The State of Dark Energy Theory

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1. What is Dark Energy?
2. Models of Dark Energy
3. General Models
4. Effective Field Theory Approaches

Outline

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2. Models of Dark Energy
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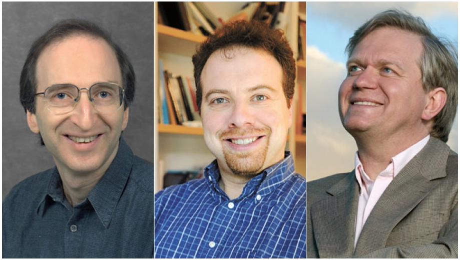
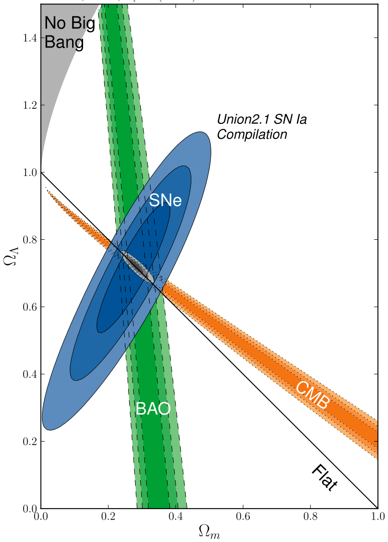
Experimental Evidence for Dark Energy

- Type Ia Supernovae



- Baryon Acoustic Oscillations
- Galaxy Cluster Abundances
- Weak Lensing

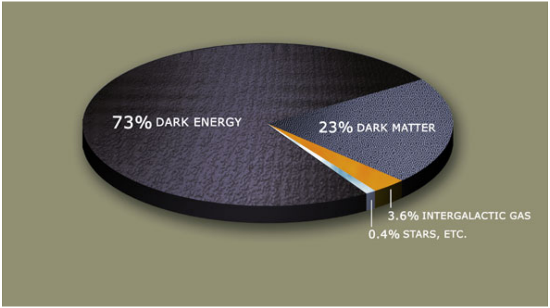
Supernova Cosmology Project
Suzuki, et al., *Ap.J.* (2011)



The Presence of Dark Energy

We observe $\ddot{a}/a > 0$, and hence postulate the existence of Dark Energy

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3}$$



The accelerated expansion of the universe $\ddot{a}/a > 0$ shows that either:

- There exists a Cosmological Constant Λ
- There exists exotic matter with $P < -\rho/3$
- We have the theory wrong

What do we know about Dark Energy?

- Causes the universe to expand equally in all directions
- Only became important in the recent cosmological history
- Has an equation of state $w = P/\rho$ consistent with -1
- Time evolution of w is consistent with $\dot{w} = 0$
- 70% of the universe is dark energy

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The Cosmological Constant

- Associated with an energy density of the vacuum, Λ
- All observations consistent with $\Lambda \sim (10^{-3} eV)^4$
- ...

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Beyond the Cosmological Constant

Lovelock's Theorem

"The only second-order, local gravitational field equations derivable from an action containing solely the 4D metric tensor (plus related tensors) are the Einstein field equations with a cosmological constant."

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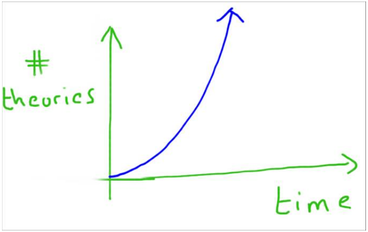
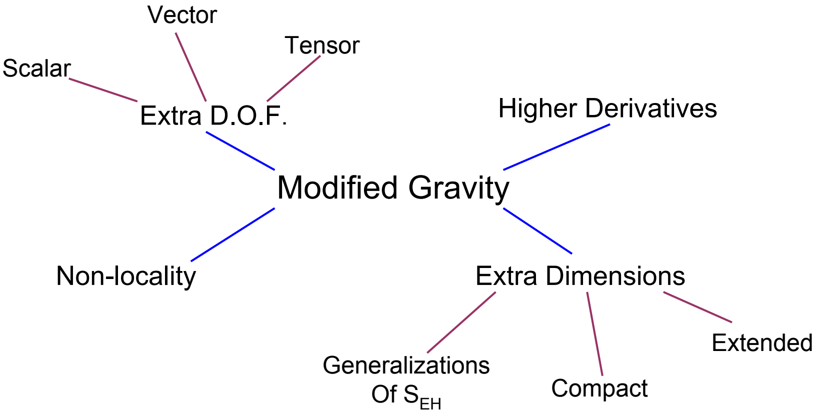
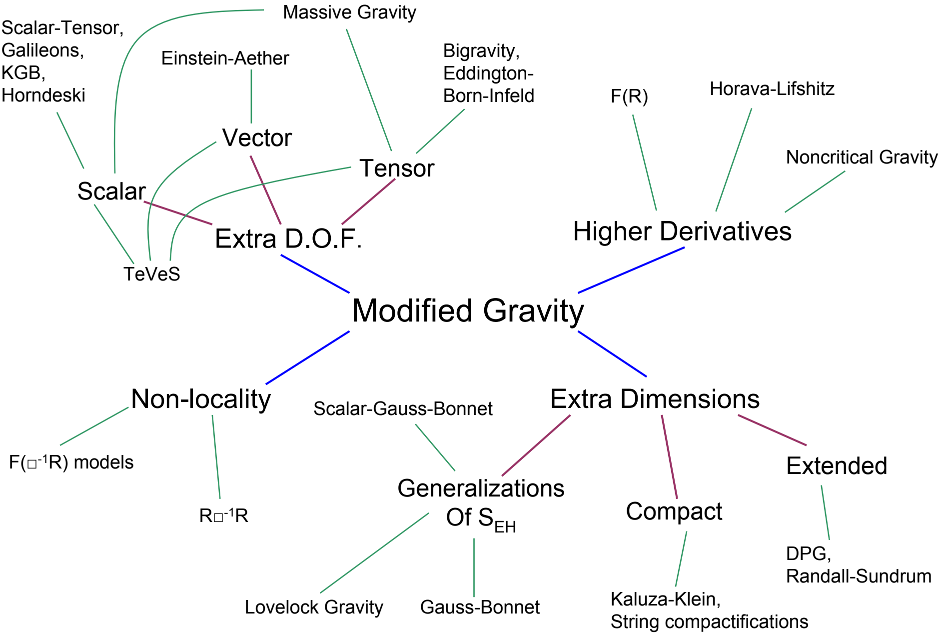
To evade this theorem requires one of the following:

- Extra degrees of freedom
- $>$ 4 dimensions
- $>$ 2nd order field equations
- Non-locality
- ~~Sacrifice the action principle~~





Courtesy of Tessa Baker



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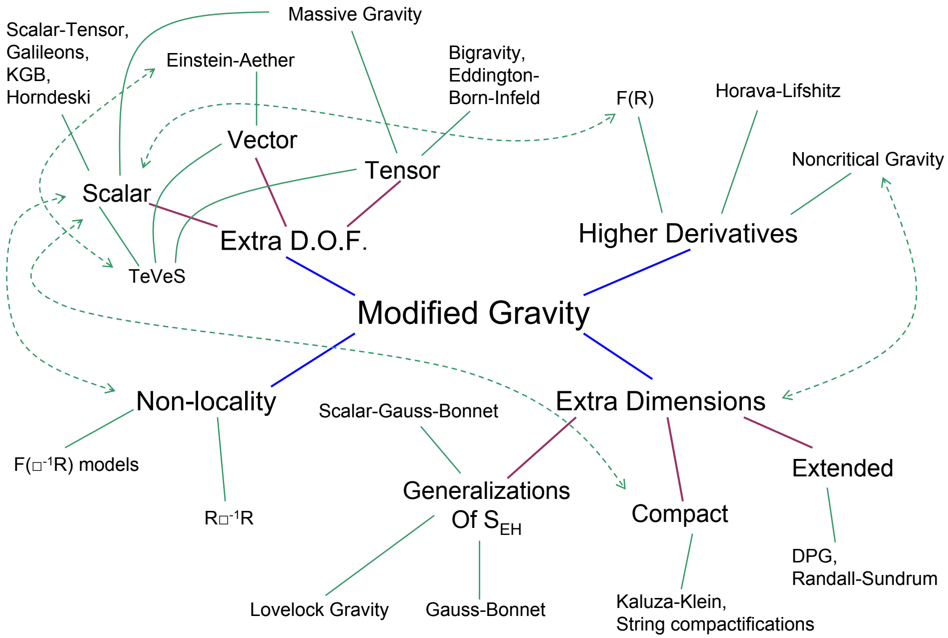

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Questions for Dark Energy Models

- Is vacuum energy vanishing? Very large? Arbitrary?
- Are fifth force/solar system constraints violated?
- Are quantum corrections dangerously large? Is there a fine-tuning problem?
- Is the cosmological history acceptable?
- Is BBN disturbed?
- What is the effect on the growth of large scale structure?

- Isotropic and Homogeneous expansion $\rightarrow$ (effective) scalar mode
- In a low-energy four-dimensional limit, almost all theories of dark energy behave as GR + scalar field(s)

Observables in Cosmology

Background

- Expansion Rate $H(t)$
- Description of background evolution is always degenerate in theory space
- Need observeables beyond the background

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Perturbations

- Matter density perturbation δ
- Matter velocity perturbation v
- Can lift degeneracy in background

Understanding Perturbations

General Relativity

- Write the metric as

$$ds^2 = -(1 + 2\psi)dt^2 + a(t)^2(1 - 2\phi)d\vec{x}^2$$

- Einstein's equations yield the following relations for the metric perturbations

$$\frac{k^2}{a^2}\psi = -4\pi G\rho(\delta + 3aHv/k)$$

$$\phi = \psi$$

Understanding Perturbations

Modified Gravity

$$\frac{k^2}{a^2}\psi = -4\pi G\rho Q(a,k)(\delta + 3aHv/k)$$
$$\phi = R(a,k)\psi$$

- Aim: Construct possible forms for Q, R (theoretical priors)

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Motivation

Why construct a general theory for dark energy?

- Systematize theoretical approaches
- Motivate new approaches
- Facilitate calculation of (model-independent) observational constraints

Towards a General Theory of Dark Energy?

Six distinct approaches have appeared in the past few years!

- Generic perturbed EOMs (Pogosian, Silvestri and Buniy 2013)
- Parameterized Post-Friedmannian (Baker *et al* 2012)
- Horndeski's Theory (Deffayet *et al* 2011, Koboyashi *et al* 2011)
- Fluid approach (Battye and Pearson 2012)
- Covariant EFT (JB and Flanagan 2012)
- Perturbative EFT (JB *et al* 2012, Gubitosi, Piazza and Vernizzi 2012)

"We don't know anything, but that's not going to stop us!"

- Silvestri, Pogosian and Buniy 1302.1193
- All perturbed equations of motion in modified gravity must take the following form:

$$\begin{aligned}\hat{A}\psi + \hat{B}\phi + \hat{C}^i \delta\phi_i &= -4\pi G a^2 \rho(\delta + 3aHv/k) \\ \hat{D}\psi + \hat{E}\phi + \hat{F}^i \delta\varphi_i &= 0 \\ \hat{H}_i\psi + \hat{K}_i\phi + \hat{L}_i^j \delta\phi_j &= 0\end{aligned}$$

(Poisson eqn, anisotropic shear eqn, EOMs for new fields)

"We don't know anything, but that's not going to stop us!"

- Prediction: $Q(a,k)$ and $R(a,k)$ can be expressed as*

$$Q(a,k) = \frac{P_1(a,k)}{P_2(a,k)}$$
$$R(a,k) = \frac{P_3(a,k)}{P_1(a,k)}$$

where $P_i(a,k)$ is a polynomial in k with coefficients that depend on a

- ANY single scalar field model: need 6 functions of a

* In the quasistatic regime

“We are going to hit this with a sledgehammer”

- Parameterized Post-Friedmannian formulation
- Baker, Ferreira and Skordis 1209.2117
- Construct completely general perturbed stress-energy tensor for arbitrary scalar fields using gauge invariance, Bianchi identity
- Results in **22** arbitrary functions of k and a (typically only 15 independent functions, all polynomials in k)
- Represents the most general formulation of (scalar) perturbation equations

“Let us be blunt”

- General covariant scalar field action with second order equations of motion
- Original model constructed by Horndeski, 1974
- Revived by Deffayet *et al* 1103.3260, Koboyashi *et al* 1105.5723 in the context of galileons
- Aim: Construct the most general local and covariant single scalar field action with second order equations of motion

"Let us be blunt"

$$S = \int d^4x\sqrt{-g}\Big[K(\phi, X) - G_3(\phi, X)\mathcal{E}_1 + G_4(\phi, X)R + G_{4,X}(\phi, X)\mathcal{E}_2 + G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{6,X}(\phi, X)\mathcal{E}_3\Big] + S_{\text{matter}}$$

where $X \equiv -\frac{1}{2}(\nabla\phi)^2$, and

$$\mathcal{E}_1 \equiv \Box\phi$$
$$\mathcal{E}_2 \equiv (\Box\phi)^2 - \nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi$$
$$\mathcal{E}_3 \equiv (\Box\phi)^3 - 3\Box\phi\nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi + 2\nabla^\mu\nabla_\alpha\phi\nabla^\alpha\nabla_\beta\phi\nabla^\beta\nabla_\mu\phi$$

- General analysis very difficult

"Let's go back to basics"

- Fluid description
- Battye and Pearson 1203.0398, JB and Pearson 1310.6033
- Cosmology is formulated as a theory of fluids; do the same for dark energy
- Construct perturbed fluid evolution equations consistent with desired symmetries

“Complete with Quantum Mechanics!”

- Effective Field Theory approach
- Weinberg 0804.4291, Park, Zurek and Watson 1003.1722, JB and Flanagan 1112.0303
- Construct a quantum theory of dark energy using the tools of Effective Field Theory (EFT)
- Aims to describe both background evolution and perturbations in a consistent theory
- Requires expansion to be potential dominated (“slow roll”)
- Observational constraints calculated by Mueller, Bean and Watson 1209.2706

"Complete with Quantum Mechanics!"

$$
\begin{aligned}
S = \int d^4x\sqrt{-g}\Bigg\{ & \frac{m_p^2}{2}R - \frac{1}{2}(\nabla\phi)^2 - U(\phi) + a_1(\nabla\phi)^4 \\
& + b_2 T(\nabla\phi)^2 + c_1 G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi \\
& + d_3\left(R^2 - 4R^{\mu\nu}R_{\mu\nu} + R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}\right) \\
& + d_4\epsilon^{\mu\nu\lambda\rho}C_{\mu\nu}{}^{\alpha\beta}C_{\lambda\rho\alpha\beta} \\
& + e_1 T^{\mu\nu}T_{\mu\nu} + e_2 T^2 + \ldots\Bigg\} \\
& + S_m\left[e^{\alpha(\phi)}g_{\mu\nu}\right]
\end{aligned}
$$

- Coefficients are functions of ϕ with specific scalings
- Parameter space is given by nine free functions

“The one field, forged in Mount Doom”

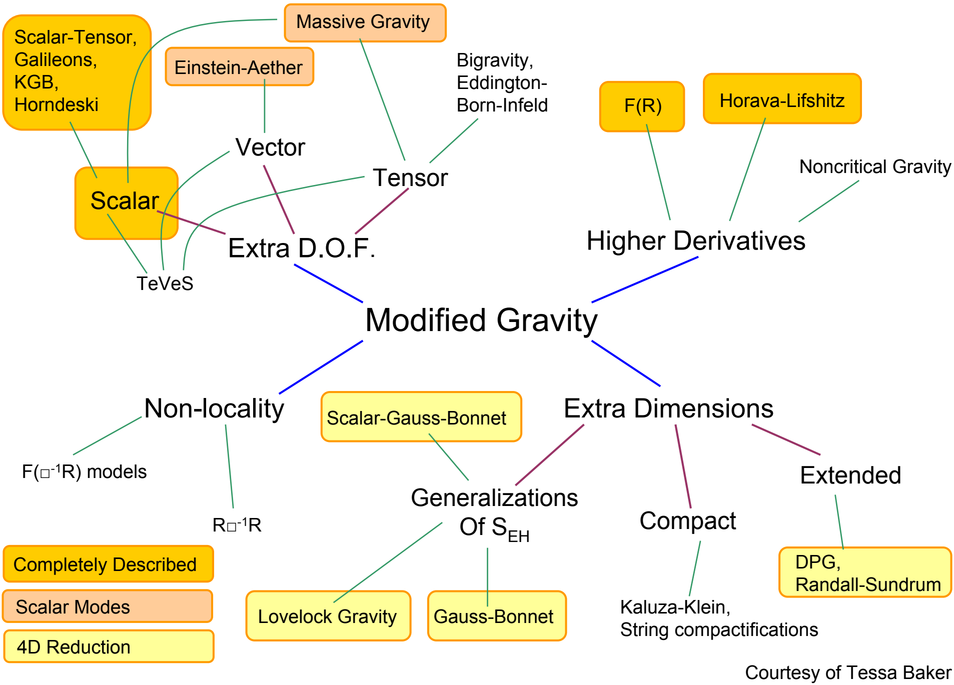
- Perturbative Effective Field Theory (single scalar)
- Gubitosi *et al.* 1210.0201, Gleyzes *et al.* 1304.4840, JB *et al.* 1211.7054, JB 1304.6712 ...
- Construct an action for scalar perturbations consistent with the symmetries of FRW
- Very powerful technique for cosmological perturbation theory

Convergence



For single scalar field models, all approaches have been shown to reduce to the same description
(some descriptions are more general than others)




Massive Gravity
Scalar-Tensor, Galileons, KGB, Horndeski
Einstein-Aether
Bigravity, Eddington-Born-Infeld
F(R)
Horava-Lifshitz
Vector
Tensor
Scalar
Noncritical Gravity
Extra D.O.F.
Higher Derivatives
TeVeS
Modified Gravity
Non-locality
Scalar-Gauss-Bonnet
Extra Dimensions
F(□⁻¹R) models
Extended
Generalizations Of S_{EH}
Compact
R□⁻¹R
Completely Described
Scalar Modes
4D Reduction
Lovelock Gravity
Gauss-Bonnet
Kaluza-Klein, String compactifications
DPG, Randall-Sundrum
Courtesy of Tessa Baker

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What is Effective Field Theory?

Idea

- Physics a long way above the energy scales of interest can be “integrated out”
- “Don’t need to understand nuclear physics to be a cook”
- Ignore massive fields above the energy range of interest
- Describe the dynamics of your appropriate particles/fields
- Anything that can happen, will happen

Effective Field Theory: Approach

Implementation

- Specify fields, symmetries, cutoff
- Construct action with all possible operators
- Arrange operators in an appropriate expansion
- EFT provides rules for scaling of coefficients, gives a handle on radiative corrections

$$\mathcal{L} = \mathcal{L}_{\text{marginal, relevant}} + \sum \frac{c_k}{\Lambda^k} O_k, \qquad c_k \sim 1$$

- Fractional errors $\propto (\sqrt{N} E/\Lambda)^n$

Why Effective Field Theory?

Benefits

- Prevent quantum instabilities (e.g., vacuum decay)
- Exclude strong-coupling (dangerously large loops)
- Fine tuning considerations give a probability measure on theory space
- Perform construction such that a match to a UV theory is possible

Challenges for Cosmological Effective Field Theory

- Cosmological models allow for large numbers of quanta in perturbative modes ($\sim m_P^2/H_0^2$)
- Evolution of background leads to difficulties comparing operators
- Expect fields to move large distances over cosmological evolution

$$\Delta\phi \sim m_P$$

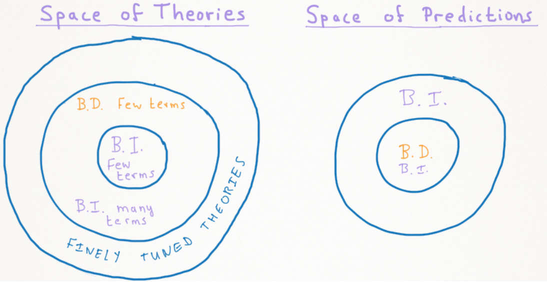
Two Classes of Effective Field Theory

Background Independent

- Works for all backgrounds, covariant expansion in $g_{\mu\nu}$, ϕ, …
- $\mathcal{L} \sim R + (\nabla\phi)^2 + (\nabla\phi)^4 + \ldots$

Background Dependent

- Expand in deviations $\delta g_{\mu\nu}$, $\delta\phi$ from our cosmological background
- Covers a larger set of theories with finitely many terms



Perturbative EFT: Construction

Idea

- Based on “EFT of Inflation” (Cheung *et al* 2008)
- Work with a single scalar field
- Choose time slicing such that $\phi = \phi(t)$ (“unitary gauge”)
- Identify objects that are invariant under remaining gauge freedom
- Construct a general action from these objects
- Arrange objects in a perturbative expansion

Perturbative EFT: Application

From the action:

- Derive background equations
- Apply EFT scaling rules
- Use Stückelberg trick to restore scalar perturbations
- Calculate equations of motion

Matter Treatment

- Assume Weak Equivalence Principle
- Work in Jordan Frame
- Matter equations are independent of gravitational model
- Depend only on ψ, ϕ (Newtonian gauge)

Unitary Gauge

Symmetries

Work with the ADM metric decomposition

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

- Unitary gauge fixes $\phi = \phi(t)$
- Residual symmetry under gauge fixing is spatial diffeomorphism invariance $x^i = x^i(\tilde{x}^j, t)$ and time reparametrization invariance $t = f(\tilde{t})$

Unitary Gauge

Objects

- The following objects are invariant under the remaining symmetries:

Functions of time: $f(t)$

Spatial metric: h_{ij}

Covariant spatial derivatives: D_i

3D gravitational invariants: R_{ijkl}, R_{ij}, R

3D volume form: ϵ^{ijk}

Lapse: N

Time derivatives: $D_t = \partial_t - \mathcal{L}_{\vec{N}}$

Unitary Gauge

Combining the Objects into an Action

- Using equivalences, integration by parts, and second order equations of motion enforces these objects are combined as

$$S = \int d^3x \, dt \, N\sqrt{h} \, F\left[t, \epsilon^{ijk}, h_{ij}, N, R_{ij}, D_i, K_{ij}\right]$$

where $K_{ij} = (D_t h_{ij})/2N$ is the extrinsic curvature tensor for surfaces of constant time

Perturbative Expansion

Combining the Objects into an Action

- Trick: rearrange action as

$$S = \int d^3x \, dt \, N\sqrt{h} \, F\left[t, \epsilon^{ijk}, h_{ij}, N - N_0, R_{ij} - 2kh_{ij}/a(t)^2, D_i, K_{ij} + H(t)h_{ij}\right]$$

$$= \int d^3x \, dt \, N\sqrt{h} \, F\left[t, \epsilon^{ijk}, h_{ij}, \delta g^{00}, \delta R_{ij}, D_i, \delta K_{ij}\right]$$

- Relies upon the symmetry of FRW to construct perturbations
- Expand order by order in perturbations
- Typical to use $g^{00} = -1/N^2$ instead of N

Action in Unitary Gauge

$$S = \int d^4x \sqrt{-g} \left\{ \frac{m_P^2}{2} R + \Lambda \right\} + S_{\text{matter}}$$

- General Relativity + cosmological constant

Action in Unitary Gauge

$$S = \int d^4x\sqrt{-g}\left\{\frac{m_P^2}{2}R + \Lambda(t) - c(t)\delta g^{00}\right\} + S_{\rm matter}$$

- Quintessence

Action in Unitary Gauge

$$S = \int d^4x \sqrt{-g} \left\{ \frac{m_P^2}{2} \Omega(t) R + \Lambda(t) - c(t) \delta g^{00} \right\} + S_{\mathrm{matter}}$$

- Non-minimal coupling

Action in Unitary Gauge

$$S = \int d^4x \sqrt{-g} \left\{ \frac{m_P^2}{2} \Omega(t) R + \Lambda(t) - c(t) \delta g^{00} + \frac{M_2^4(t)}{2} (\delta g^{00})^2 \right\} + S_{\text{matter}}$$

- k-essence

Action in Unitary Gauge

$$S = \int d^4x\sqrt{-g}\left\{\frac{m_P^2}{2}\Omega(t)R + \Lambda(t) - c(t)\delta g^{00} + \frac{M_2^4(t)}{2}(\delta g^{00})^2 - \frac{\bar{M}_1^3(t)}{2}\delta g^{00}\delta K^i_i\right\} + S_{\text{matter}}$$

- Galileon/Kinetic Braiding

Action in Unitary Gauge

$$S = \int d^4x\sqrt{-g}\left\{\frac{m_P^2}{2}\Omega(t)R + \Lambda(t) - c(t)\delta g^{00} + \frac{M_2^4(t)}{2}(\delta g^{00})^2 - \frac{\bar{M}_1^3(t)}{2}\delta g^{00}\delta K^i{}_i - \frac{\bar{M}_2^2(t)}{2}(\delta K^i{}_i)^2 - \frac{\bar{M}_3^2(t)}{2}\delta K^i{}_j\delta K^j{}_i + \frac{\hat{M}^2(t)}{2}\delta g^{00}\delta R^{(3)}\right\} + S_{\mathrm{matter}}$$

- Horndeski's general scalar field theory

Action in Unitary Gauge

$$
\begin{aligned}
S = \int d^4x\sqrt{-g}\Bigg\{ & \frac{m_P^2}{2}\Omega(t)R + \Lambda(t) - c(t)\delta g^{00} + \frac{M_2^4(t)}{2}(\delta g^{00})^2 \\
& - \frac{\bar{M}_1^3(t)}{2}\delta g^{00}\delta K^i{}_i - \frac{\bar{M}_2^2(t)}{2}(\delta K^i{}_i)^2 - \frac{\bar{M}_3^2(t)}{2}\delta K^i{}_j\delta K^j{}_i \\
& + \frac{\hat{M}^2(t)}{2}\delta g^{00}\delta R^{(3)} + m_2^2(t)h^{ij}\partial_i g^{00}\partial_j g^{00} \Bigg\} \\
& + S_{\text{matter}}
\end{aligned}
$$

- Hořava-Lifshitz Gravity

Background Evolution

Linear Terms

Background evolution comes from linear terms:

$$S \supset \int d^4x\sqrt{-g}\left\{\frac{m_P^2}{2}\Omega(t)R + \Lambda(t) - c(t)\delta g^{00}\right\} + S_{\text{matter}}$$

The Friedmann and acceleration equations become:

$$3m_P^2\Omega\left[H^2 + \frac{k_0}{a^2} + H\frac{\dot{\Omega}}{\Omega}\right] = \rho_m - \Lambda + 2c$$

$$m_P^2\Omega\left[3H^2 + 2\dot{H} + \frac{k_0}{a^2} + \frac{\ddot{\Omega}}{\Omega} + 2H\frac{\dot{\Omega}}{\Omega}\right] = -\Lambda - P_m$$

Can replace $\Lambda(t)$ and $c(t)$ with $H(t)$.

Example: Application to Horndeski

EFT Description

$$
\begin{aligned}
S = \int d^4x\sqrt{-g}\Bigg\{ & \frac{m_P^2}{2}\Omega(t)R + \Lambda(t) - c(t)\delta g^{00} + \frac{M_2^4(t)}{2}(\delta g^{00})^2 \\
& - \frac{\bar{M}_1^3(t)}{2}\delta g^{00}\delta K^i{}_i \\
& - \frac{\bar{M}_2^2(t)}{2}\left(\delta K^i{}_i{}^2 - \delta K^i{}_j\delta K^j{}_i + 2\delta g^{00}\delta R^{(3)}\right)\Bigg\} \\
& + S_{\text{matter}}[g_{\mu\nu}]
\end{aligned}
$$

- Only five functions of time needed! H, Ω, M_2^4, $\bar{M}_1^3$ and $\bar{M}_2^2$

What can we use this approach for?

Investigating:

- Effective stress-energy tensor, scalar equation of motion
- $Q(a, k)$, $R(a, k)$
- Effective Newtonian constant
- Speed of sound of perturbations
- Stability
- Tensor modes

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- $Q(a,k)$, $R(a,k)$
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- Tensor modes

- Expresses $Q(a,k)$, $R(a,k)$ in terms of coefficients in the action
- Reduces two functions of scale and time to a few functions of time
- Stronger theoretical prior on theory space of modified gravity models

Benefits and Limitations

Benefits

- Time dependence arises from a small number of coefficients in the action
- General parametrization of theory space
- Allows for model-independent constraints
- Approach uses an action principle
- Quantum analysis possible (in principle; not yet performed)

Limitations

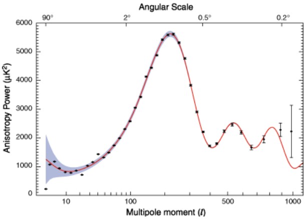
- Cannot predict background evolution
- Only applies to single (effective) scalar field models
- Requires $\phi(t)$ to be strictly monotonic
- Linear regime only

Present State of the Theory

- Growth function has been investigated (Baker *et al* 1310.1086, Piazza *et al* 1312.6111)
- Theoretical work constraining $\Omega(t)$ undertaken (Frusciante *et al* 1310.6026)
- Implementation of theory in CAMB (Silvestri *et al* 1312.5742, JB *et al* (in progress))

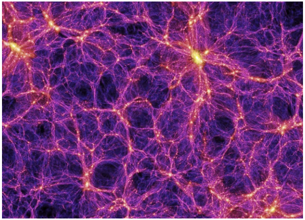
Future Work

- Understand behavior of coefficients in known models
- Investigate technical details regarding regime of validity of the theory (put the Quantum in the EFT)
- Compare theory space to observations



Large-Angle CMB: $\dot{\phi} + \dot{\psi}$

CMB and Galaxy Weak Lensing: $\phi + \psi$



Growth of Structure: ψ

Conclusions

- Theory space of dark energy is large
- Perturbative EFT construction yields a powerful general description to map model-independent constraints onto a large number of theories

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