born Corrections

Update

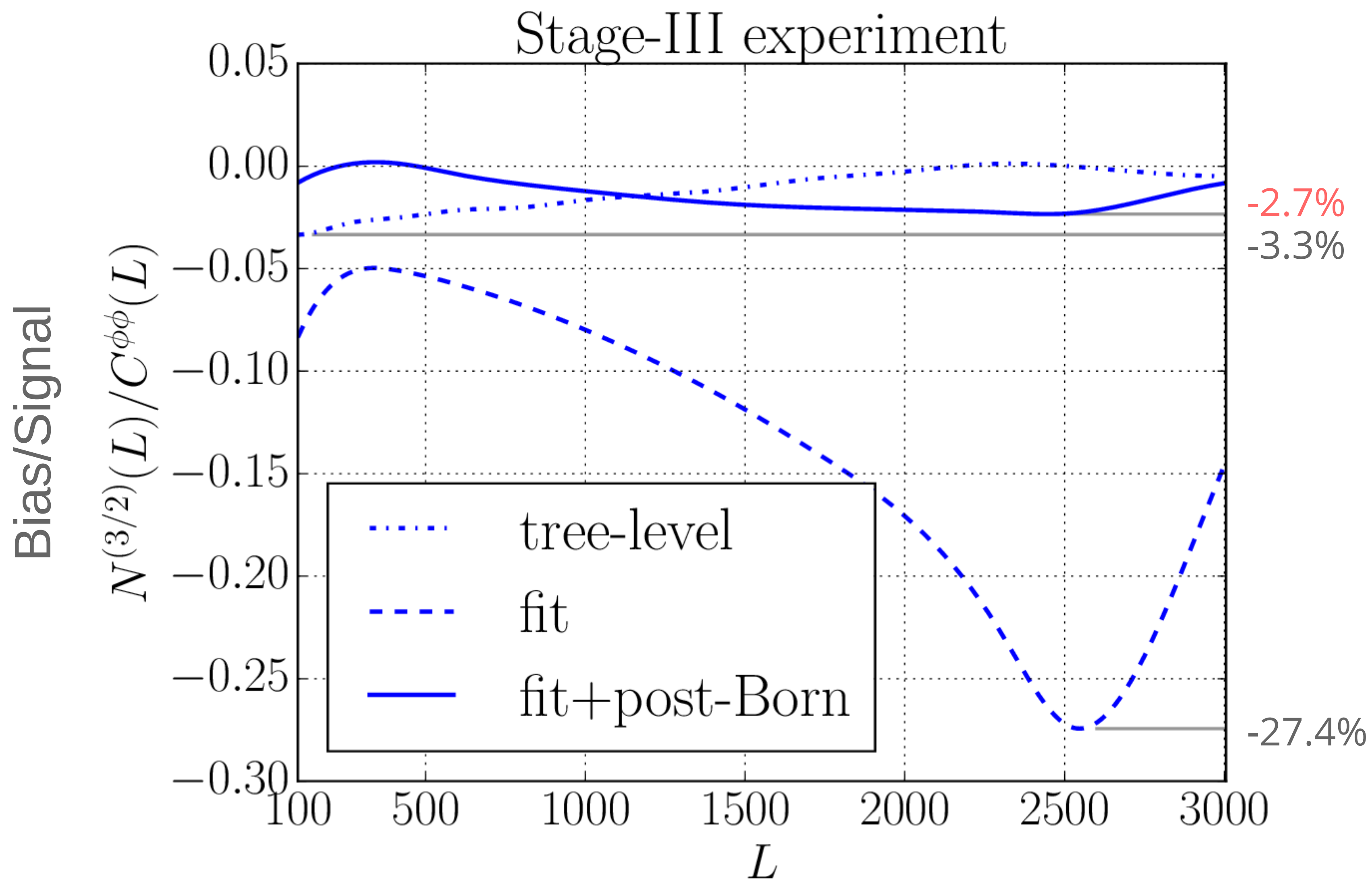
Post-Born corrections → non-negligible contributions to Bispectrum

Pratten&Lewis 2016



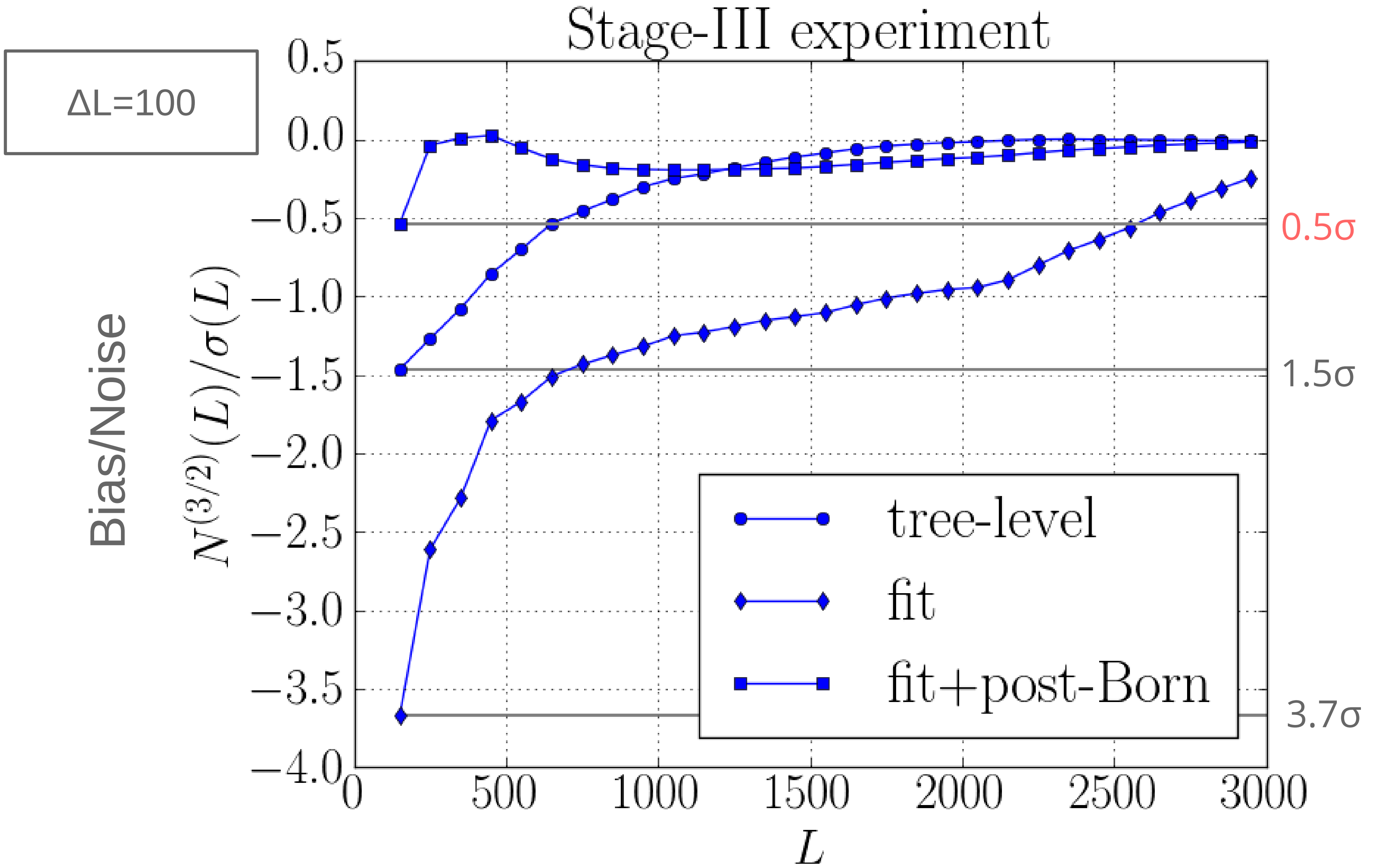
Results

Updated TT,TT Bias/Signal



Results

Updated TT,TT Bias/Noise



Results

test against ray-traced lensing simulations

- Evaluation of bias is numerically involved
- Neglected, tightly coupled, terms might not be negligible
- Possibly non-negligible higher order contributions
- Possible shortcomings of bispectrum model

independent test with simulations

VB/Sherwin

(Jia Liu, Colin Hill, Marcel Schmittfull,

Andrea Petri)

Results

test against ray-traced lensing simulations

- **10 000 fully non-linear** convergence fields
  $\leftarrow$ ray-tracing through an N-body simulation
  many lens planes $\rightarrow$ include post-Born corrections

- **10 000 Gaussian** realizations of the convergence
  with same (measured) power spectrum as N-body result

- Lens same background CMB with Gaussian and Non-Gaussian simulations and add same noise maps
  $\rightarrow$ **cancels cosmic variance**

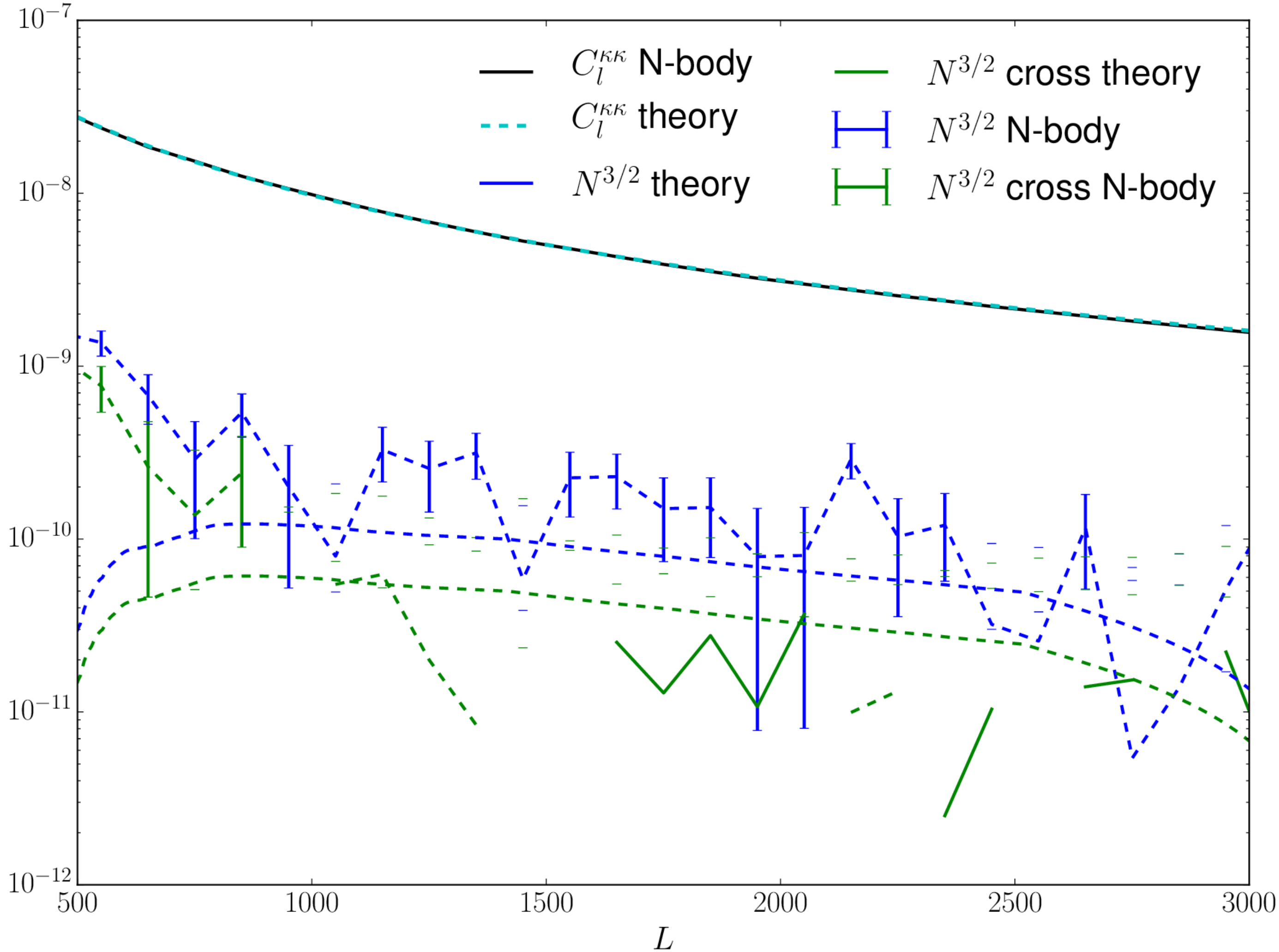
- Reconstruct lensing power spectrum from lensed maps

➔ **Compare the residuals of the reconstructions**

Results

test against ray-traced lensing simulations

bias in simulation vs theory



VB, Sherwin, Liu, Hill in prep.

Results

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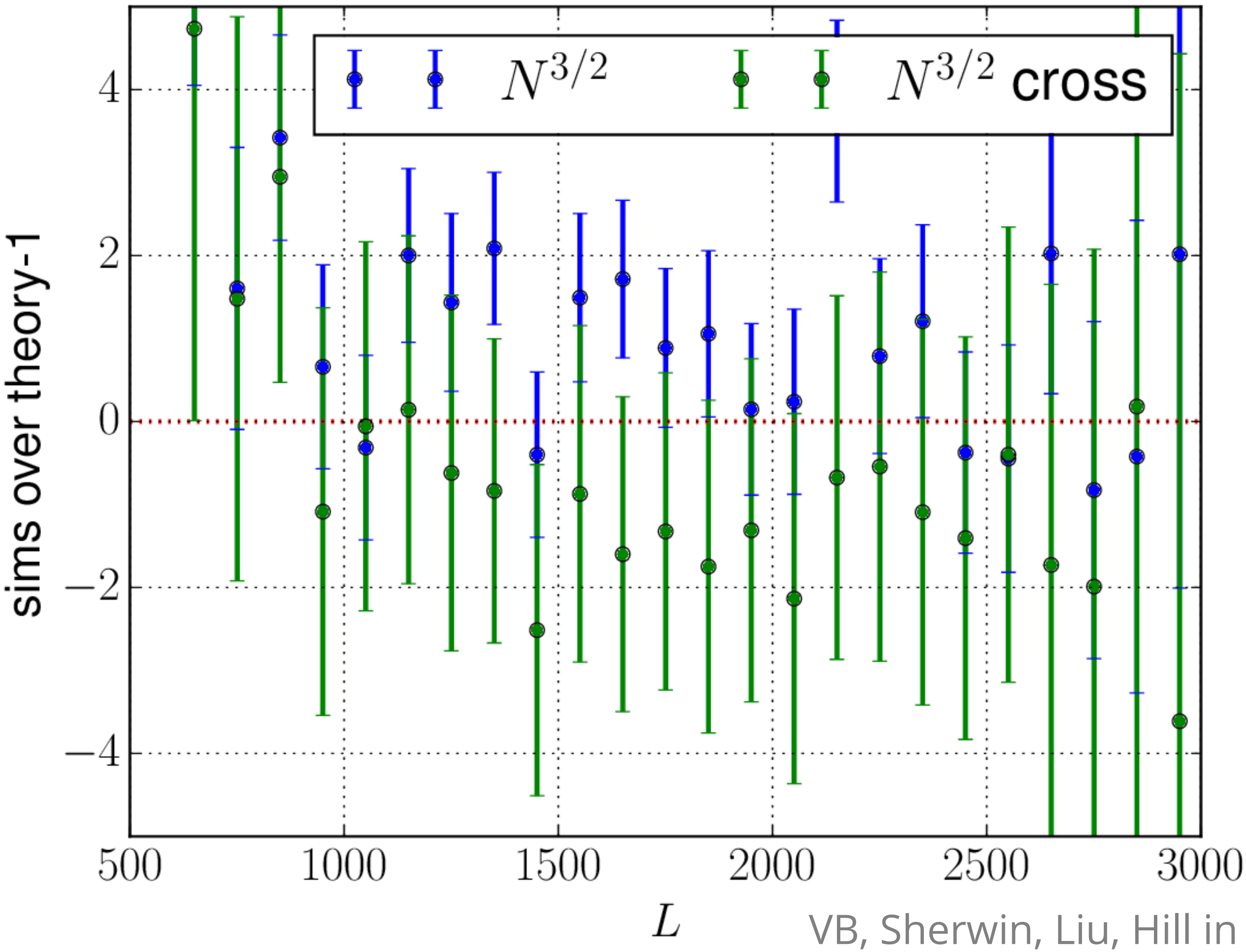

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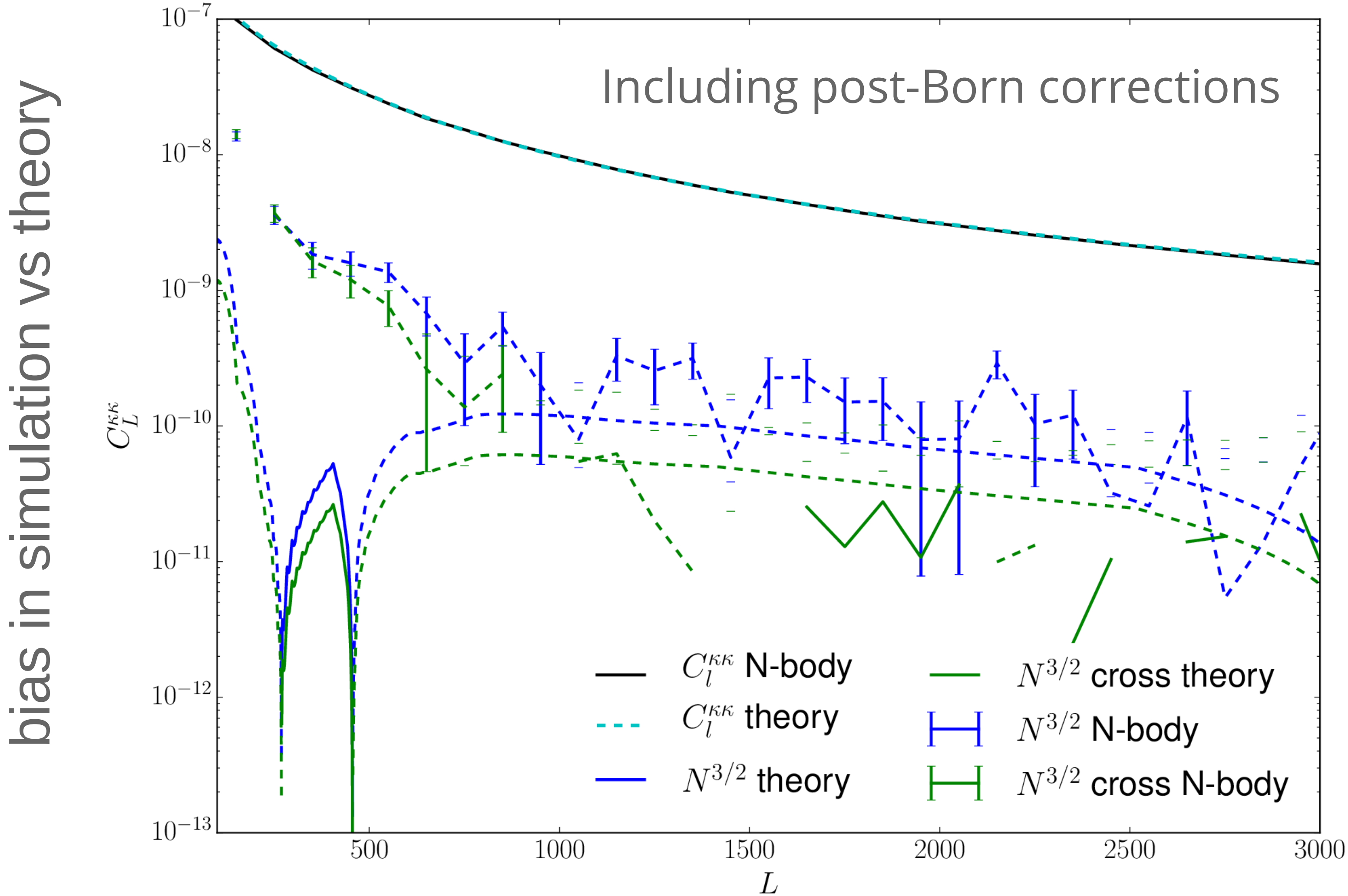

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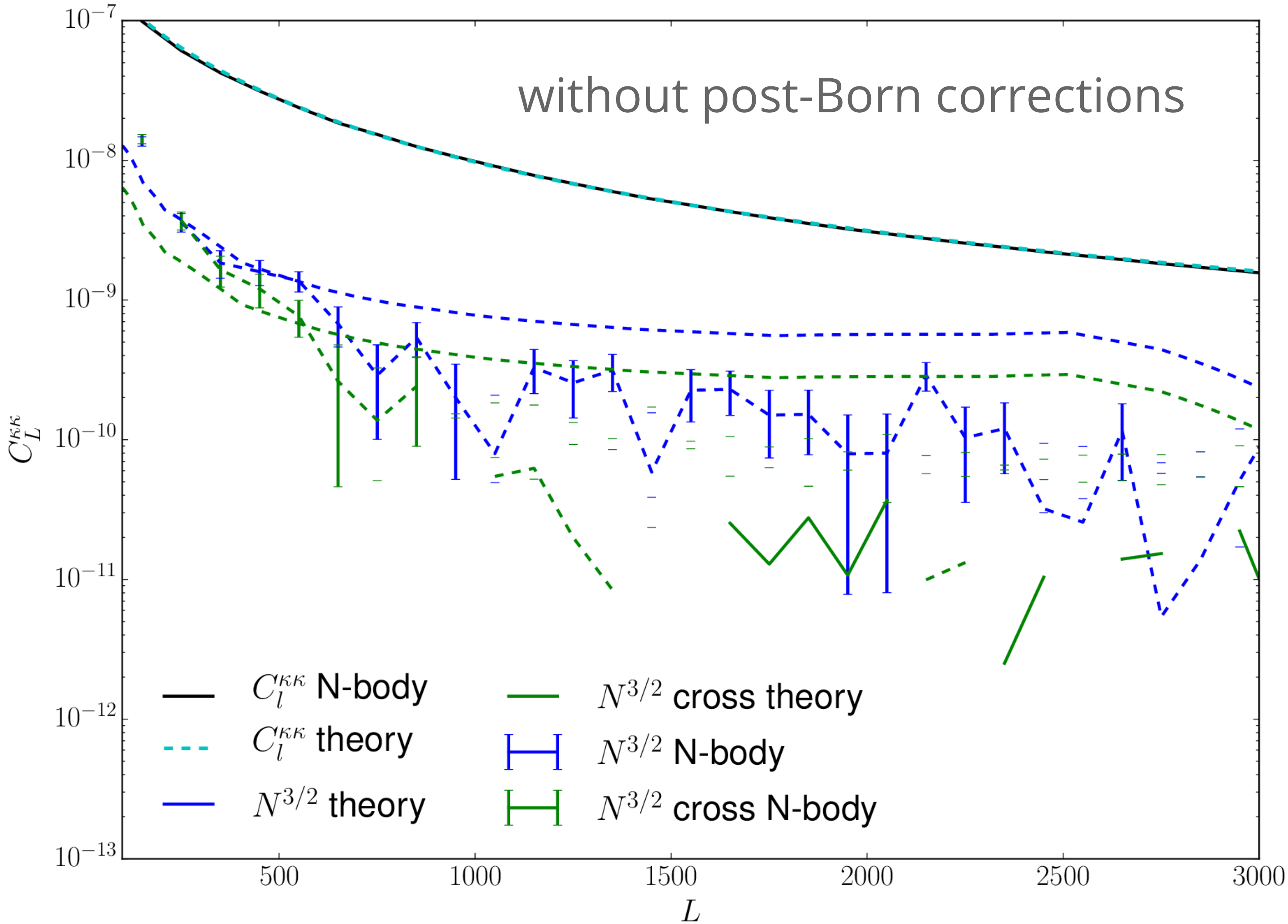
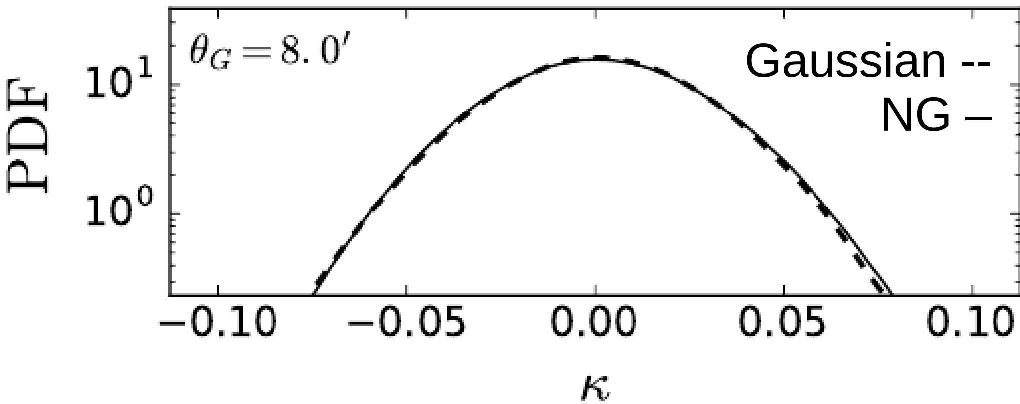


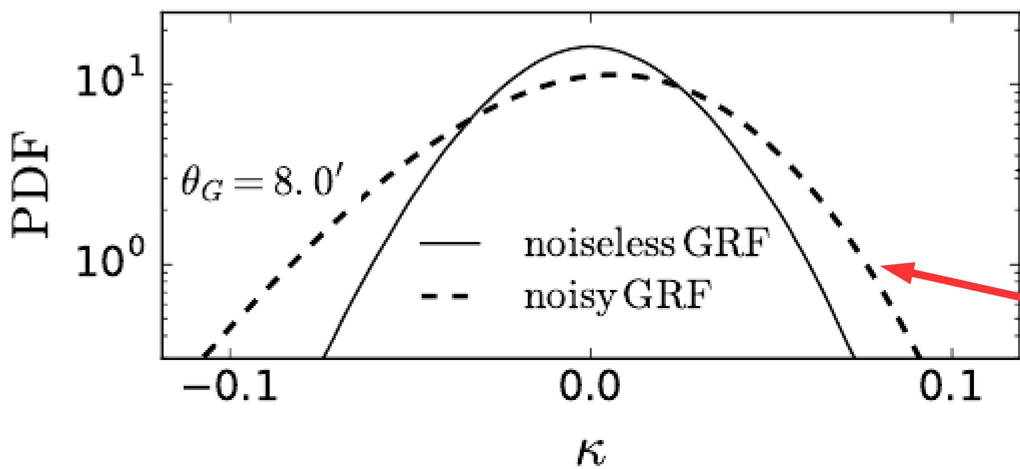
VB, Sherwin, Liu, Hill in prep.

Non-Gaussianity in 1-Point PDF

Liu, Hill, Sherwin, Petri, VB, Haiman 2016



Gaussian random fields (GRF) vs N-body



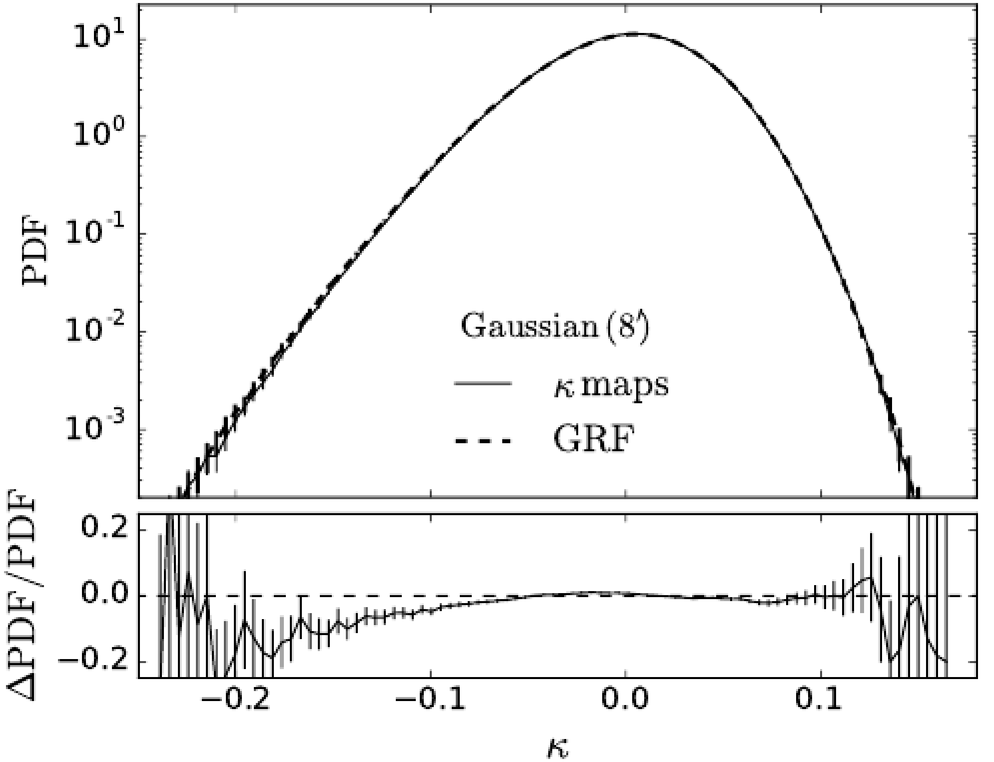
Gaussian random fields (GRF) before vs after reconstruction

Bias from reconstruction

Non-Gaussianity in 1-Point PDF

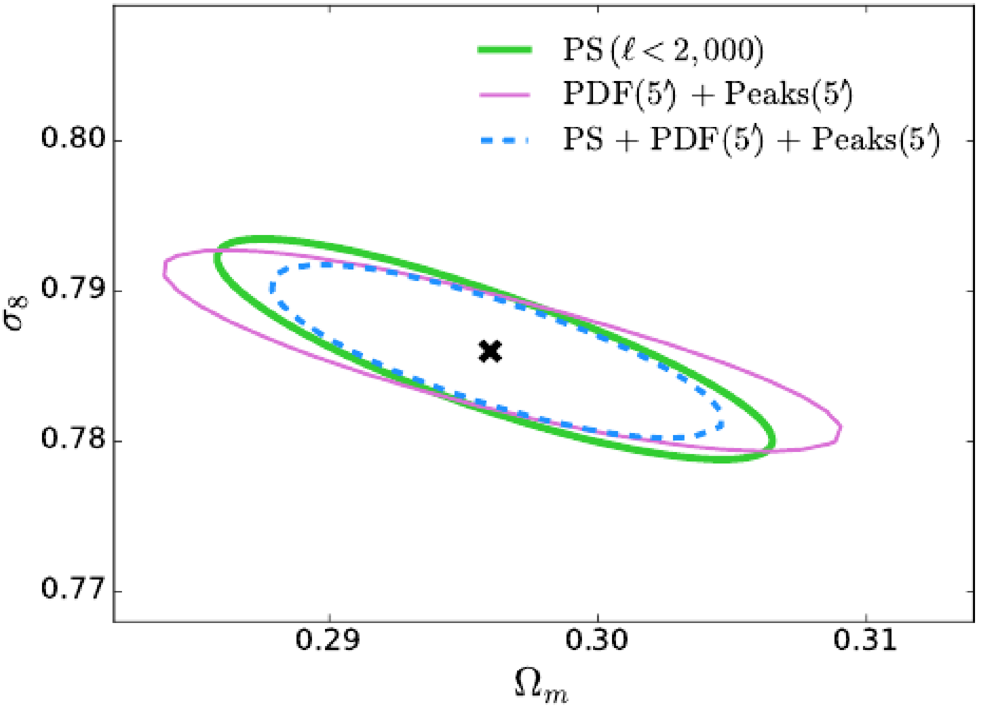
Gaussian vs Non-Gaussian PDF after reconstruction

← 9 σ difference for Wiener filtered maps



Liu, Hill, Sherwin, Petri, VB, Haiman 2016

additional constraints from PDF and Peaks



Conclusions

- The **CMB lensing potential** is **non-Gaussian**

  ← non-Gaussianity of the large-scale structure

  ← correlated deflections (post-Born corrections)

- The **bispectrum** of the lensing potential induces a **bias** to measurements of the lensing power spectrum
- For temperature-based reconstruction the bias from LSS alone is of order **~3σ for a Stage-III** experiment
- **Post-Born** corrections seem to reduce its magnitude to **<1σ** for single bandpower → but cumulative effect matters!
- Its exact magnitude still needs to be **confirmed with simulations**

  but preliminary results look promising

Outlook

- Evaluation of the bias for **polarization-based reconstruction**

  (probably reduced for EB-EB but possibly similar for EE-EE)

- Evaluation of **bias in cross-correlation** measurements with other tracers of large-scale structure

- Higher-order statistics are now becoming detectable

  (PDF, peak counts, bispectrum)

  → need to characterize biases/noise in measurements of these statistics

Outlook

- Evaluation of the bias for **polarization-based reconstruction**

As measurement precision of CMB lensing increases we need a more and more refined theoretical modeling

→ lots of work still ahead to make full use of the upcoming data

(PDF, peak counts, bispectrum)

→ need to characterize biases/noise in measurements of these statistics

Backups

- CMB Lensing Theory
- CMB Lensing Reconstruction
- Lensing potential bispectrum

CMB lensing potential

$$C_L^{\phi\phi} = \int_0^{\chi_*} \mathrm{d}\chi \frac{W(\chi)^2}{\chi^2} \frac{\gamma(\chi)^2}{(L/\chi)^4} P_\delta(L/\chi;\chi) \qquad \gamma(\chi) \equiv \frac{3}{2}\frac{H_0^2\,\Omega_{\mathrm{m}0}}{c^2 a(\chi)}$$

CMB lensing parameter constraints

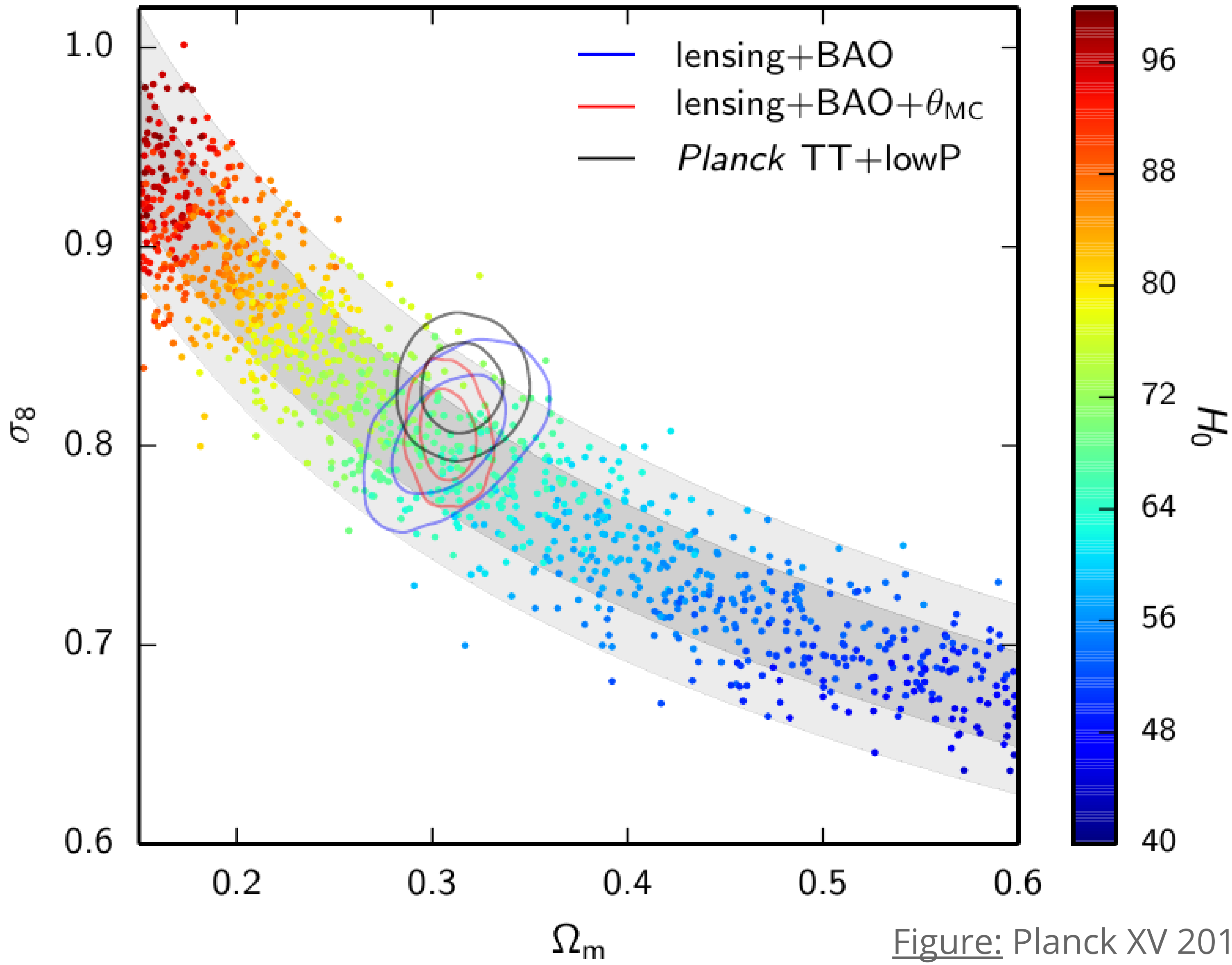


Figure: Planck XV 2015

CMB lensing parameter constraints

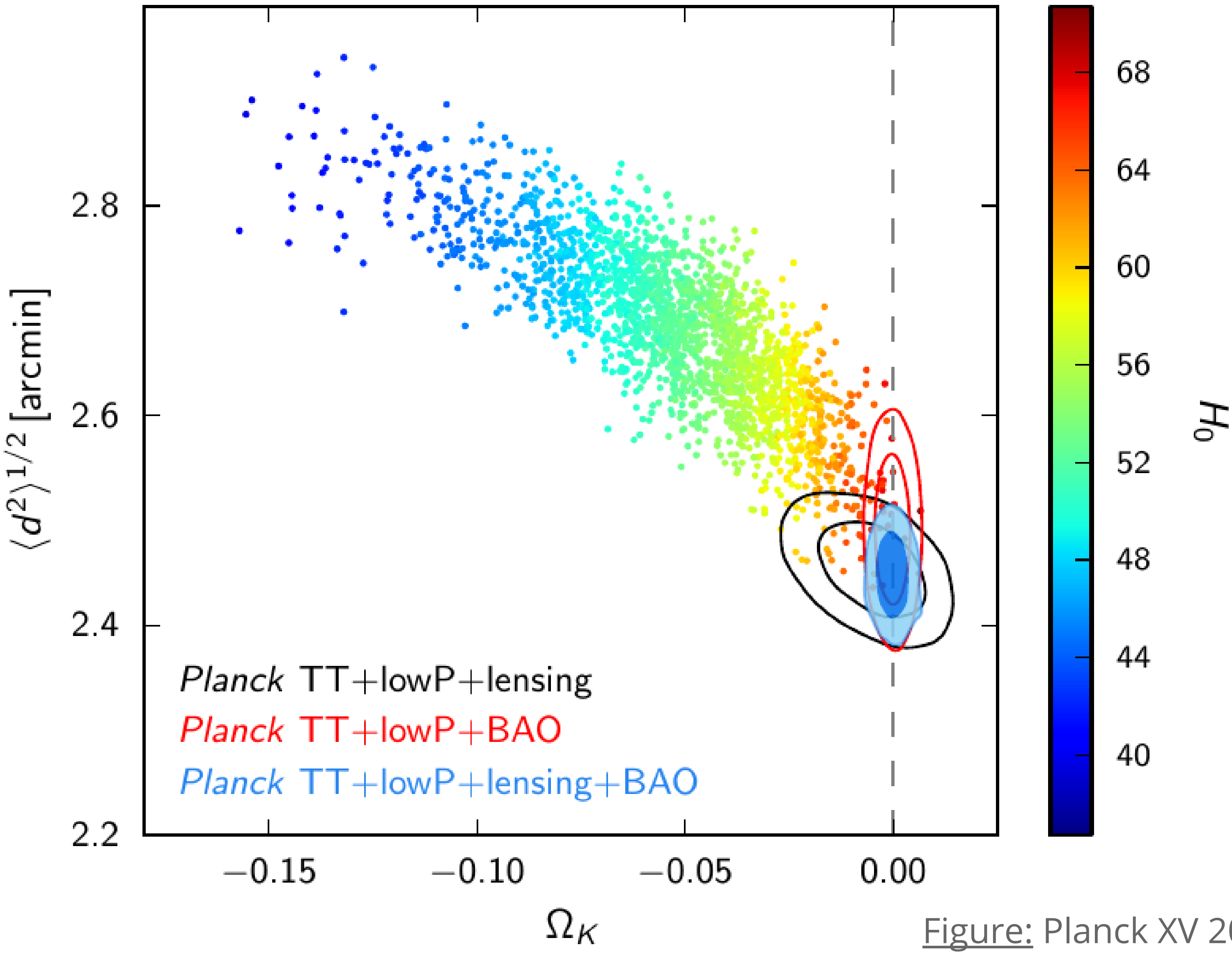


Figure: Planck XV 2015

Planck constraints on sum of neutrino masses

$$\sum m_\nu < 0.72 \text{ eV} \quad \textit{Planck} \text{ TT+lowP}\,; \tag{54a}$$

$$\sum m_\nu < 0.21 \text{ eV} \quad \textit{Planck} \text{ TT+lowP+BAO}\,; \tag{54b}$$

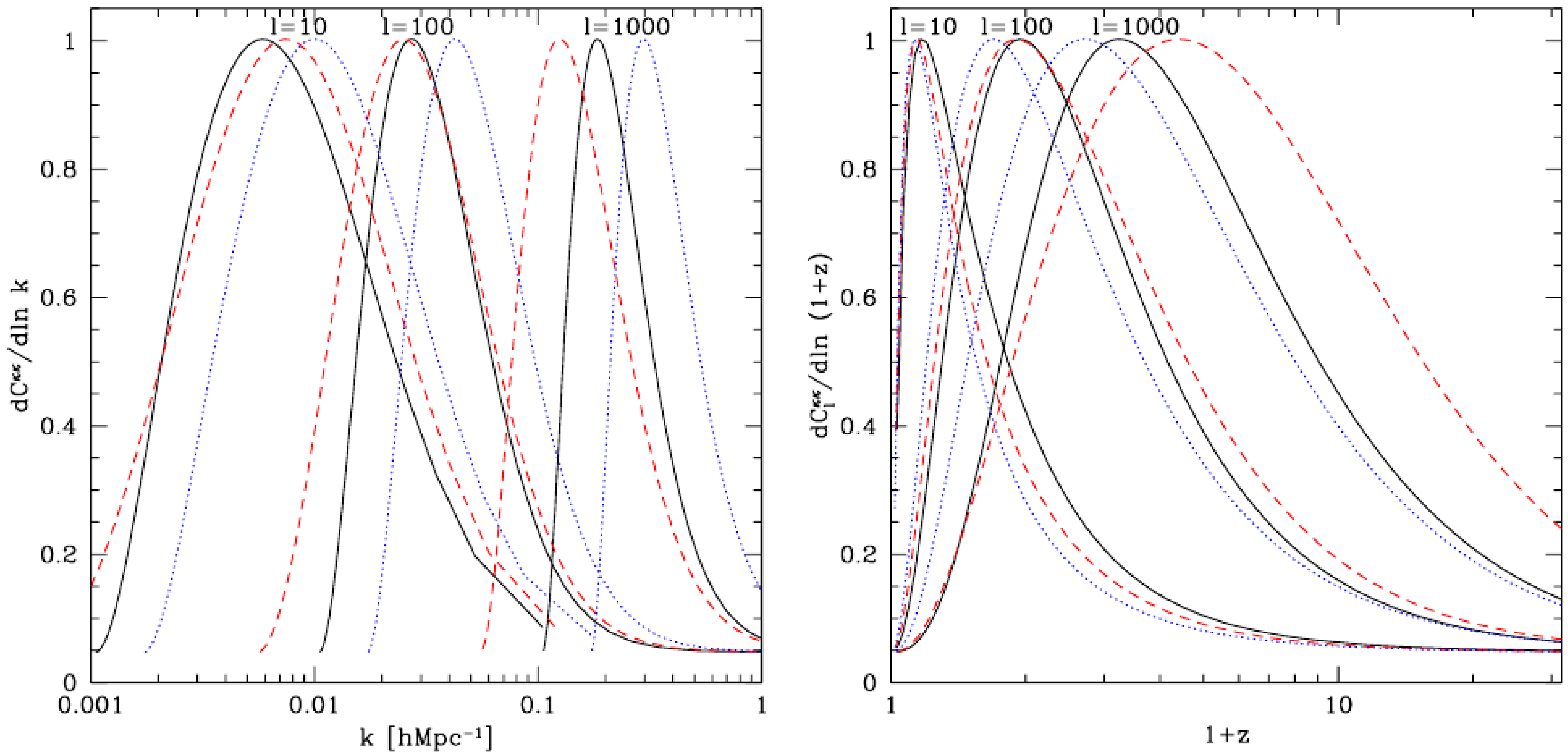
$$\sum m_\nu < 0.49 \text{ eV} \quad \textit{Planck} \text{ TT, TE, EE+lowP}\,; \tag{54c}$$

$$\sum m_\nu < 0.17 \text{ eV} \quad \textit{Planck} \text{ TT, TE, EE+lowP+BAO}\,. \tag{54d}$$

Planck XIII 2015

Convergence Power Spectrum scale and redshift dependence

Figure: Zaldarriaga & Seljak 1998



CMB lensing reconstruction

Quadratic estimator (Hu & Okamoto 2002)

$$\hat{\phi}(\mathbf{L}) = A_L \int_{\mathbf{l}} g(\mathbf{l},\mathbf{L}) \tilde{T}_{\rm expt}(\mathbf{l}) \tilde{T}^*_{\rm expt}(\mathbf{l}-\mathbf{L})$$

Power spectrum estimate ← mean square estimator

$$\langle \hat{\phi}(\mathbf{L}) \hat{\phi}^*(\mathbf{L}') \rangle_{(\phi,T)} = A_L^2 \int_{\mathbf{l}_1} \int_{\mathbf{l}_2} g(\mathbf{l}_1,\mathbf{L})\, g(\mathbf{l}_2,\mathbf{L}') \langle \tilde{T}_{\rm expt}(\mathbf{l}_1)\, \tilde{T}_{\rm expt}(\mathbf{L}-\mathbf{l}_1)\, \tilde{T}_{\rm expt}(-\mathbf{l}_2)\, \tilde{T}_{\rm expt}(\mathbf{l}_2-\mathbf{L}') \rangle$$

CMB lensing reconstruction

Quadratic estimator

Weight

$$g(\mathbf{l},\mathbf{L}) = \frac{(\mathbf{L}-\mathbf{l})\cdot\mathbf{L}C^{\tilde{T}\tilde{T}}_{|\mathbf{L}-\mathbf{l}|} + \mathbf{l}\cdot\mathbf{L}C^{\tilde{T}\tilde{T}}_{l}}{2C^{\tilde{T}}_{l,\mathrm{expt}}C^{\tilde{T}\tilde{T}}_{|\mathbf{L}-\mathbf{l}|,\mathrm{expt}}}$$

Normalization

$$A_L^{-1} = 2\int_{\mathbf{l}} g(\mathbf{l},\mathbf{L})\mathbf{l}\cdot\mathbf{L}C^{\tilde{T}\tilde{T}}_{l}$$

Gaussian variance

$$\sigma^2(L) = \frac{1}{f_{\mathrm{sky}}}\frac{2}{(2L+1)}\left(N_L^{(0)} + C_L^{\phi\phi} + N_L^{(1)}\right)^2$$

CMB lensing reconstruction

Representative experiment	Stage-IV(CMB-S4)	Stage-III(AdvancedACT-like)	Planck
θ_{FWHM}[arcmin]	1.0	1.4	7.0
σ_N^{TT}[μKarcmin]	1.0	6.0	30.0
f_{sky}	0.5	0.4	0.63

Lensing potential bispectrum

Weighted projection of LSS bispectrum

$$B_\phi(\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}_3) = -\int_0^{\chi_*} \mathrm{d}\chi\, \chi^2 W(\chi)^3 \frac{\gamma(\chi)^3}{(l_1 l_2 l_3)^2} B_\delta(l_1/\chi, l_2/\chi, l_3/\chi; \chi)$$

LSS bispectrum in Eulerian perturbation theory at leading order

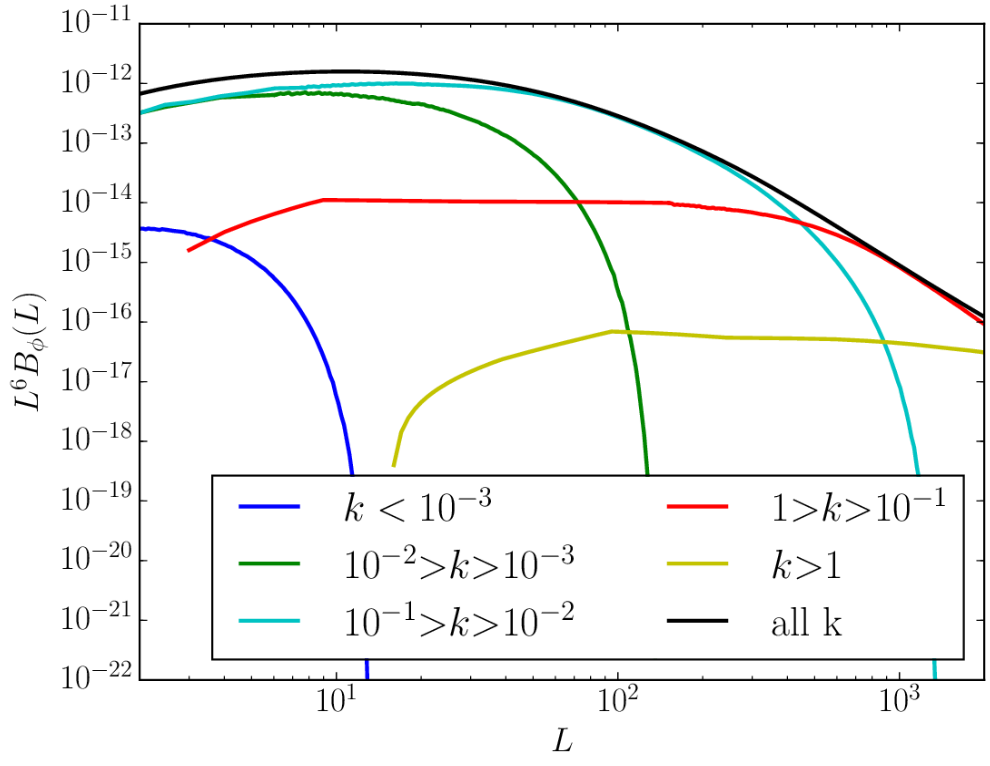
$$B_\delta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3; \eta) = 2\, F_2(\mathbf{k}_1, \mathbf{k}_2) P_\delta(k_1, \eta) P_\delta(k_2, \eta) + (1 \leftrightarrow 3) + (2 \leftrightarrow 3)$$

$$F_2(\mathbf{k}_i, \mathbf{k}_j) = F_2(\mathbf{k}_i, \mathbf{k}_j) = \frac{5}{7} + \frac{1}{2}\left(\frac{k_i}{k_j} + \frac{k_j}{k_i}\right) \hat{\mathbf{k}}_i \cdot \hat{\mathbf{k}}_j + \frac{2}{7}(\hat{\mathbf{k}}_i \cdot \hat{\mathbf{k}}_j)^2$$

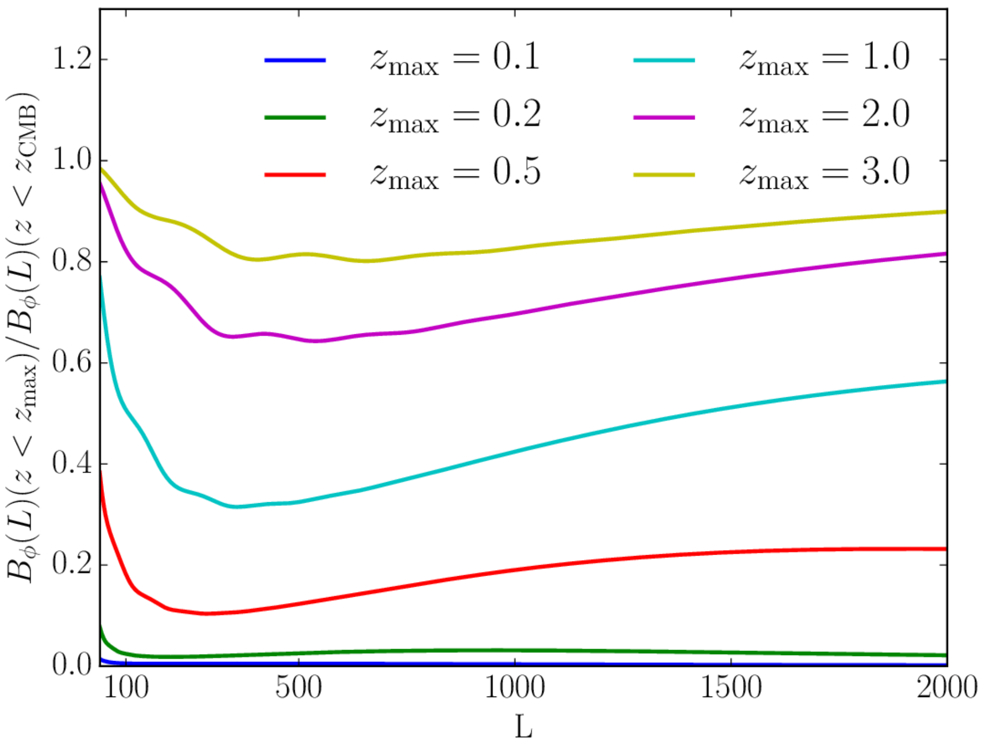
Lensing potential bispectrum

- Contributions to equilateral configuration

→ From different LSS modes



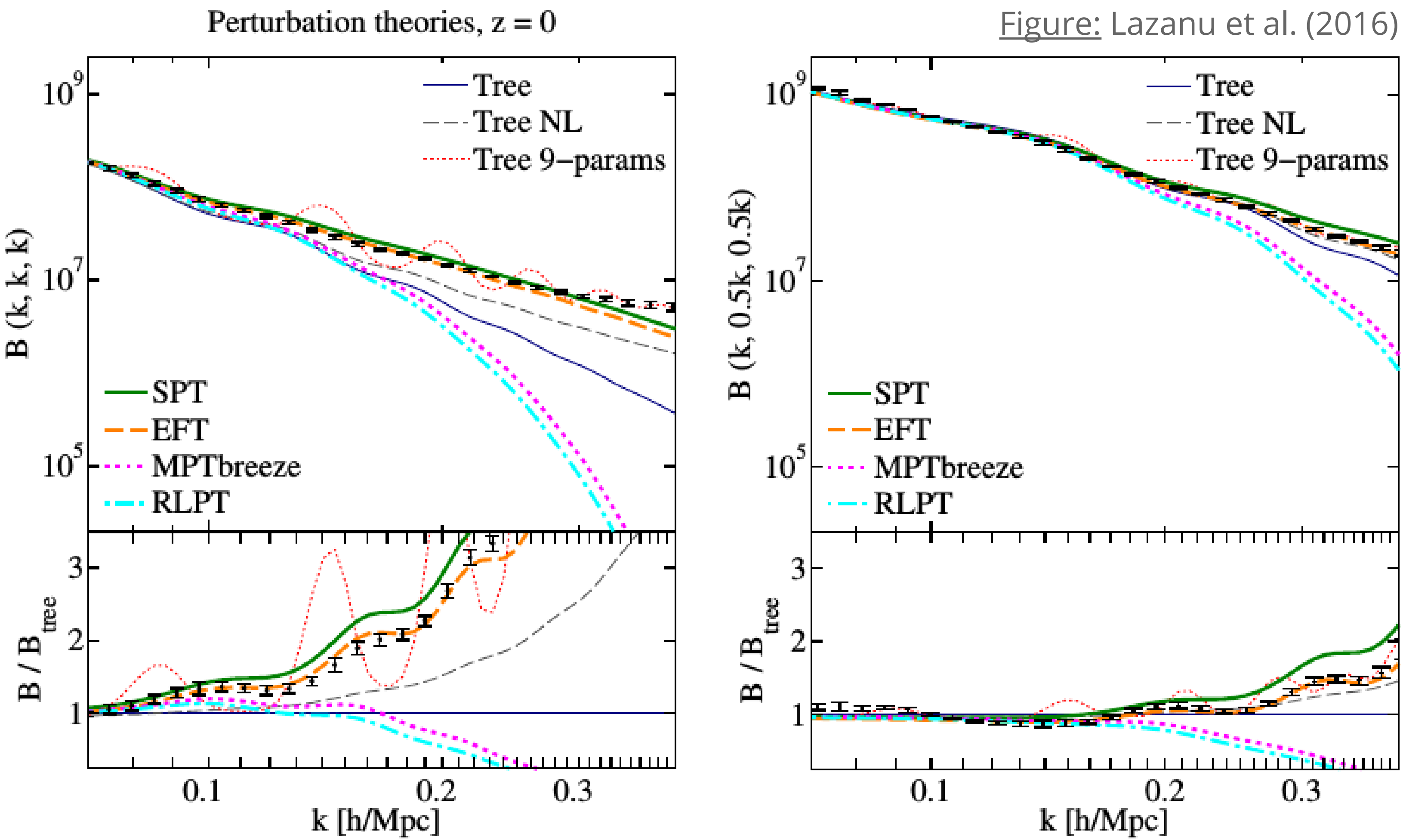
→ From different redshifts



Bispectrum of large-scale structure

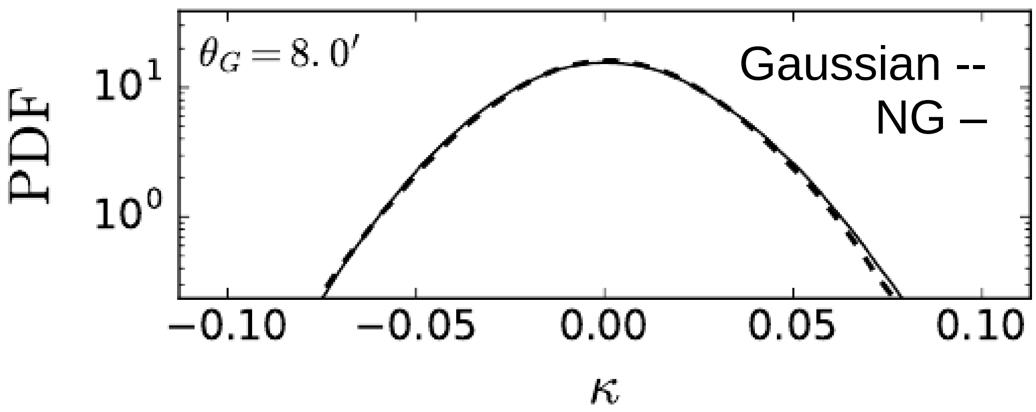
- No exact analytical model for LSS bispectrum
  - Standard perturbation theory at leading order (tree-level)

Figure: Lazanu et al. (2016)

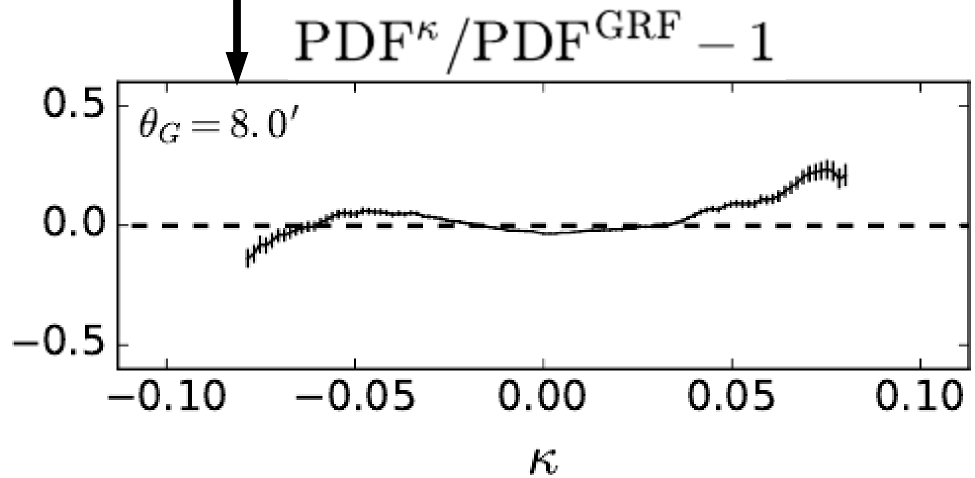


1-Point PDF

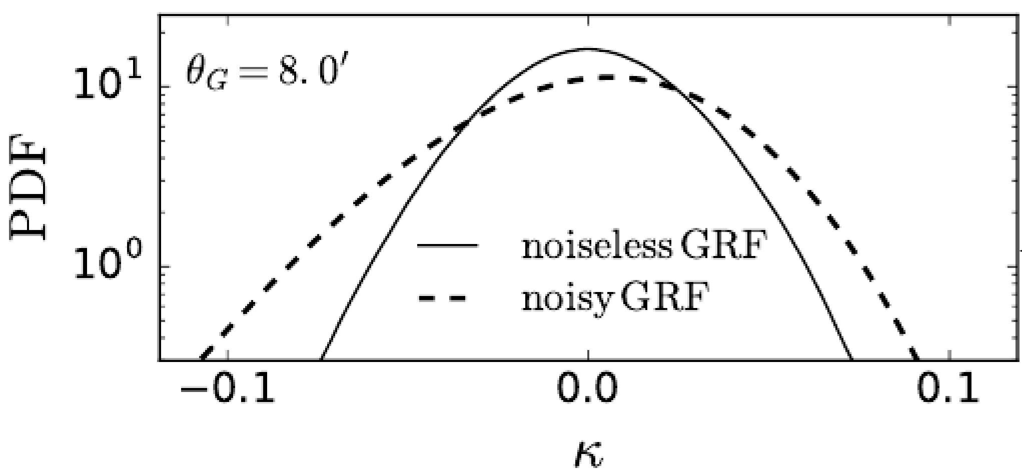
Liu, Hill, Sherwin, Petri, VB, Haiman 2016



Before reconstruction



Bias from reconstruction



Gaussian random fields (GRF) before vs after reconstruction

Bispectrum of large-scale structure

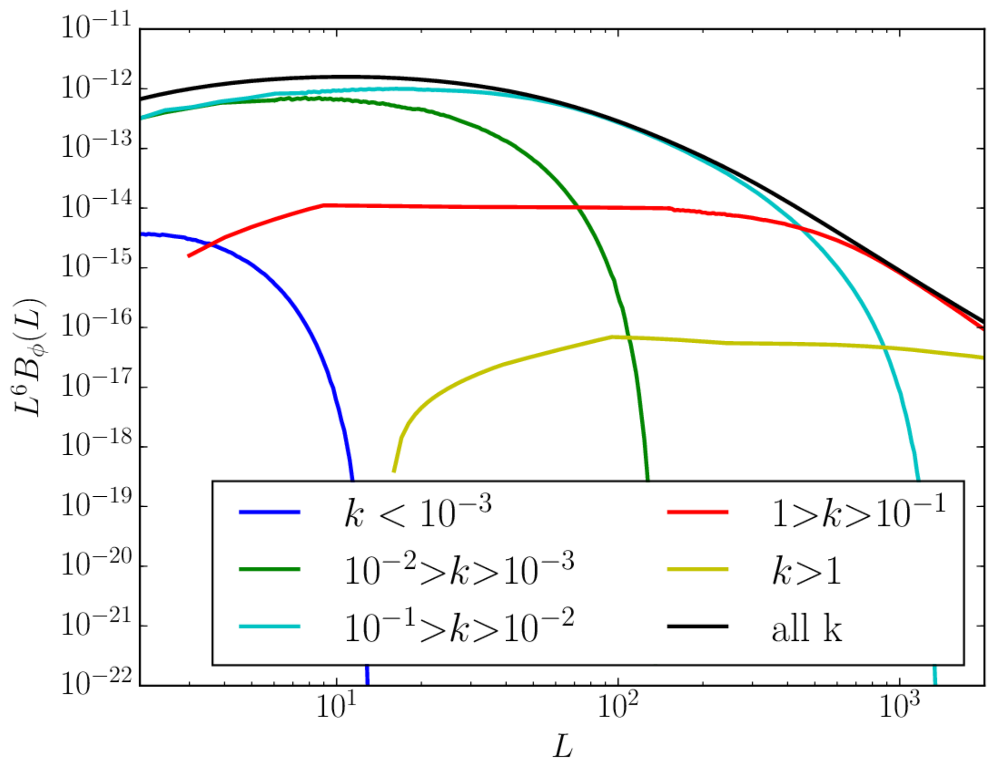
- No exact analytical model for LSS bispectrum
- Eulerian perturbation theory at leading-order (tree-level)

$$B_\delta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3; \eta) = 2\, F_2(\mathbf{k}_1, \mathbf{k}_2) P_\delta(k_1, \eta) P_\delta(k_2, \eta) + (1 \leftrightarrow 3) + (2 \leftrightarrow 3)$$

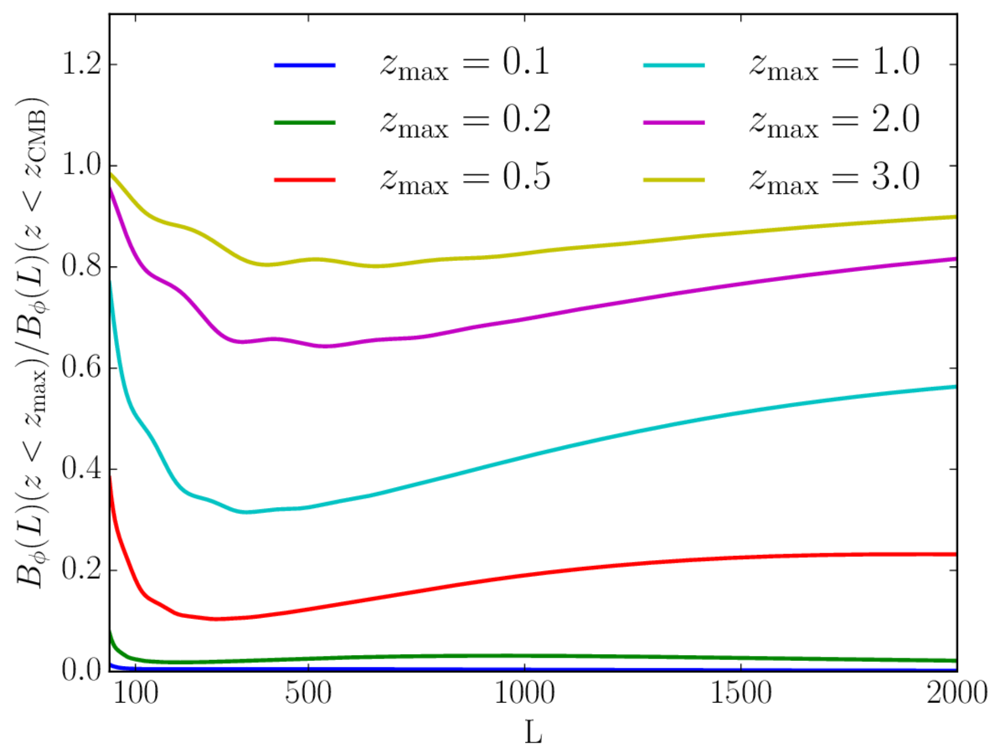
Lensing potential bispectrum

- Contributions to equilateral configuration

→ From different LSS modes

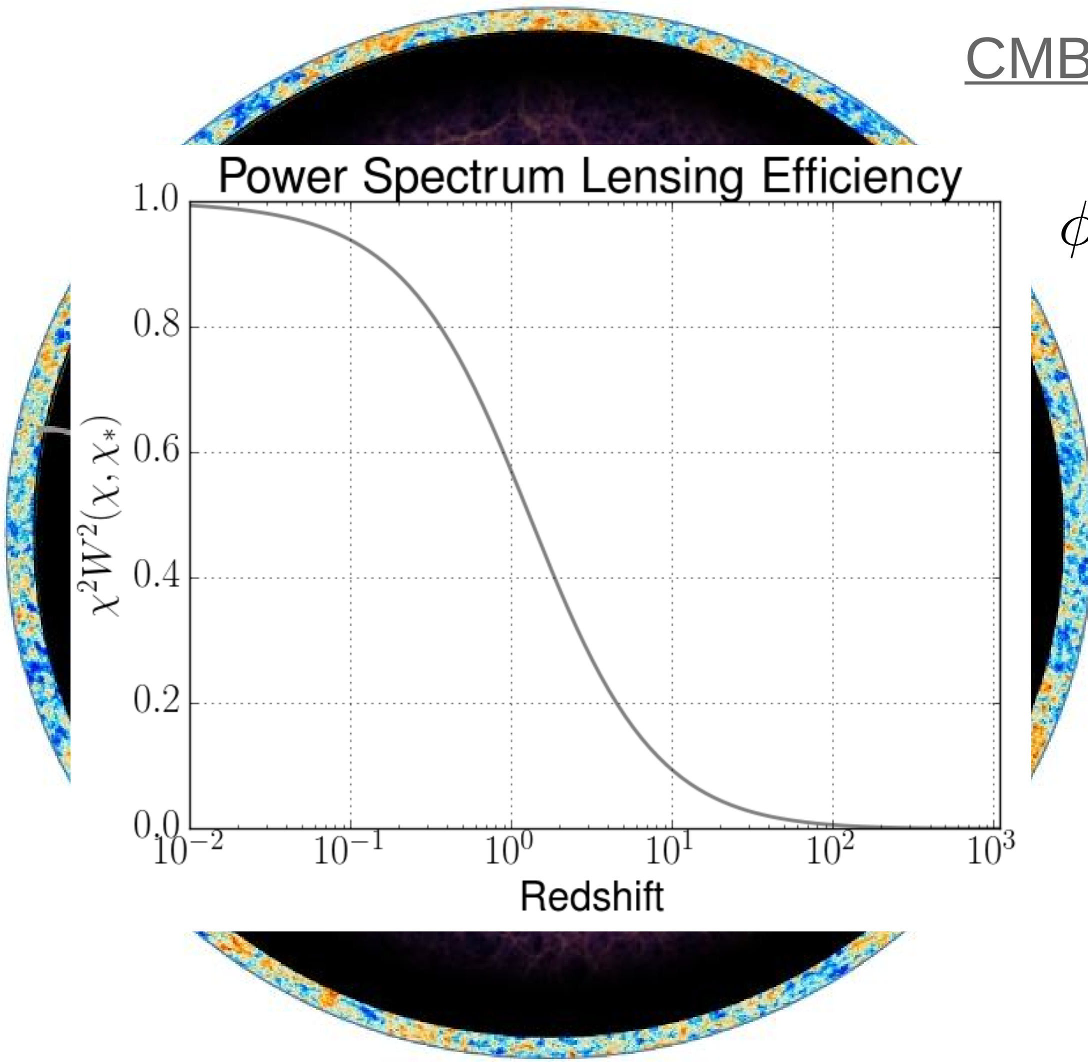


→ From different redshifts



VB, Schmittfull, Sherwin 2016

Lensing of the cosmic microwave background



CMB lensing potential

$$\phi(\mathbf{x}) = -2 \int_0^{\chi_*} \mathrm{d}\chi \, W(\chi) \, \psi(\mathbf{x}, \chi)$$

$$W(\chi) = \frac{f_k(\chi_* - \chi)}{f_k(\chi_*) f_k(\chi)}$$

Power spectrum

$$C_L^{\phi\phi} \leftarrow P_\delta(L/\chi; \chi)$$