

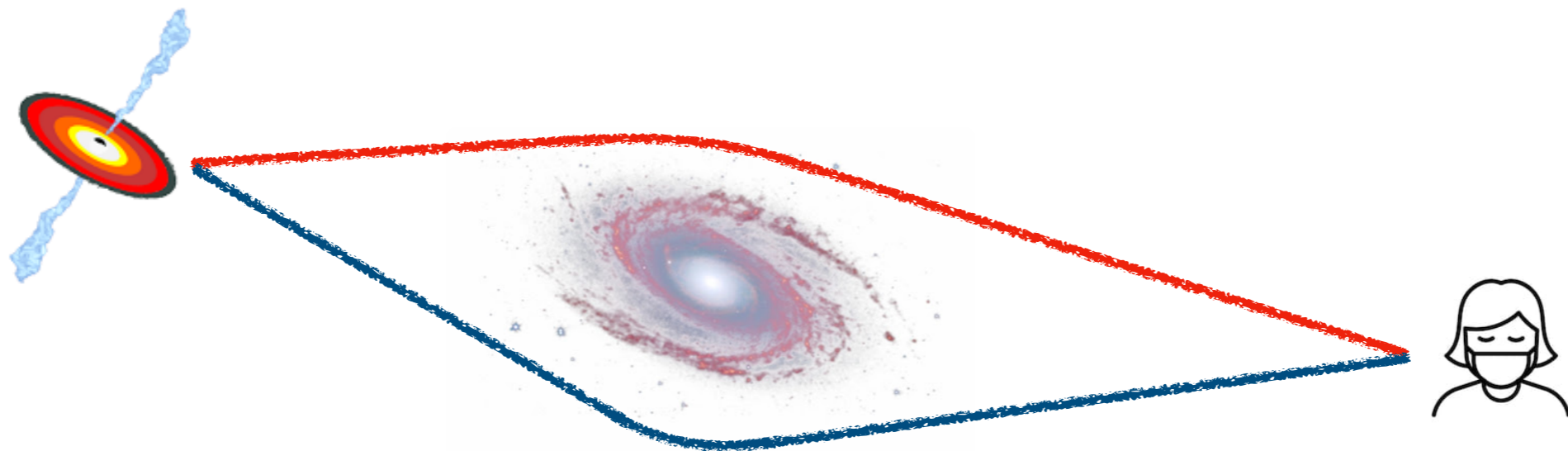
Gravitational lensing measurements of the Hubble parameter: challenges and opportunities

Kfir Blum (Weizmann Institute)

KB, Castorina, Simonović, 2001.07182

KB, Teodori, 2105.10873

Teodori, KB, Castorina, Simonović, Soreq, 2201.05111



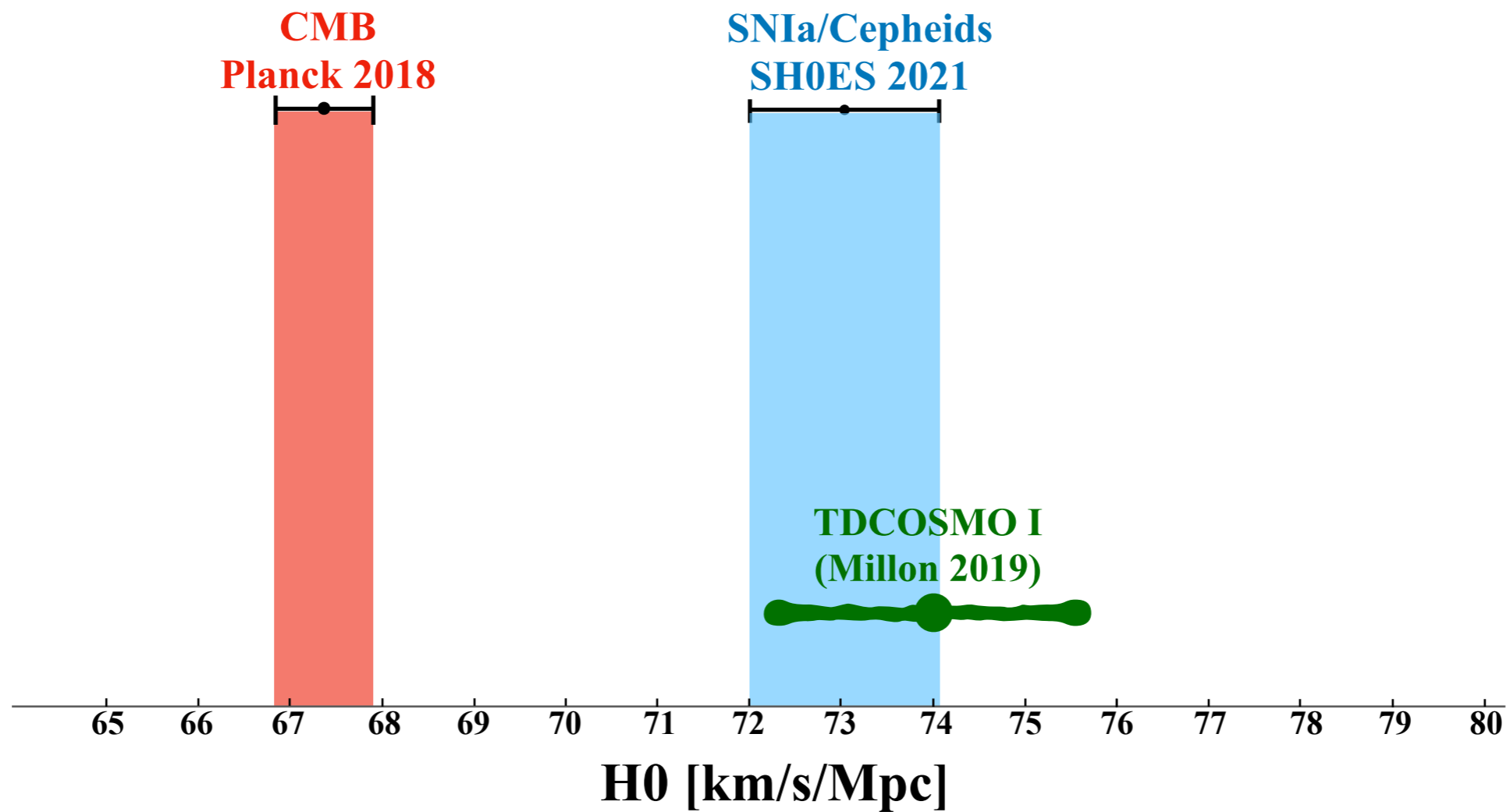
H0 tension

SNIa and **gravitational lensing** agree?

TDCOSMO

<http://www.tdcosmo.org/projects.html>

- H0LiCOW
- COSMOGRAIL
- STRIDES
- SHARP
- COSMICLENs



H0 tension

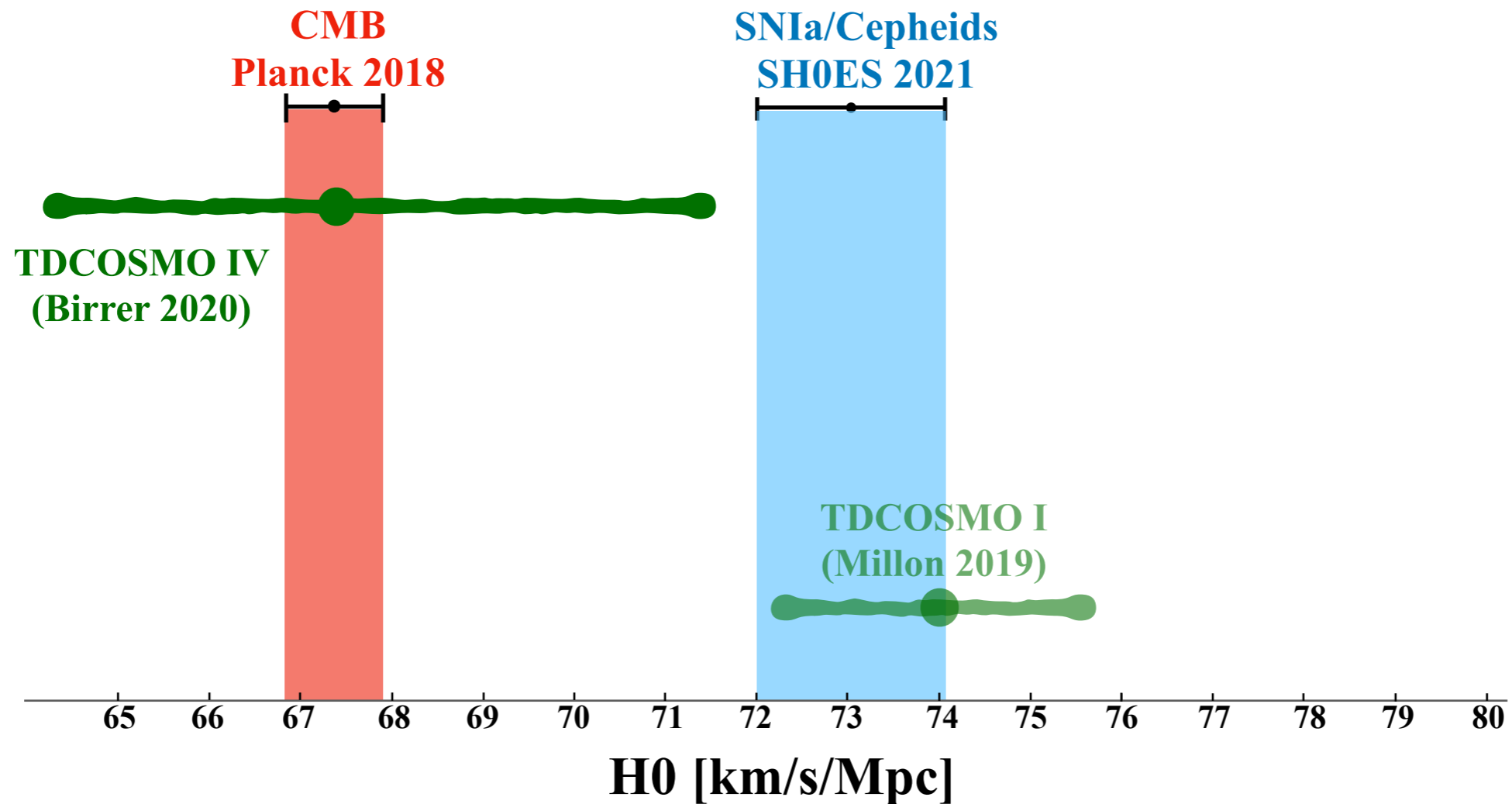
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TDCOSMO

<http://www.tdcosmo.org/projects.html>

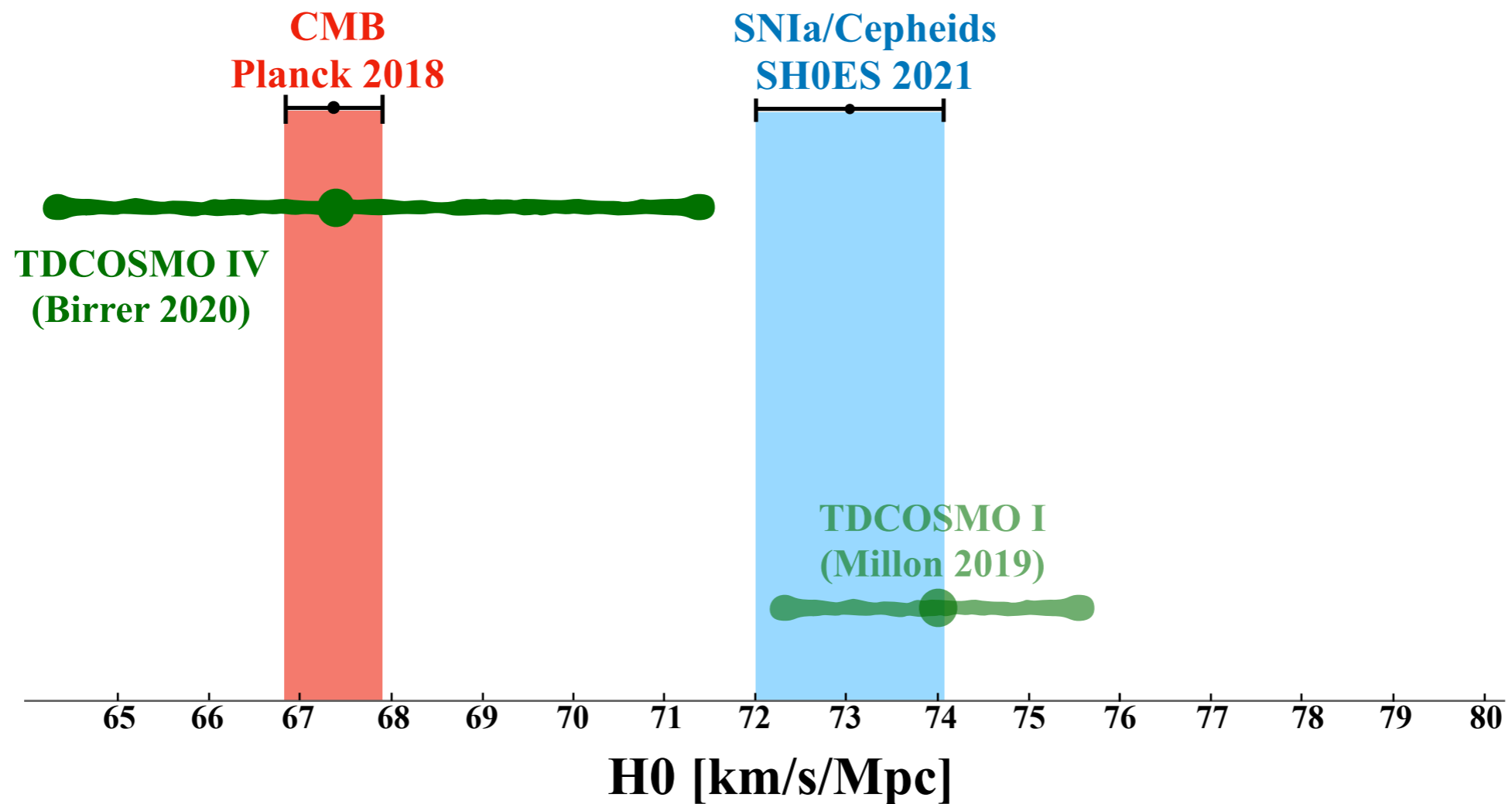
- H0LiCOW
- COSMOGRAIL
- STRIDES
- SHARP
- COSMICLENs

...lensing out of the game?



Plan:

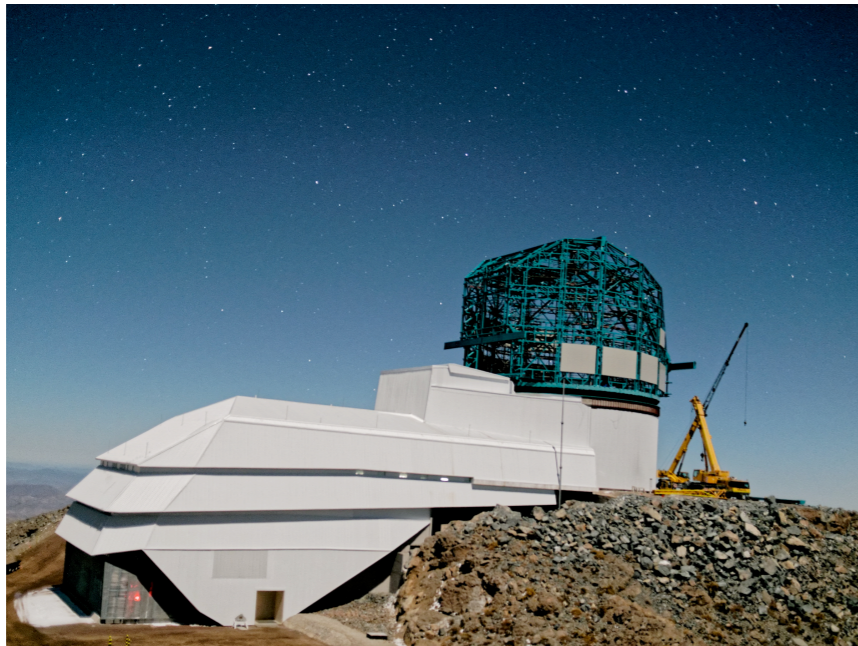
1. Recap: how lensing measures H_0
2. Challenges: modeling degeneracy (TDCOSMO I \rightarrow IV)
3. Opportunities: galactic structure



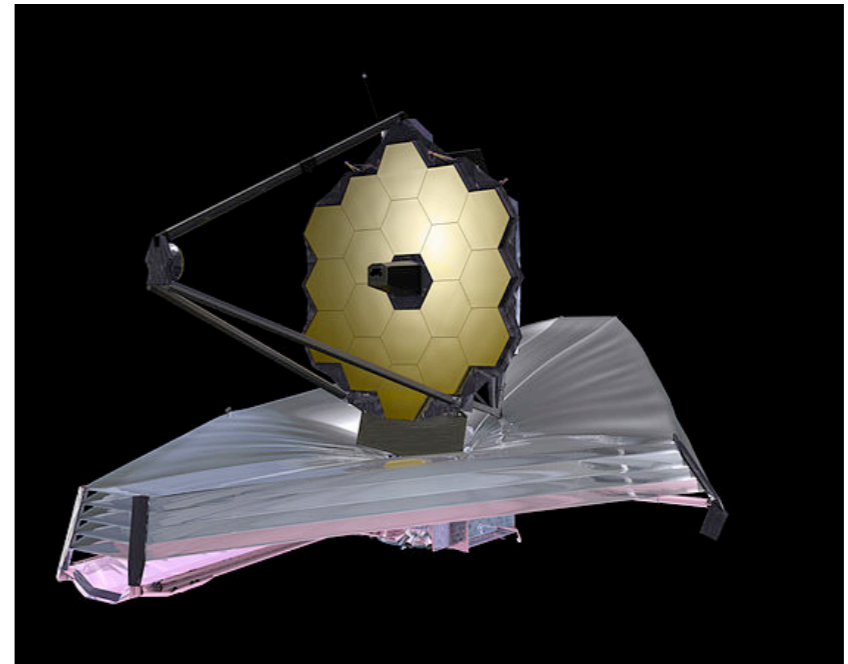
Plan:

1. Recap: how lensing measures H_0
2. Challenges: modeling degeneracy (TDCOSMO I \rightarrow IV)
3. **Opportunities: galactic structure**

LSST: 100's of strongly lensed variable quasars
Oguri, Marshall, 1001.2037



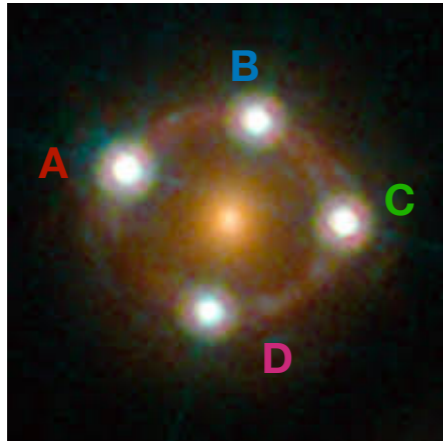
JWST: improved kinematics
Yıldırım, Suyu, Halkola, 1904.07237
Birrer, Treu, 2008.06157



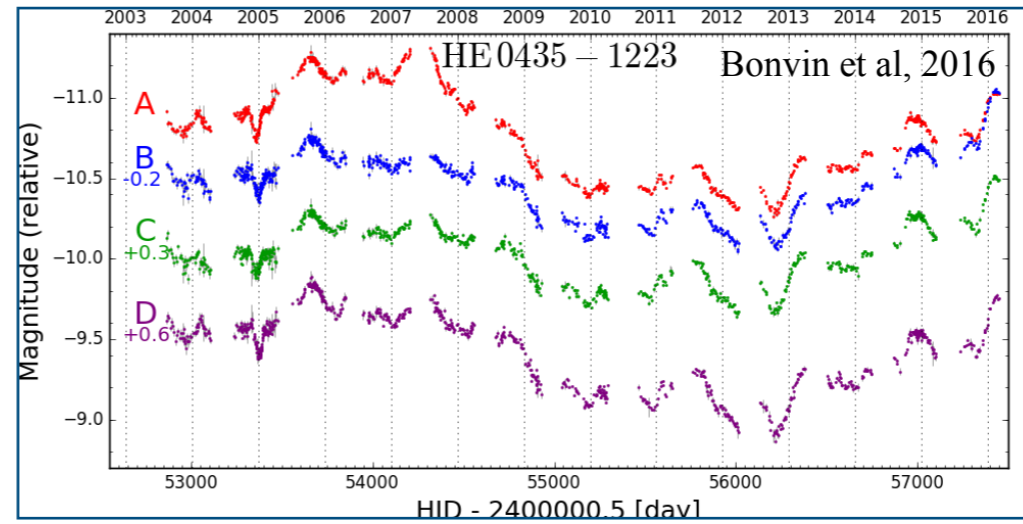
1. Recap: how lensing measures H_0

Observables:

- Extended source image



- Time delay Δt

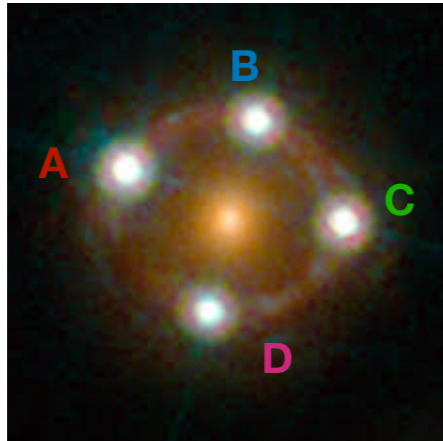


1. Recap: how lensing measures H0

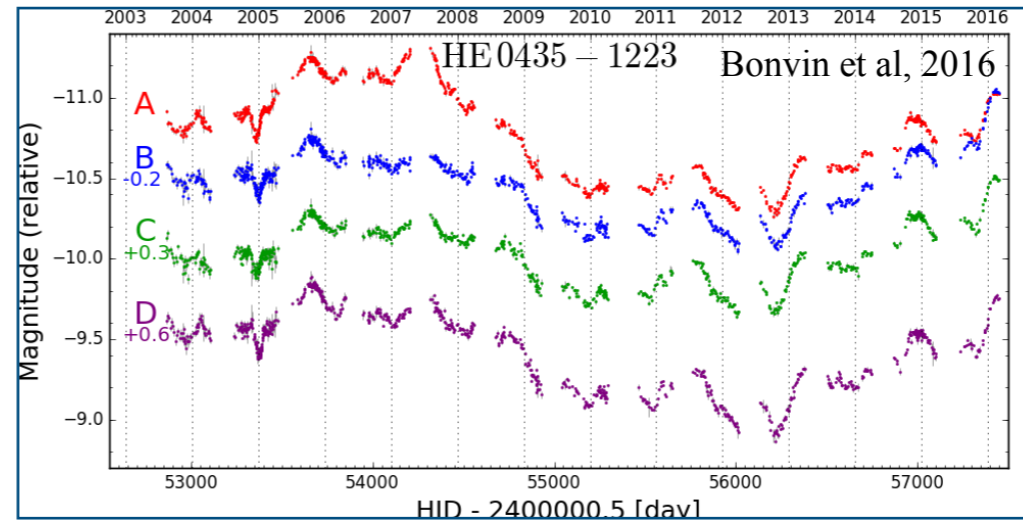
Observables:

- Extended source image

$$\vec{\theta} = \vec{\beta} + \vec{\alpha}(\vec{\theta})$$

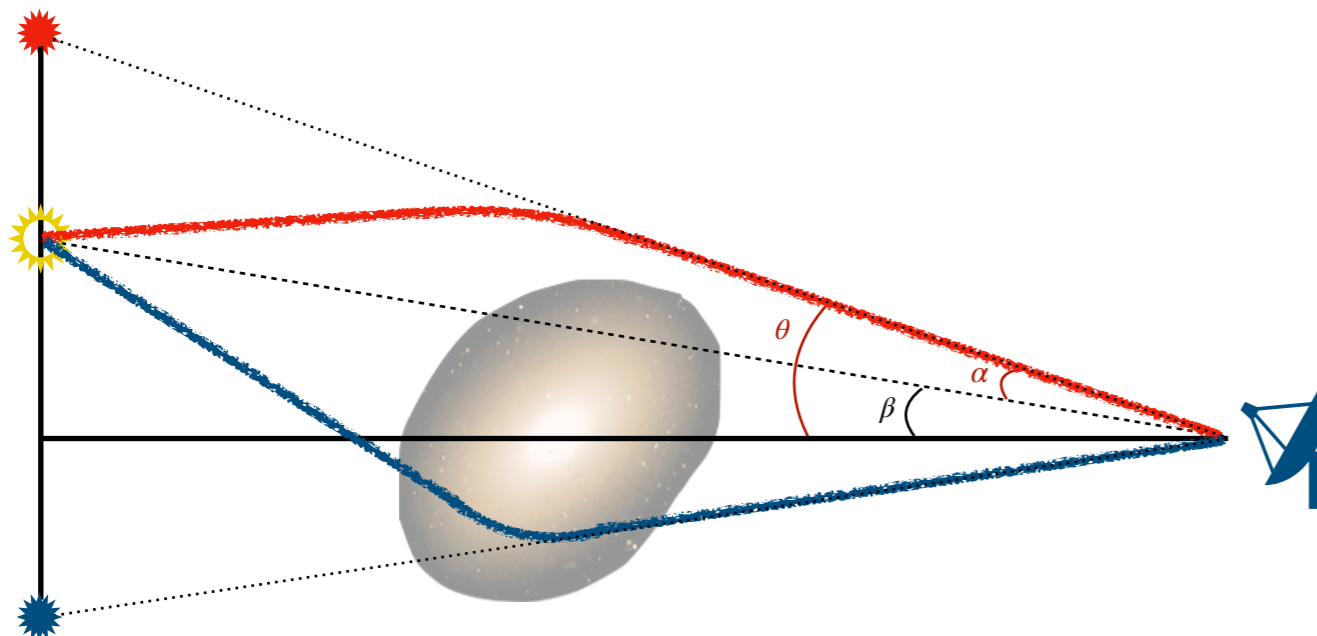


- Time delay Δt



$$\kappa(\vec{\theta}) = \frac{\Sigma(\vec{\theta})}{\Sigma_{\text{crit}}} = \frac{1}{2} \vec{\nabla}_{\theta} \cdot \vec{\alpha} = \frac{1}{2} \vec{\nabla}_{\theta}^2 \psi$$

$$\Sigma_{\text{crit}} = \frac{d_A(z_s, 0)}{4\pi G d_A(z_l, 0) d_A(z_s, z_l)}$$



1. Recap: how lensing measures H0

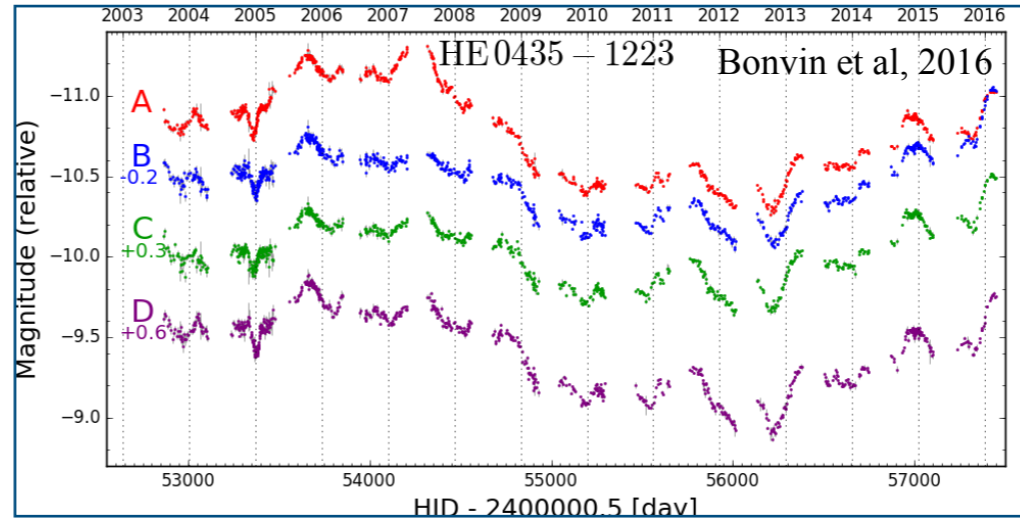
Observables:

- Extended source image

$$\vec{\theta} = \vec{\beta} + \vec{\alpha}(\vec{\theta})$$



- Time delay $\Delta t_{AB} = D_{\Delta t} \Delta \tau_{AB}$

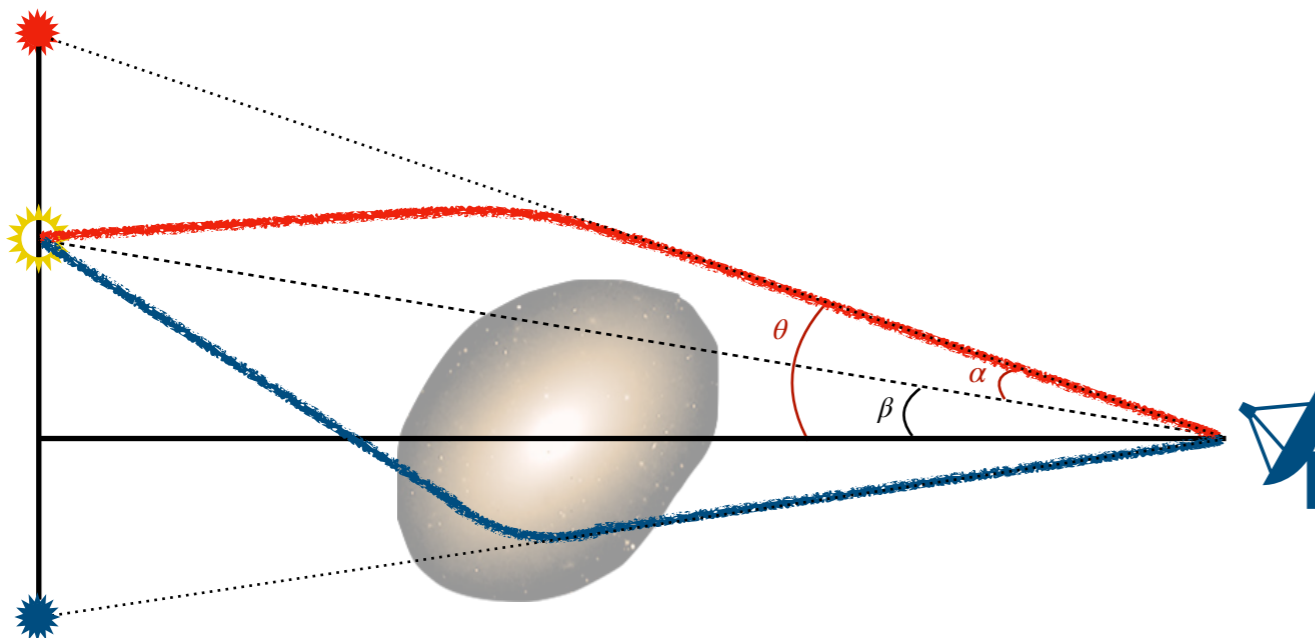


$$\kappa(\vec{\theta}) = \frac{\Sigma(\vec{\theta})}{\Sigma_{\text{crit}}} = \frac{1}{2} \vec{\nabla}_{\theta} \cdot \vec{\alpha} = \frac{1}{2} \vec{\nabla}_{\theta}^2 \psi$$

$$\Sigma_{\text{crit}} = \frac{d_A(z_s, 0)}{4\pi G d_A(z_l, 0) d_A(z_s, z_l)}$$

$$\tau(\vec{\theta}) = \frac{\vec{\theta}^2}{2} - \vec{\beta} \cdot \vec{\theta} - \psi(\vec{\theta})$$

$$D_{\Delta t} = (1 + z_l) \frac{d_A(z_l, 0) d_A(z_s, 0)}{d_A(z_s, z_l)}$$

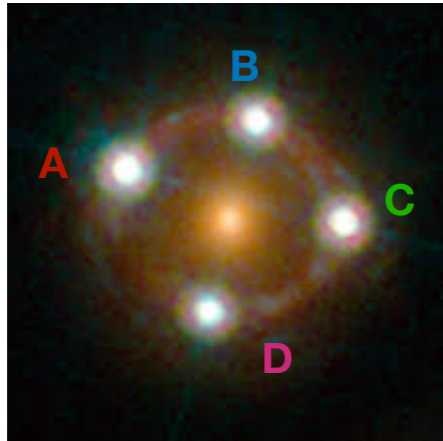


1. Recap: how lensing measures H0

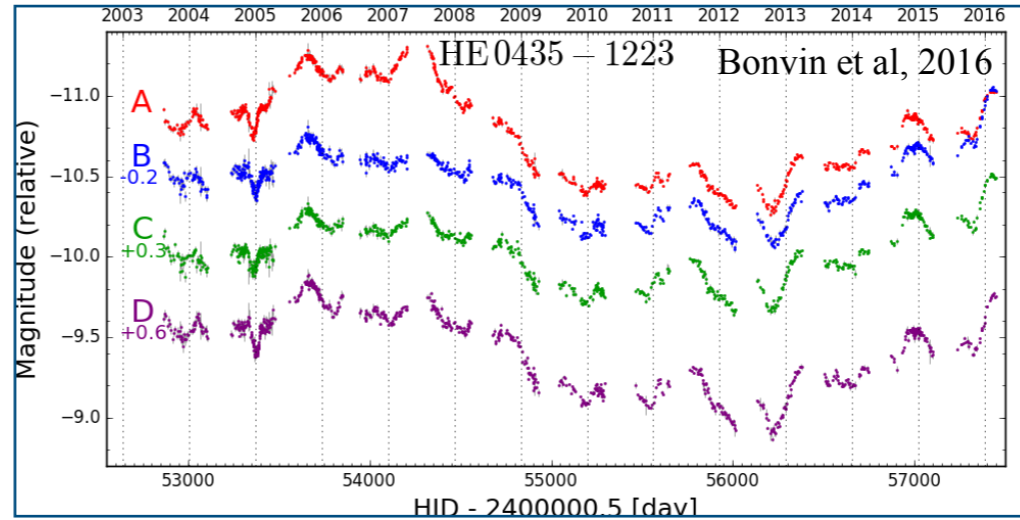
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$$D_{\Delta t} = (1 + z_l) \frac{d_A(z_l, 0) d_A(z_s, 0)}{d_A(z_s, z_l)}$$

- From the image, reconstruct a model $\kappa(\vec{\theta}), \vec{\beta} \rightarrow \tau(\vec{\theta})$
- Given the model and Δt , extract $D_{\Delta t} = \frac{\Delta t}{\Delta \tau} \propto \frac{1}{H_0}$


2. Challenges: modeling degeneracy


$$\begin{aligned}\vec{\theta} &= \vec{\beta} + \vec{\alpha} \\ &= \vec{\beta}_\lambda + \vec{\alpha}_\lambda \\ &= \lambda \vec{\beta} + \lambda \vec{\alpha} + (1 - \lambda) \vec{\theta} \\ &= \lambda (\vec{\beta} + \vec{\alpha} - \vec{\theta}) + \vec{\theta} \\ &= \vec{\theta}\end{aligned}$$

$$\kappa(\vec{\theta}) = \frac{\Sigma(\vec{\theta})}{\Sigma_{\text{crit}}} = \frac{1}{2} \vec{\nabla}_\theta \cdot \vec{\alpha} = \frac{1}{2} \vec{\nabla}_\theta^2 \psi$$

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2. Challenges: modeling degeneracy



$$\begin{aligned}
 \vec{\theta} &= \vec{\beta} + \vec{\alpha} \\
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 \end{aligned}$$

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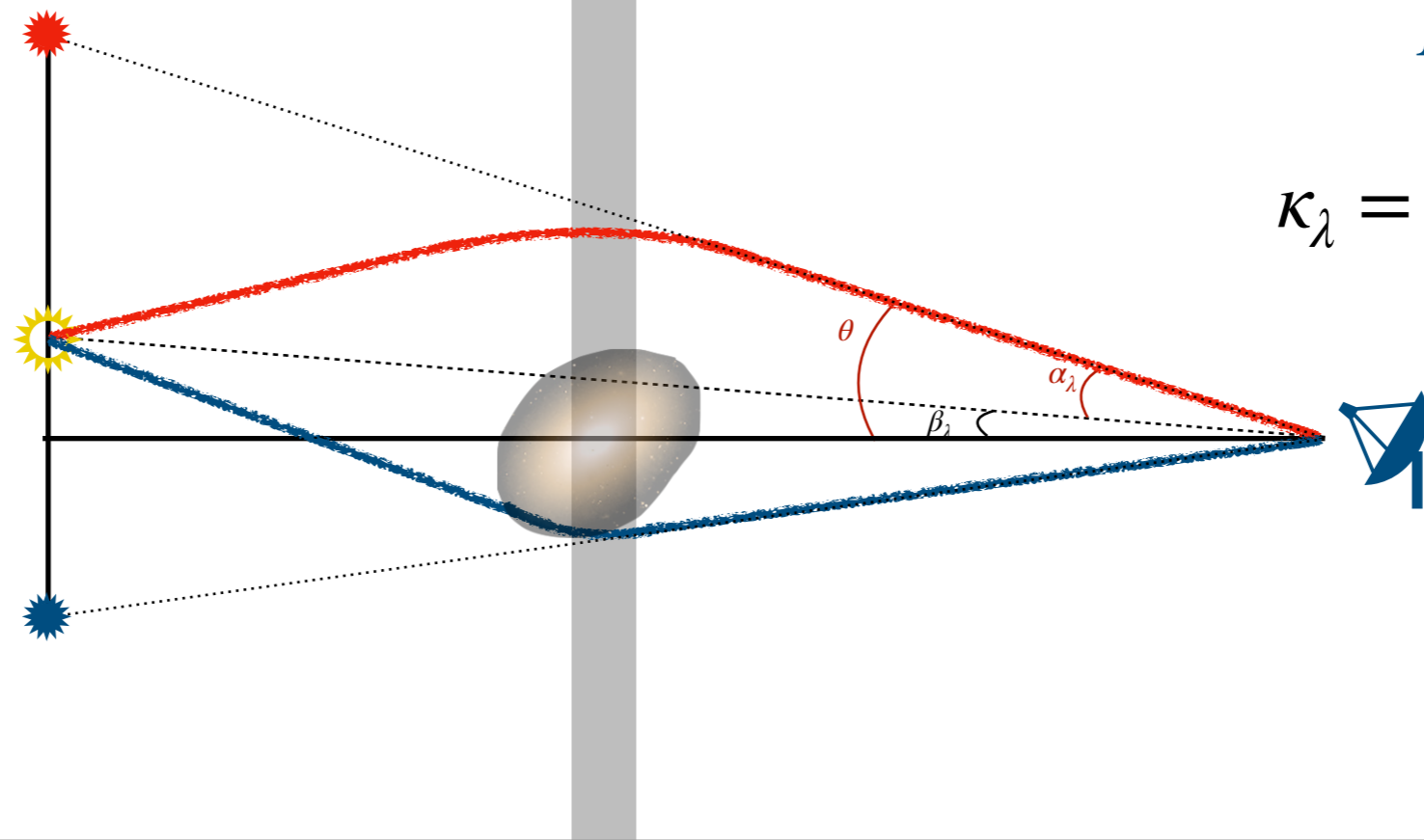
$$\tau(\vec{\theta}) = \frac{\vec{\theta}^2}{2} - \vec{\beta} \cdot \vec{\theta} - \psi(\vec{\theta})$$

$$\vec{\theta} = \vec{\beta} + \vec{\alpha}(\vec{\theta}) \quad \leftarrow \quad \vec{\nabla}_\theta \tau(\vec{\theta}) = 0$$

Imaging + time delay **cannot** measure H_0 .

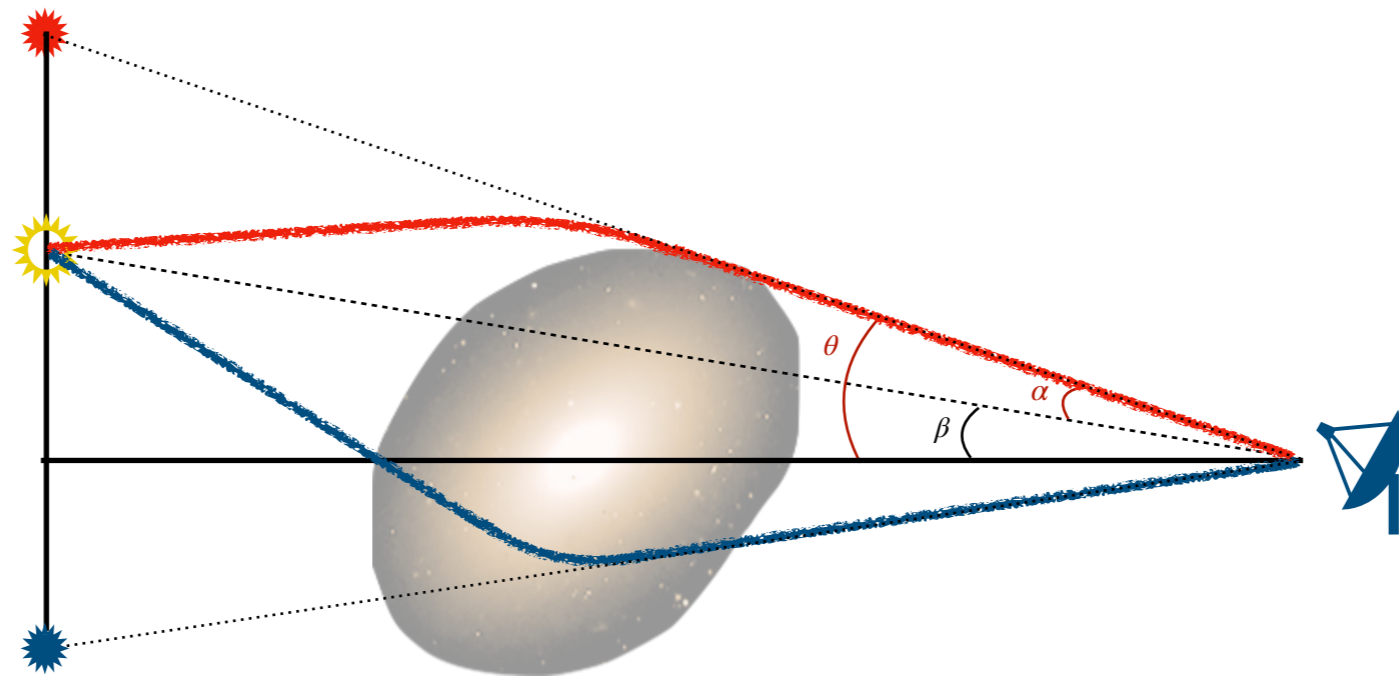
It cannot access the **overall normalization** of $\tau(\vec{\theta})$,
and the normalization of $\tau(\vec{\theta})$ is **essentially** H_0 .

$$D_{\Delta t} = \frac{\Delta t}{\Delta \tau} \propto \frac{1}{H_0}$$



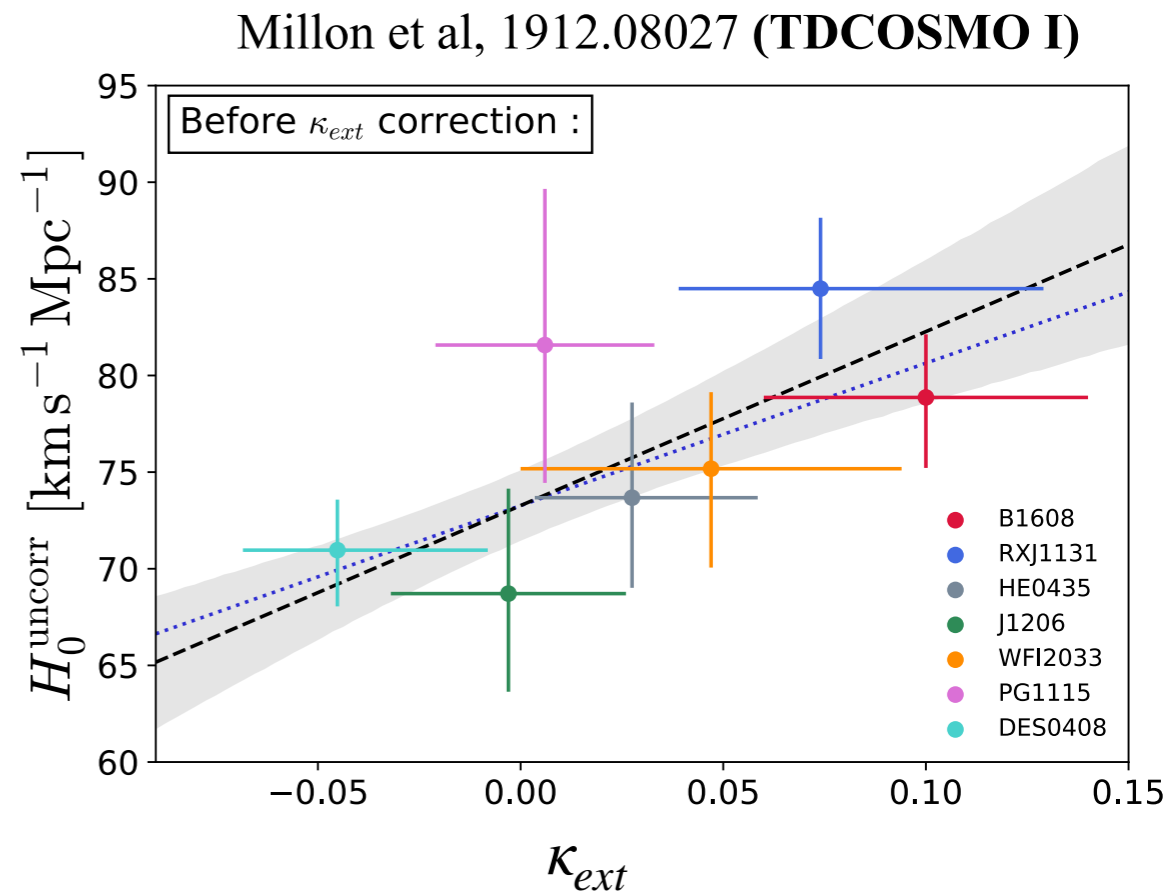
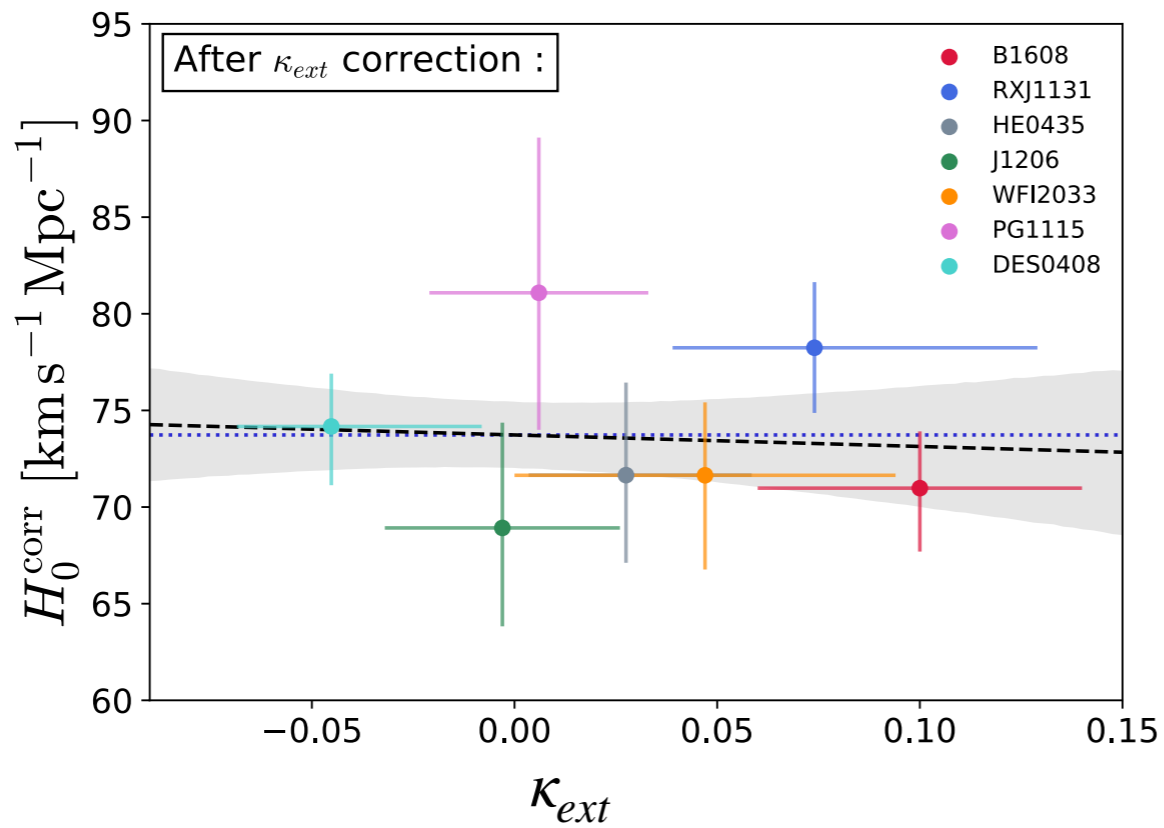
$$H_{0\lambda} = \lambda H_0$$

$$\kappa_\lambda = \lambda \kappa + (1 - \lambda)$$

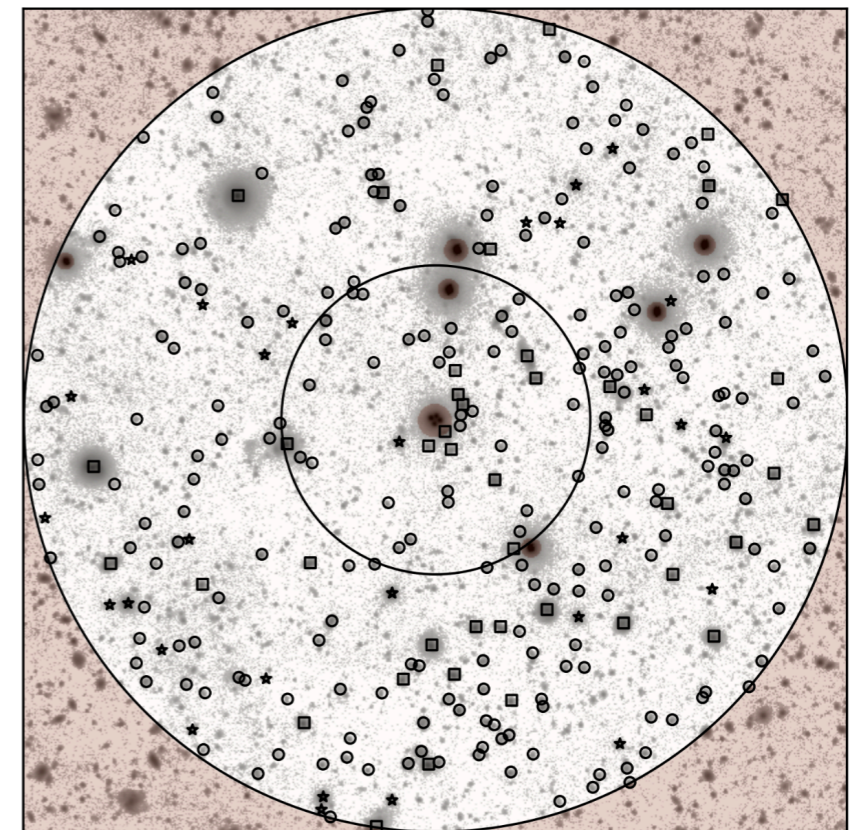
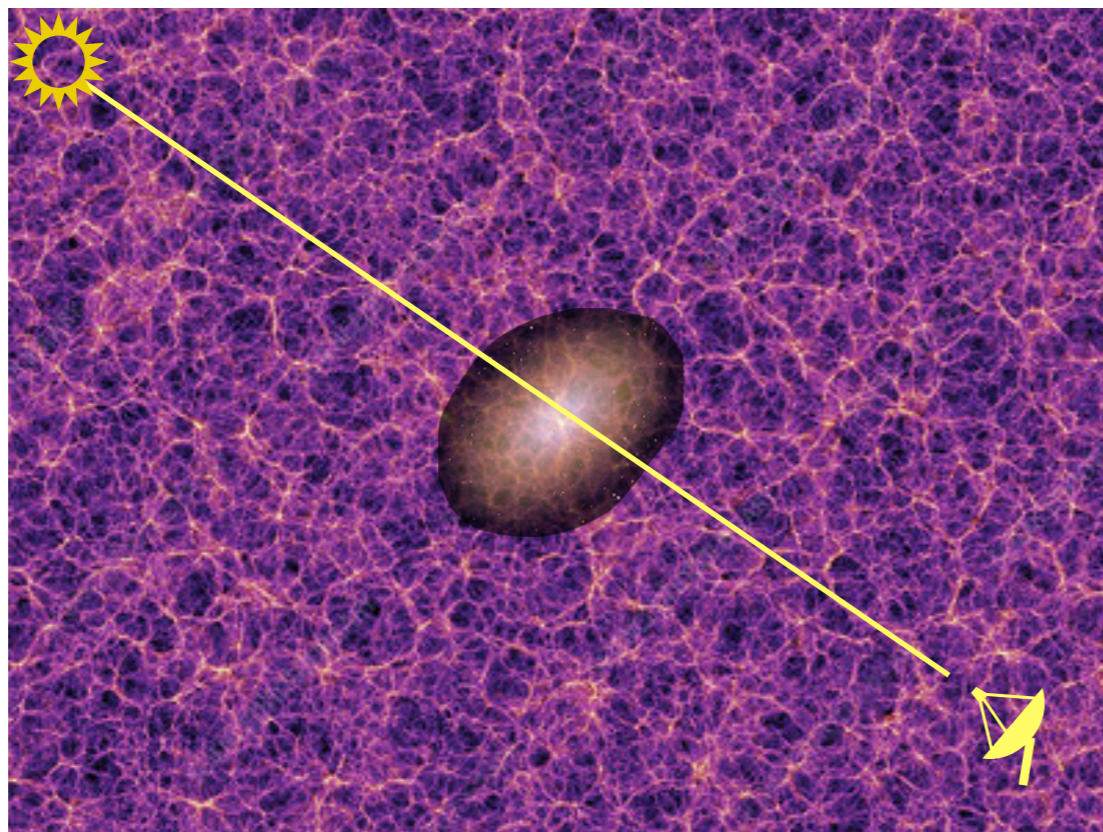


$$H_0$$

$$\kappa$$

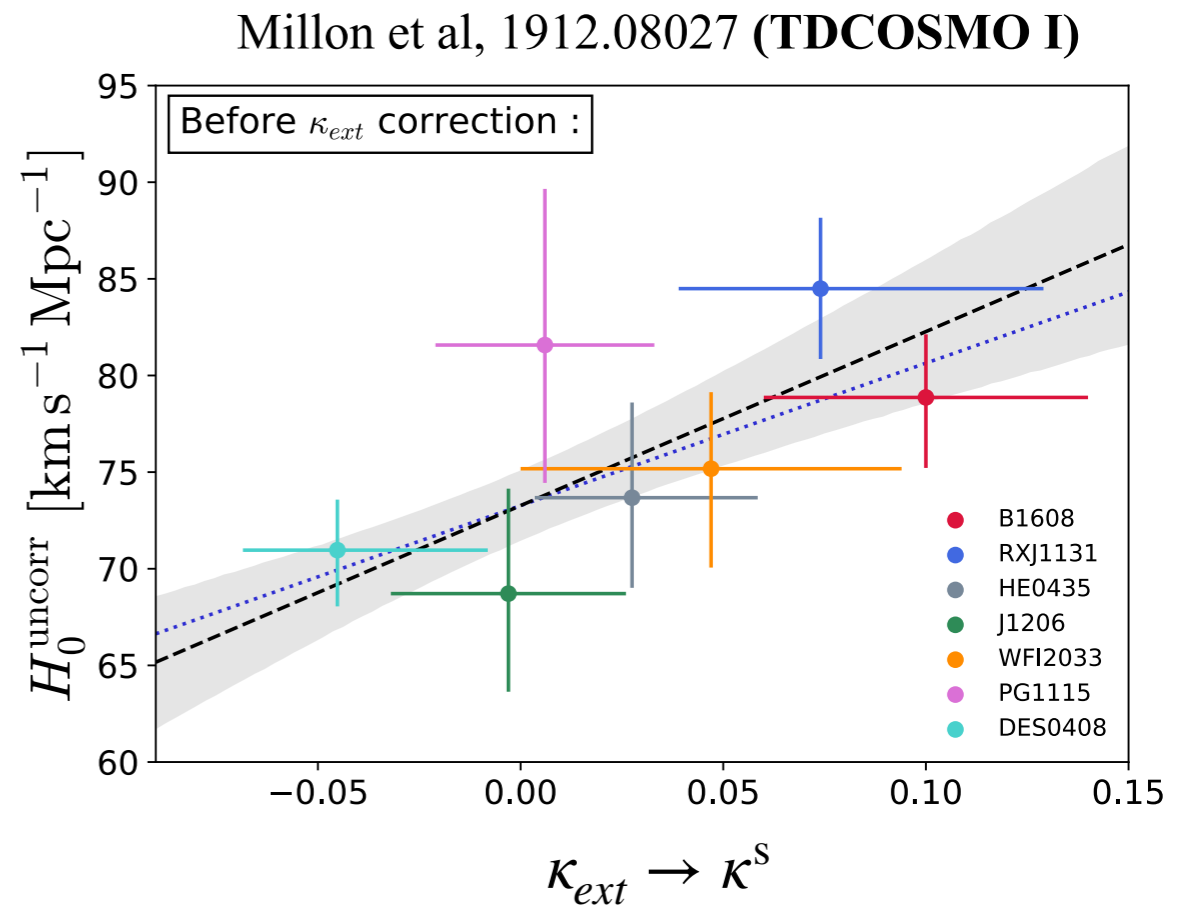
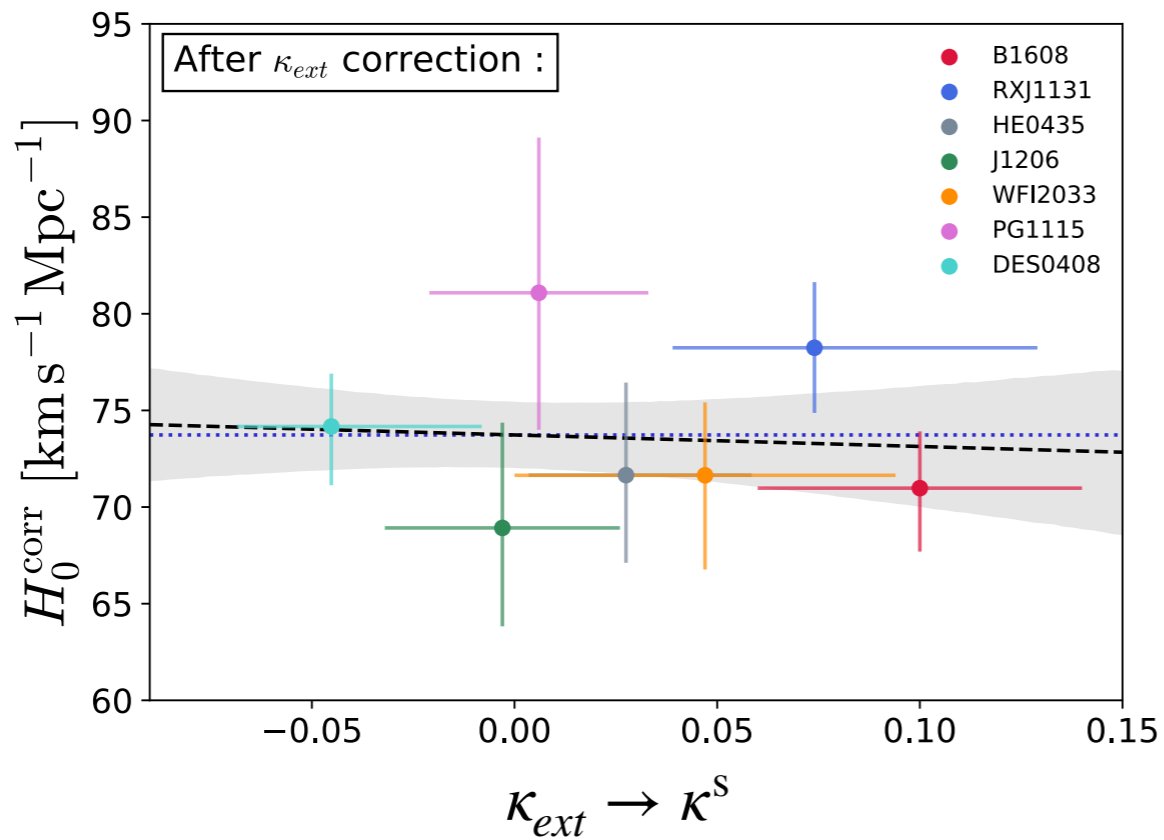


Modelling
external
convergence

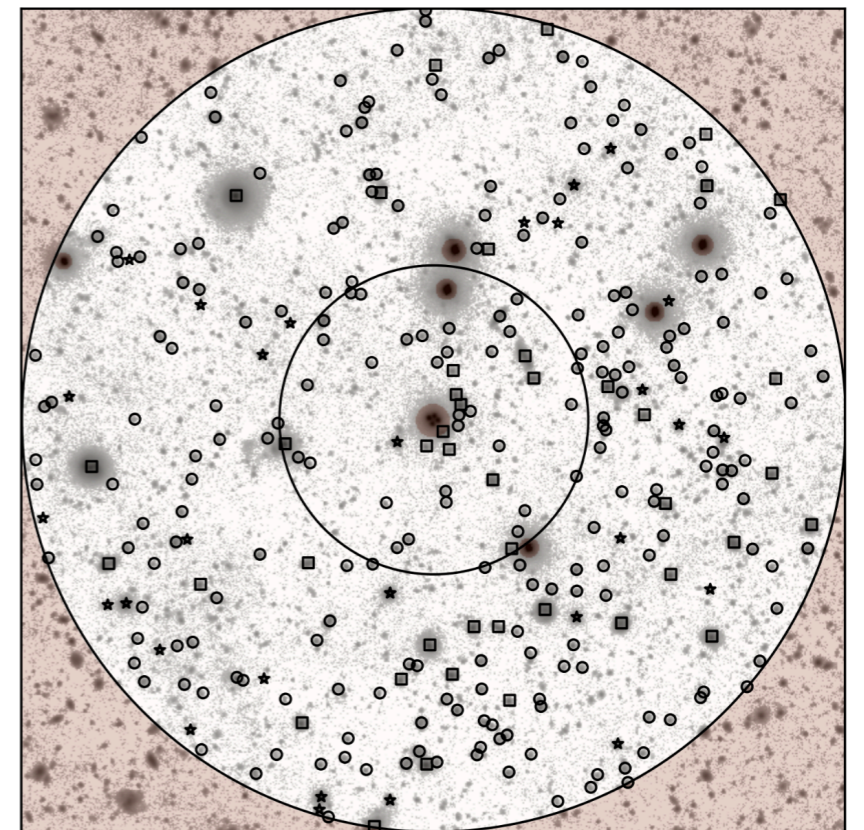
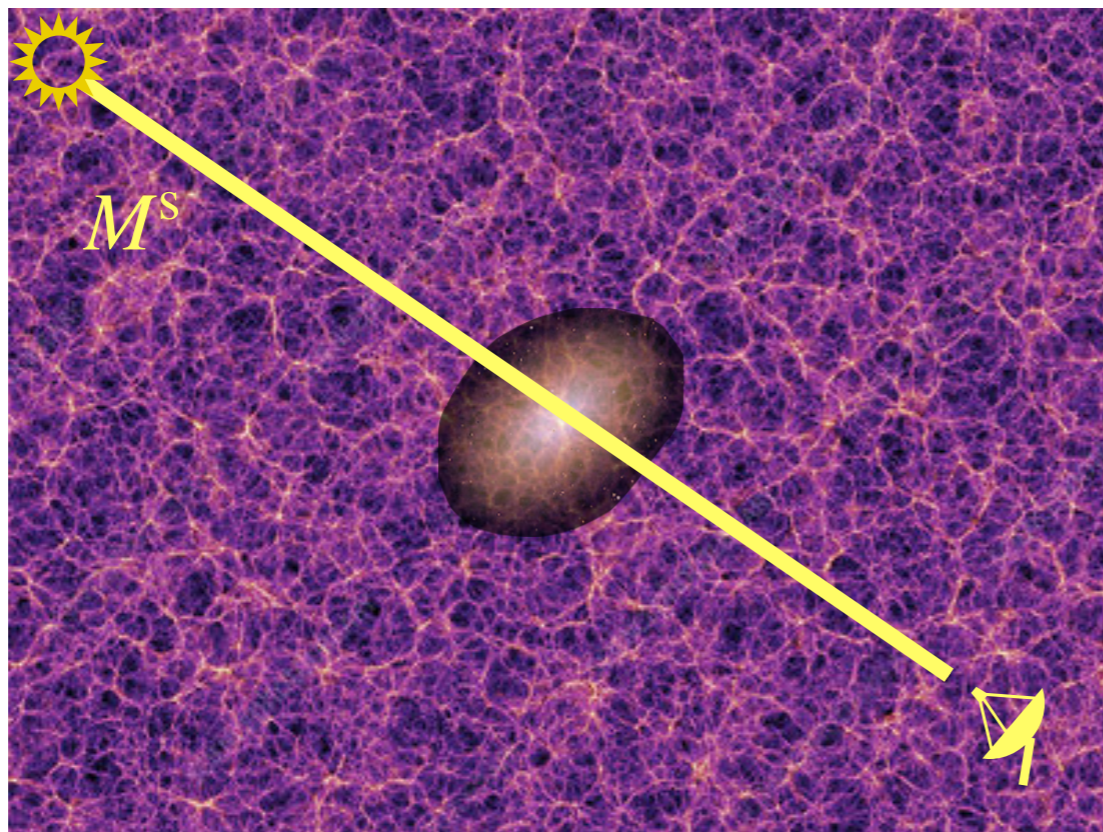


Rusu et al, 1607.01047 (H0LiCOW III)

Some comments on external convergence



Modelling
external
convergence



Rusu et al, 1607.01047 (H0LiCOW III)

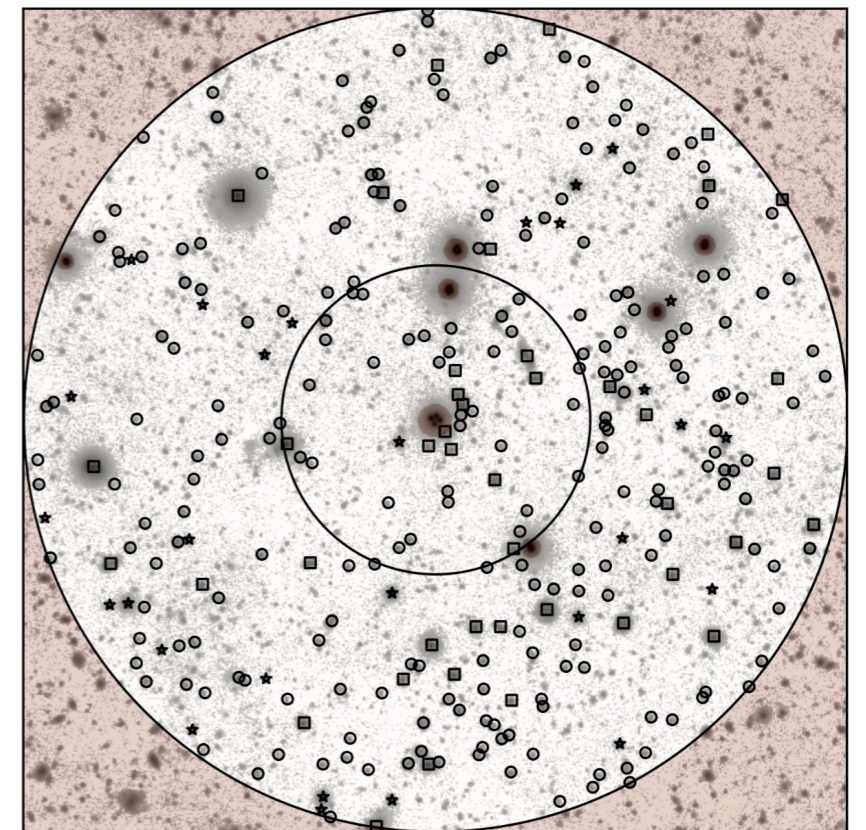
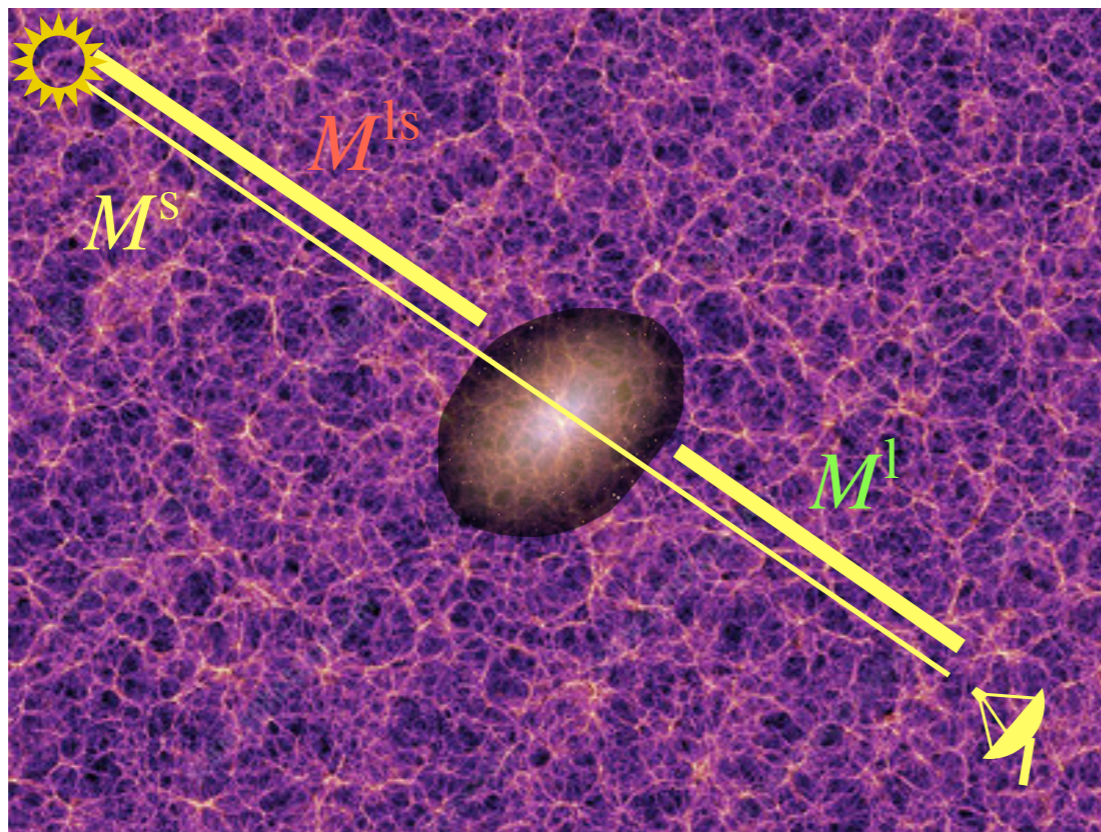
$$M(0, \eta_s) = M^s = \kappa^s - \Gamma^s$$

$$M(\eta_l, \eta_s) = M^{ls} = \kappa^{ls} - \Gamma^{ls}$$

$$M(0, \eta_l) = M^l = \kappa^l - \Gamma^l$$

$$M_{ij}(\eta_1, \eta_2) \approx 2 \int_{\eta_1}^{\eta_2} d\eta' \frac{(\eta_2 - \eta')(\eta' - \eta_1)}{(\eta_2 - \eta_1)\eta'^2} \partial_i \partial_j \Phi_t(0, \eta')$$

$$\tau(\vec{\theta}) = \frac{1}{2} \vec{\theta}^T (1 - M^s - M^l + M^{ls}) \vec{\theta} - \vec{\beta}^T (1 - M^l + M^{ls}) \vec{\theta} - \psi((1 - M^l) \vec{\theta})$$



Rusu et al, 1607.01047 (H0LiCOW III)

Imaging degeneracy:

$\lambda_s, \lambda_{ls}, \lambda_l$

$$M^s - 1 \mapsto \lambda_s (M^s - 1)$$

$$M^{ls} - 1 \mapsto \lambda_{ls} (M^{ls} - 1)$$

$$M^l - 1 \mapsto \lambda_l (M^l - 1)$$

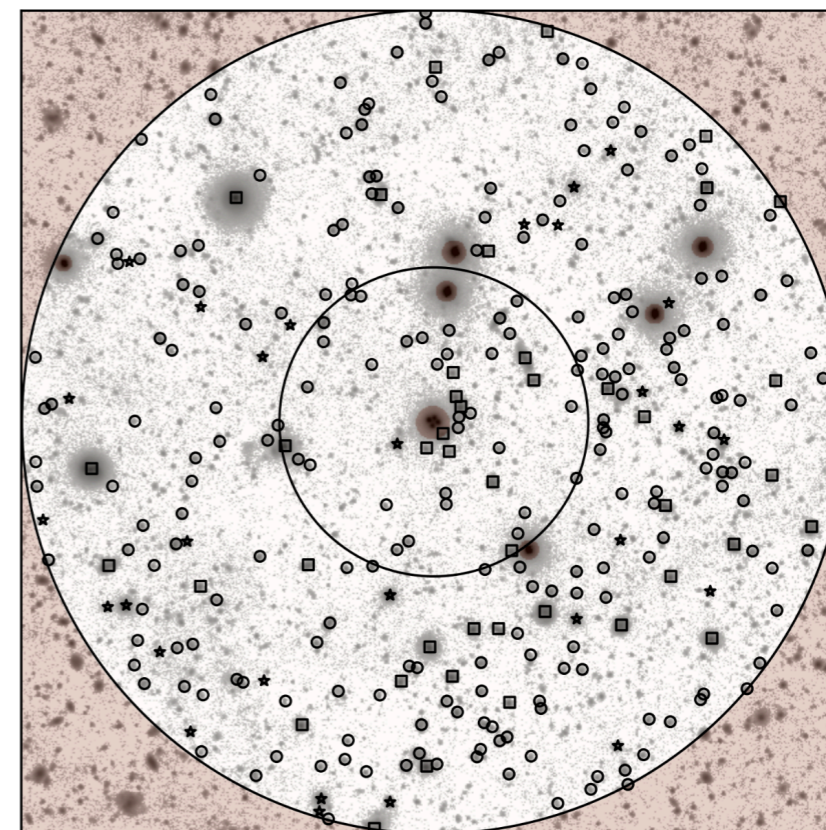
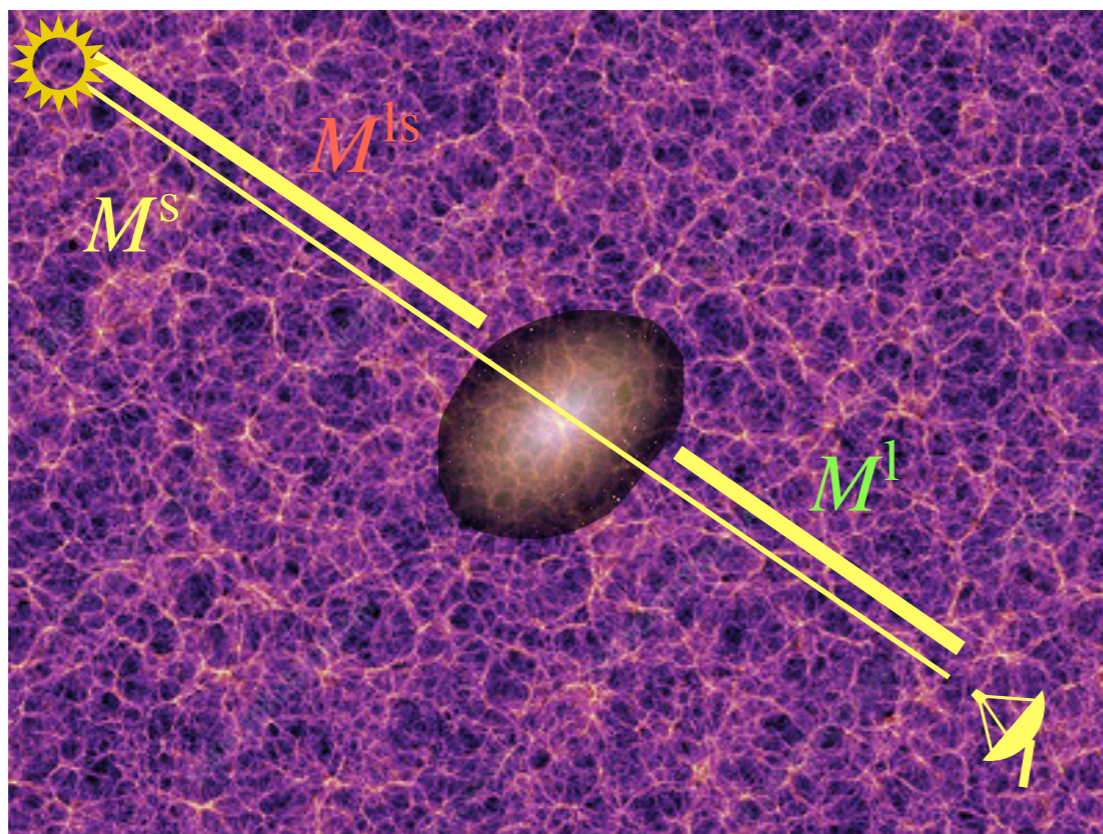
$$\psi(\vec{\theta}) \mapsto \lambda_s \lambda_{ls}^{-1} \lambda_l \psi(\lambda_l^{-1} \vec{\theta})$$

$$\vec{\beta} \mapsto \lambda_s \vec{\beta}$$

$$\tau(\vec{\theta}) \mapsto \lambda_s \lambda_{ls}^{-1} \lambda_l \tau(\vec{\theta})$$

$$H_0^{\text{uncorr}} \mapsto \lambda_s \lambda_{ls}^{-1} \lambda_l H_0$$

$$\tau(\vec{\theta}) = \frac{1}{2} \vec{\theta}^T (1 - M^s - M^l + M^{ls}) \vec{\theta} - \vec{\beta}^T (1 - M^l + M^{ls}) \vec{\theta} - \psi((1 - M^l) \vec{\theta})$$



Rusu et al, 1607.01047 (H0LiCOW III)

$$\kappa^s \mapsto \lambda_s \kappa^s + 1 - \lambda_s, \quad \Gamma^s \mapsto \lambda_s \Gamma^s$$

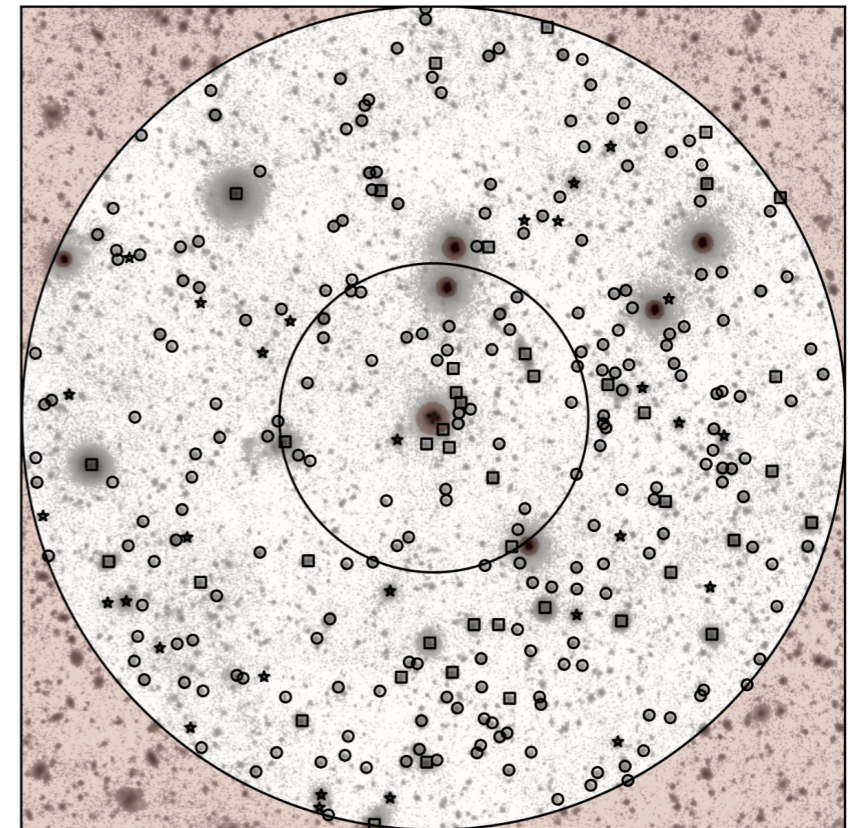
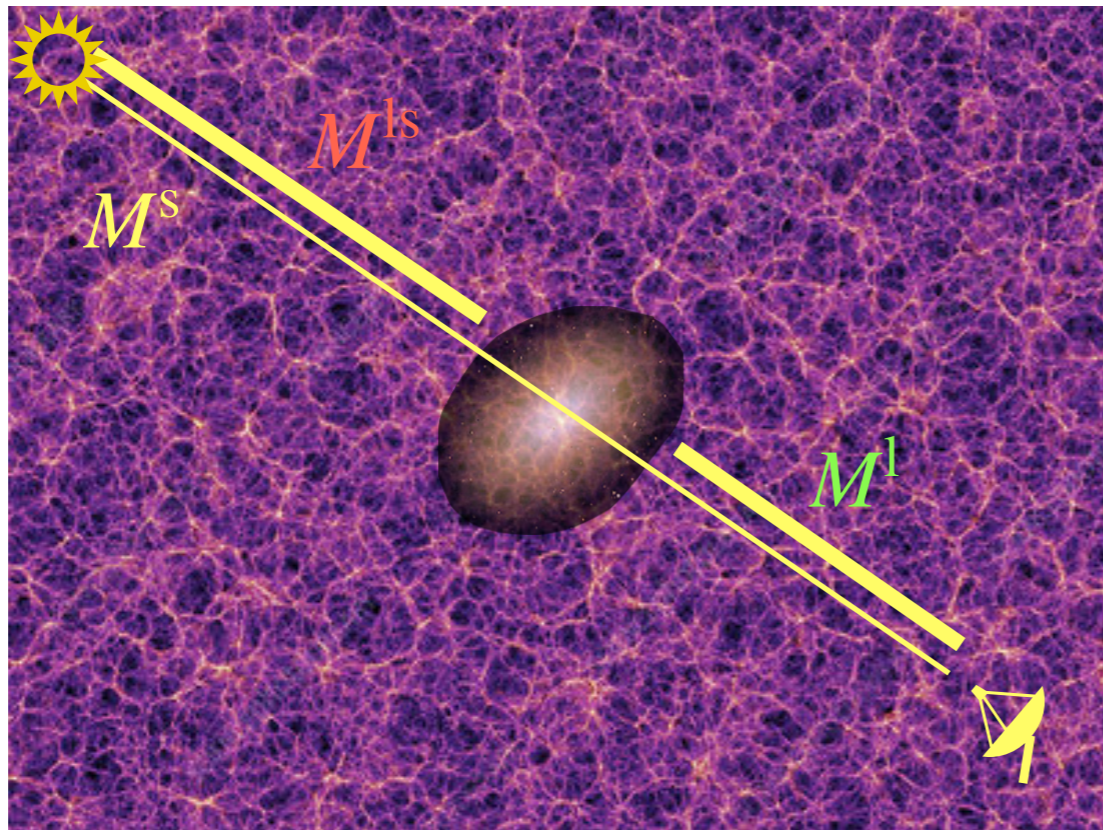
Imaging degeneracy: $\kappa^{ls} \mapsto \lambda_{ls} \kappa^{ls} + 1 - \lambda_{ls}, \quad \Gamma^{ls} \mapsto \lambda_{ls} \Gamma^{ls}$

$$\lambda_s, \lambda_{ls}, \lambda_l \quad \kappa^l \mapsto \lambda_l \kappa^l + 1 - \lambda_l, \quad \Gamma^l \mapsto \lambda_l \Gamma^l$$

$$\tau(\vec{\theta}) \mapsto \lambda_s \lambda_{ls}^{-1} \lambda_l \tau(\vec{\theta})$$

$$H_0^{\text{uncorr}} \mapsto \lambda_s \lambda_{ls}^{-1} \lambda_l H_0$$

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Rusu et al, 1607.01047 (H0LiCOW III)

Weak lensing degeneracy:

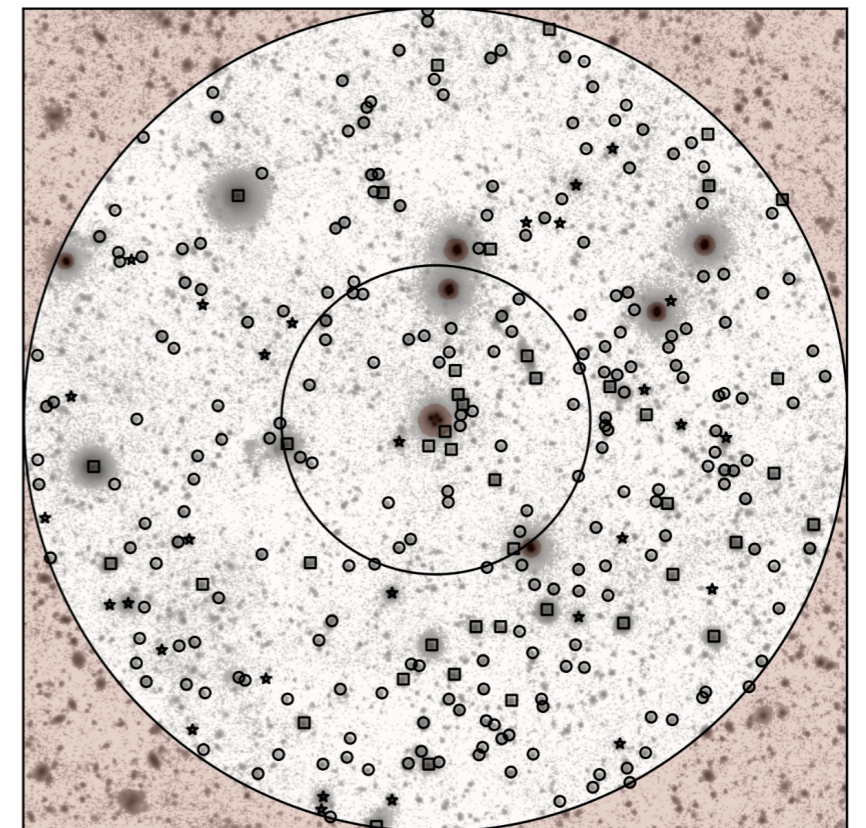
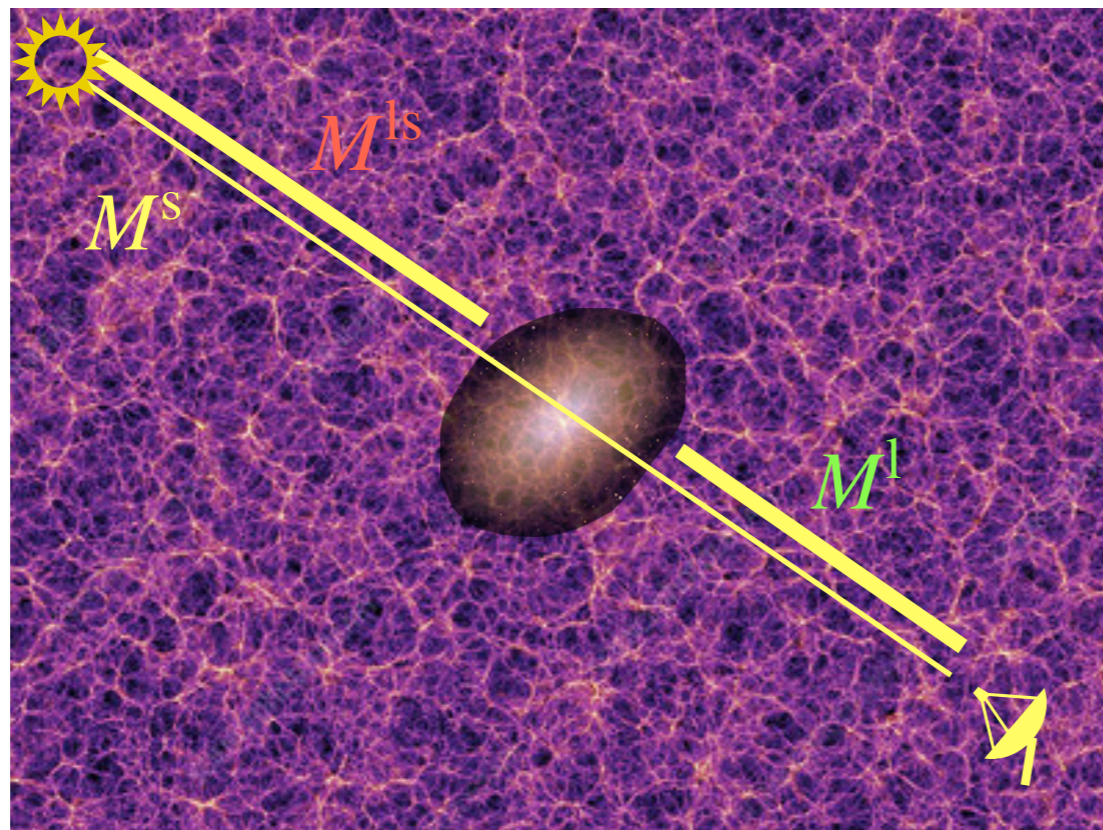
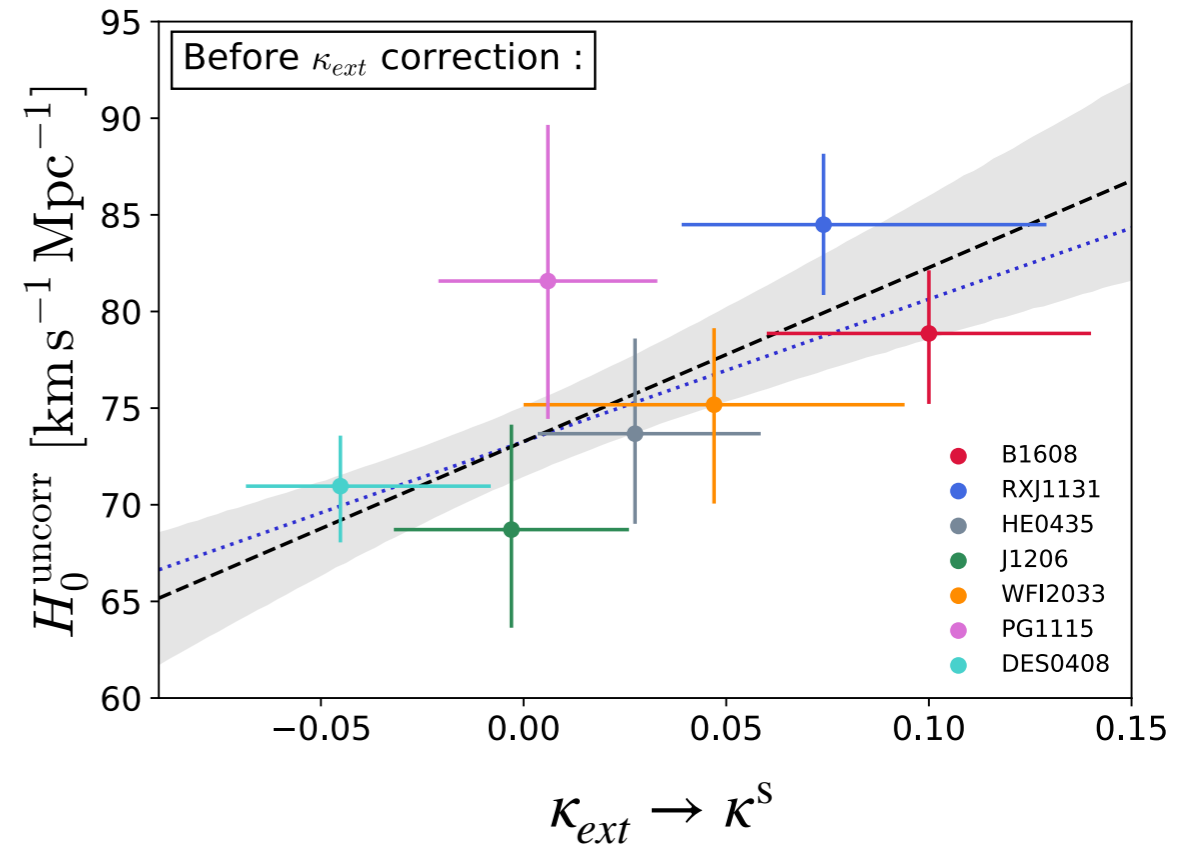
$$H_0^{\text{uncorr}} = \frac{1 - \kappa^{\text{ls}}}{1 - \kappa^{\text{l}}} \frac{1}{1 - \kappa^{\text{s}}} H_0$$

Weak lensing correction in H0LiCOW / TDCOSMO is a little bit off.

Birrer* et al, 2007.02941 (TDCOSMO IV)

Teodori, et al, 2201.05111

Millon et al, 1912.08027 (TDCOSMO I)



Rusu et al, 1607.01047 (H0LiCOW III)

* Thanks to Simon Birrer for catching a mistake in our draft!

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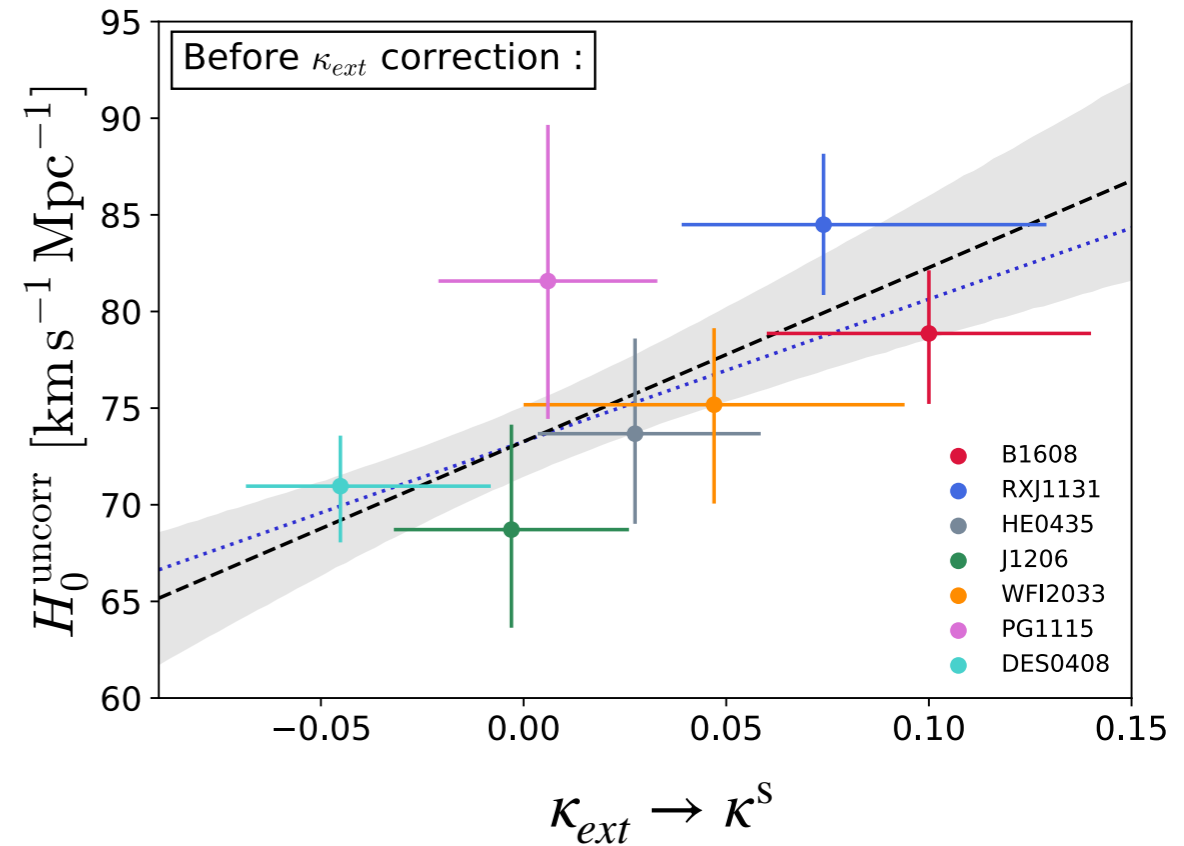
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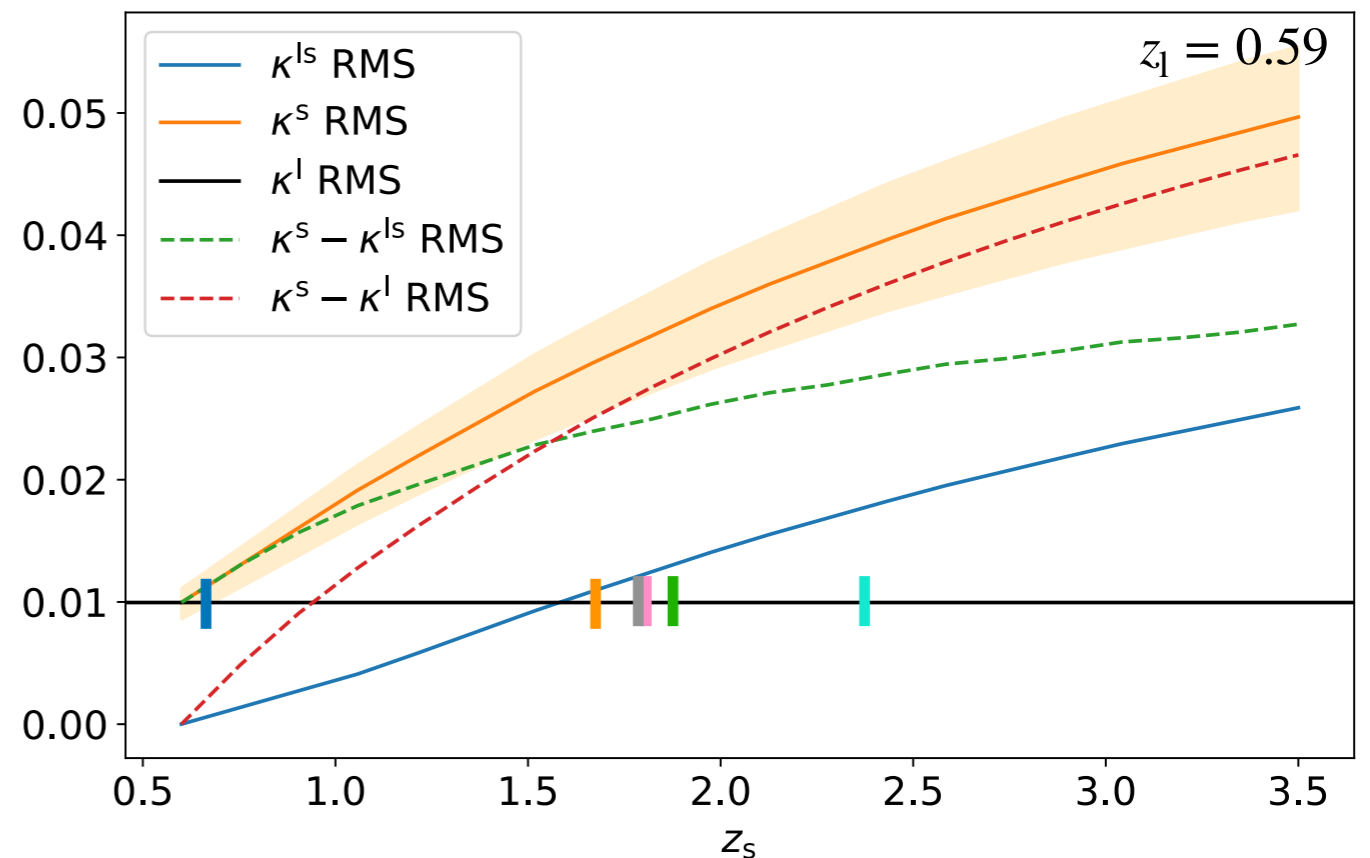
Birrer et al, 2007.02941 (TDCOSMO IV)

Teodori, et al, 2201.05111

Millon et al, 1912.08027 (TDCOSMO I)



We can try to roughly estimate this, but a real estimate probably needs ray-tracing calibrated to characteristic field of the individual lens



Weak lensing degeneracy:

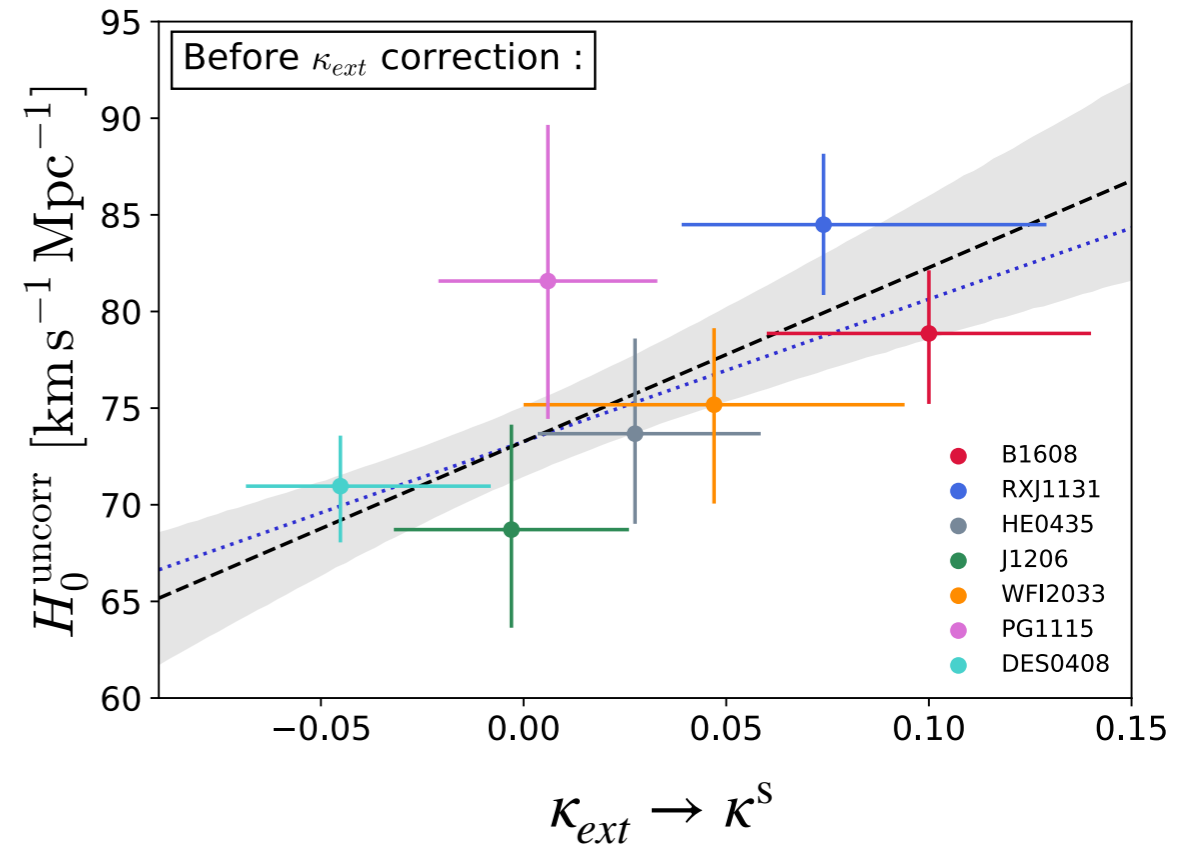
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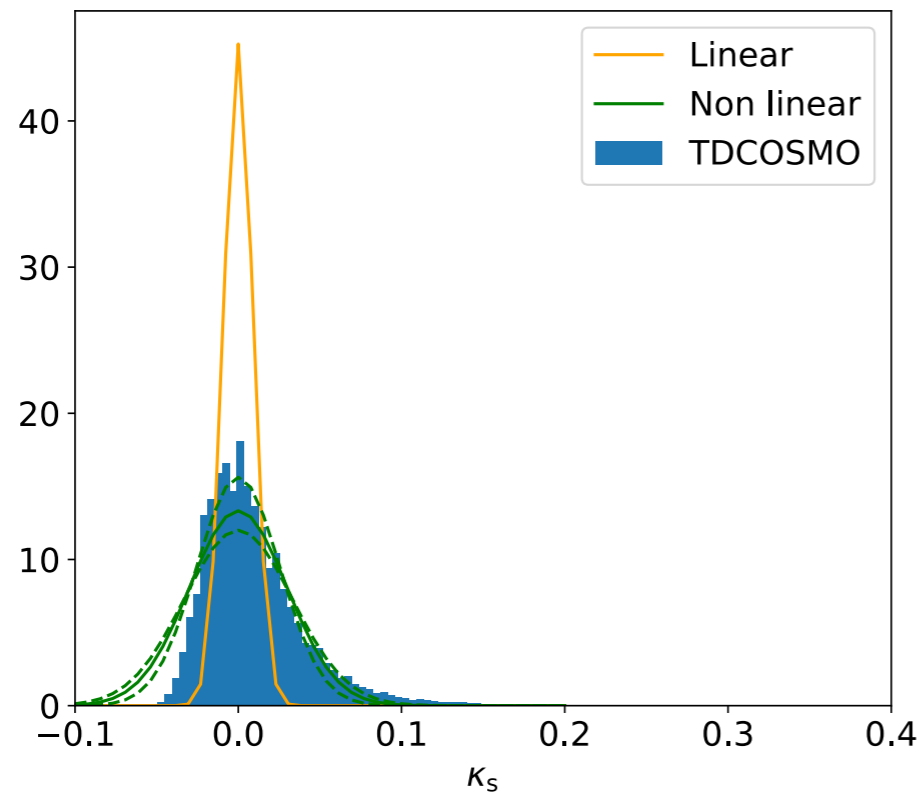
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Teodori, et al, 2201.05111

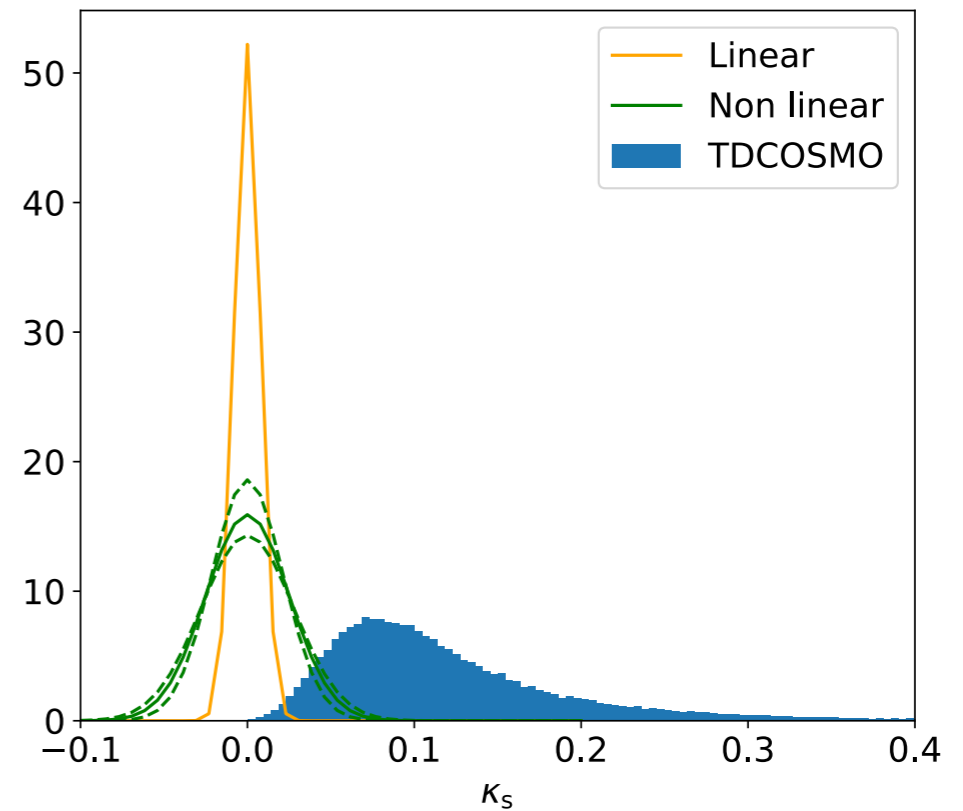
Millon et al, 1912.08027 (TDCOSMO I)



HE0435-1223



B1608+656



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Weak lensing correction in H0LiCOW / TDCOSMO
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Joint imaging+kinematics likelihood also a bit off:

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Suppose truth intrinsic lens is a power law: $\vec{\alpha}(\vec{\theta}) = \left(\frac{\theta}{\tilde{\theta}_E}\right)^{1-\gamma_{\text{PL}}} \vec{\theta}$

Imaging analyses model the observed deflection:

$$\vec{\alpha}^{\text{model}}(\vec{\theta}) = \frac{(1 - \kappa^{\text{ls}})(1 - \kappa^{\text{l}})^{2-\gamma_{\text{PL}}}}{1 - \kappa^{\text{s}}} \left(\frac{\theta}{\tilde{\theta}_E}\right)^{1-\gamma_{\text{PL}}} \vec{\theta} = \left(\frac{\theta}{\theta_E}\right)^{1-\gamma_{\text{PL}}} \vec{\theta}$$

At the same time, stellar kinematics measures:

$$\begin{aligned} \sigma^2(\theta) &= 2G\Sigma_{\text{crit}}d_A(z_1, 0) \frac{\sqrt{\pi}\Gamma\left(\frac{\gamma_{\text{PL}}}{2}\right)}{\Gamma\left(\frac{\gamma_{\text{PL}}-1}{2}\right)} \tilde{\theta}_E^{\gamma_{\text{PL}}-1} \theta^{2-\gamma_{\text{PL}}} \\ &= \frac{1 - \kappa^{\text{s}}}{(1 - \kappa^{\text{ls}})(1 - \kappa^{\text{l}})^{2-\gamma_{\text{PL}}}} \frac{d_A(z_s, 0)}{d_A(z_s, z_1)} J(\theta_E, \gamma_{\text{PL}}) \end{aligned}$$

Weak lensing degeneracy:

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$$= \frac{1 - \kappa^{\text{s}}}{(1 - \kappa^{\text{ls}})(1 - \kappa^{\text{l}})^{2-\gamma_{\text{PL}}}} \frac{d_A(z_s, 0)}{d_A(z_s, z_1)} J(\theta_E, \gamma_{\text{PL}})$$

Instead, post-processing weak lensing correction applied in TDCOSMO I and IV for the kinematics, was equivalent to setting:

$$\sigma^2 \rightarrow (1 - \kappa^{\text{ext}}) \sigma^2$$

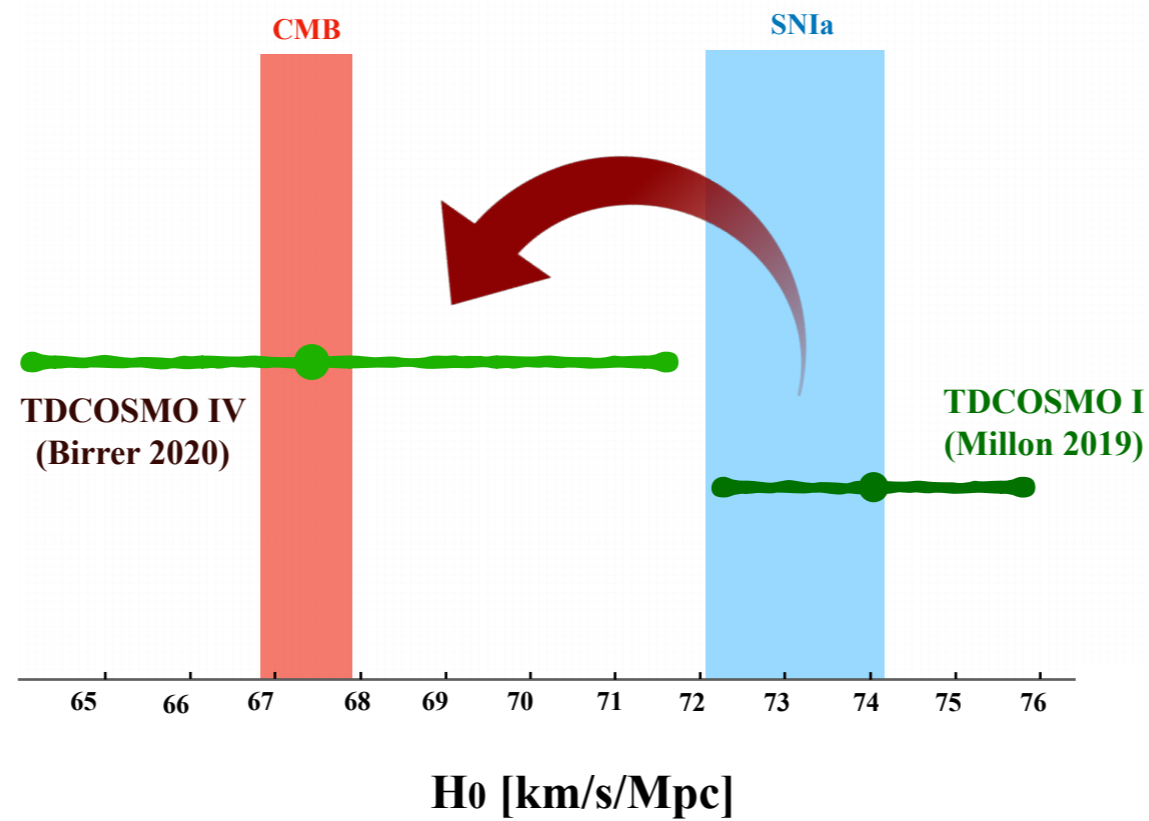
With: $1 - \kappa^{\text{ext}} \rightarrow \frac{(1 - \kappa^{\text{s}})(1 - \kappa^{\text{l}})}{1 - \kappa^{\text{ls}}}$

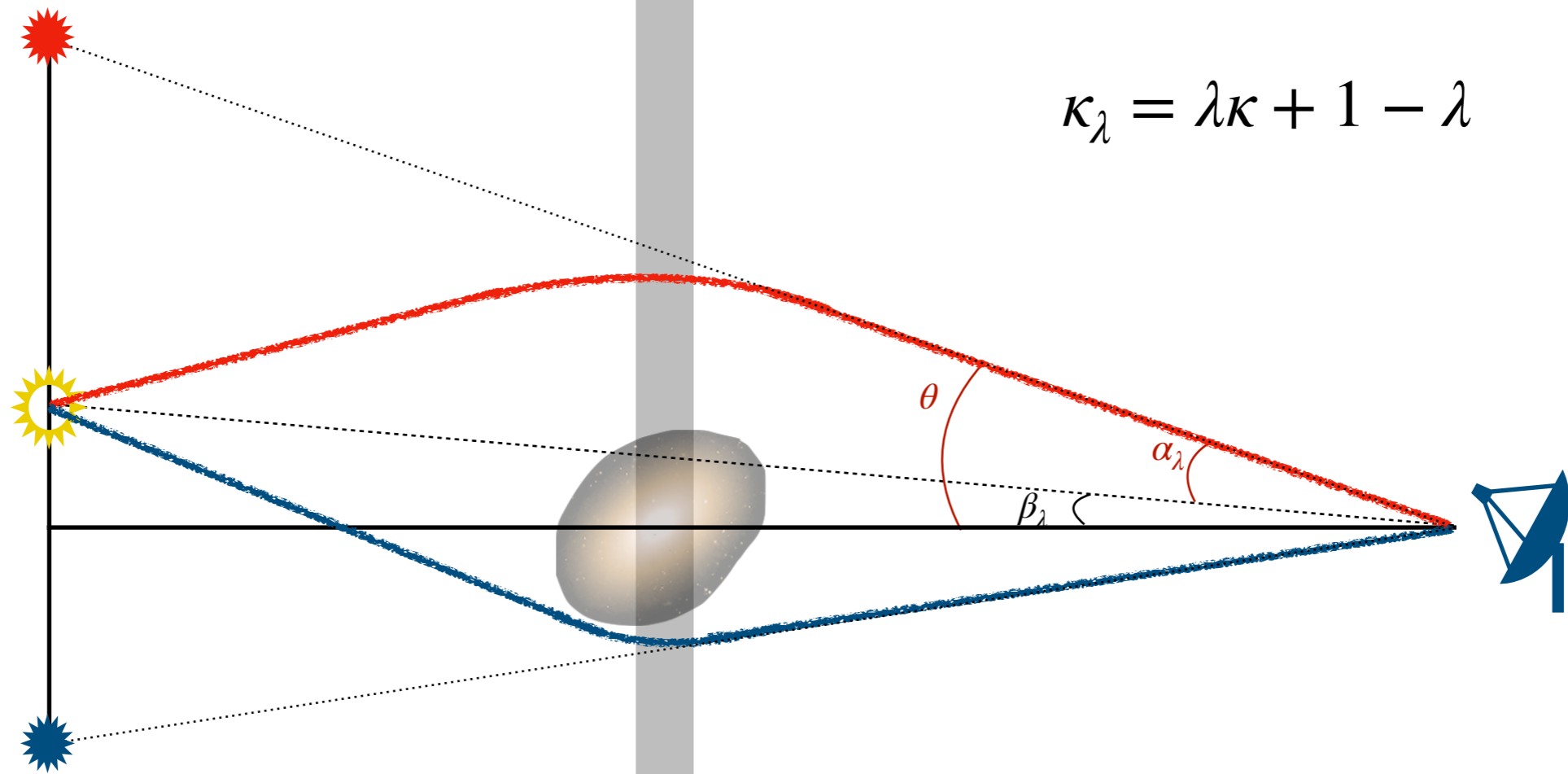
Along with: $H_0^{\text{inferred}} = (1 - \kappa^{\text{ext}}) H_0^{\text{model}}$

Leading to kinematics-induced bias:

$$\frac{H_0^{\text{inferred}}}{H_0} \approx 1 - (3 - \gamma_{\text{PL}})\kappa^{\text{l}}$$

Potentially bigger problem: ``internal convergence''

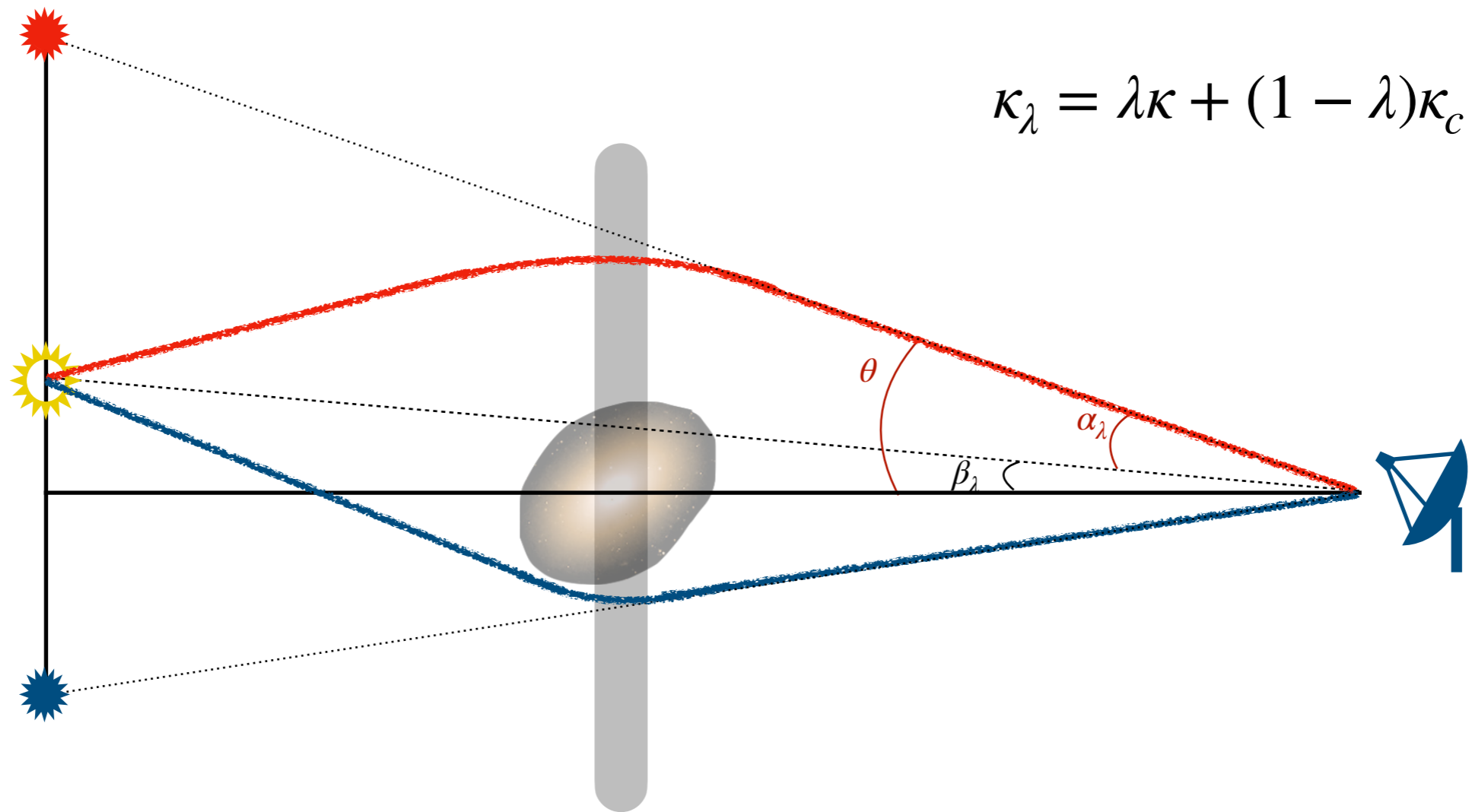




$$\kappa_{\lambda} = \lambda\kappa + 1 - \lambda$$

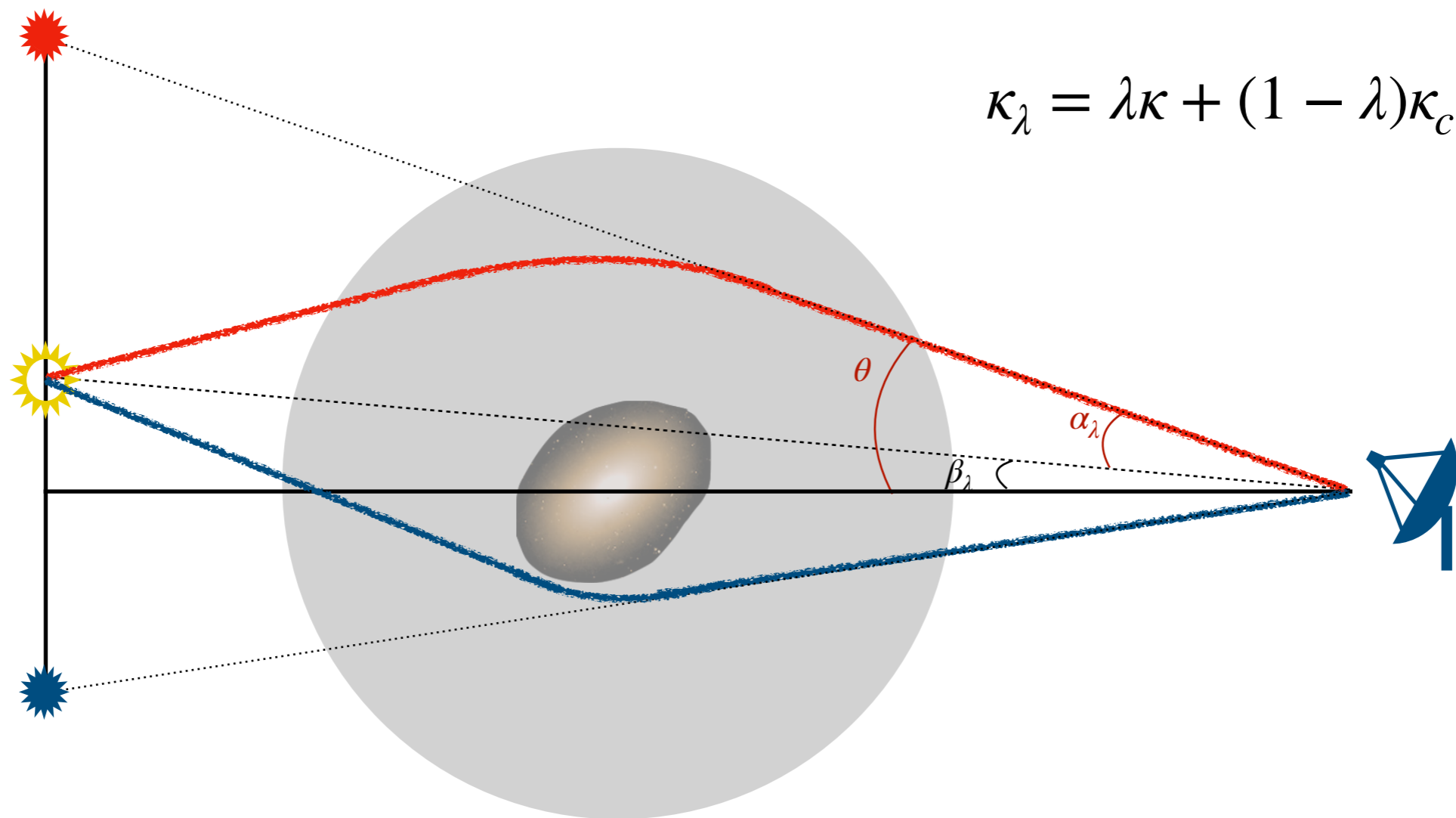
Internal vs. External Convergence

Internal vs. External Mass Sheet Degeneracy



Internal vs. External Convergence

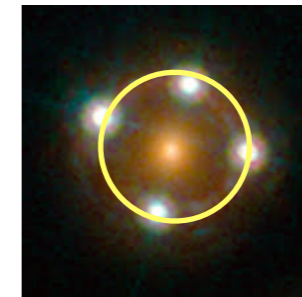
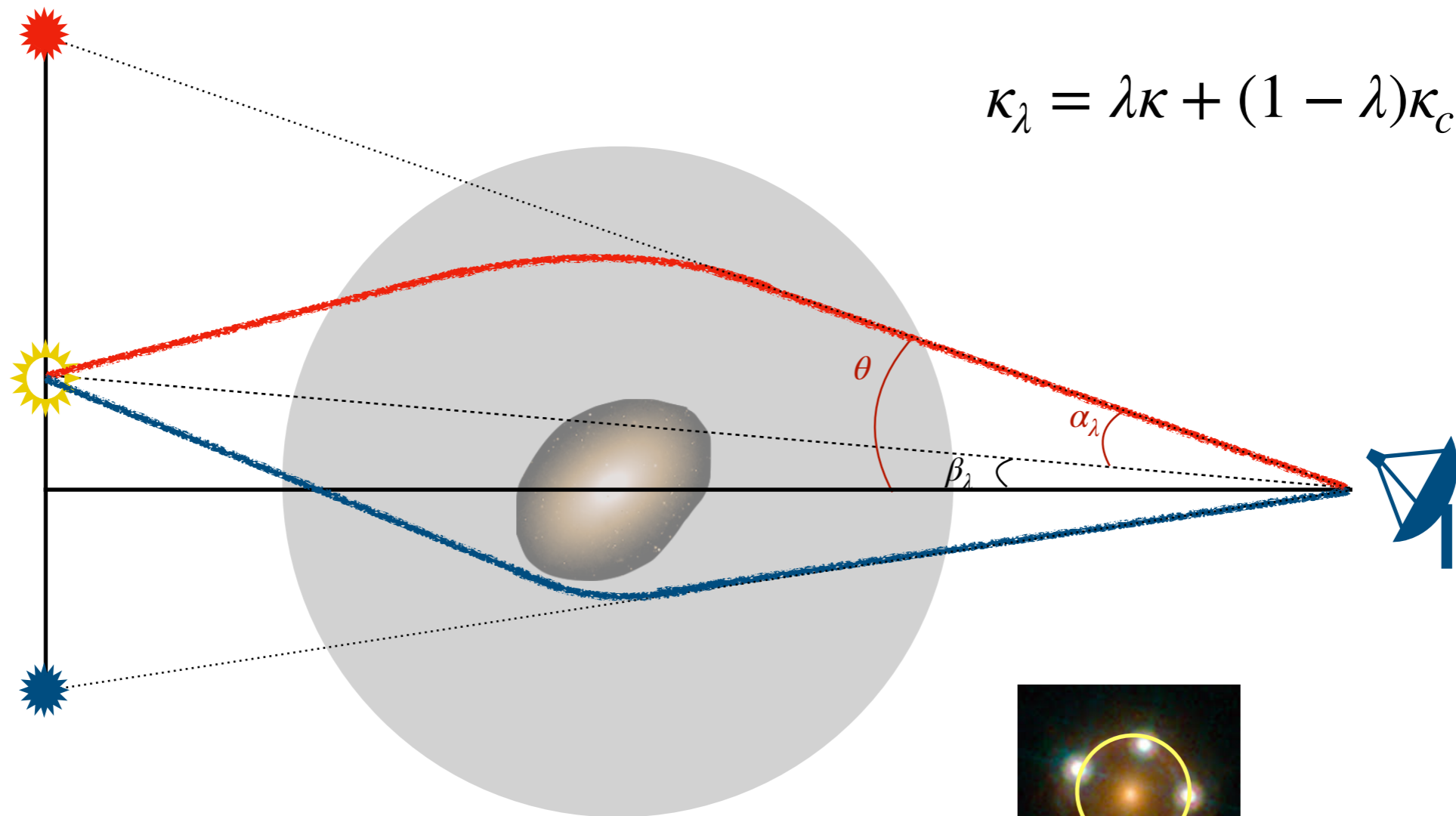
Internal vs. External Mass Sheet Degeneracy



Internal vs. External Convergence

Internal vs. External Mass Sheet Degeneracy

A core component in lens halos could explain lensing H0 tension.



Example:

$$\rho_c(r) = \frac{1}{\sqrt{\pi}\Gamma\left(\frac{3}{2}\right)} \frac{\Sigma_c R_c^3}{(R_c^2 + r^2)^2}$$

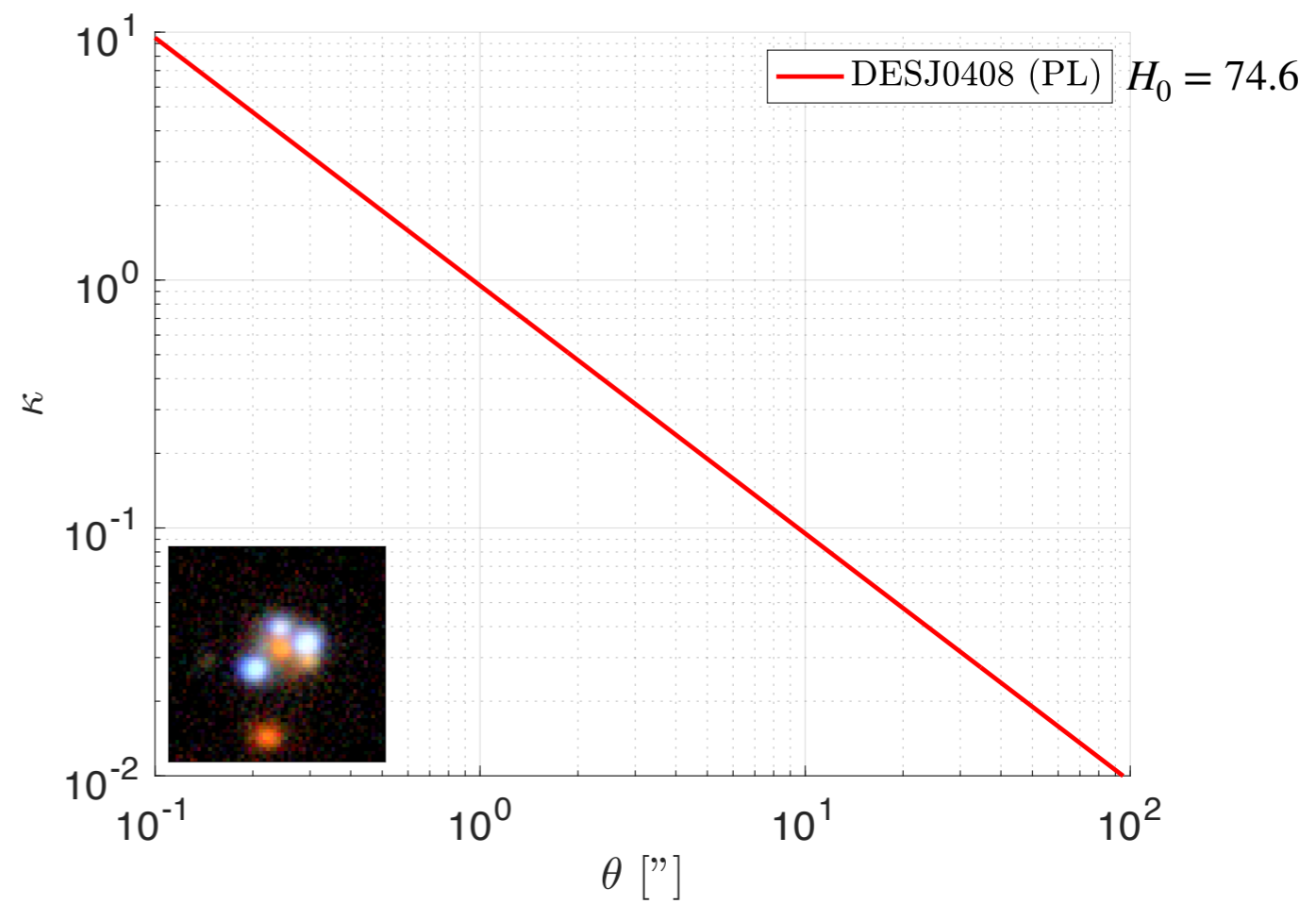
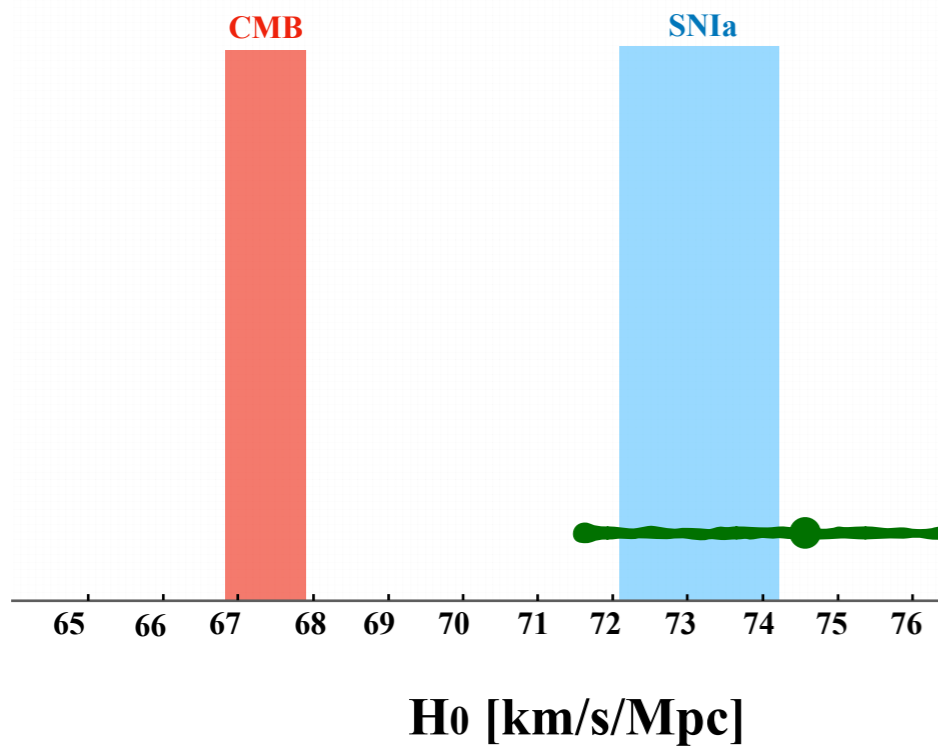
$$\kappa_c(\theta) = \left(1 + \frac{\theta^2}{\theta_c^2}\right)^{-\frac{3}{2}}$$



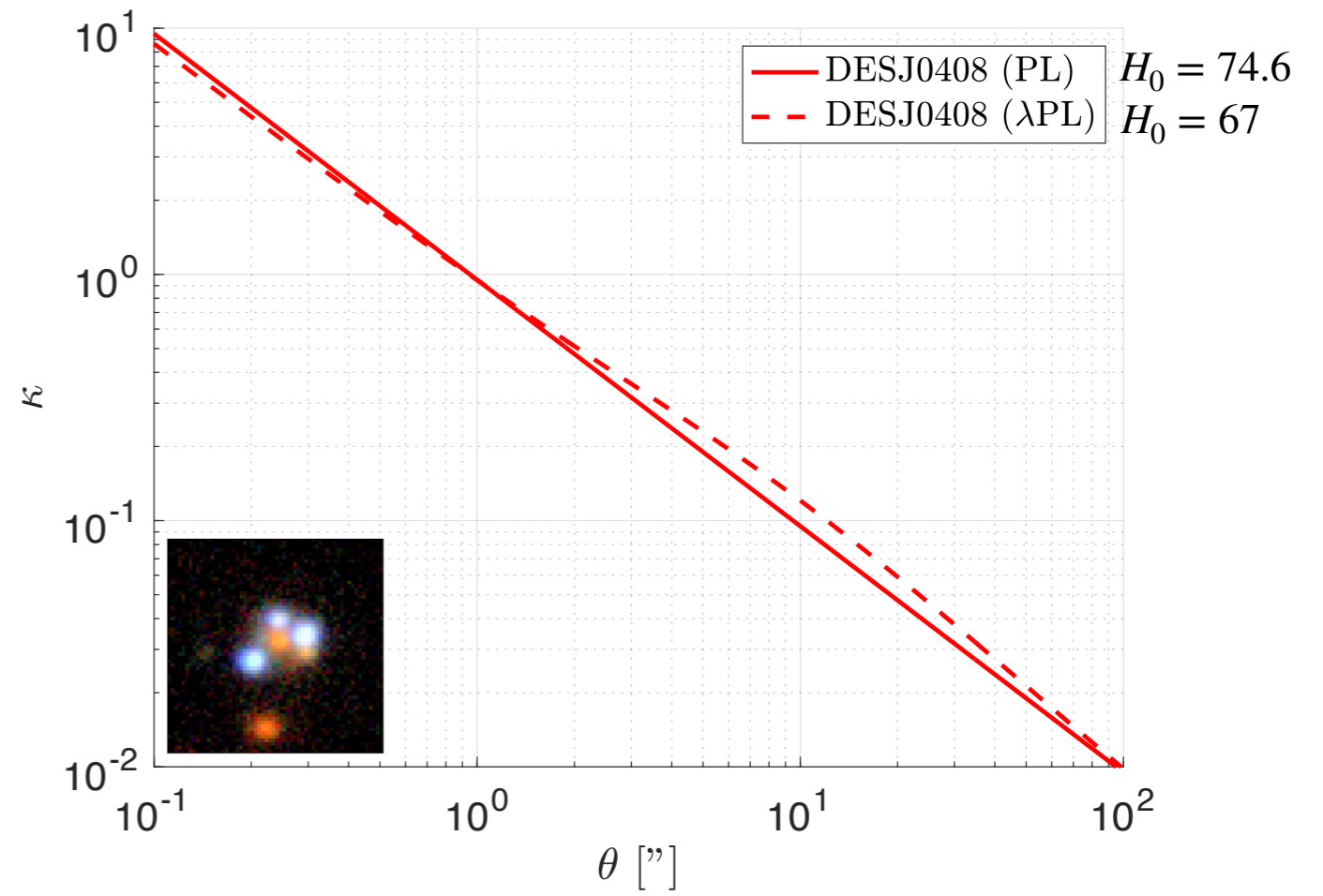
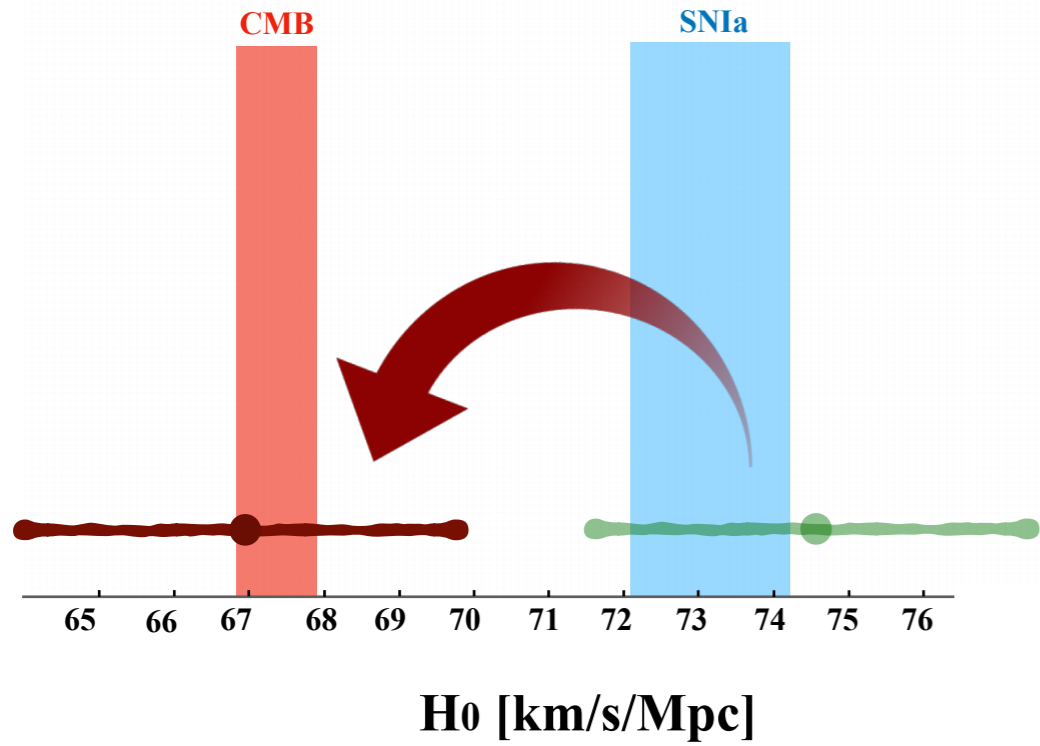
$$\theta_{E\lambda} = \theta_E(1 + \delta)$$

$$\delta = -\frac{3}{4(\gamma-1)} \frac{1-\lambda}{\lambda} \frac{\theta_E^2}{\theta_c^2} + \mathcal{O}\left(\frac{\theta_E^4}{\theta_c^4}\right)$$

2. Challenges: modeling degeneracy

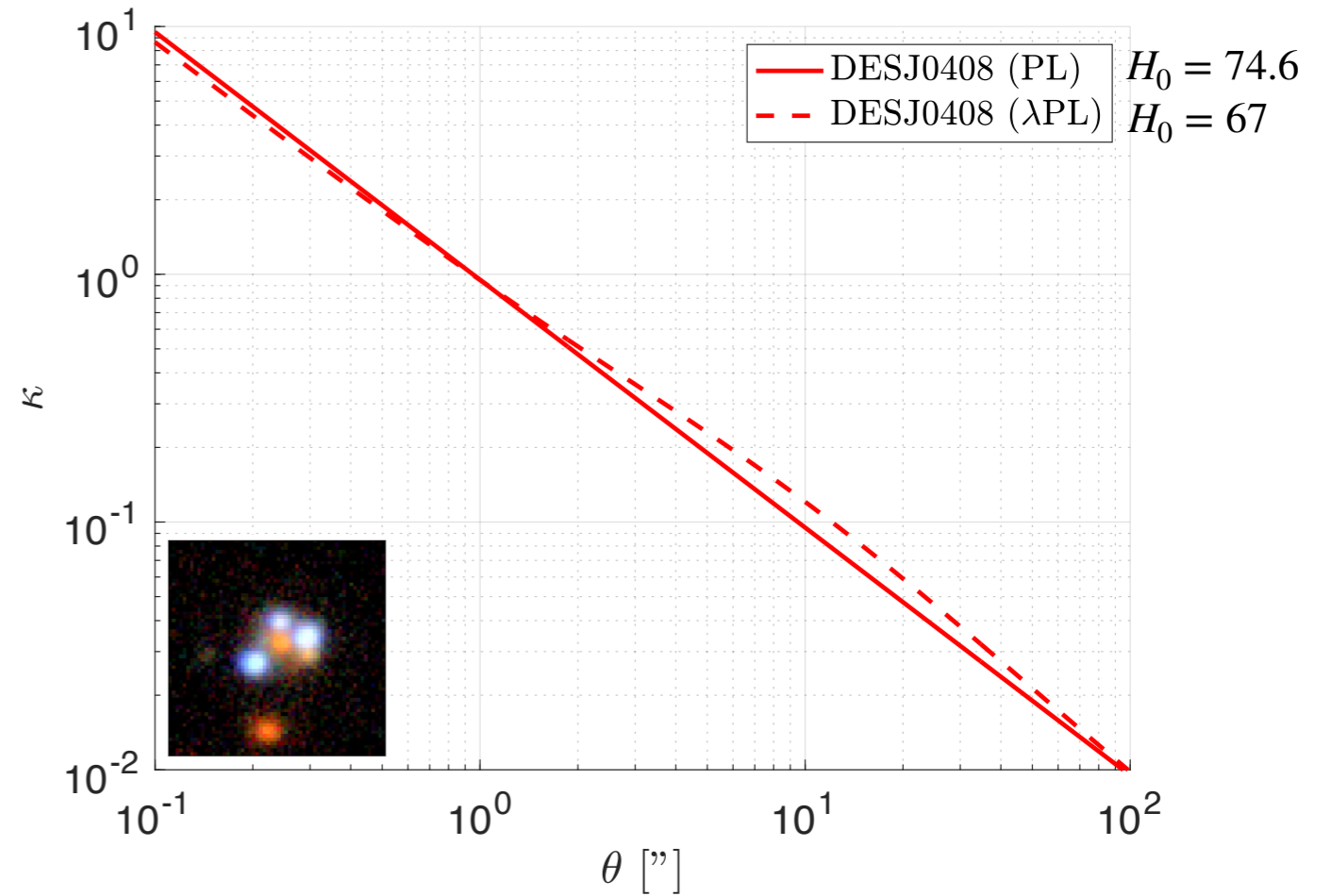
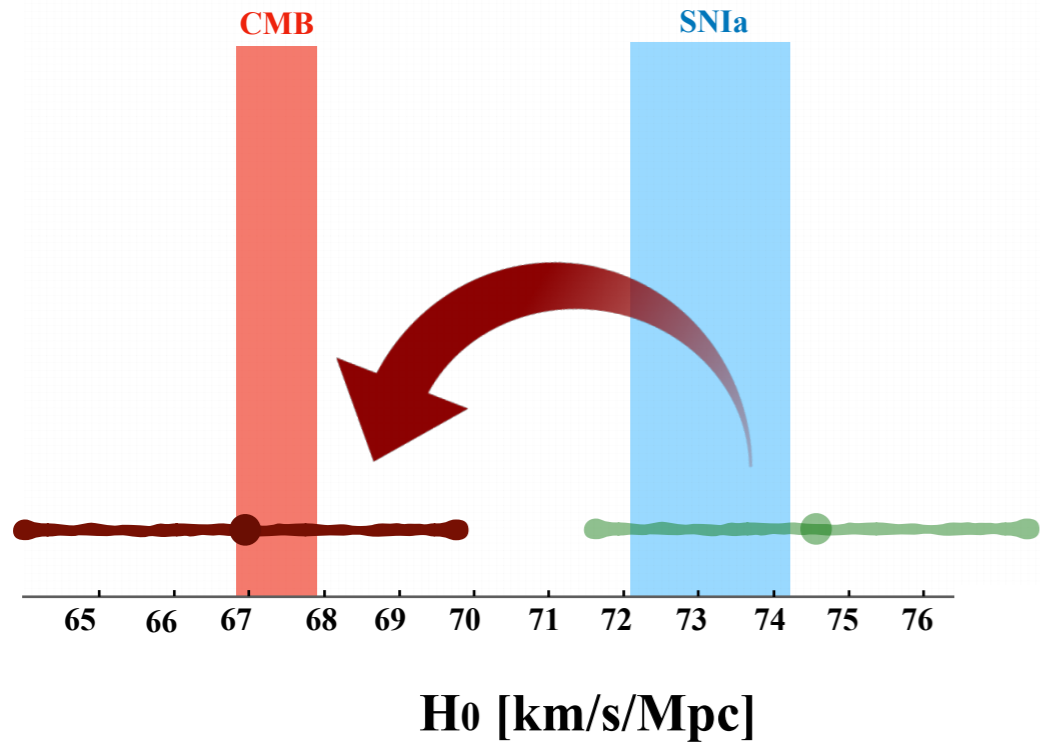


2. Challenges: modeling degeneracy



3. Opportunities: galactic structure

What do we learn about galaxies if we add CMB/LSS prior?



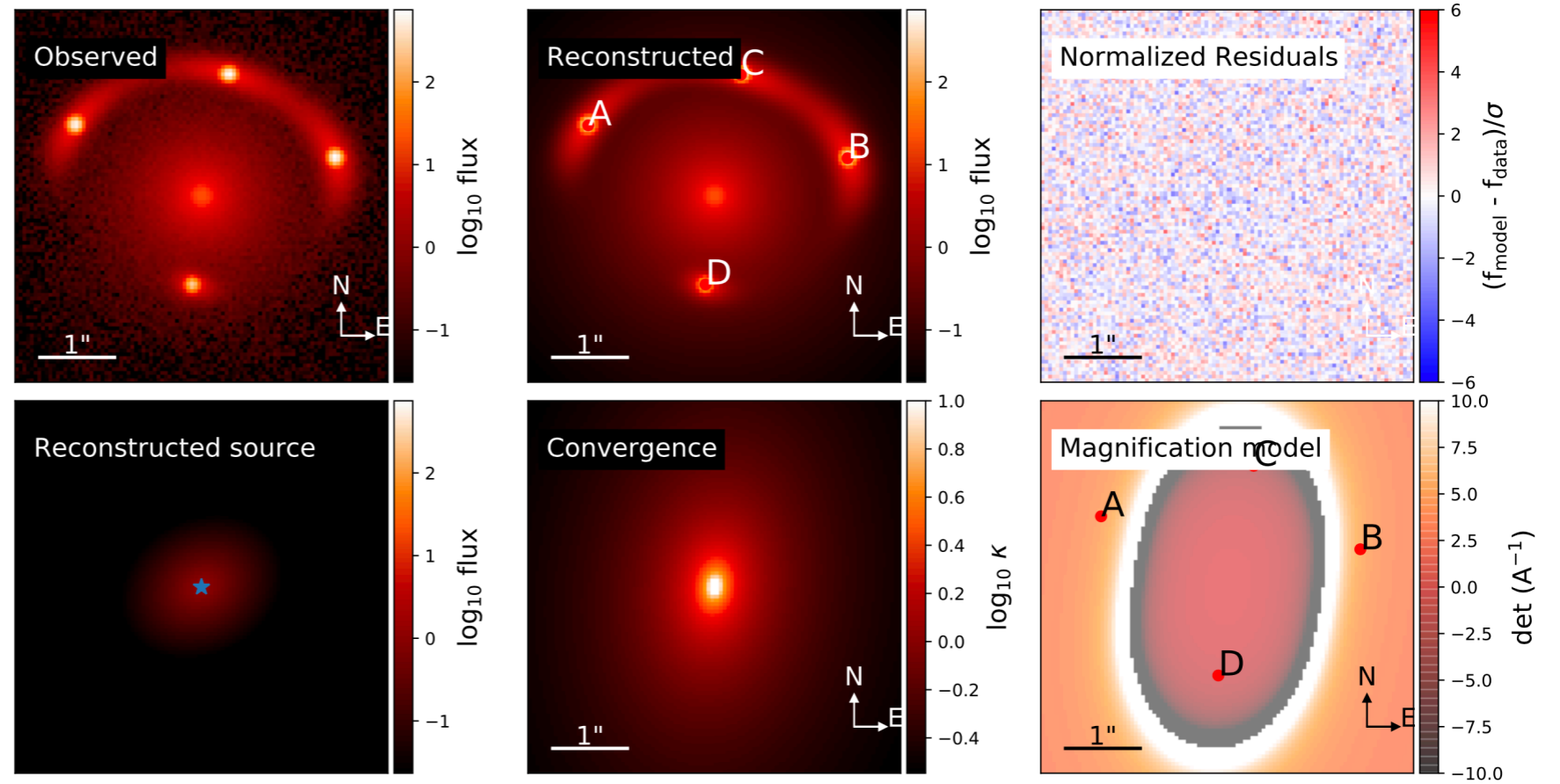
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What do we learn about galaxies if we add CMB/LSS prior?

Expect evidence for core component, reflecting precision on H_0

KB, Teodori, 2105.10873



3. Opportunities: galactic structure

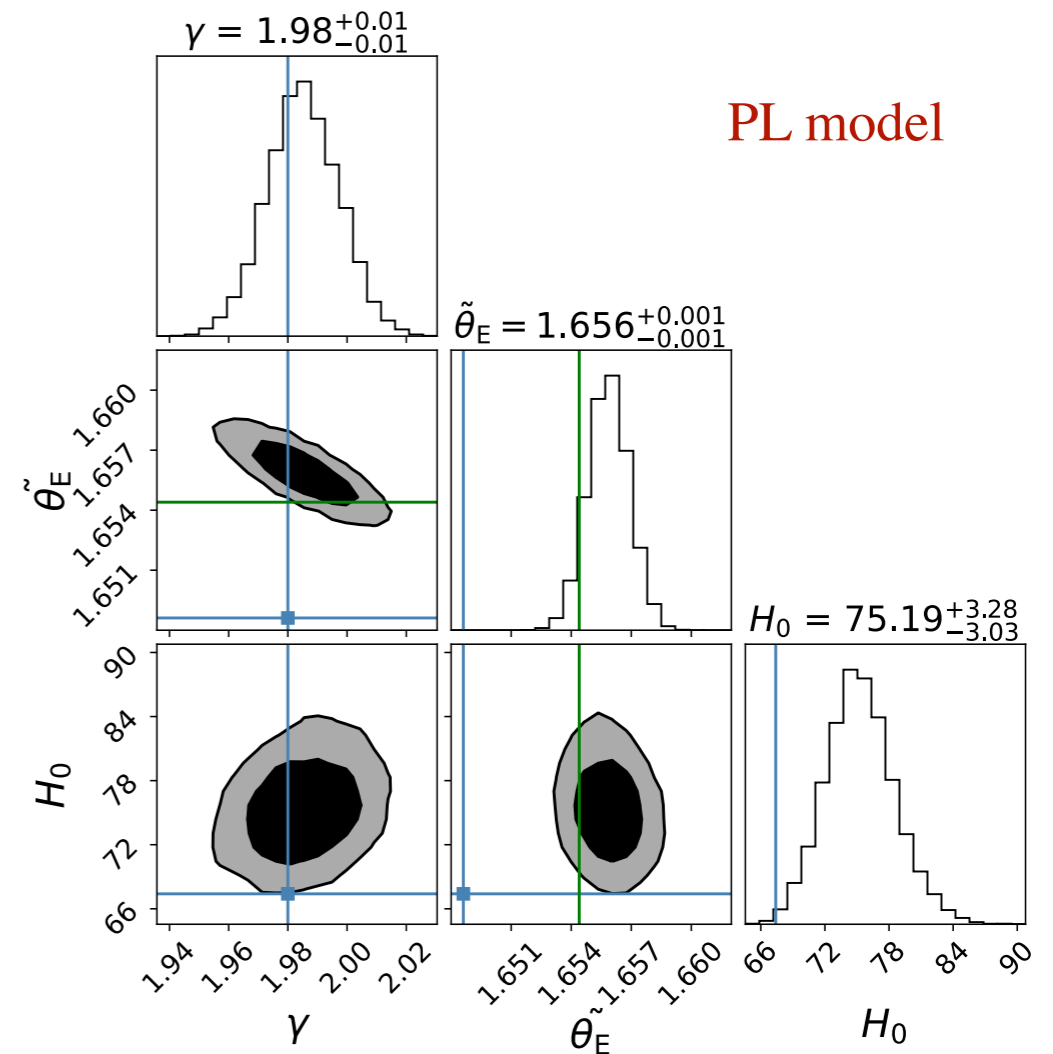
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Expect evidence for core component, reflecting precision on H_0

KB, Teodori, 2105.10873

Mock inference using
power-law model.

Truth has $H_0=67.4$ km/s/Mpc,
and a 10% core!

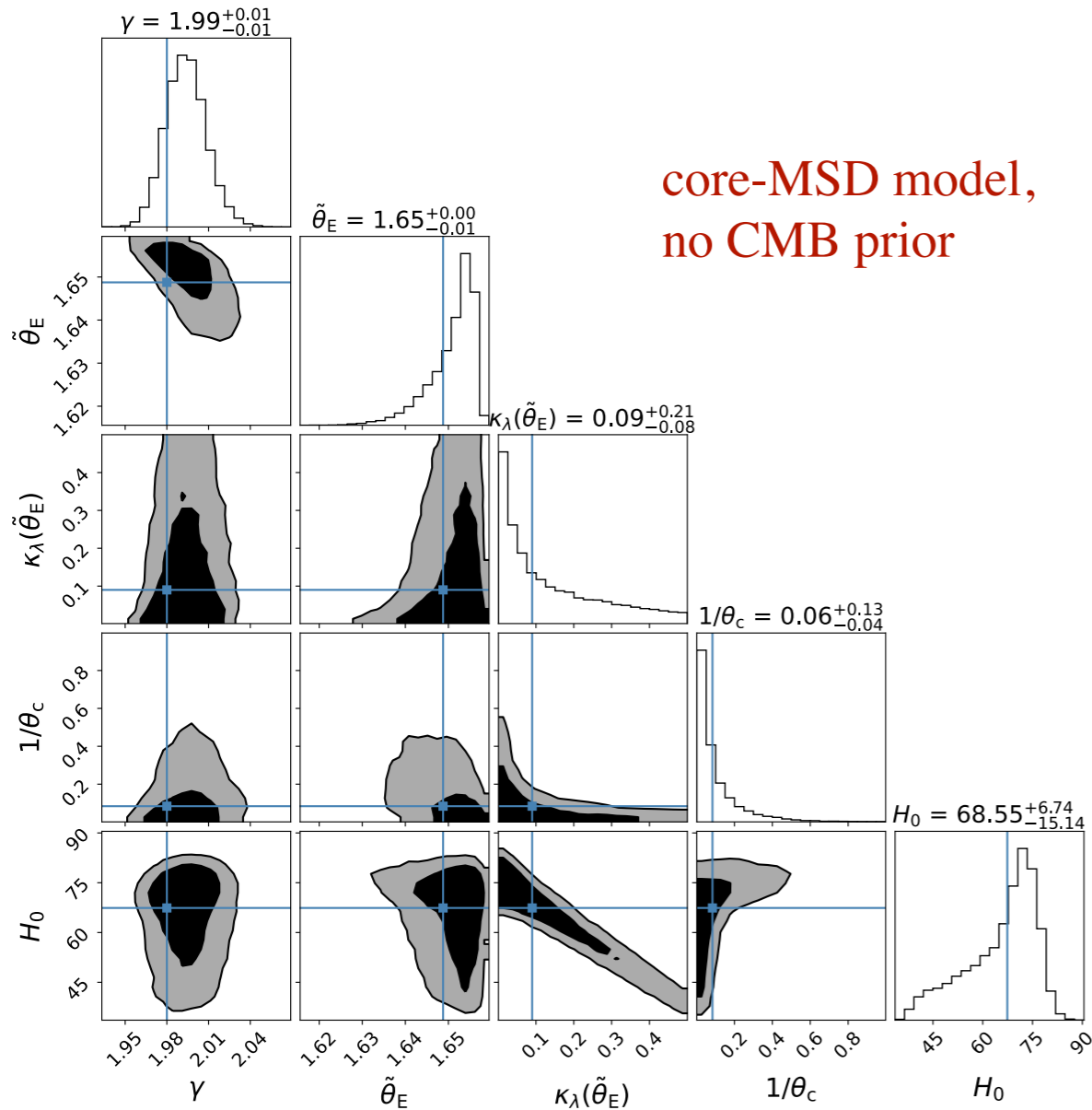


3. Opportunities: galactic structure

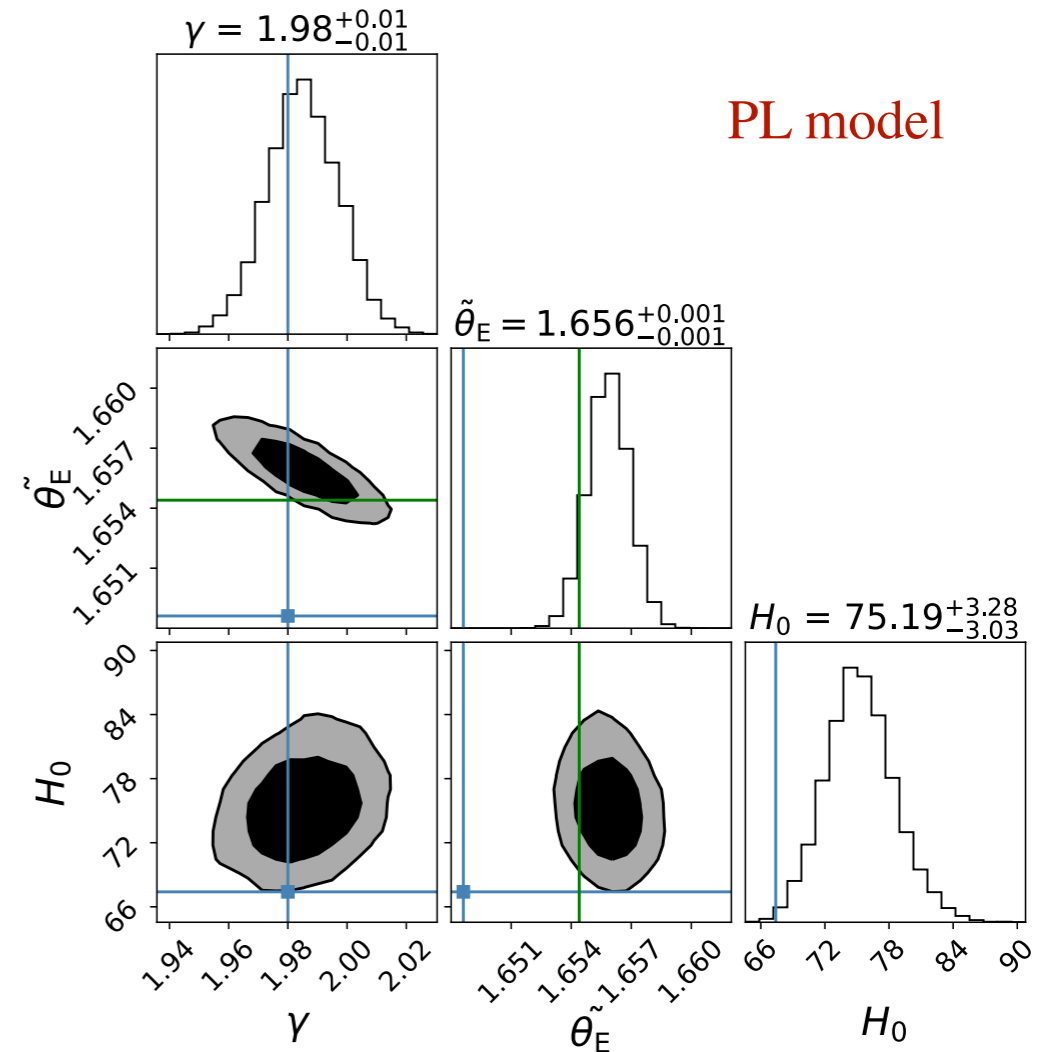
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KB, Teodori, 2105.10873



core-MSD model,
no CMB prior



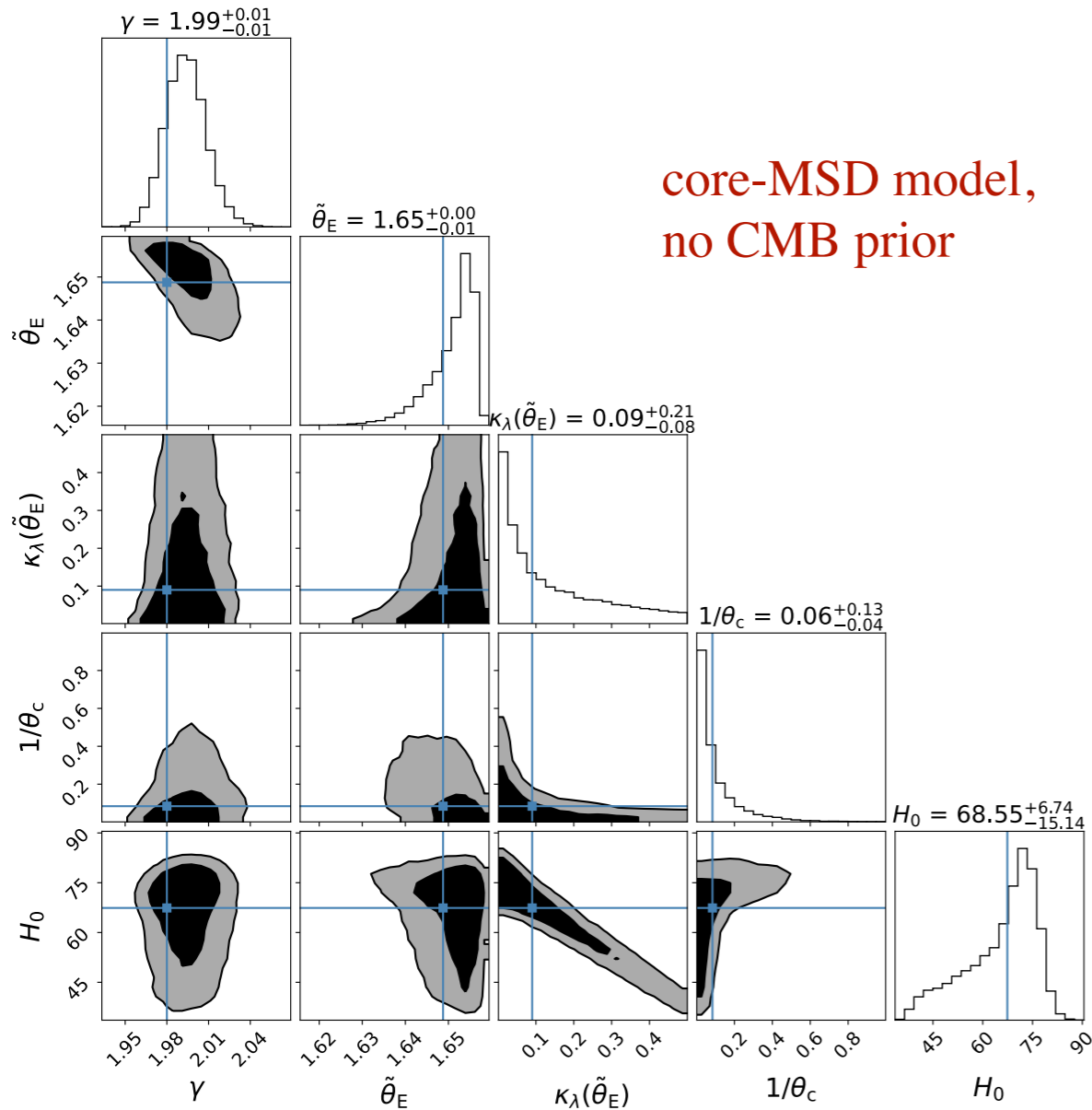
PL model

3. Opportunities: galactic structure

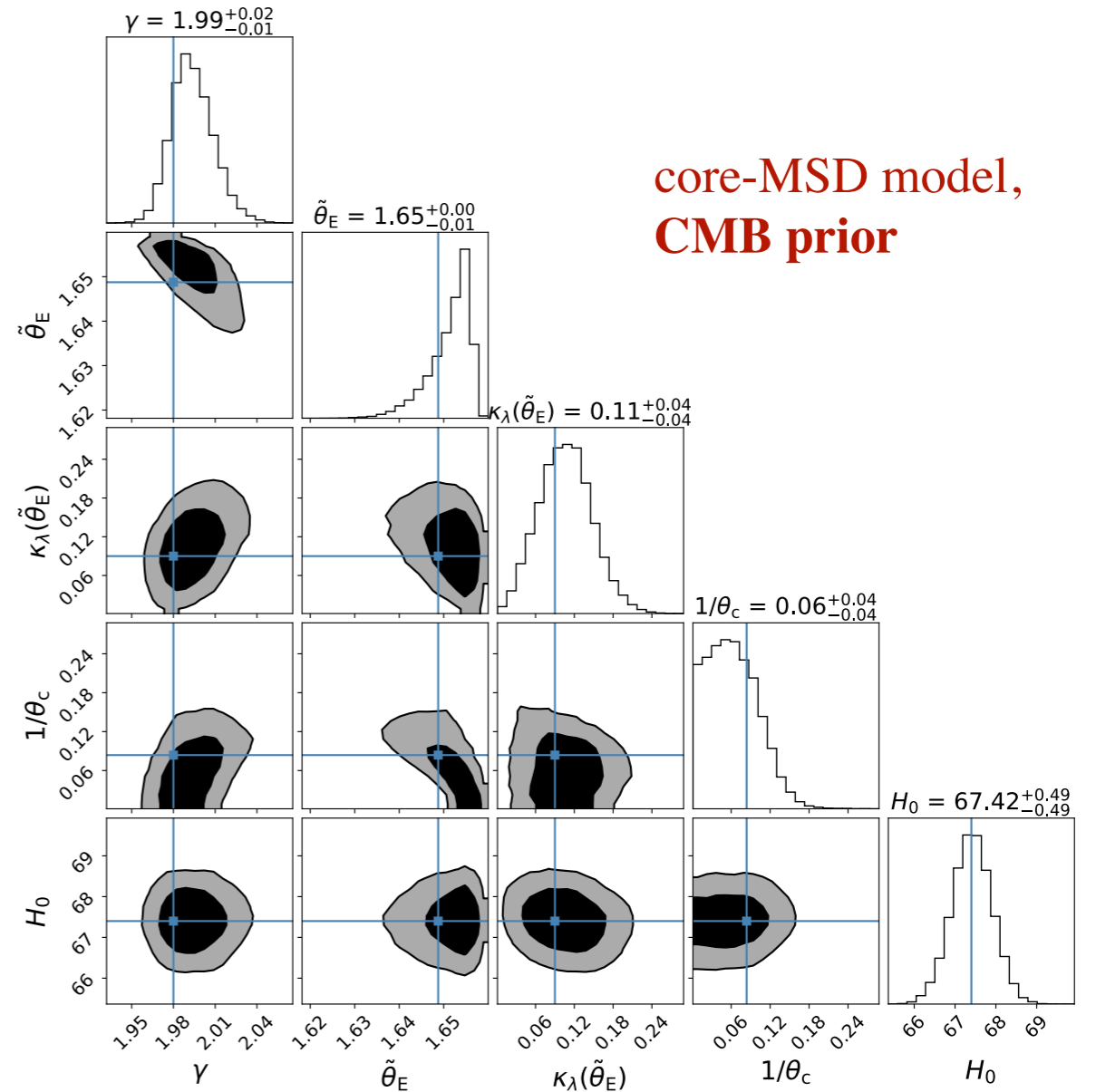
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core-MSD model,
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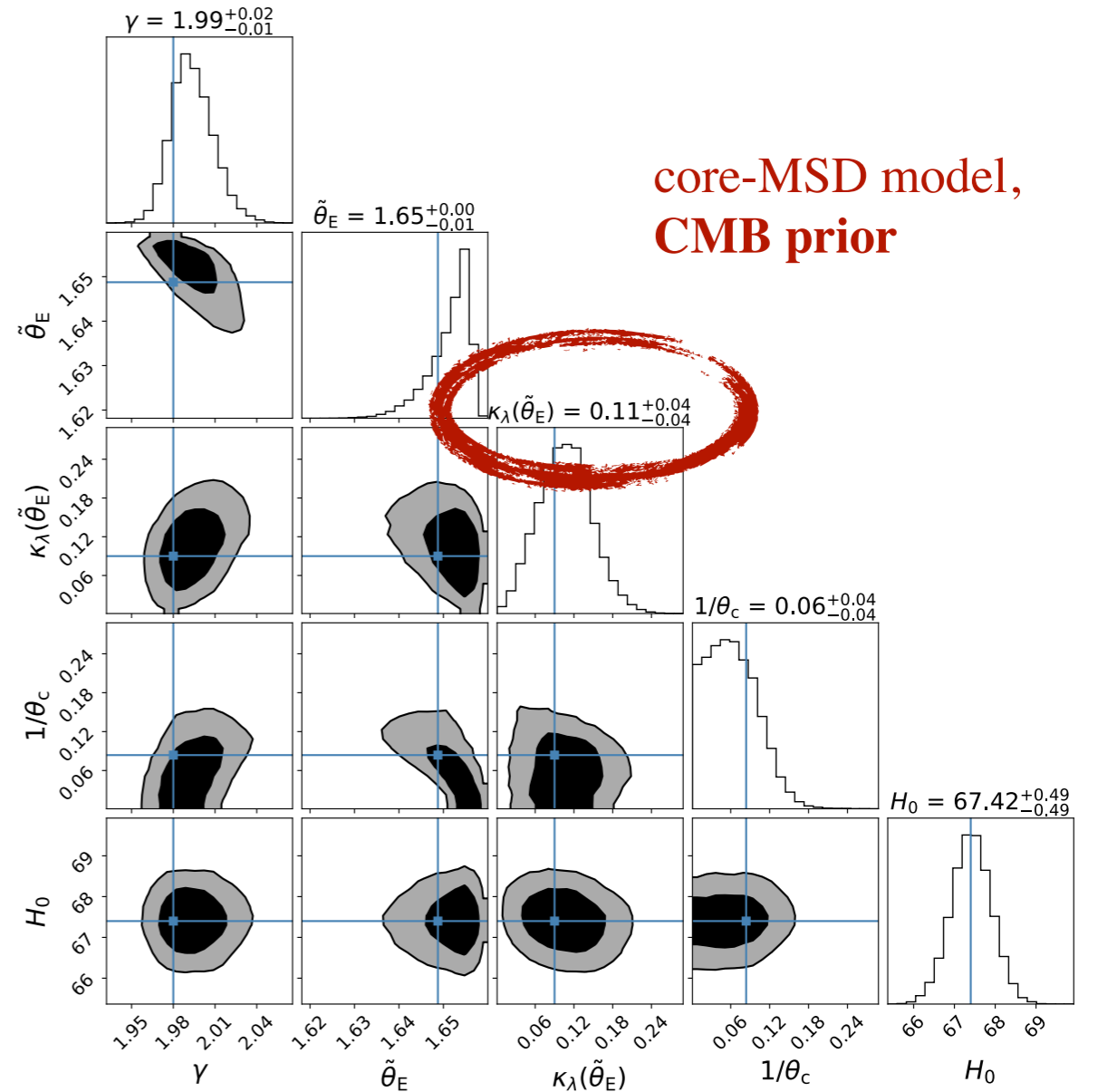
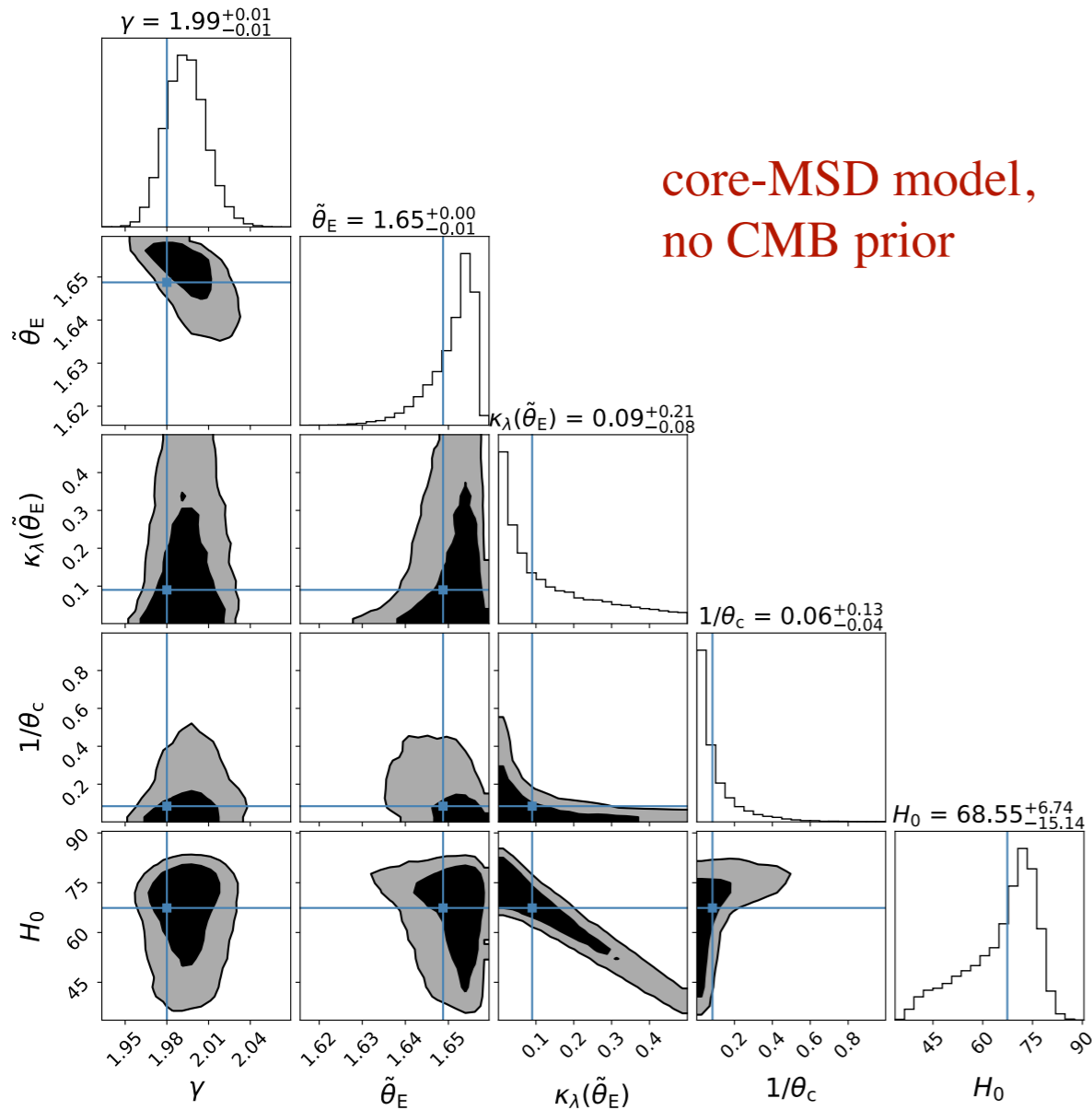
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What do we learn about galaxies if we add CMB/LSS prior?

Expect evidence for core component, reflecting precision on H_0

KB, Teodori, 2105.10873

Could reach $>2\sigma$ per system



A step towards covering internal MSD (no CMB prior) :

Birrer et al 2020 (**TDCOSMO IV**)

Data sets	H_0 [km s ⁻¹ Mpc ⁻¹]	$\lambda_{\text{int},0}$	α_λ	$\sigma(\lambda_{\text{int}})$	a_{ani}	$\sigma(a_{\text{ani}})$	$\sigma_{\sigma^{\text{P}},\text{sys}}$
TDCOSMO-only	$74.5^{+5.6}_{-6.1}$	$1.02^{+0.08}_{-0.09}$	$0.00^{+0.07}_{-0.07}$	$0.01^{+0.03}_{-0.01}$	$2.32^{+1.62}_{-1.17}$	$0.16^{+0.50}_{-0.14}$	-
TDCOSMO + SLACS _{IFU}	$73.3^{+5.8}_{-5.8}$	$1.00^{+0.08}_{-0.08}$	$-0.07^{+0.06}_{-0.06}$	$0.07^{+0.09}_{-0.05}$	$1.58^{+1.58}_{-0.54}$	$0.15^{+0.47}_{-0.13}$	-
TDCOSMO + SLACS _{SDSS}	$67.4^{+4.3}_{-4.7}$	$0.91^{+0.05}_{-0.06}$	$-0.04^{+0.04}_{-0.04}$	$0.02^{+0.04}_{-0.01}$	$1.52^{+1.76}_{-0.70}$	$0.28^{+0.45}_{-0.25}$	$0.06^{+0.02}_{-0.02}$
TDCOSMO + SLACS _{SDSS+IFU}	$67.4^{+4.1}_{-3.2}$	$0.91^{+0.04}_{-0.04}$	$-0.07^{+0.03}_{-0.04}$	$0.06^{+0.08}_{-0.04}$	$1.20^{+0.70}_{-0.27}$	$0.18^{+0.50}_{-0.15}$	$0.06^{+0.02}_{-0.02}$

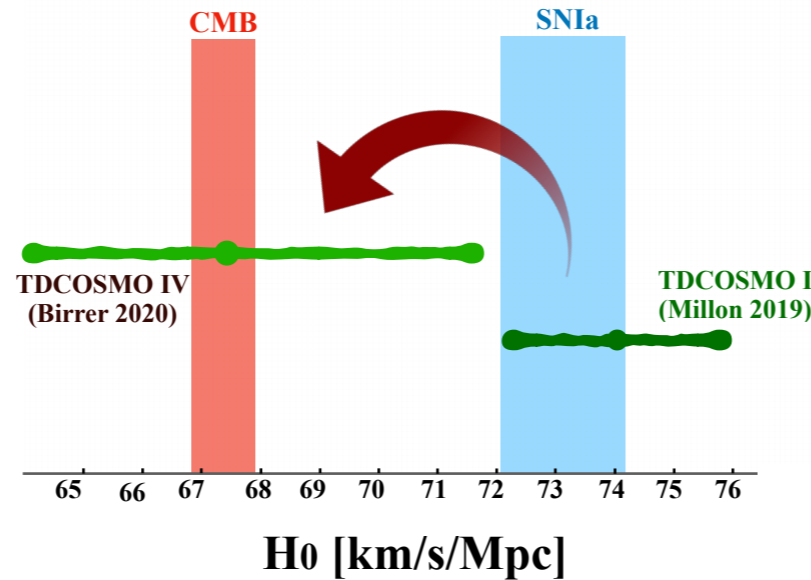
KB, Castorina, Simonovic 2020

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Birrer et al 2020 (TDCOSMO IV)

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TDCOSMO + SLACS _{SDSS}	67.4 ^{+4.3} _{-4.7}	0.91 ^{+0.05} _{-0.06}	-0.04 ^{+0.04} _{-0.04}	0.02 ^{+0.04} _{-0.01}	1.52 ^{+1.76} _{-0.70}	0.28 ^{+0.45} _{-0.25}	0.06 ^{+0.02} _{-0.02}
TDCOSMO + SLACS _{SDSS+IFU}	67.4 ^{+4.1} _{-3.2}	0.91 ^{+0.04} _{-0.04}	-0.07 ^{+0.03} _{-0.04}	0.06 ^{+0.08} _{-0.04}	1.20 ^{+0.70} _{-0.27}	0.18 ^{+0.50} _{-0.15}	0.06 ^{+0.02} _{-0.02}



Hint from SLACS kinematics?

(but see comments in KB, Teodori, 2105.10873)

KB, Castorina, Simonovic 2020

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“Internal MSD” may require a **non-minimal** density profile.

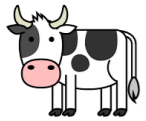


Why would galaxies be non-minimal?



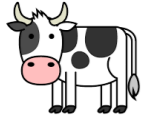
Why should galaxies have a core component?

``Internal MSD'' may require a **non-minimal** density profile.



Why would galaxies be non-minimal?

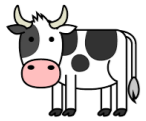
Why not? (What is dark matter?)



Why should galaxies have a core component?

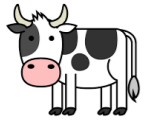
Can think of several reasons.

“Internal MSD” may require a **non-minimal** density profile.



Why would galaxies be non-minimal?

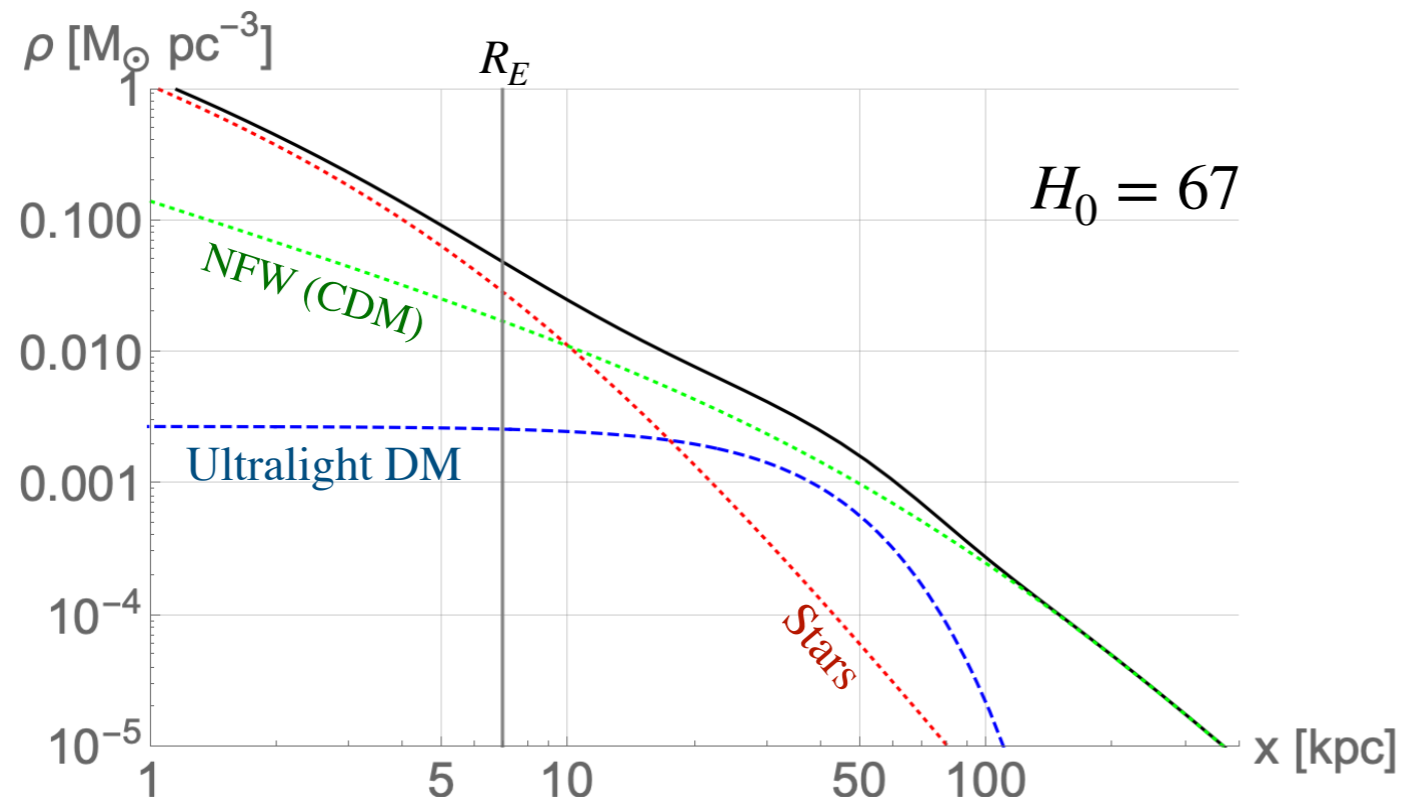
Why not? (What is dark matter?)



Why should galaxies have a core component?

Can think of several reasons.

Dark matter not boring NFW? ...a little bit of ultralight dark matter?



KB, Teodori, 2105.10873

20 % of total DM, $m = 2.5 \times 10^{-25}$ eV

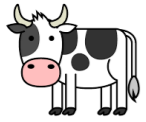
Dynamical relaxation consistent at O(1).
Cosmology OK.

“Internal MSD” may require a **non-minimal** density profile.



Why would galaxies be non-minimal?

Why not? (What is dark matter?)



Why should galaxies have a core component?

Can think of several reasons.

“Missing” baryons?



X-ray: NASA/CXC/SAO/S.Randall et al., Optical: SDSS
Werner & Mernier, 2001.10023

Mass models of TDCOSMO lens systems: stars/DM ~ 0.05 .

Much below cosmological baryon/DM ratio.
This is typical, puzzle of missing baryons.

Missing baryons probably in extended CGM.

What is the convergence due to the CGM?
— What is the radial scale of the CGM?

Should have enough mass to make an effect, *if mostly within ~ 50 kpc.*

Summary

Lensing H_0 sensitive to galaxy profile at few % level:
Feature in the galaxy profile, or breakdown of Λ CDM?

Weak lensing: include all segments of line of sight.
Lacking in published results. Likely \sim % bias on H_0 .

Teodori, et al, 2201.05111

Adding a core to a density profile is an approximate MSD.
10% core explains the lensing H_0 tension?

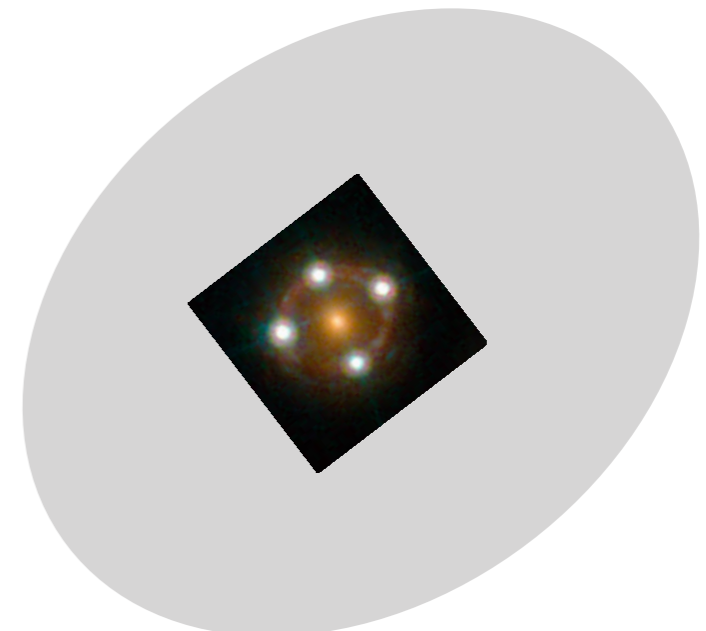
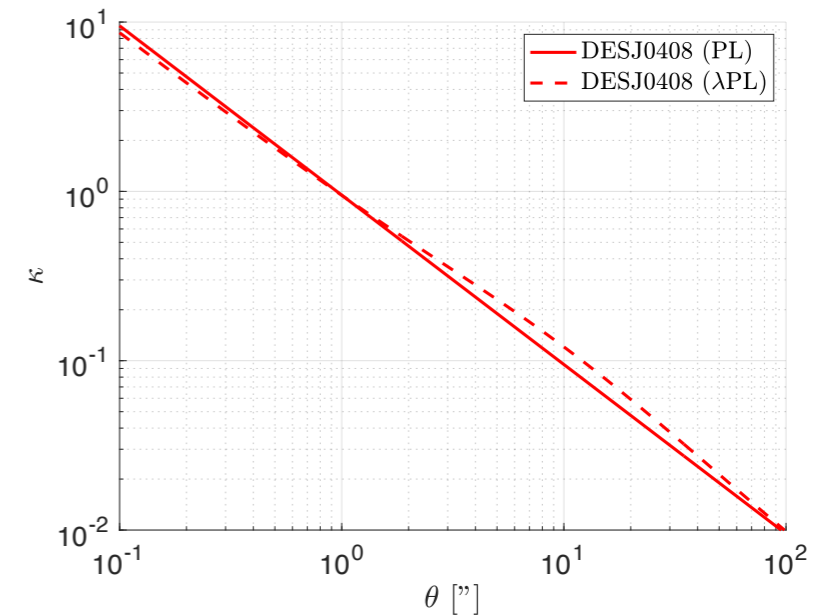
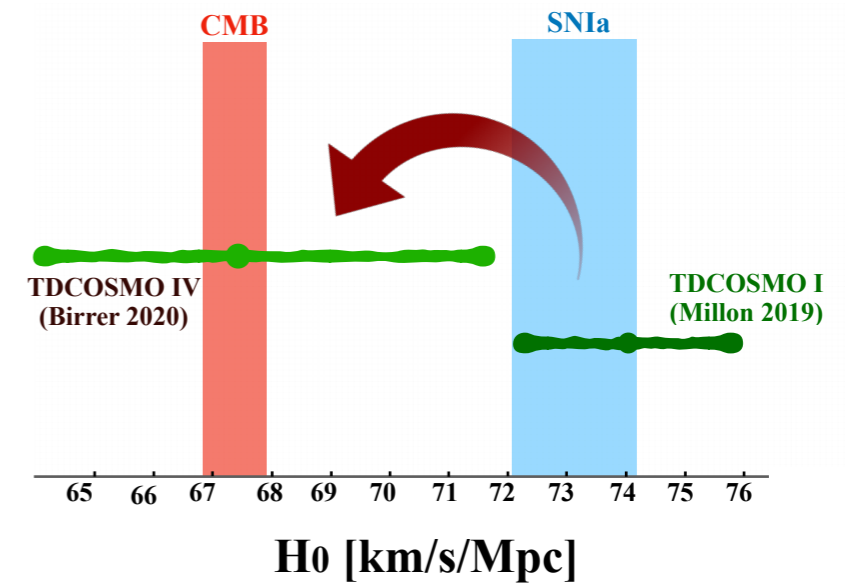
KB, Castorina, Simonović 2001.07182

Could point to interesting dark matter dynamics.
If we go there, may as well adopt CMB (or SNIa!) prior on H_0 .

Ultralight DM (axion-like):

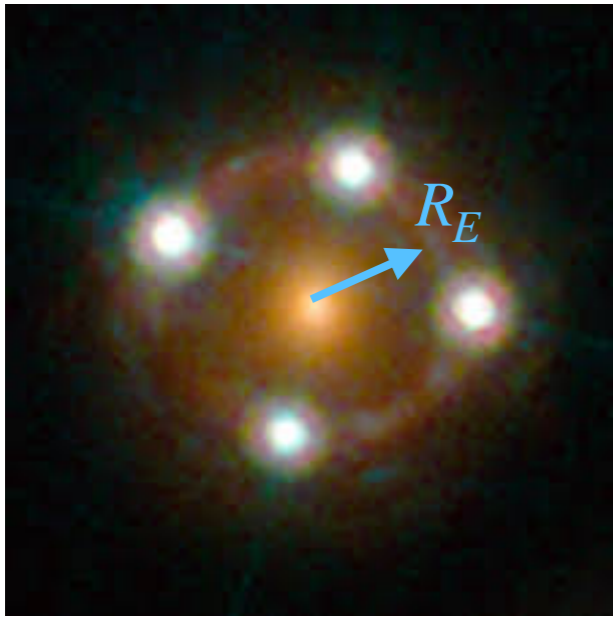
Vanilla vacuum misalignment. Dynamically makes a core.
Correct ballpark to solve lensing H_0 tension, if
Dynamical relaxation consistent at O(1) level. $10^{-25} \text{ eV} \lesssim m \lesssim 10^{-24} \text{ eV}$

KB, Teodori 2105.10873

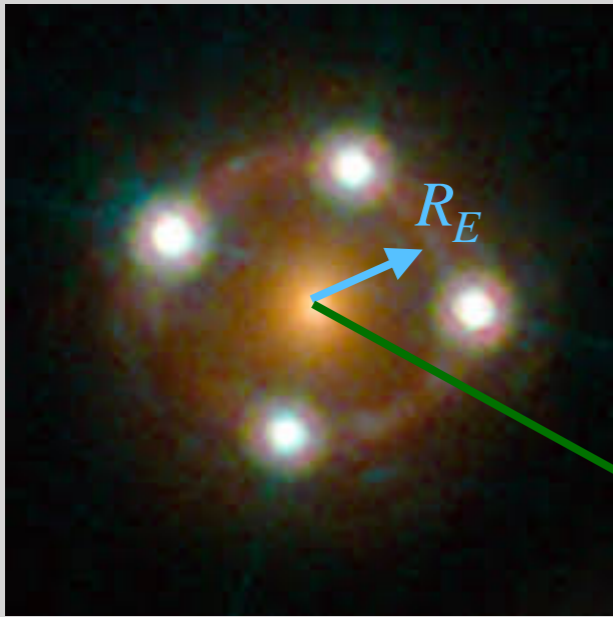


Thank you!

Xtra

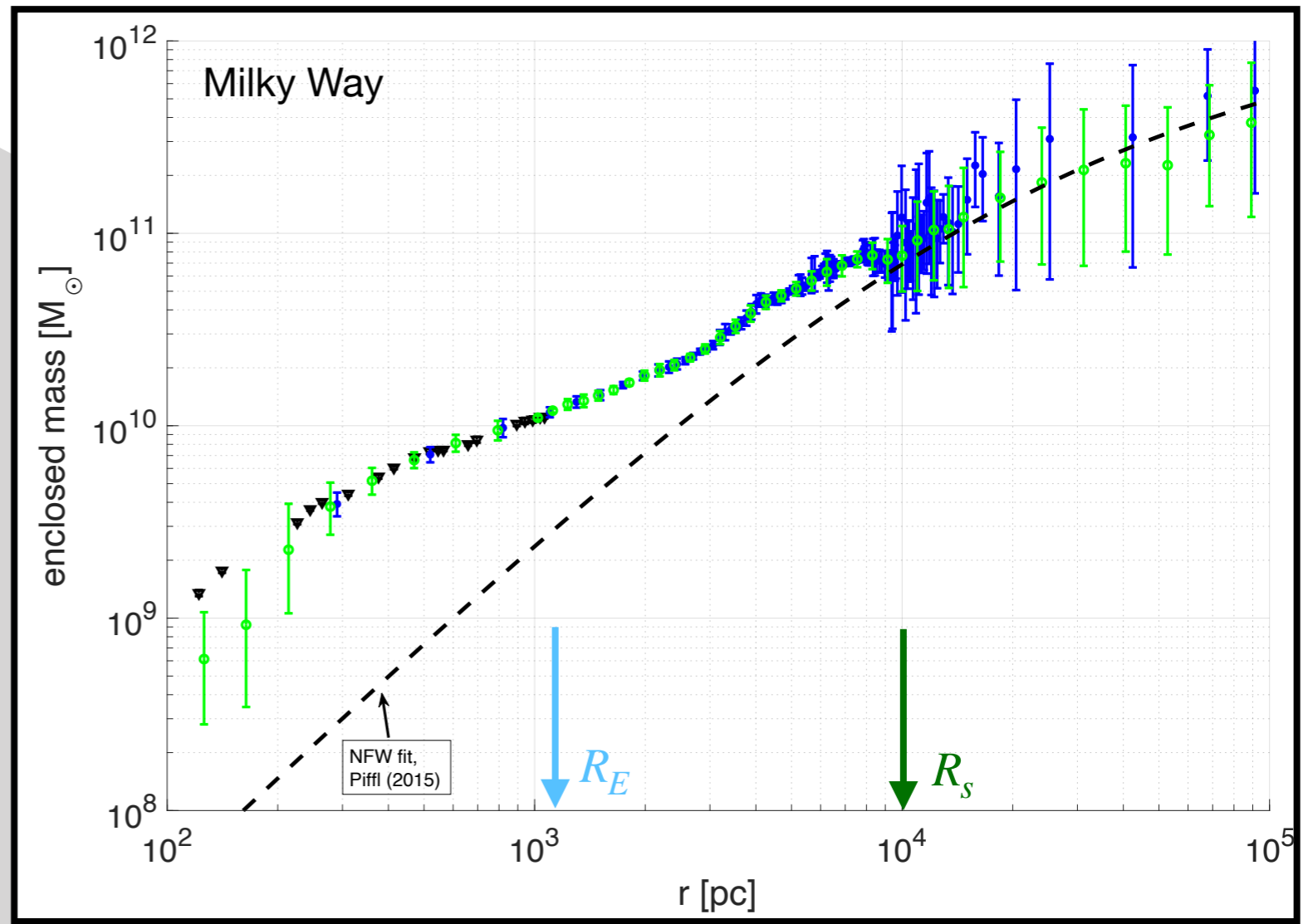
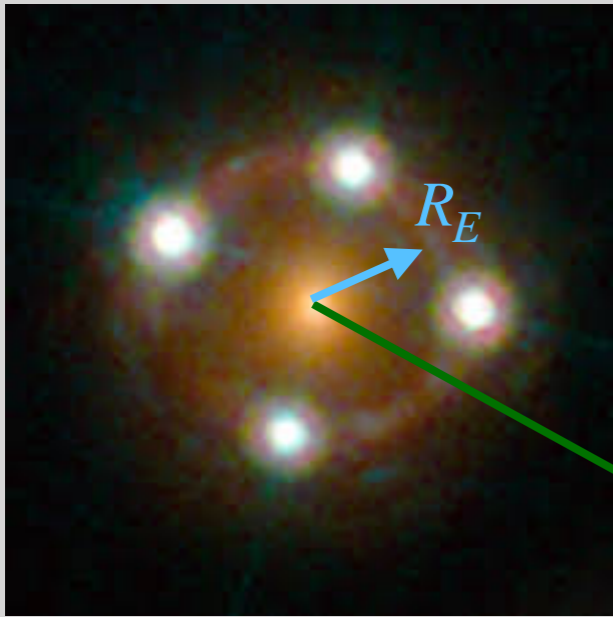


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R_s

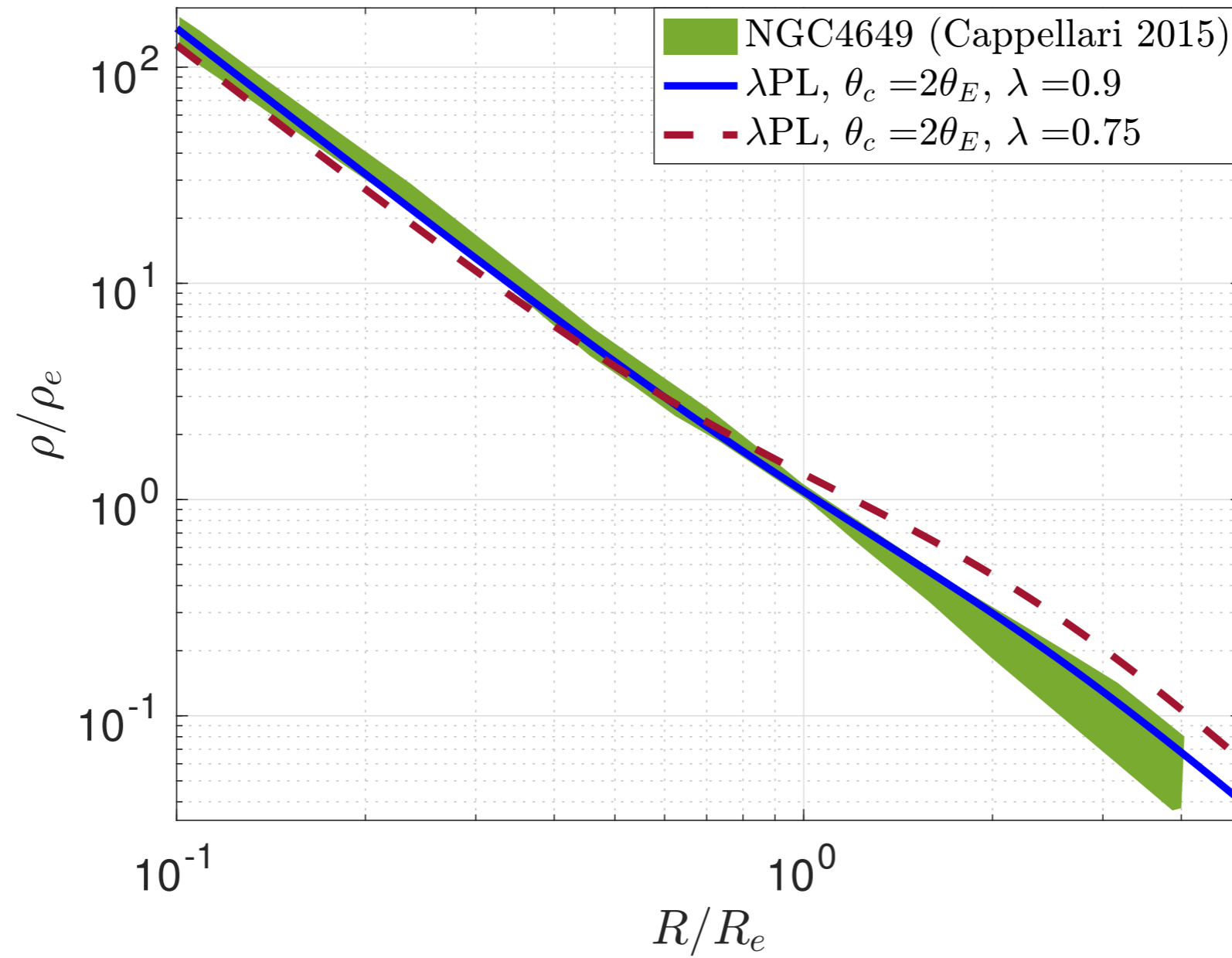
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R_s

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Stellar kinematics (of *other* elliptical galaxies)

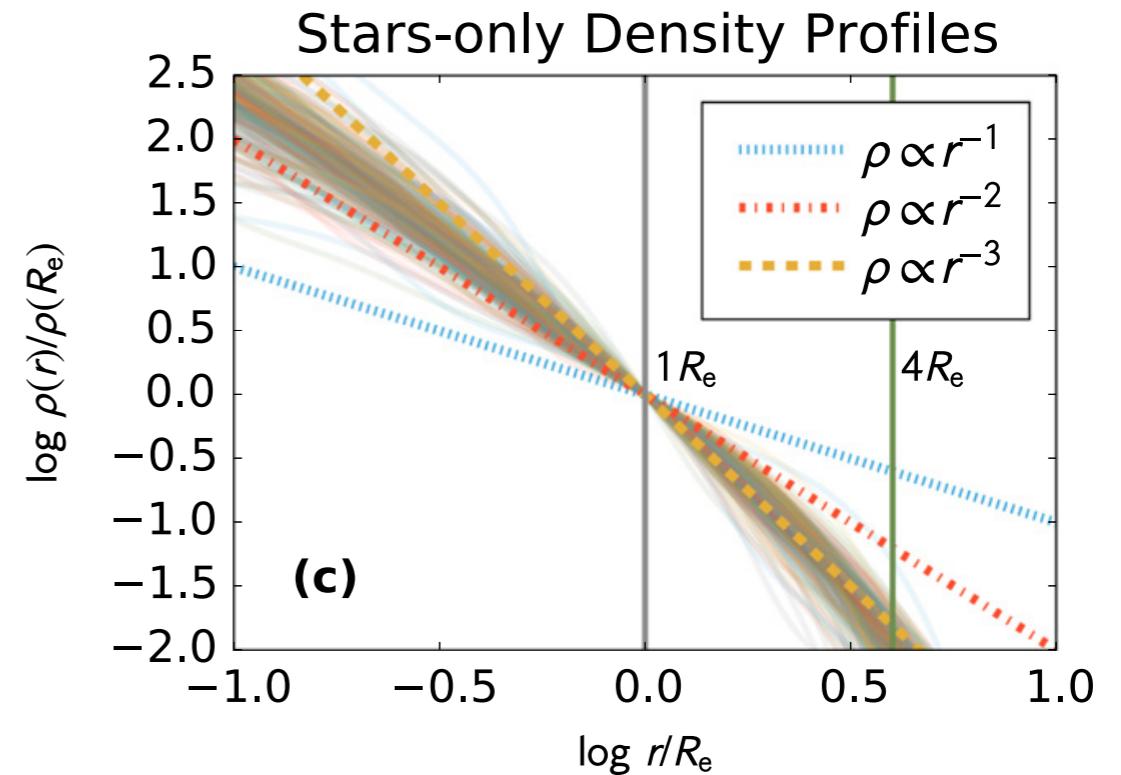
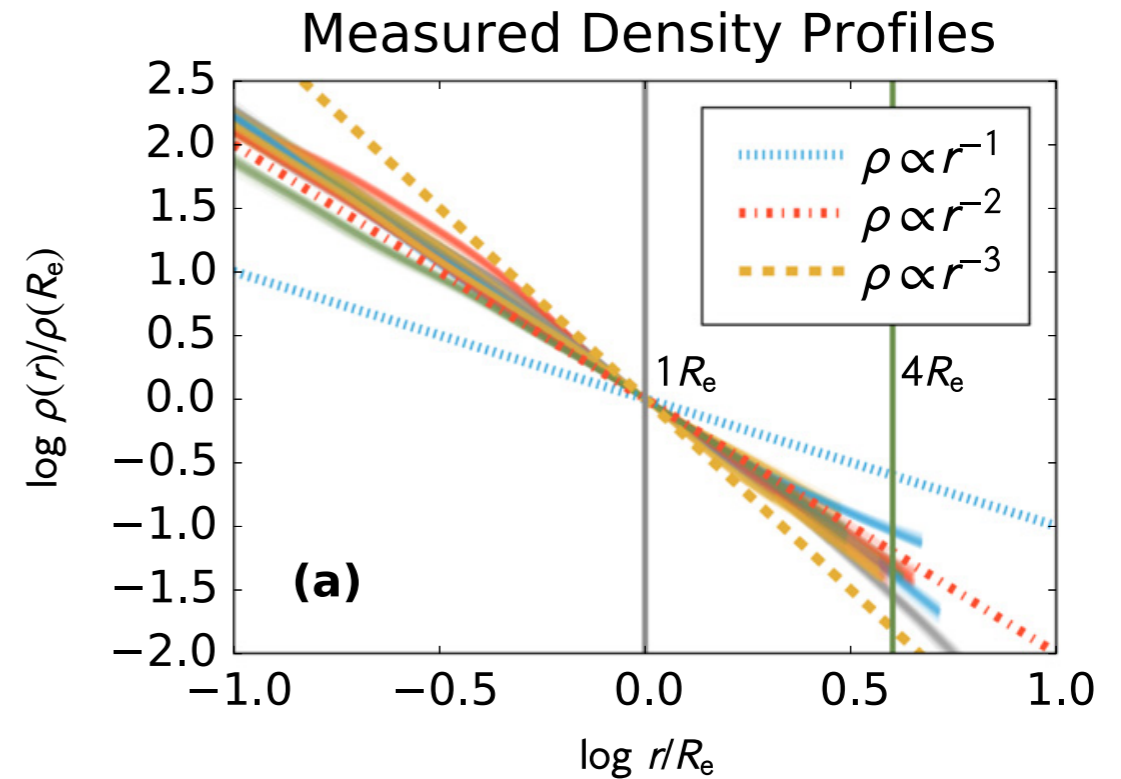


Stellar kinematics (of *other* elliptical galaxies)

Cappellari et al, 1504.00075

$$\rho_{\text{DM}}(r) = \rho_s \left(\frac{r}{r_s} \right)^\alpha \left(\frac{1}{2} + \frac{1}{2} \frac{r}{r_s} \right)^{-\alpha-3}. \quad (3)$$

Our models have seven free parameters. Some are poorly constrained but are not of interest here. They are just “nuisance parameters,” marginalized out to derive the total mass profiles studied here. The parameters are (i) the inclination i ; (ii) the anisotropy $\beta_z \equiv 1 - \sigma_z^2 / \sigma_R^2$, with σ_z and σ_R the stellar dispersion in cylindrical coordinates, for the MGE Gaussians with $\sigma_j < R_e$; (iii) the anisotropy for the remaining Gaussians at larger radii; (iv) the stellar $(M/L)_{\text{stars}}$; (v) the break radius of the dark halo, constrained to be $10 < r_s < 50$ kpc; (vi) the halo density ρ_s at r_s ; and (vii) the dark halo slope α for $r \ll r_s$.

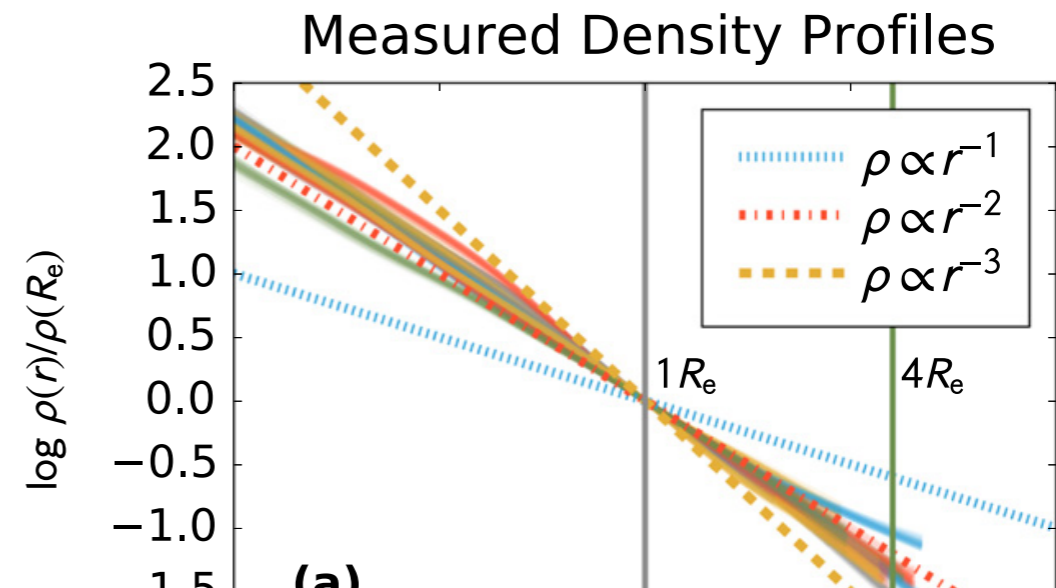


Stellar kinematics (of *other* elliptical galaxies)

Cappellari et al, 1504.00075

$$\rho_{\text{DM}}(r) = \rho_s \left(\frac{r}{r_s} \right)^\alpha \left(\frac{1}{2} + \frac{1}{2} \frac{r}{r_s} \right)^{-\alpha-3}. \quad (3)$$

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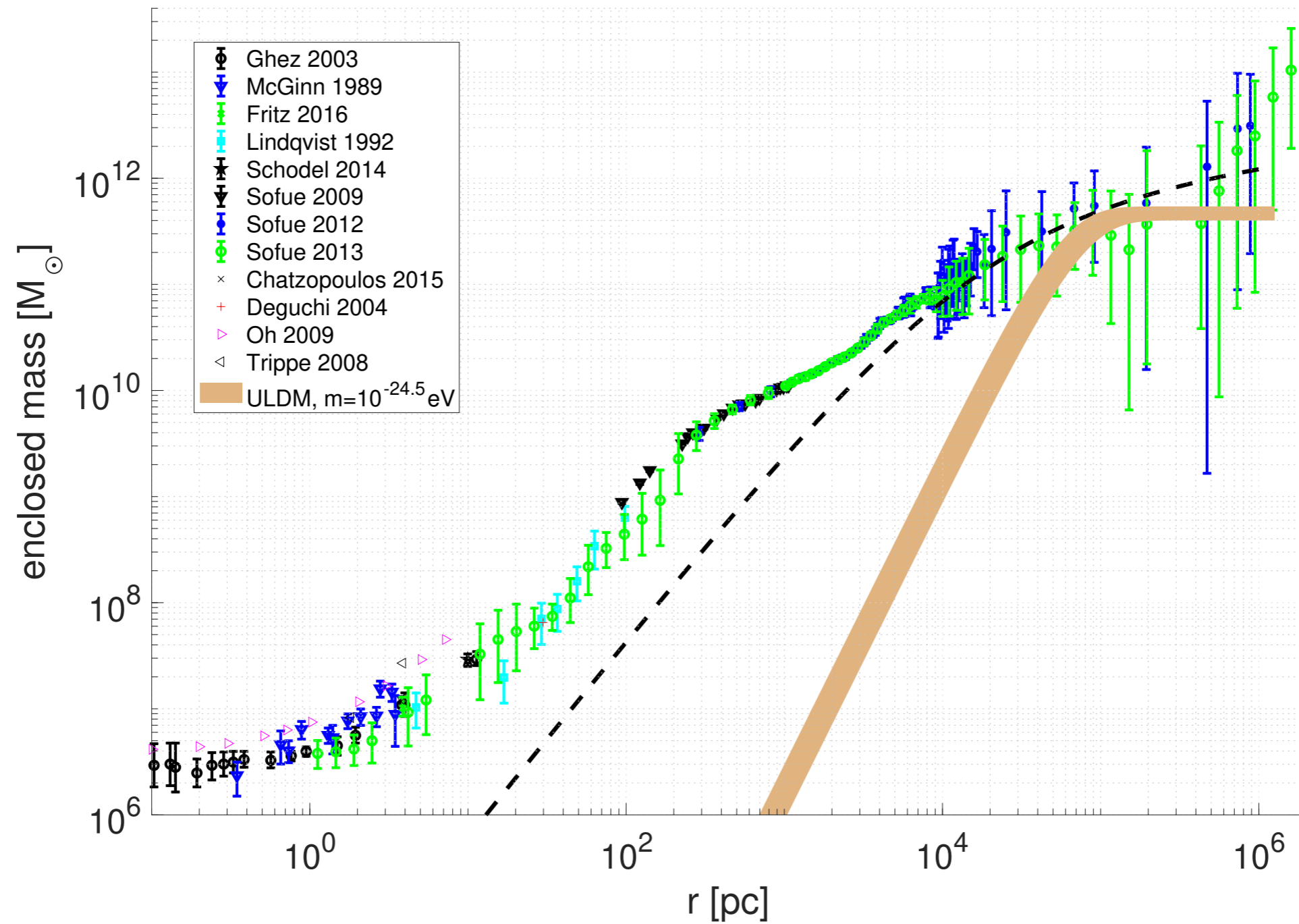


A friend:

...a cored structure of the kind you propose would be difficult to exclude from measurements of the stellar kinematics. Part of the reason is the mass profile-velocity anisotropy degeneracy. Another part is simply that no one has tried: most modelers fit the system to a small number of components (stars, gas, dark matter, central black hole) with constant mass-to-light ratio and none of these look like the core you propose. It would be straightforward for some of the modelers to try adding cores.

I suppose some critics will say that your **cores are ad hoc**, but I think they are **less ad hoc than most of the modifications to cosmology needed to explain the Hubble discrepancy!**

Stellar kinematics (of *other* galaxies, e.g., Milky Way)



Ultralight dark matter (ULDM)

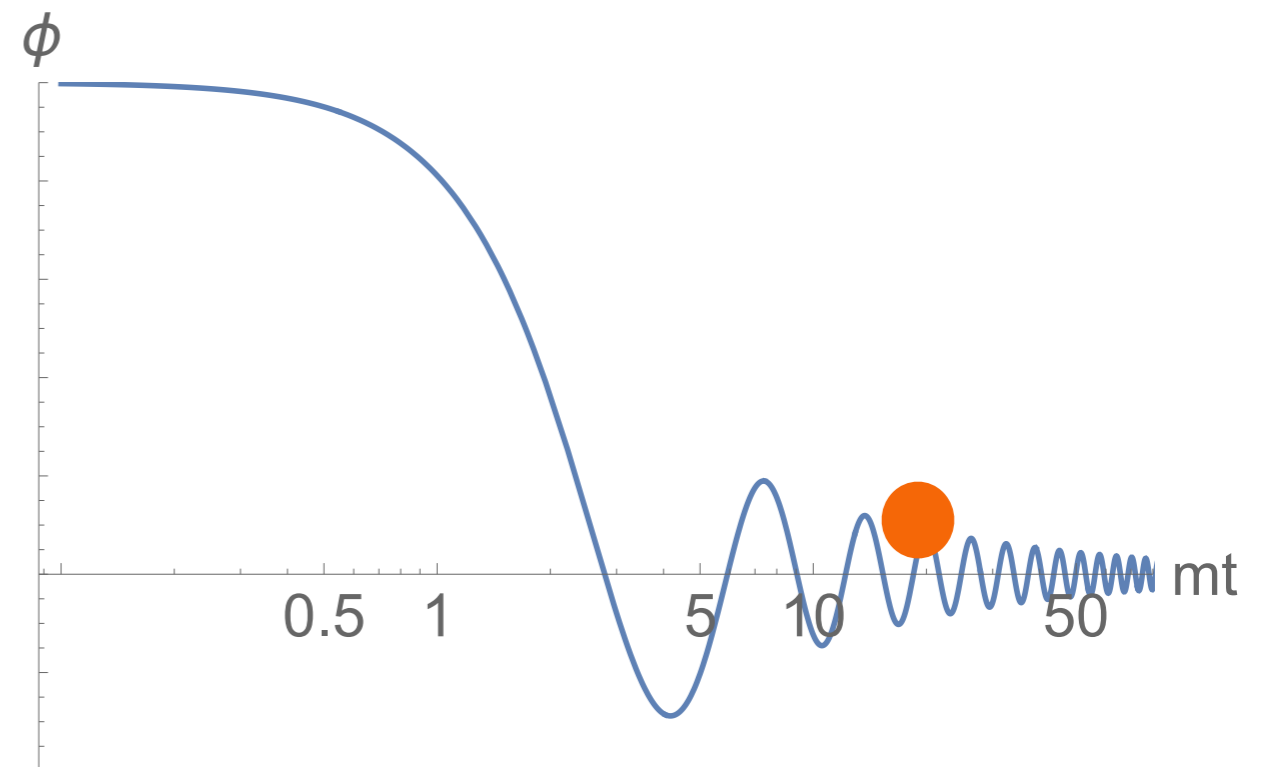
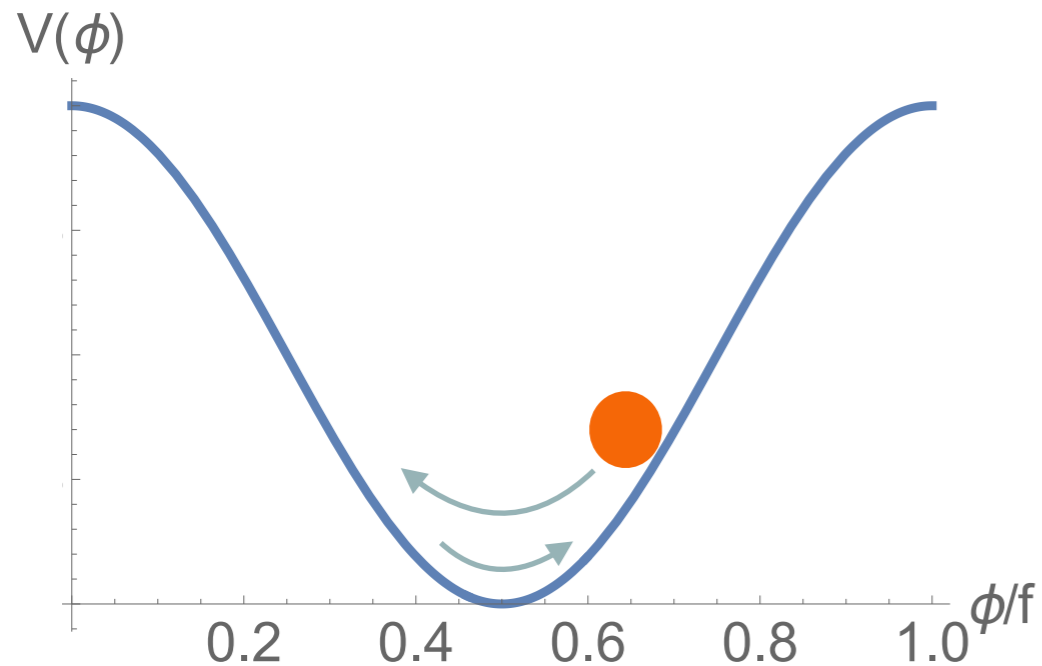
Light fields (Goldstone bosons) feature in many models. Svrcek & Witten 2006; Arvanitaki et al 2010

Cosmology: field initially displaced from minimum of the potential, starts to oscillate when $t \sim 1/m$.

When $t \gg 1/m$, correct equation of state for dark matter.

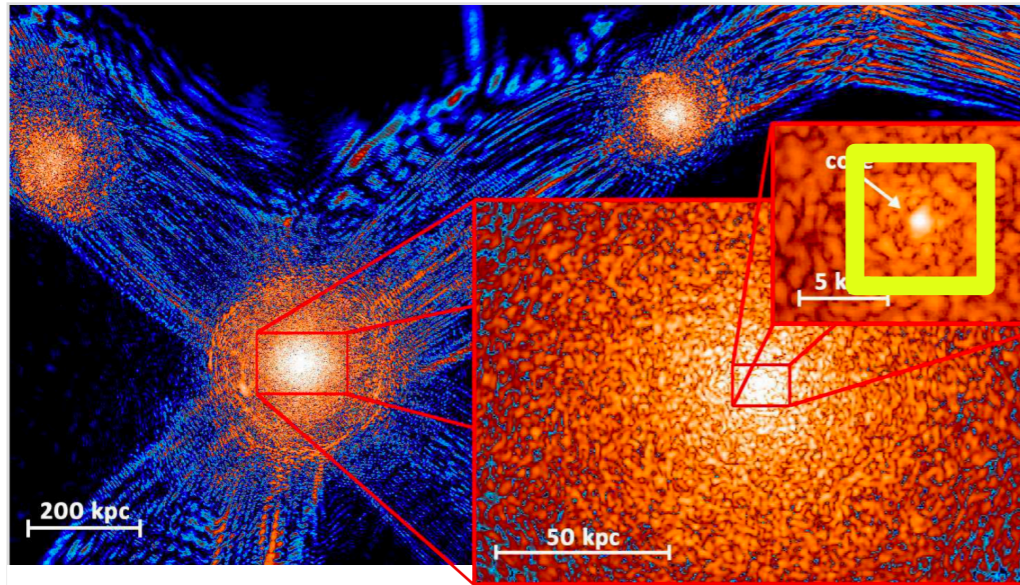
Contribution to energy density today:

$$\Omega_m \approx 0.3 \left(\frac{m}{10^{-21} \text{ eV}} \right)^{\frac{1}{2}} \left(\frac{f}{10^{17} \text{ GeV}} \right)^2$$

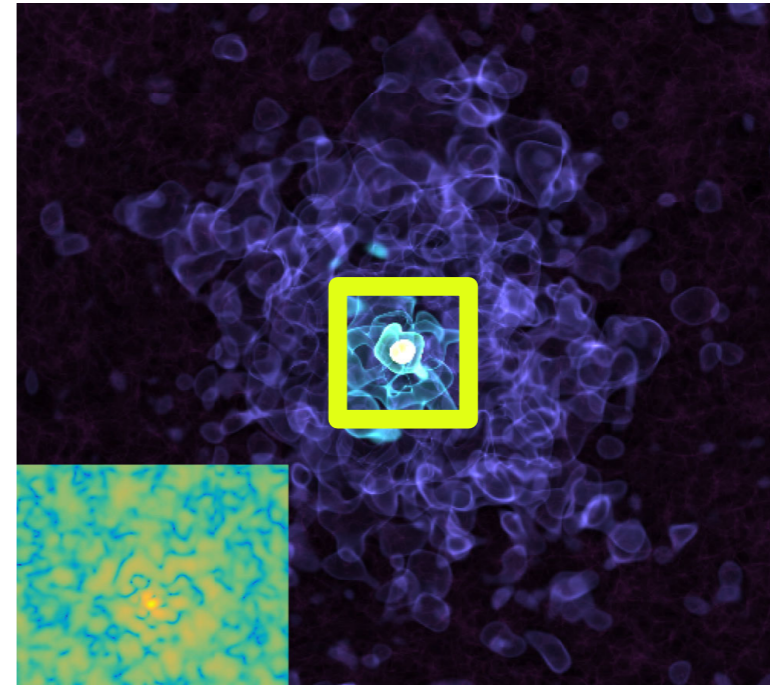


ULDM in galaxies

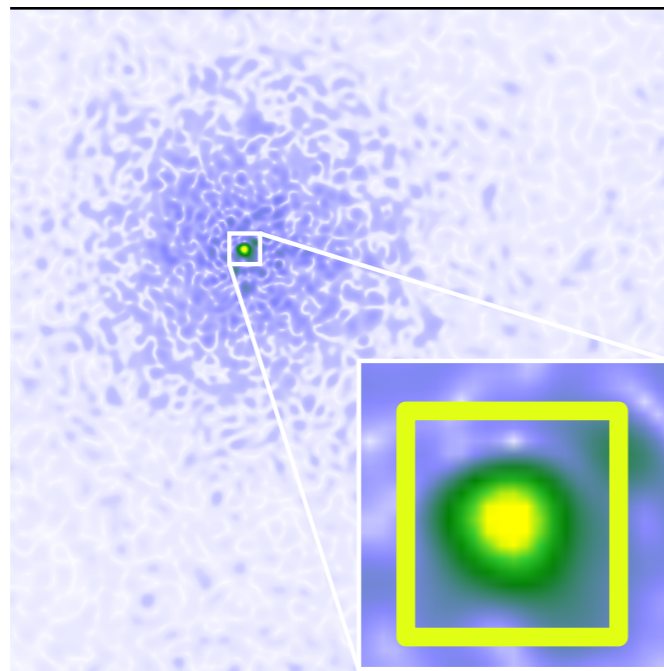
Inner part of simulated galaxies forms a **core**



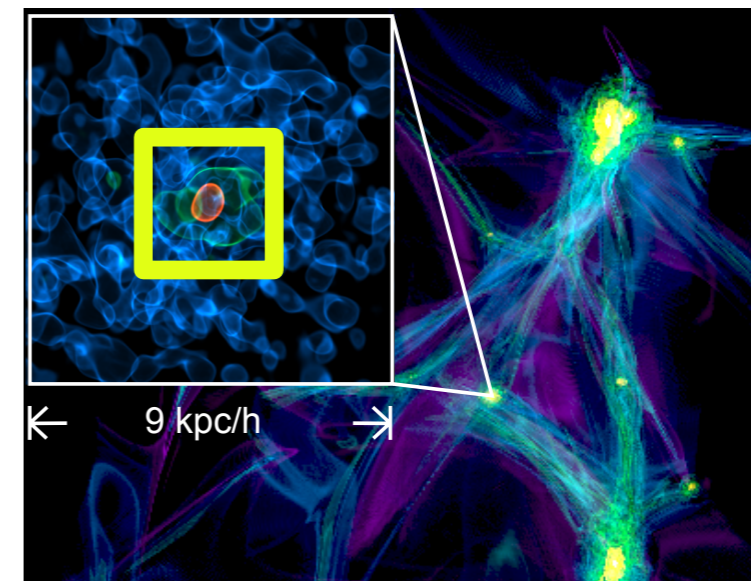
Schive et al 2014



Mocz et al 2017



Levkov et al 2018



Veltmaat et al 2018

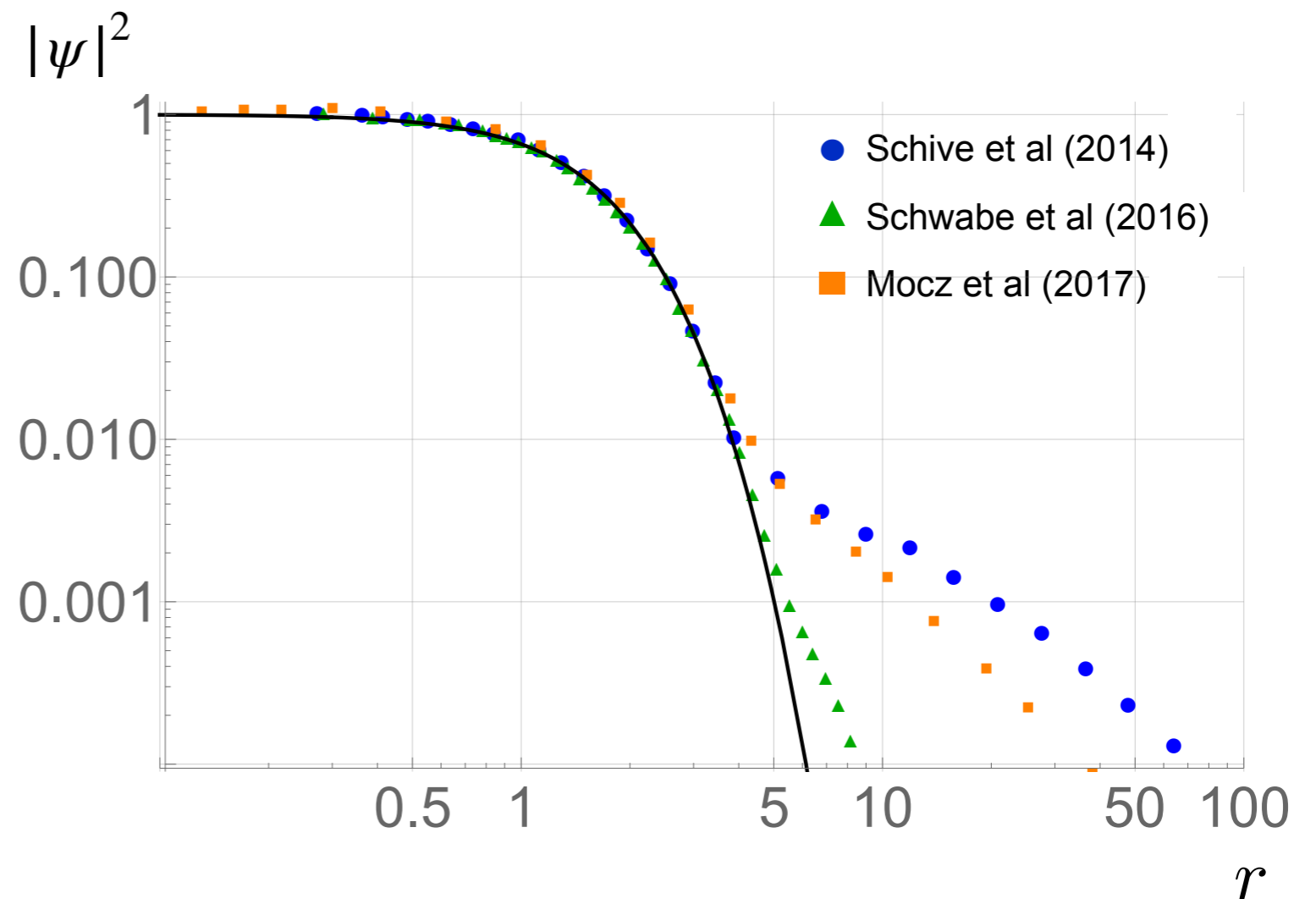
Self-gravitating ULDM / **Nonrelativistic limit**

Free scalar field $\phi(x, t) = \frac{1}{\sqrt{2m}} e^{-imt} \psi(x, t) + cc$

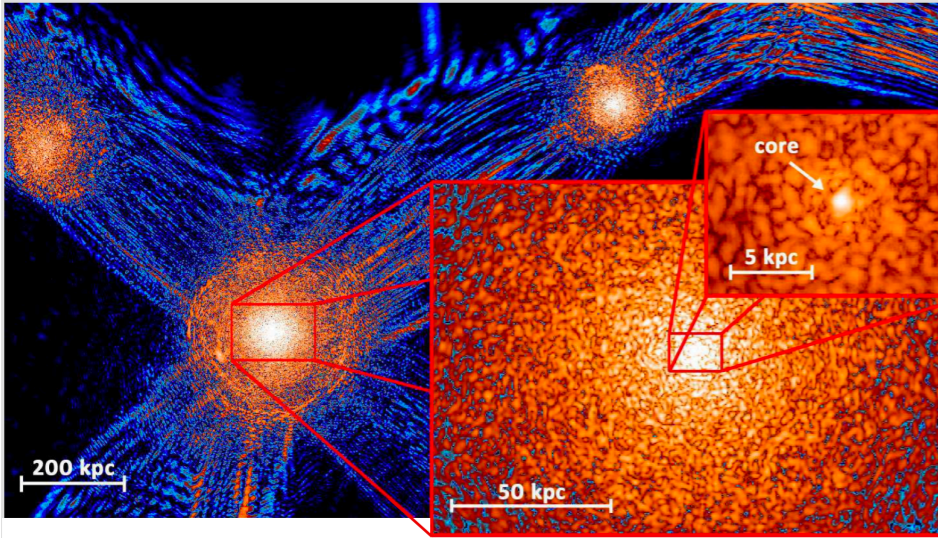
On scales of order de Broglie wavelength: coherent ground state

Numerical simulations
find the ground state.

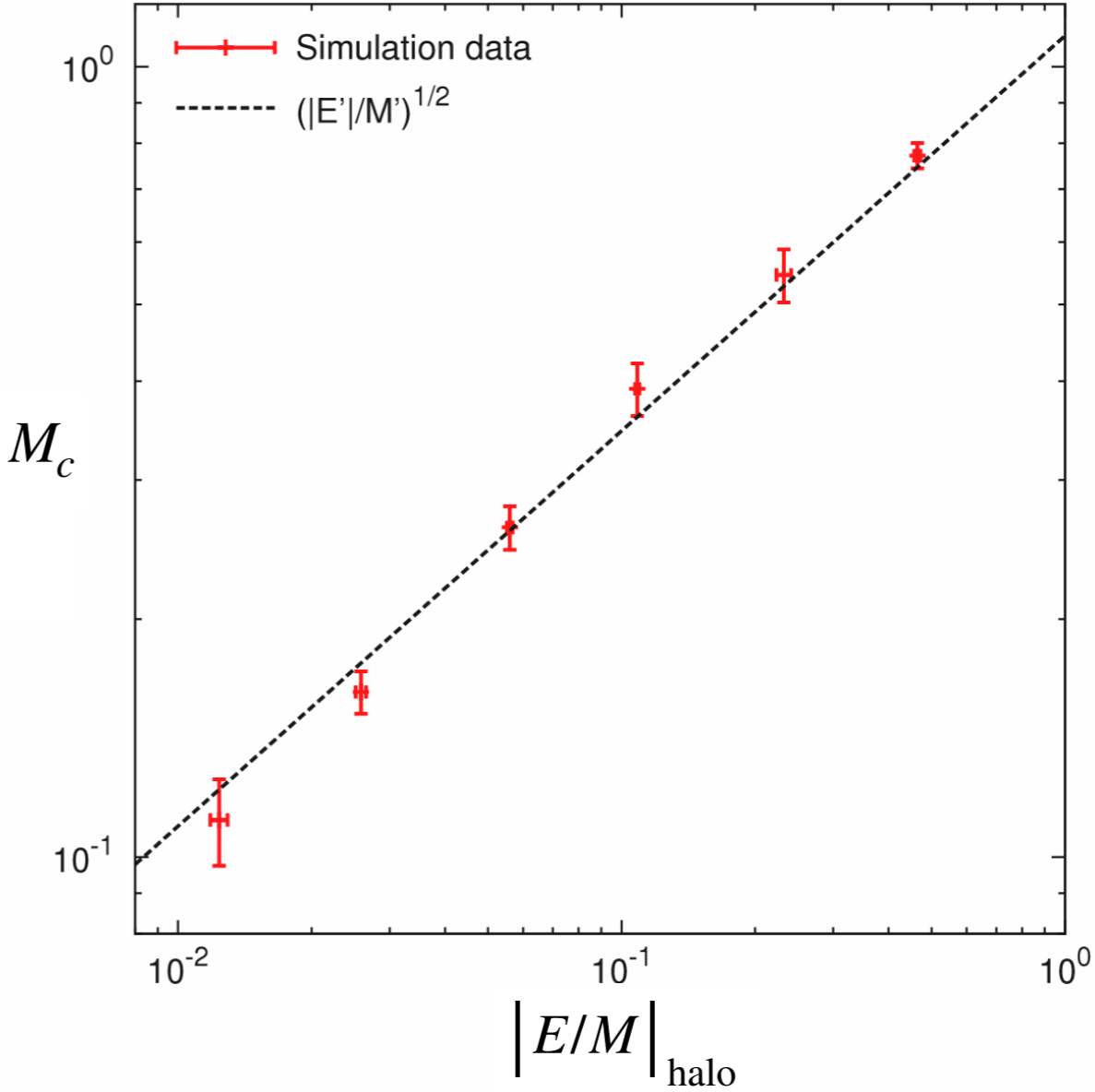
Bar, Blas, KB, Sibiryakov 2018



Core — halo relation: empirical evidence



Schive et al 2014; Veltmaat et al 2018



Core — halo relation: theoretical insight

Bar, Blas, KB, Sibiriyakov 2018

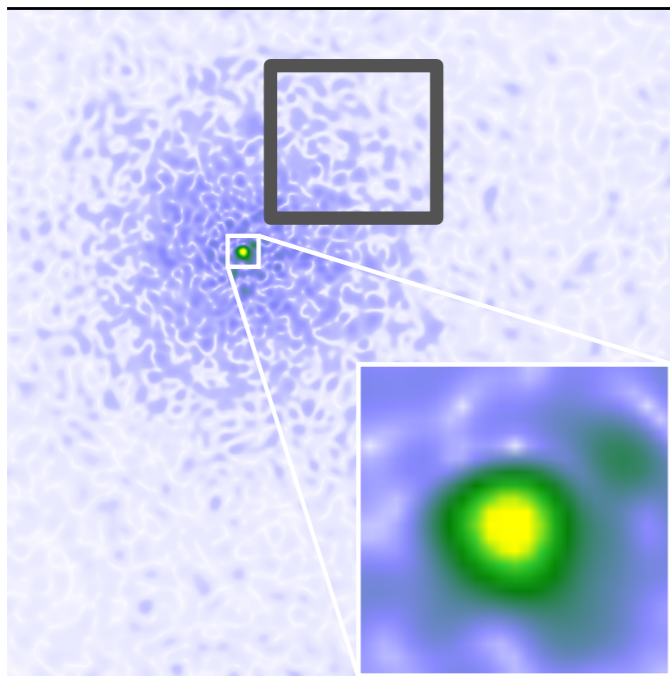
Bar, KB, Sato, Eby 2019

$$\frac{K}{M} \Big|_{\text{core}} = \frac{K}{M} \Big|_{\text{halo}}$$

Dynamical relaxation:

Hui et al 2017; Bar-Or, Tremaine 2018

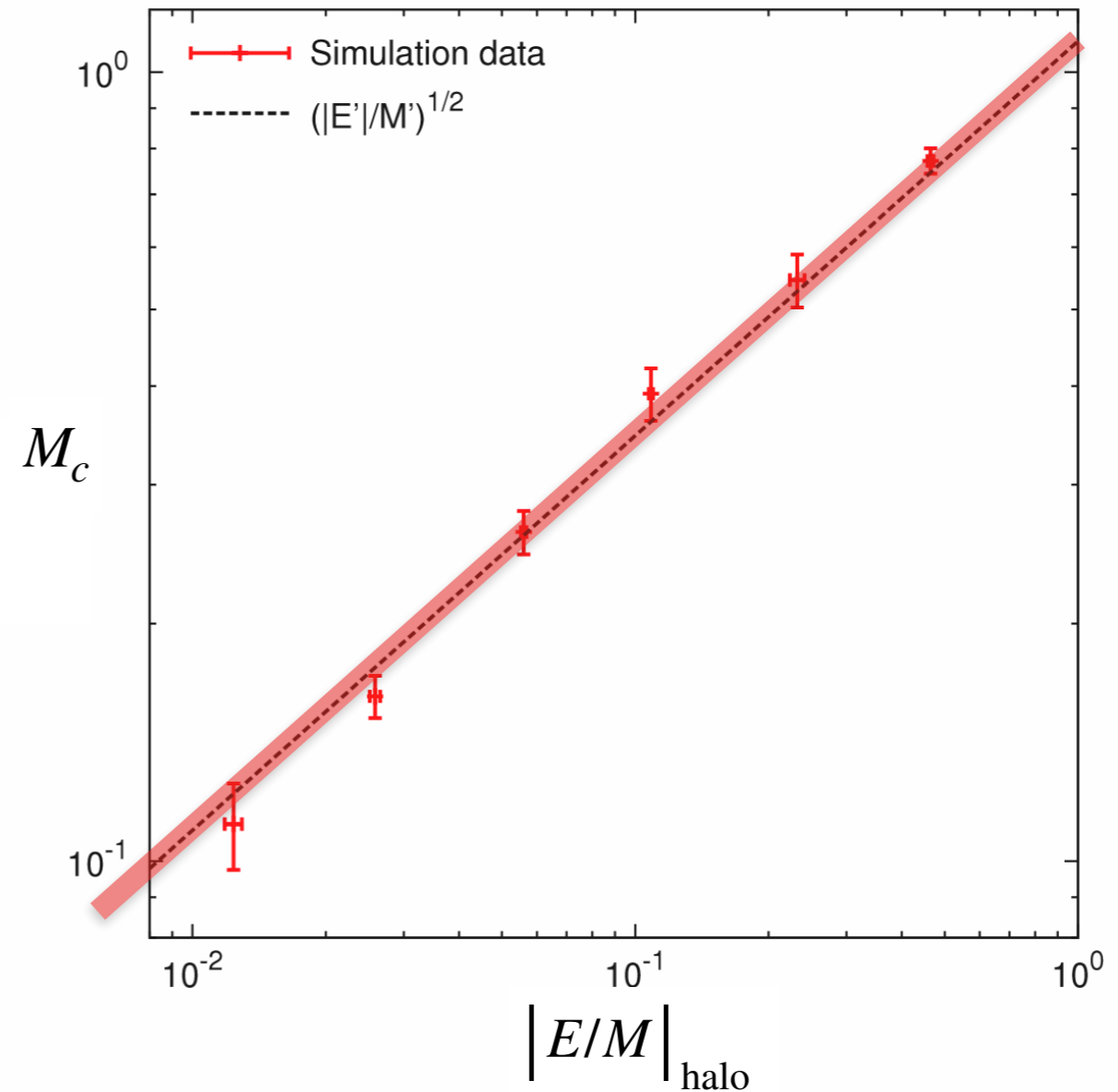
Bar, KB, Lacroix, Panci 2019



Levkov et al 2018

$$\tau \sim \frac{\sqrt{2}}{12\pi^3} \frac{m^3 \sigma^6}{G^2 \rho^2 \ln \Lambda}$$

Schive et al 2014; Veltmaat et al 2018



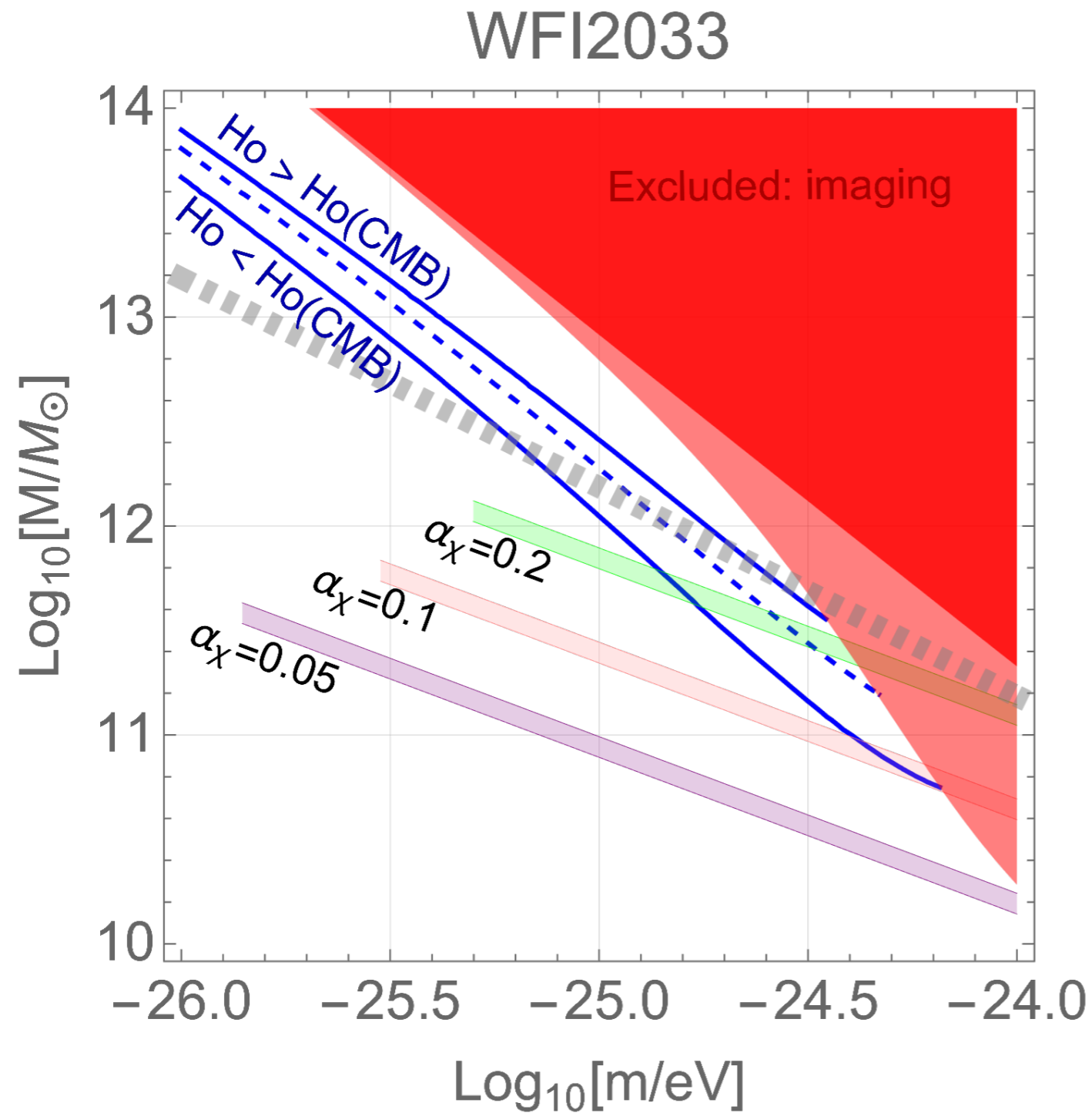
Also:

Eggemeier, Niemeyer 2019,

Chen et al 2020,

Schwabe et al 2020

ULDM as a solution of the lensing H₀ tension?

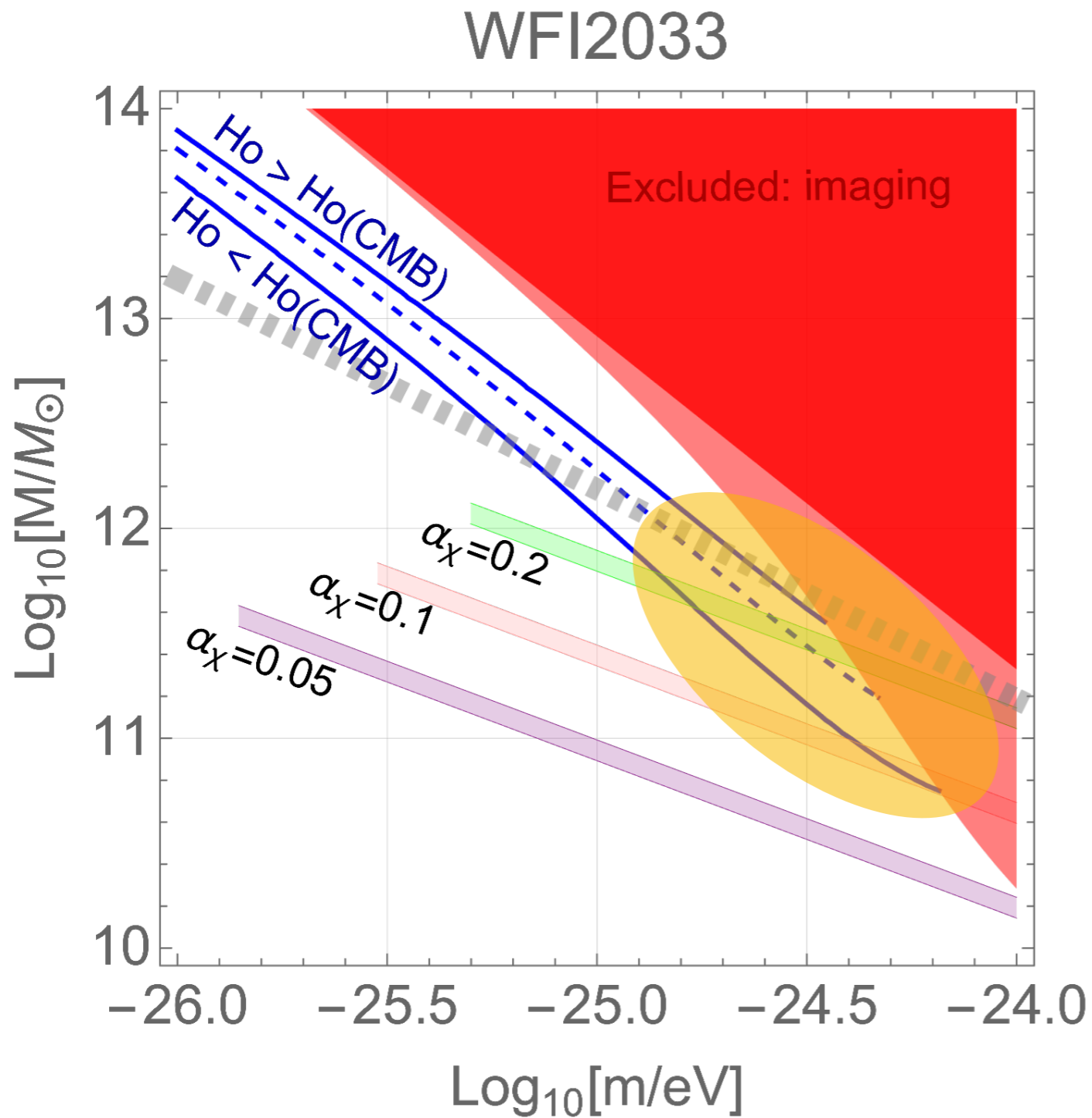


Dynamical relaxation
consistent at O(1),
can become a bottleneck:

$$\tau \sim \frac{\sqrt{2}}{12\pi^3} \frac{m^3 \sigma^6}{G^2 \rho^2 \ln \Lambda}$$

(But see Eggemeier, Niemeyer 2019,
Chen et al 2020, Schwabe et al 2020;
for effect of background density.)

ULDM as a solution of the lensing H₀ tension?

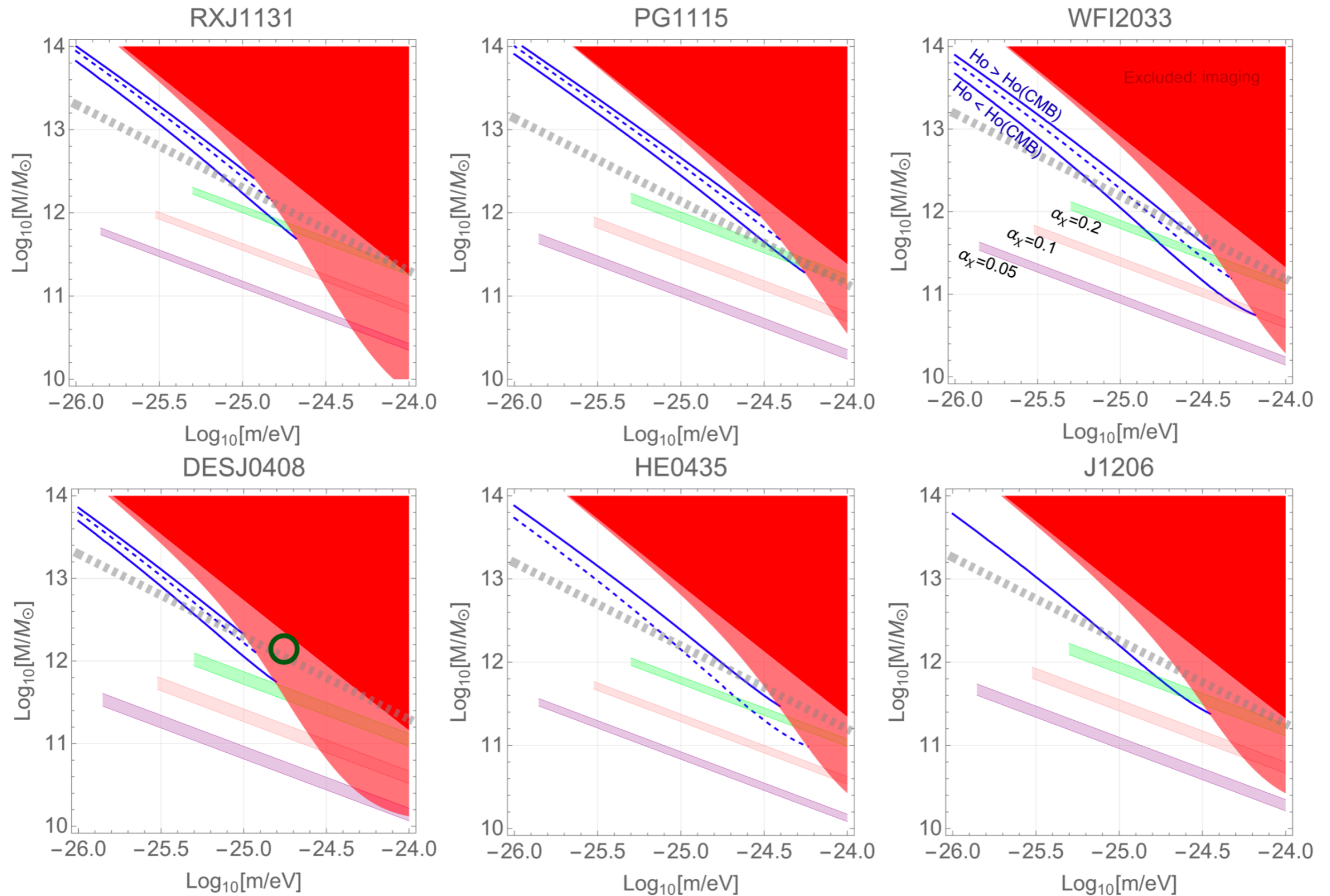


Dynamical relaxation
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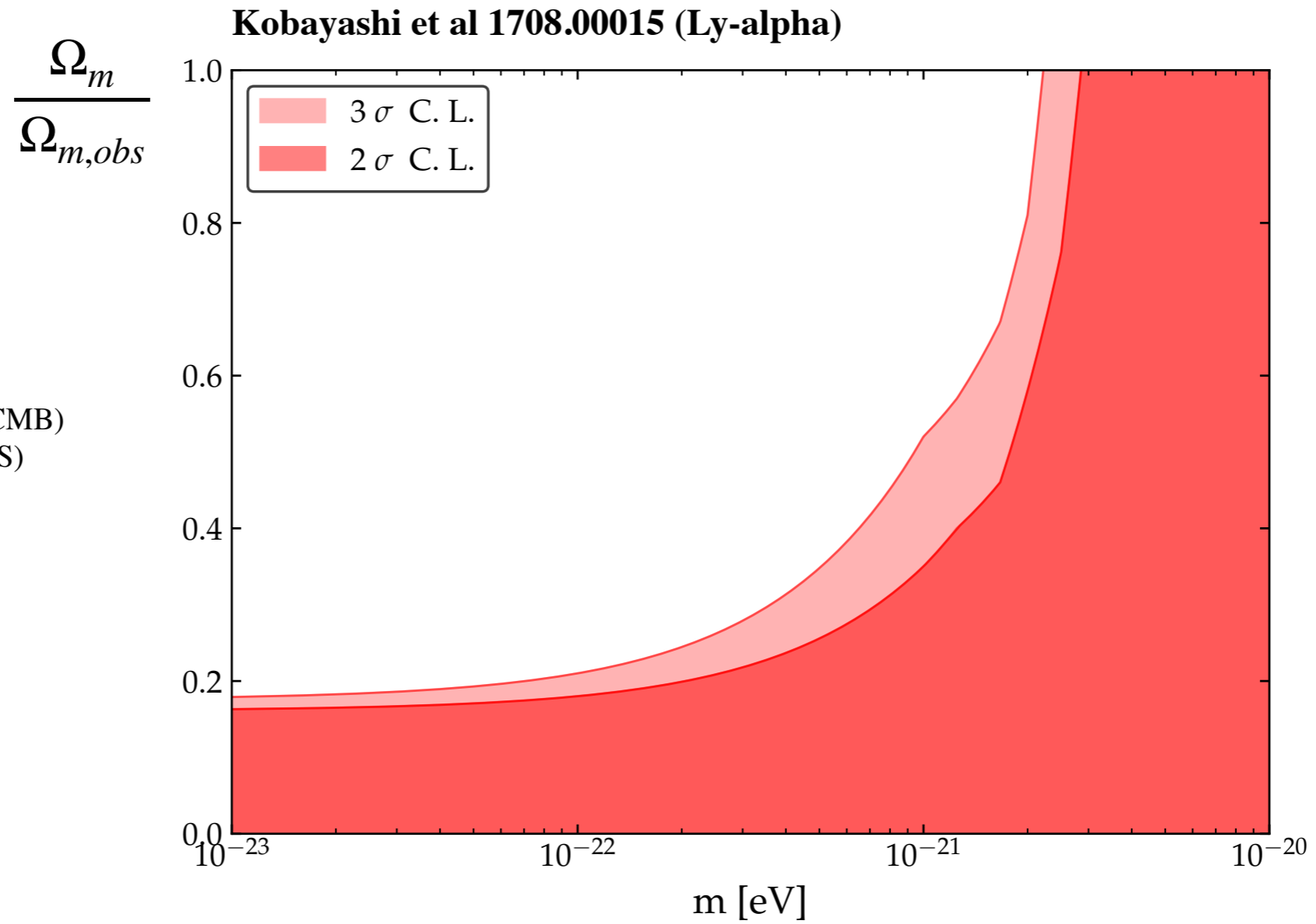
$$\tau \sim \frac{\sqrt{2}}{12\pi^3} \frac{m^3 \sigma^6}{G^2 \rho^2 \ln \Lambda}$$

(But see Eggemeier, Niemeyer 2019,
Chen et al 2020, Schwabe et al 2020;
for effect of background density.)

ULDM as a solution of the lensing H_0 tension?



Cosmological constraints: ULDM can only make up a fraction of the DM



Also:
 Hlozek, Marsh, Grin 1708.05681 (CMB)
 Lague et al, 2104.07802 (CMB+LSS)

$$\frac{\Omega_m}{\Omega_{m,obs}} \approx \left(\frac{m}{10^{-21} \text{ eV}} \right)^{\frac{1}{2}} \left(\frac{f}{10^{17} \text{ GeV}} \right)^2 \approx 0.1 \left(\frac{m}{10^{-25} \text{ eV}} \right)^{\frac{1}{2}} \left(\frac{f}{3 \times 10^{17} \text{ GeV}} \right)^2$$

Weak lensing degeneracy:

$$H_0^{\text{uncorr}} = \frac{1 - \kappa^{\text{ls}}}{1 - \kappa^{\text{l}}} \frac{1}{1 - \kappa^{\text{s}}} H_0$$

Weak lensing correction in H0LiCOW / TDCOSMO is probably *a little bit off*.

Birrer et al, 2007.02941 (TDCOSMO IV)

Teodori, et al, 2201.05111

