Minimally Parametric Constraints on P(k) from Lyman-α

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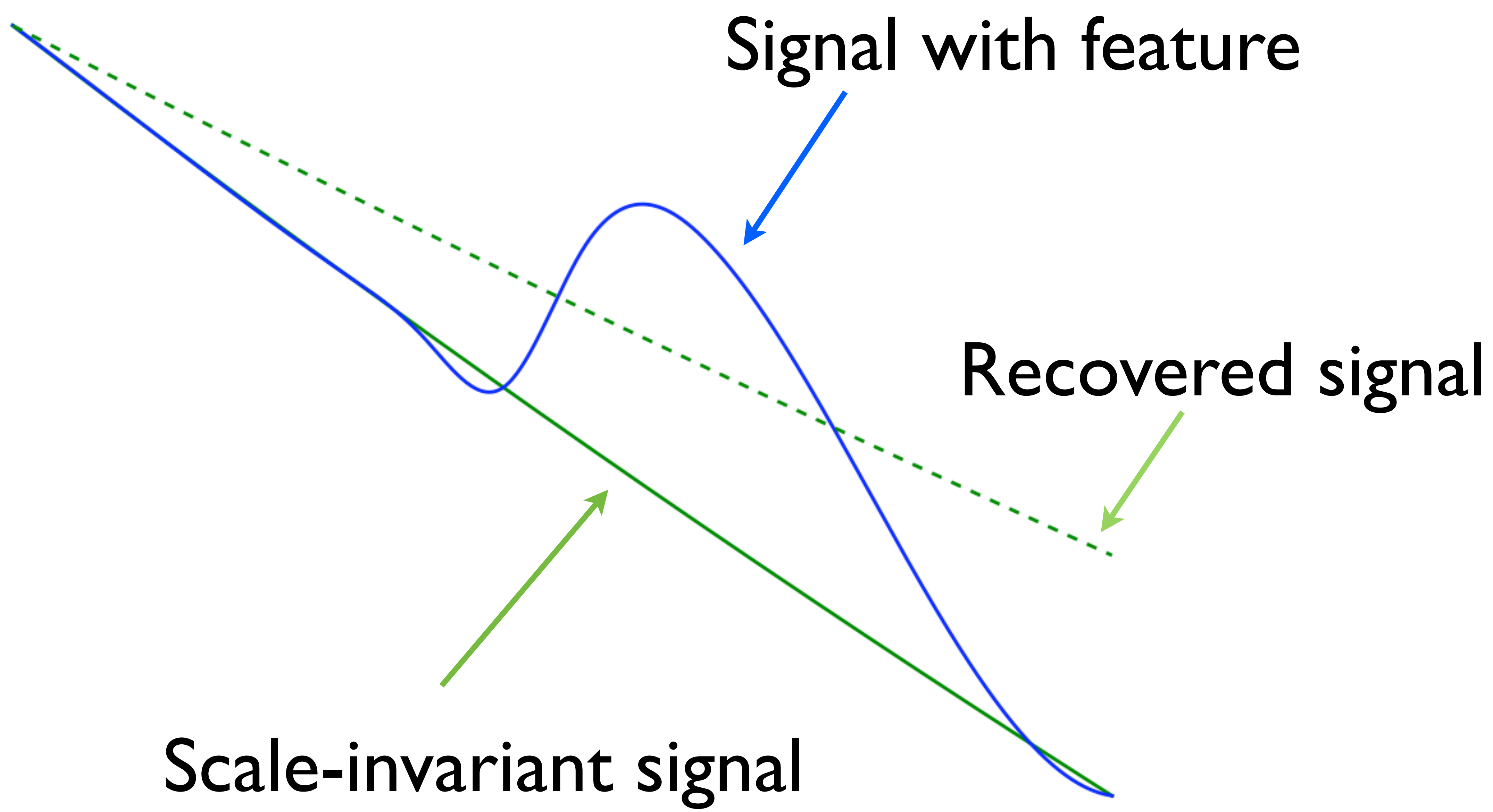
Bird et al (2010), arxiv:1010.1519

Motivation

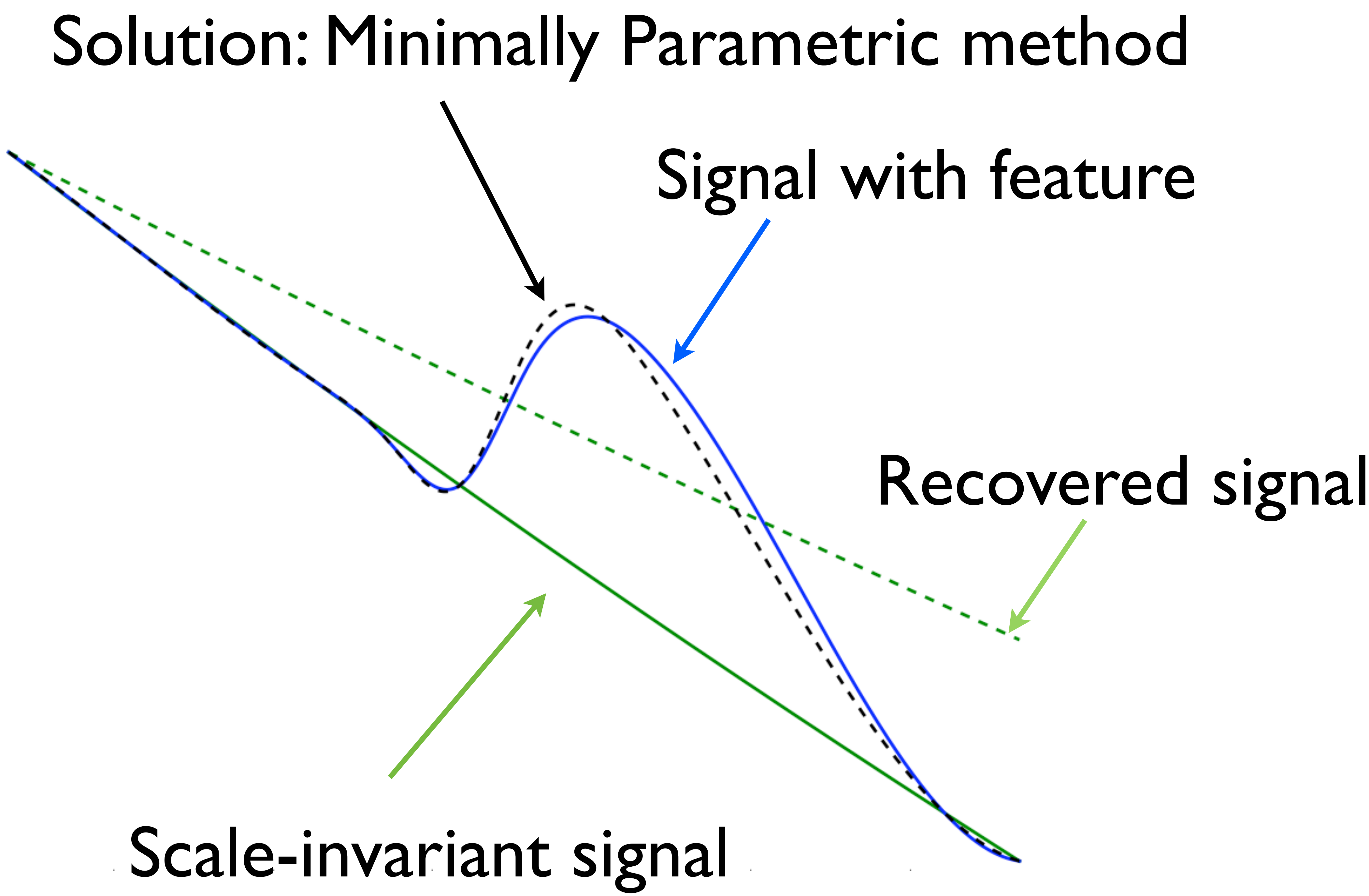
- Lyman-α currently only direct probe of small-scale power spectrum.
- Minimally Parametric tests scale-invariant models

Motivation

Local feature may bias recovered parameters



Motivation



Need to ensure robustness

Lyman-α forest

Neutral hydrogen clouds scatter quasar light

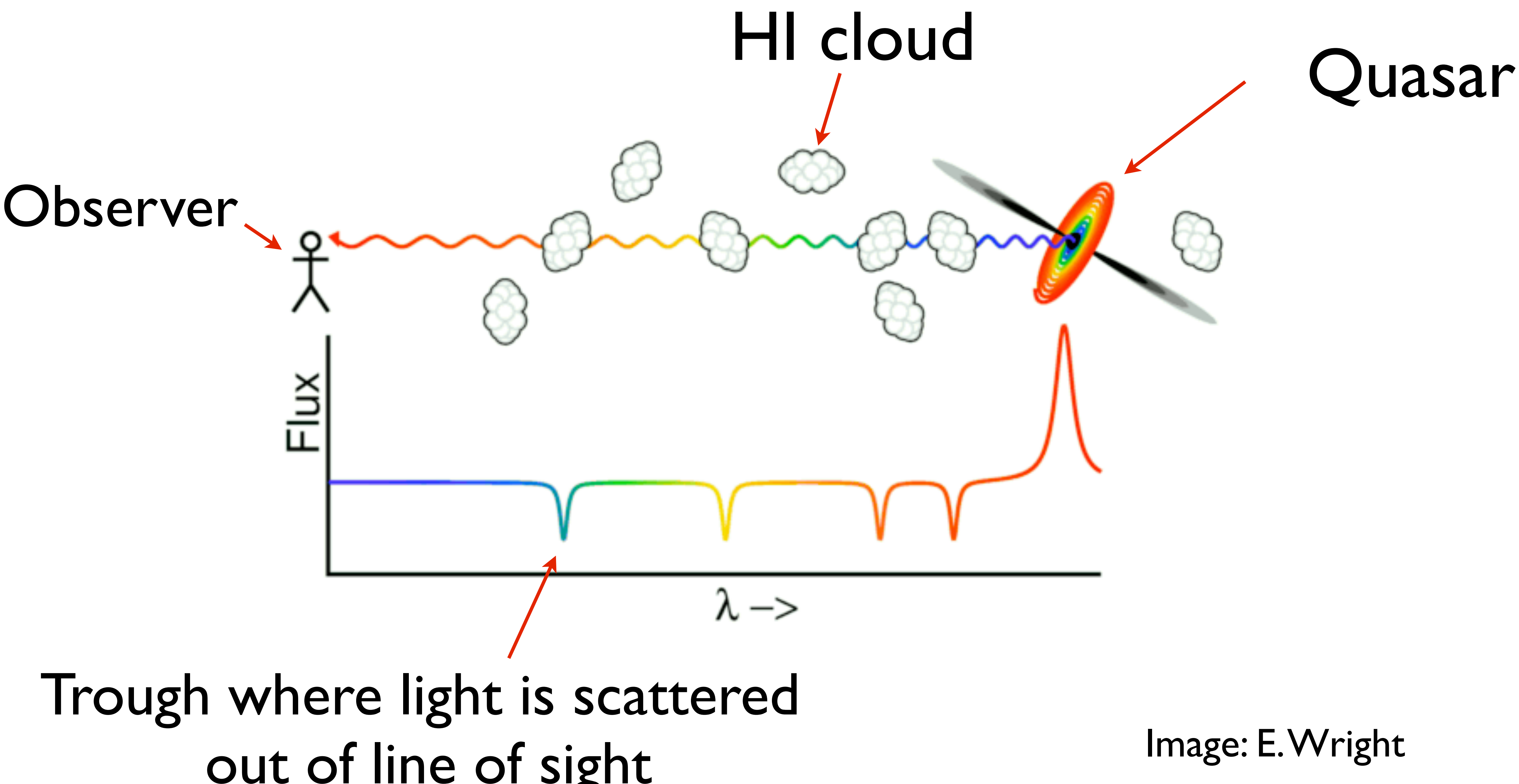
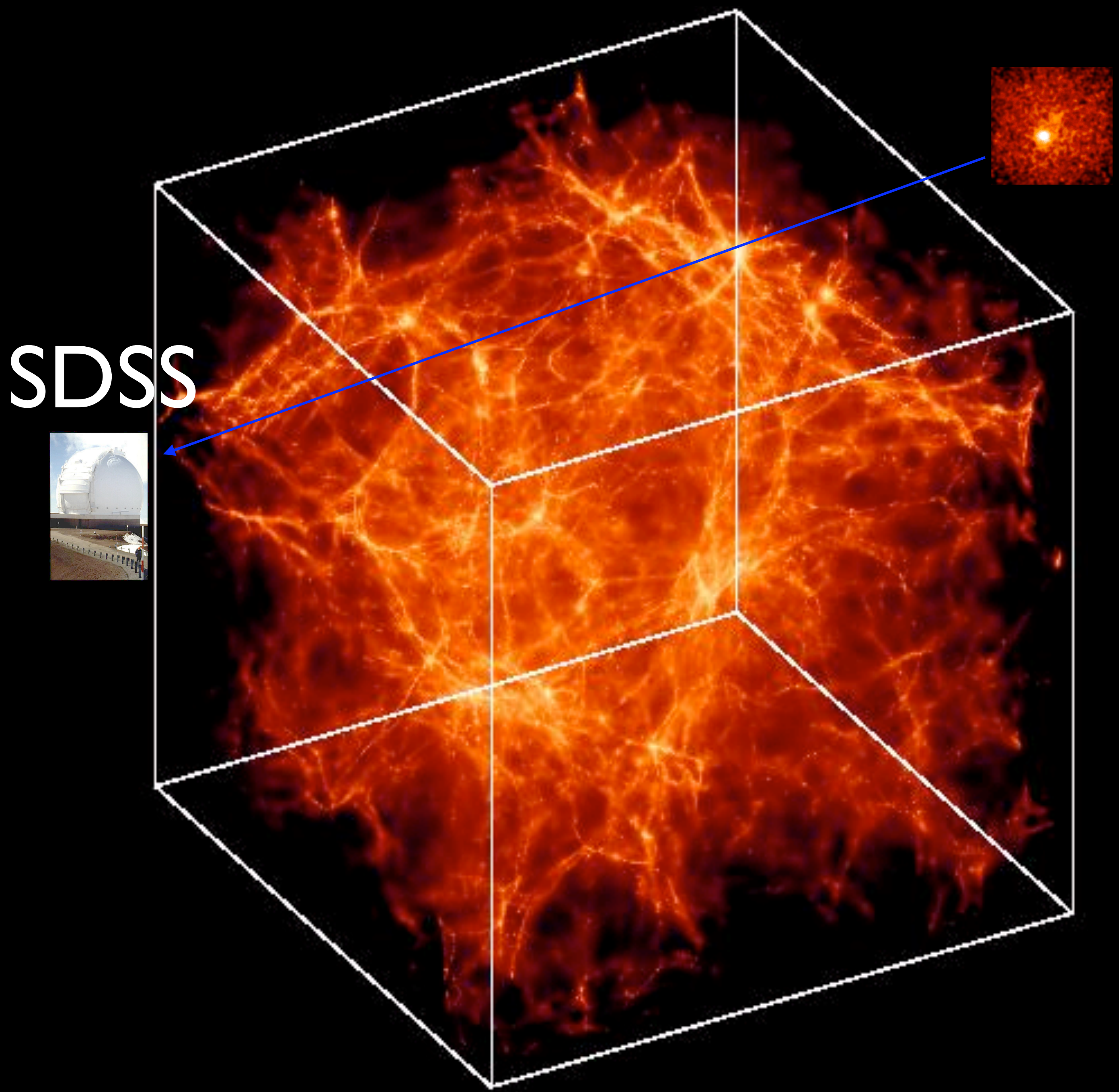


Image: E. Wright

Lyman-α forest

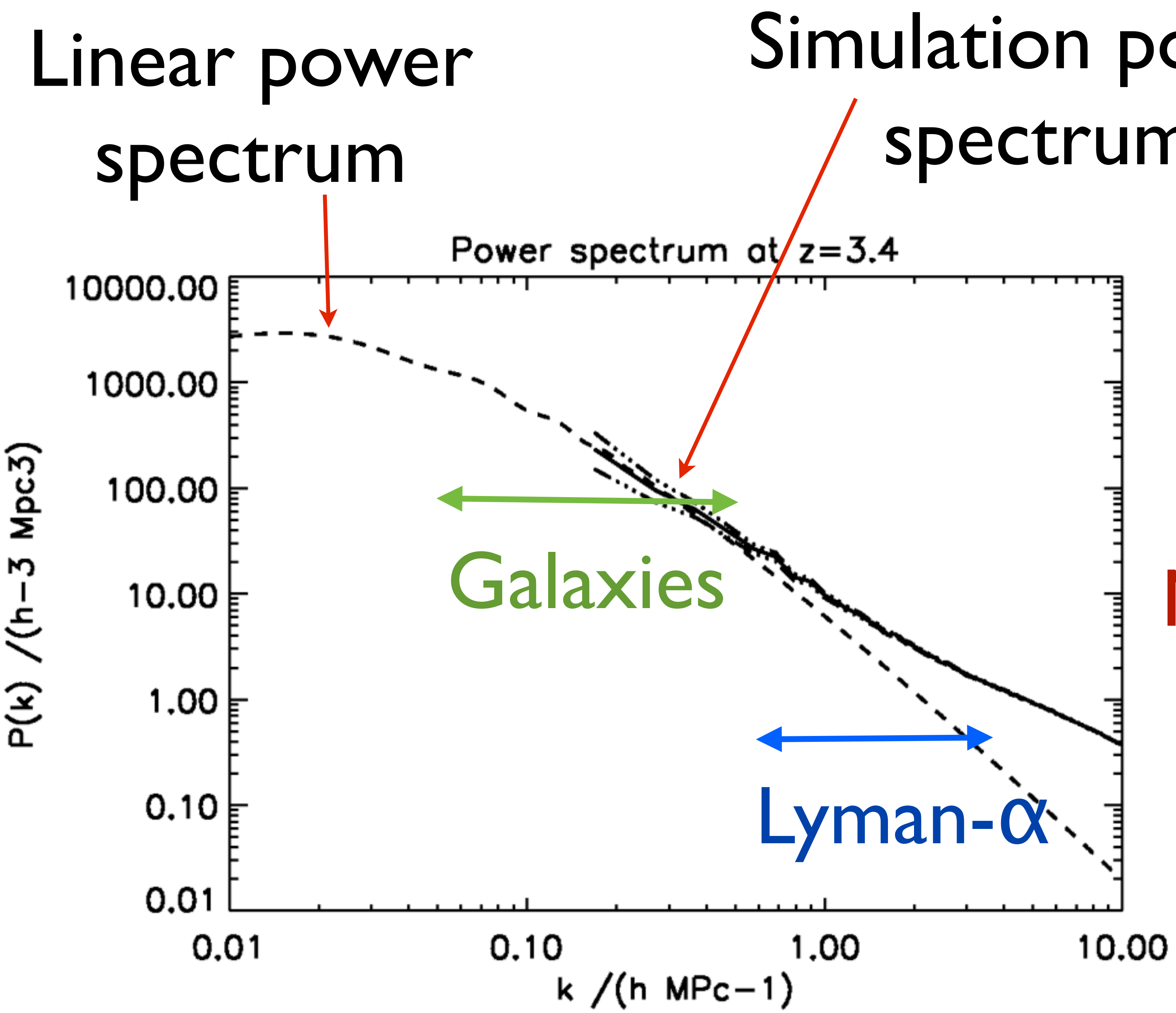


At redshift 2-4
80% of baryonic mass
in hydrogen clouds

Traces matter density
along line of sight from
quasar

Image: M. Viel

Lyman-α forest



Lyman-α probes smallest scales

Mildly nonlinear physics

Lyman-α forest

Absorption with optical depth, τ

$$\mathrm{Flux} = \exp(-\tau)$$

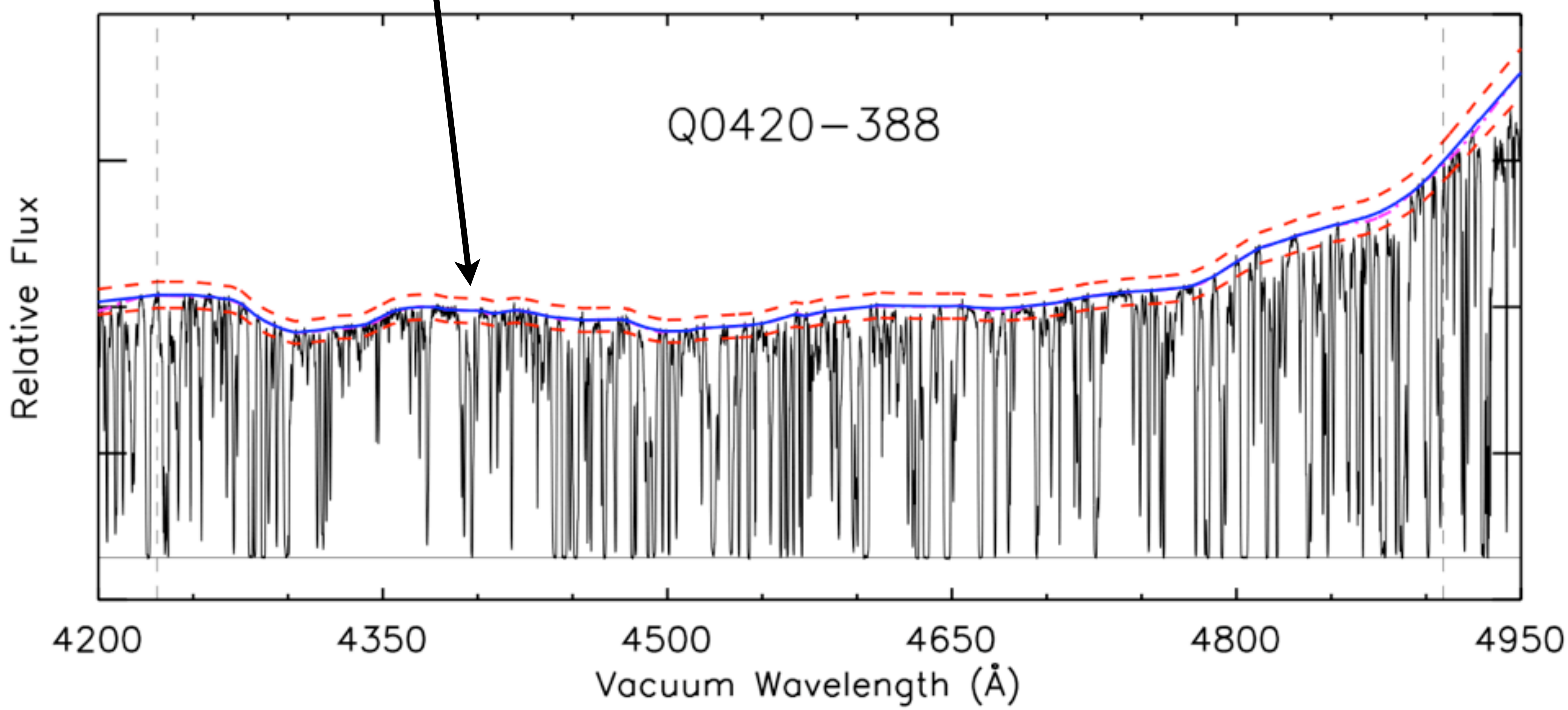


Image: Kim et al

Lyman-α forest

Observable: Flux Power spectrum

Fairly insensitive to small-scale structure

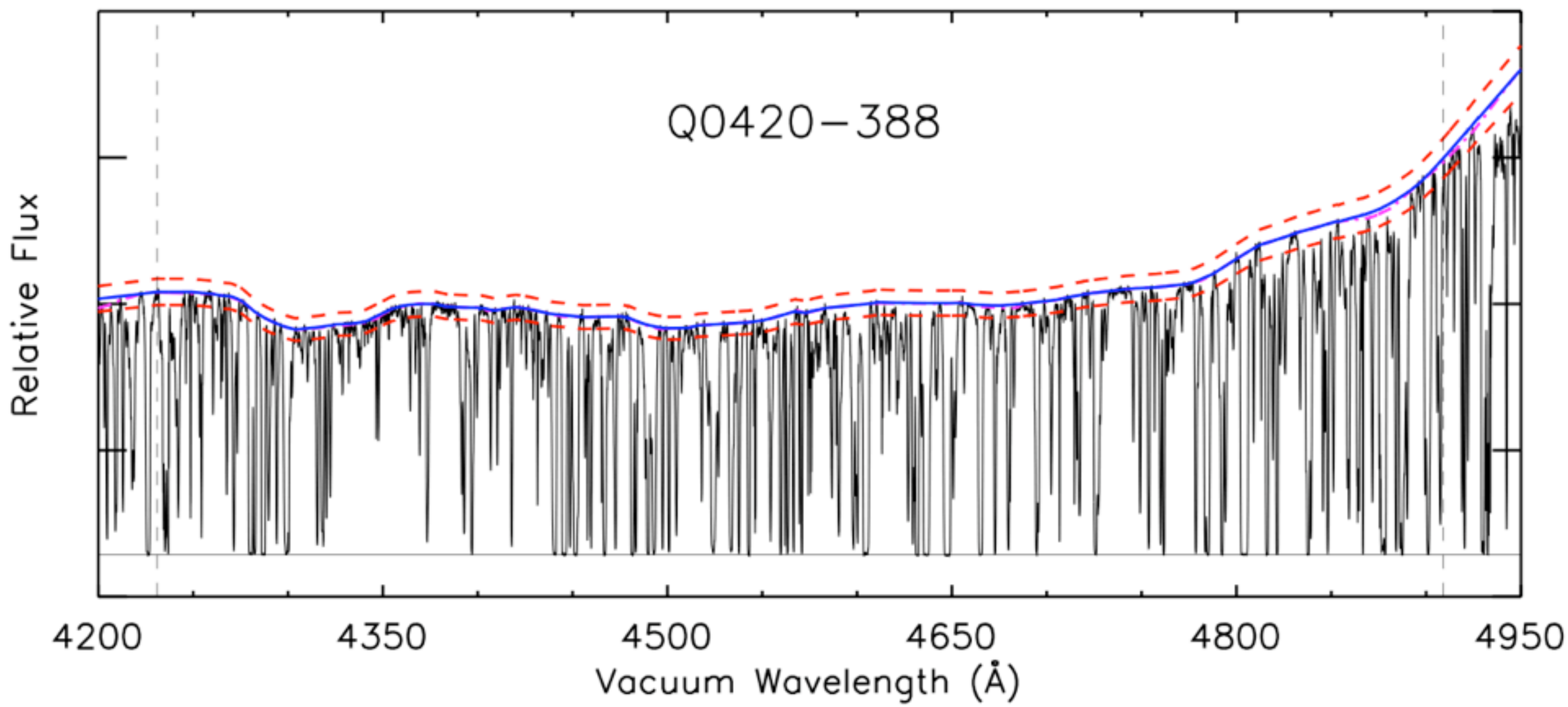
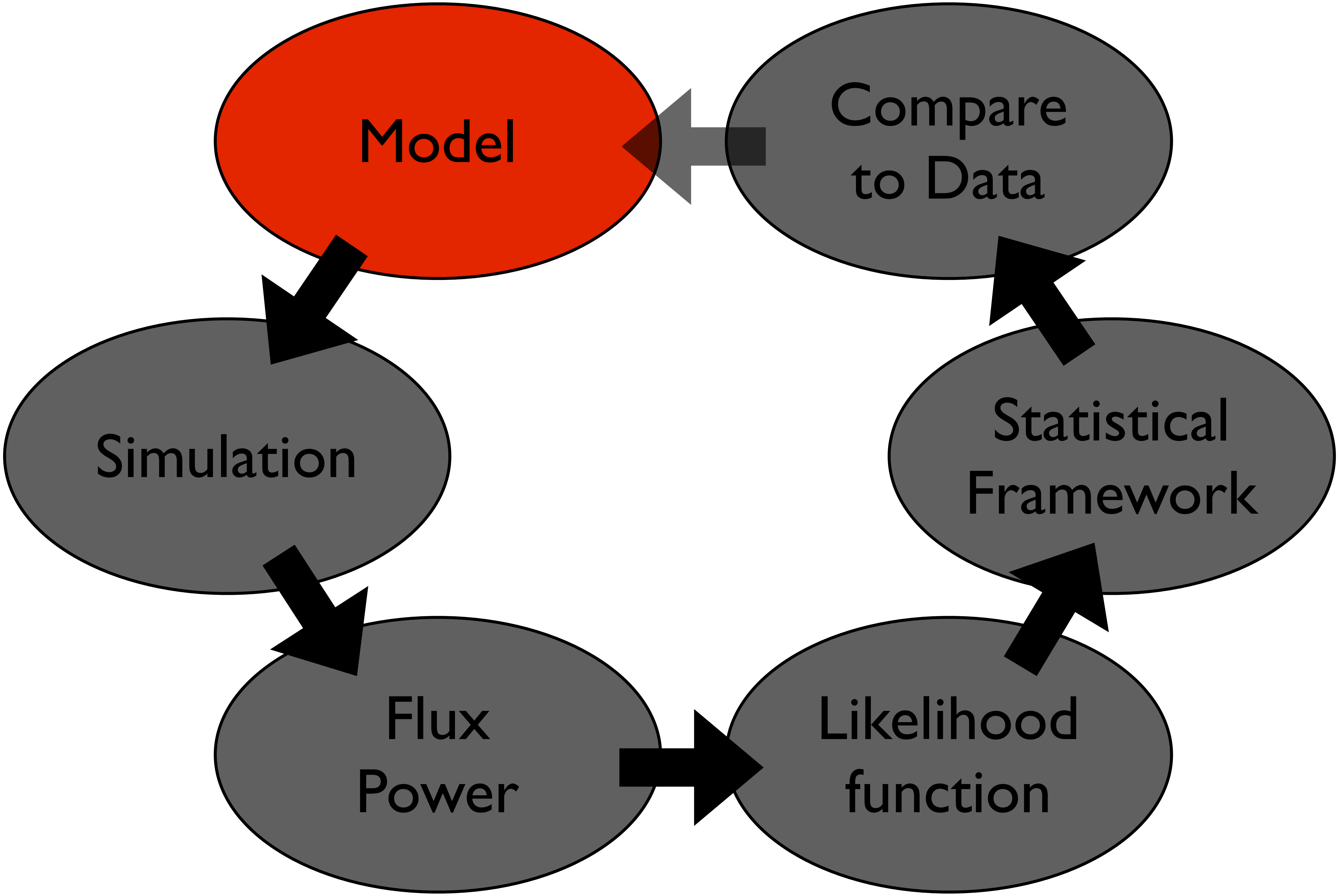


Image: Kim et al


Model
Compare to Data
Simulation
Statistical Framework
Flux Power
Likelihood function

Power Spectrum Reconstruction

Power law primordial power spectrum:

$$P(k) = A_s \left(\frac{k}{k_0}\right)^{n_s - 1}$$

Do parameter estimation.

Power Spectrum Reconstruction

~~Power law primordial power spectrum:~~

$$\cancel{P(k) = A_s \left(\frac{k}{k_0}\right)^{n_s-1}}$$

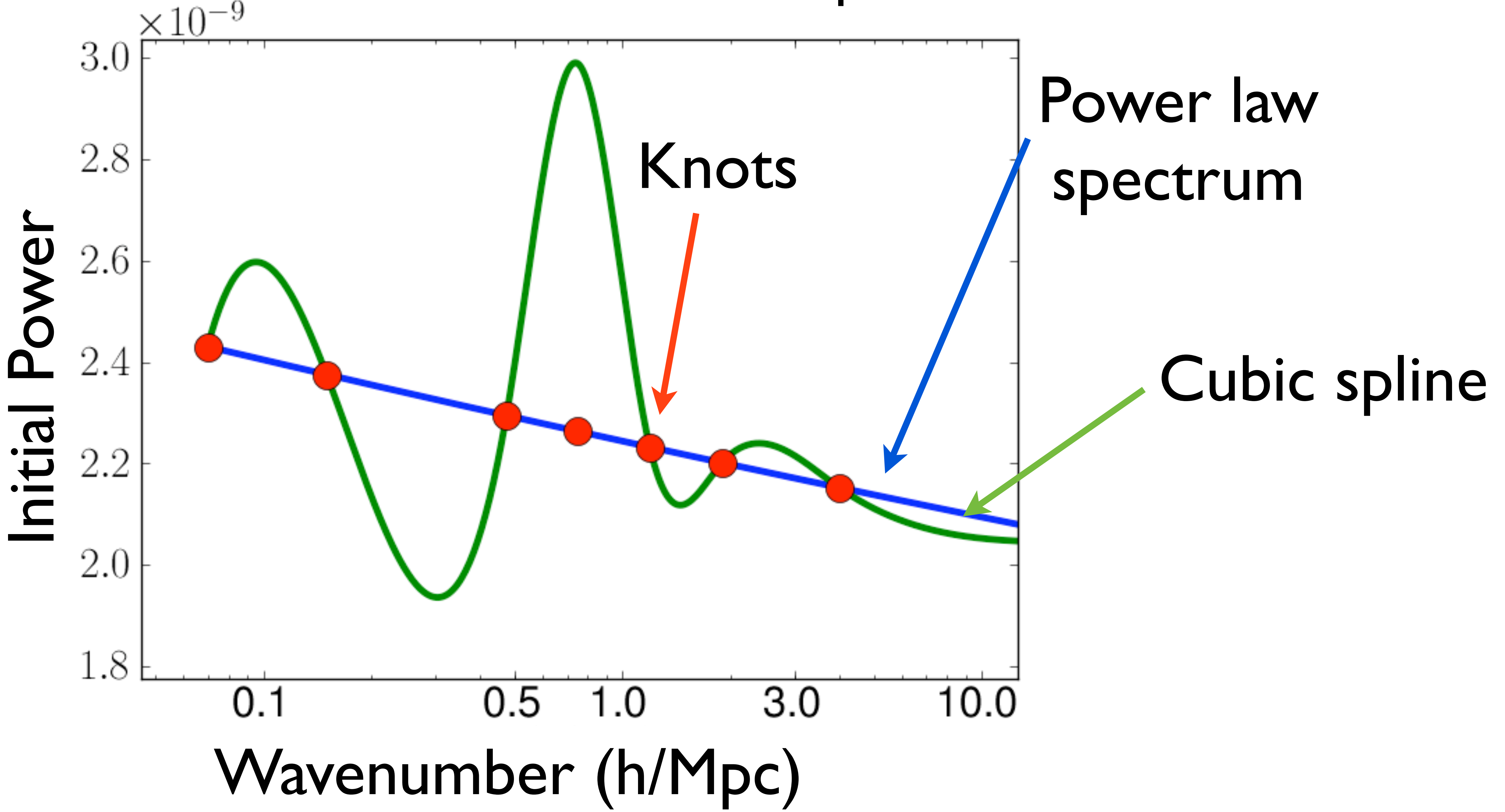
~~Do parameter estimation.~~

Power Spectrum Reconstruction

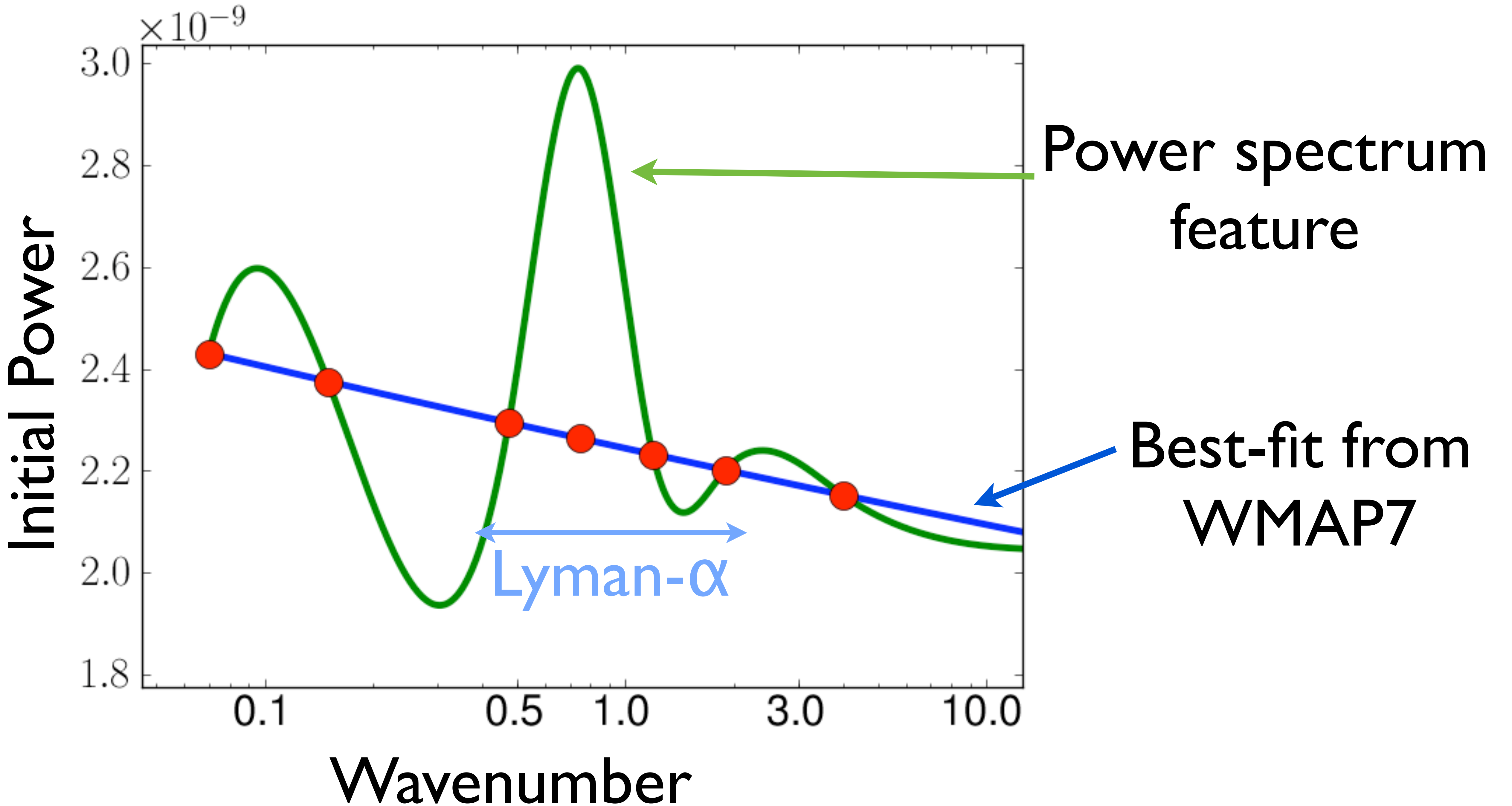
- Replace power law form with cubic spline.
- Replace parameter estimation with reconstruction

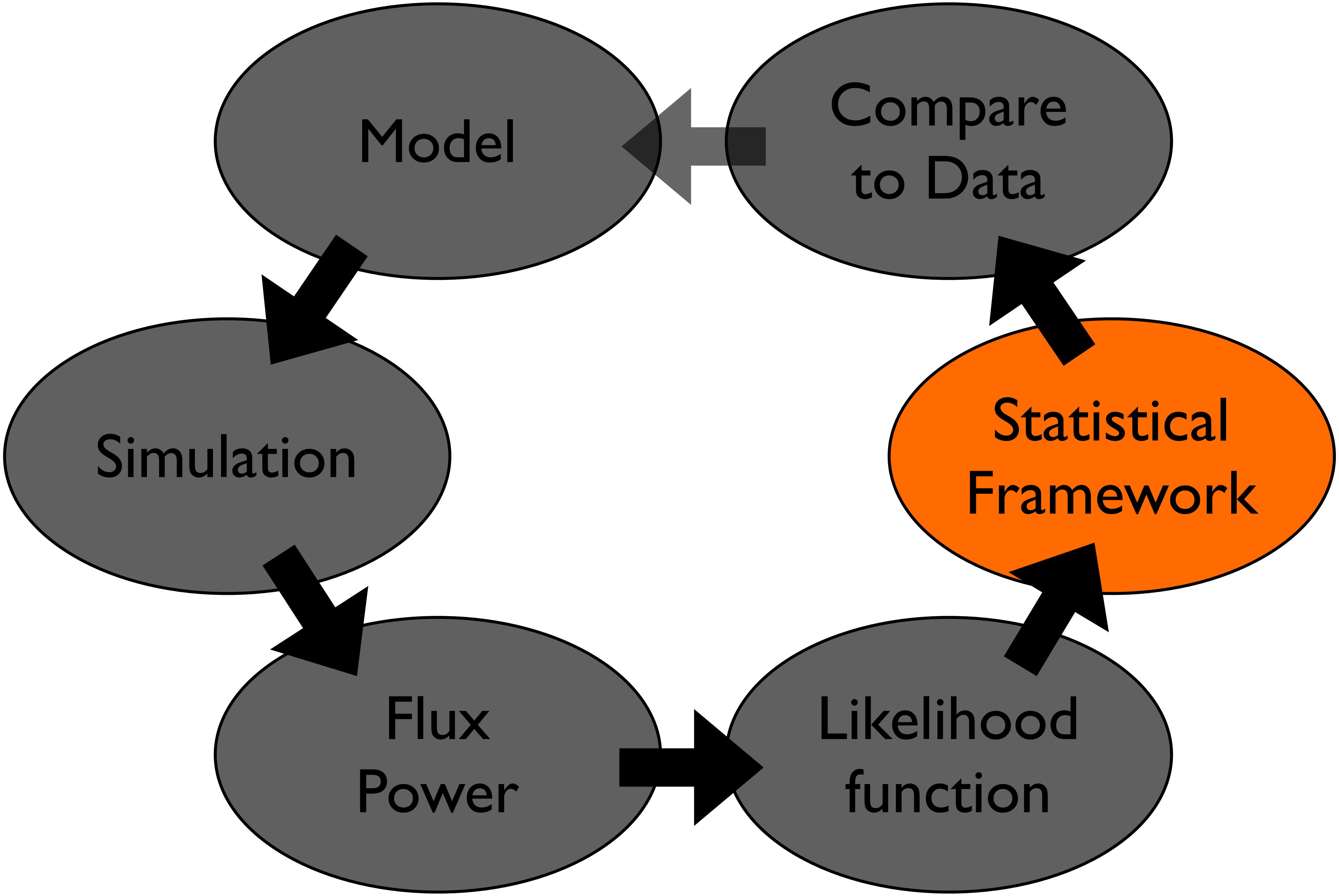
Reconstruction

Fit with cubic spline.



Reconstruction




Model
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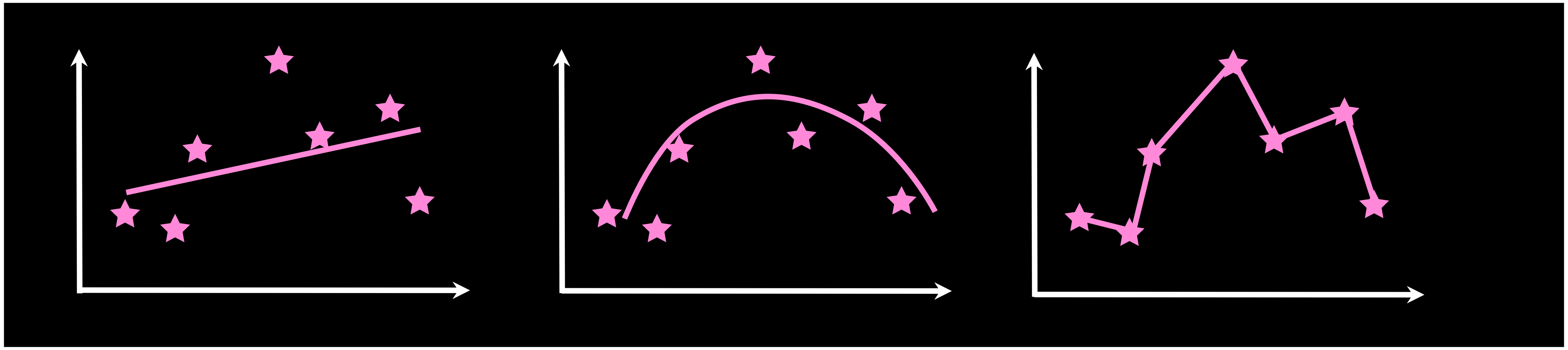
Power
Likelihood
function

Reconstruction

We need to fit the signal, but NOT the noise

Use cross-validation: similar to jack-knifing.

Which is best?



Power Spectrum

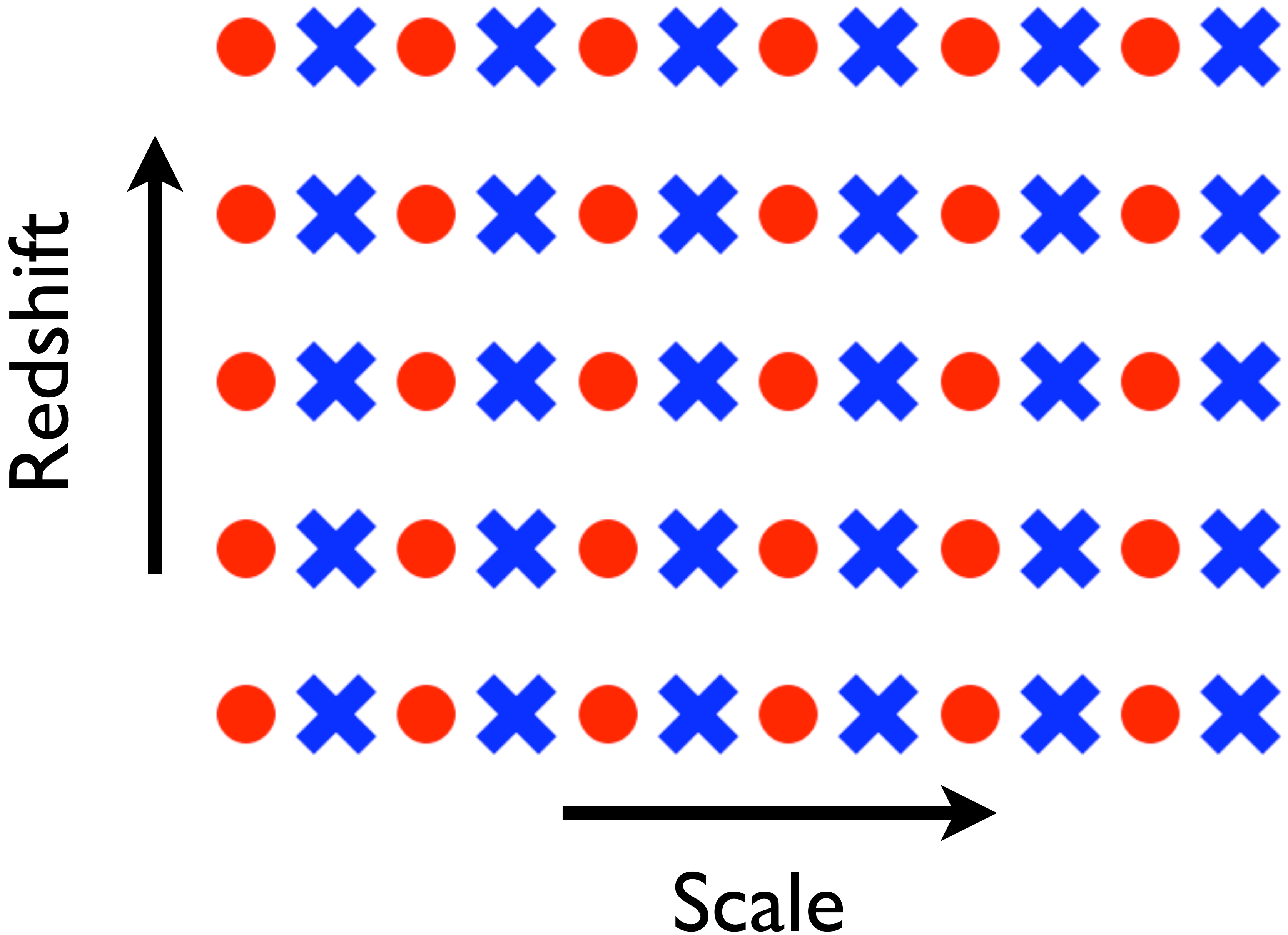
Noise is extra small-scale variation.

Likelihood to penalise "wiggly" shapes:

$$\log \mathcal{L} = \log \mathcal{L}(\mathrm{Data}|P(k)) + \lambda \int_k dk (P''(k))^2$$

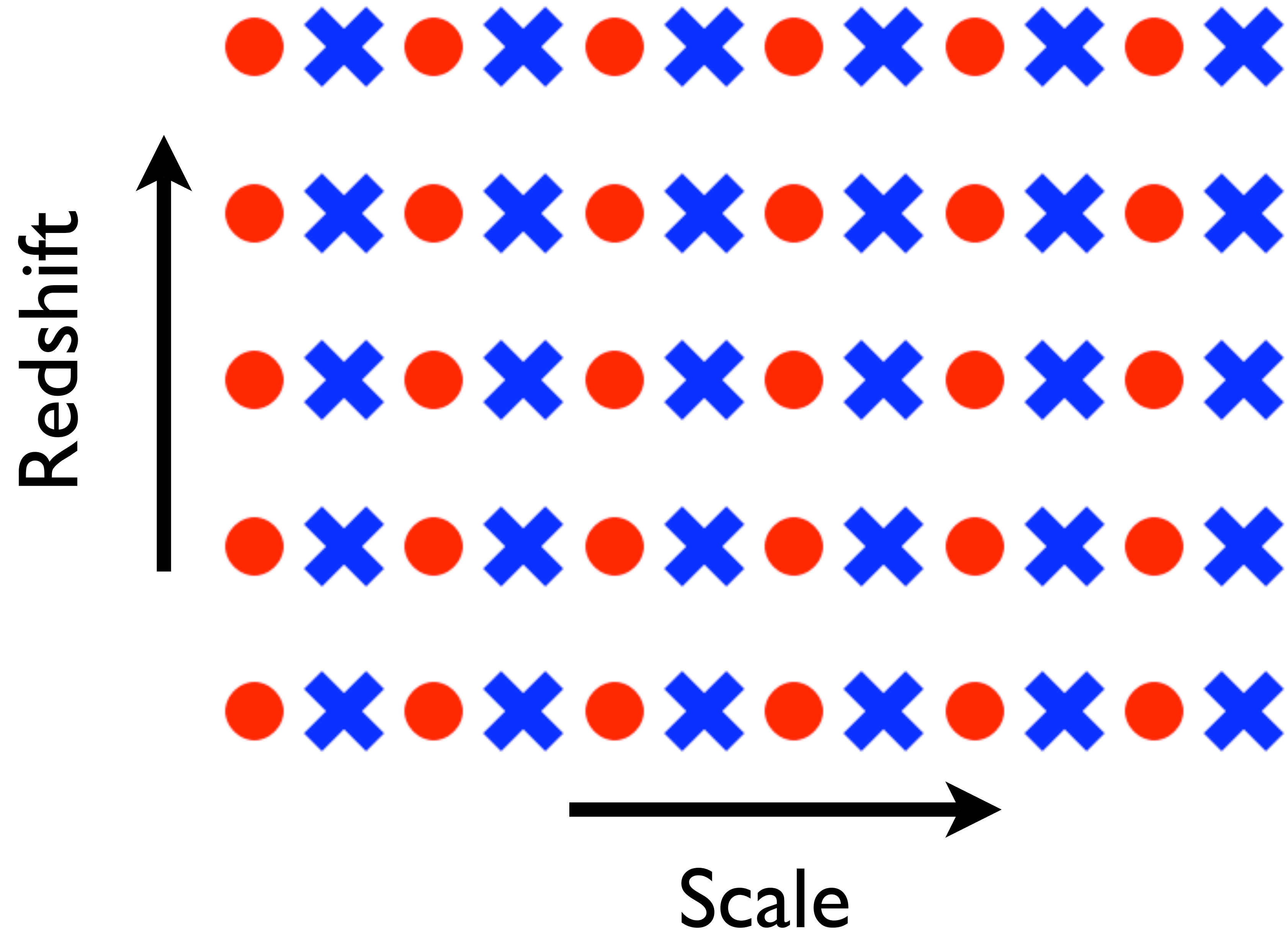
Cross-validation to choose penalty most accepted by data

Cross-Validation



Divide data into: training set (crosses) and
validation set (circles)

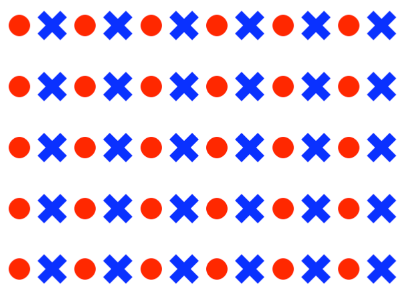
Cross-Validation



Training set should predict validation set

Cross-Validation

1. Pick penalty.
2. Find best fit to training set
3. Predict validation set from best-fit
4. Find penalty which best predicts validation set



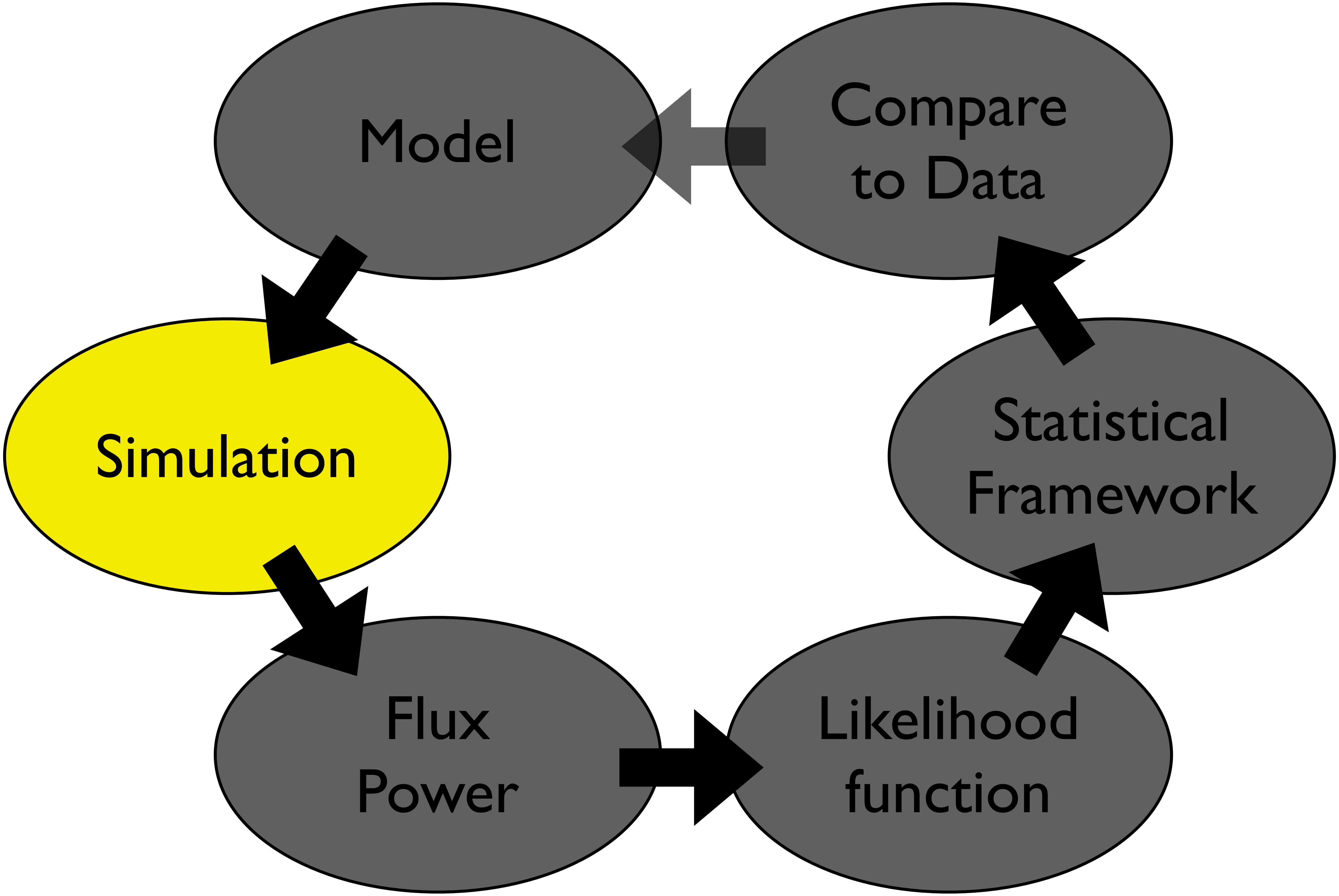
$$\log \mathcal{L} = \log \mathcal{L}(\mathrm{Data}|P(k)) + \lambda \int_k dk (P''(k))^2$$

Parameter Estimation

- Assume data Gaussian: $N(\mu,\sigma)$
- Find μ, σ in best agreement with data

Minimally Parametric

- Choose some form $F(\mu,\sigma)$
- Find μ, σ in agreement with training data
- Check how well $F(\mu,\sigma)$ predicts validation data


Model
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Statistical Framework
Flux Power
Likelihood function

Motivation

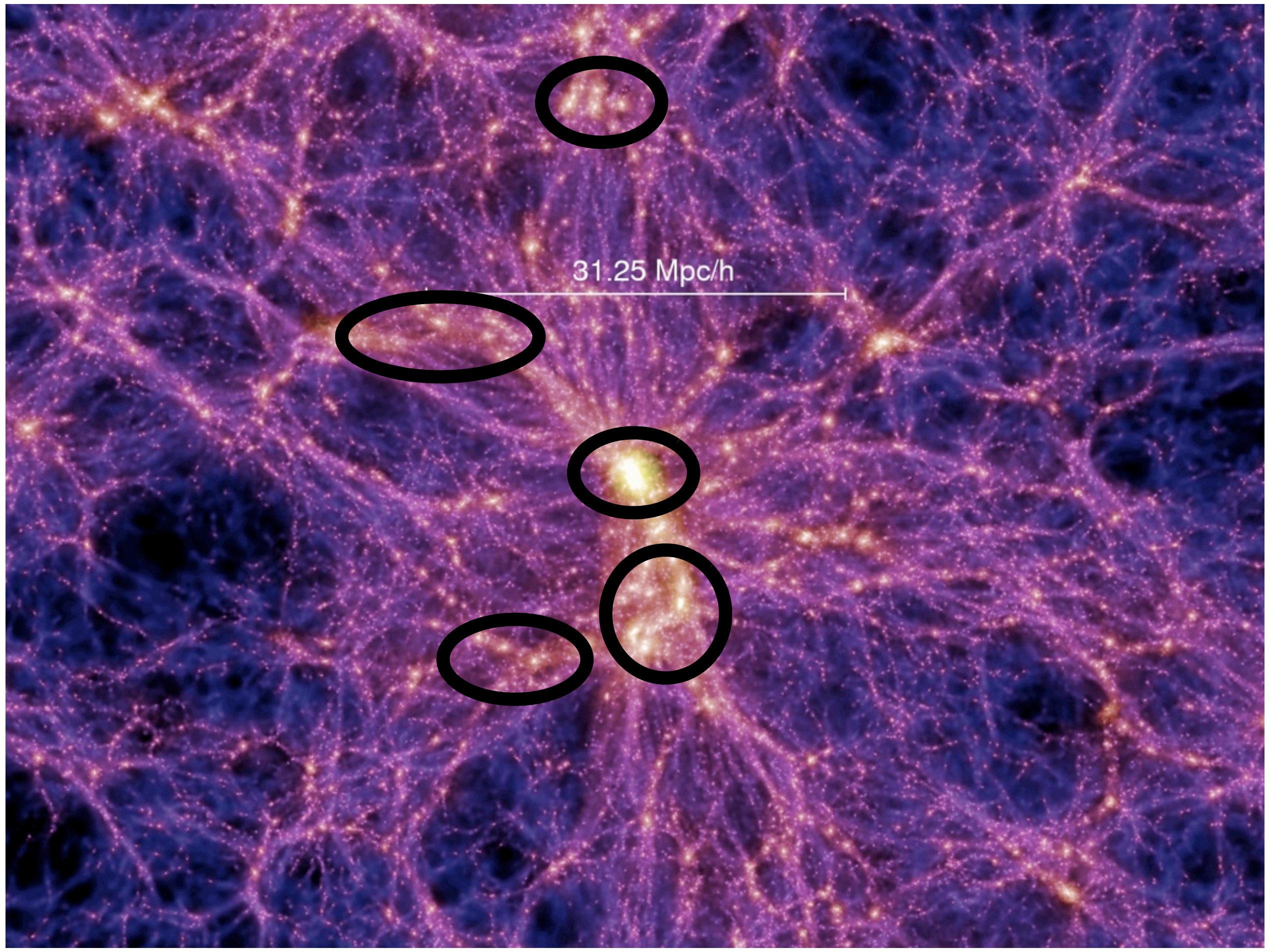
Why do we need new simulations?

- Structure nonlinear
- Need to construct a map between P(k) and flux statistics: depends on baryonic physics
- Previous map assumed scale-invariance

Simulation Setup

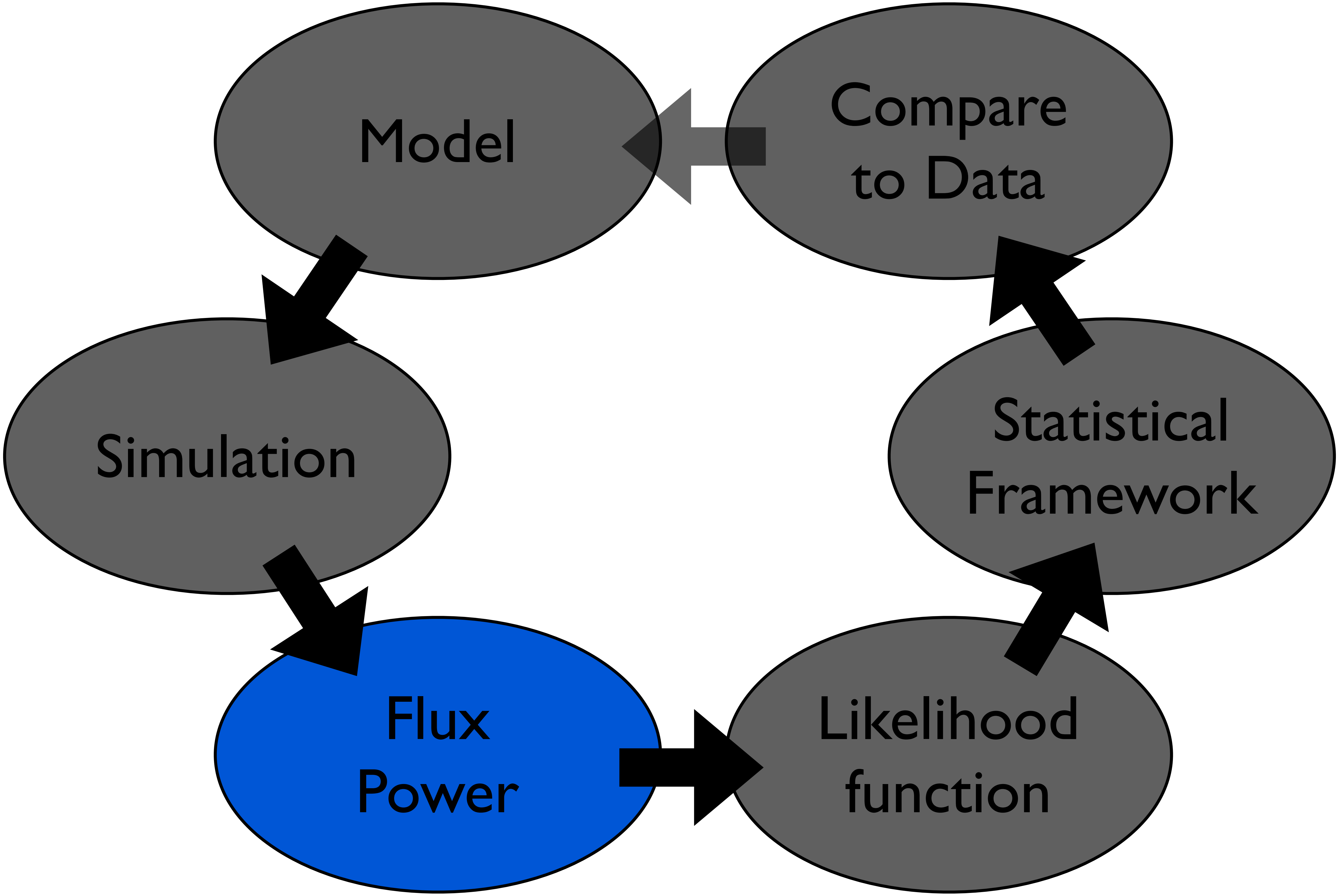
- 30+ hydrodynamic simulations using GADGET-II.
- 60 Mpc box, 2x400^3 particles
- 400^3 dark matter particles - collisionless
- 400^3 baryons - with cooling

Important Trick



- Dense regions have many slow collisions
- Do not influence the Lyman-α forest
- Save time by making dense regions "stars"

Image: Millennium Simulation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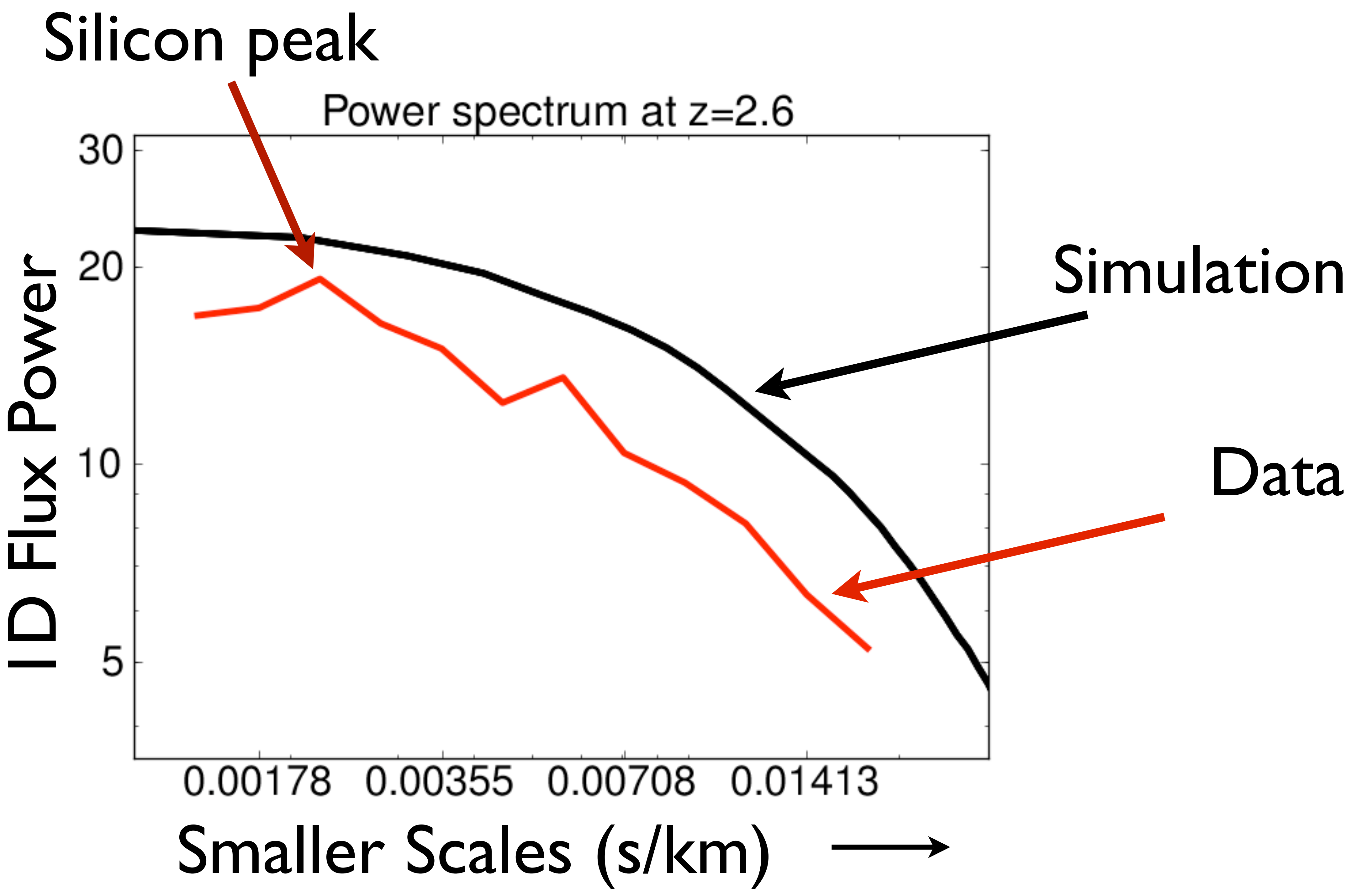
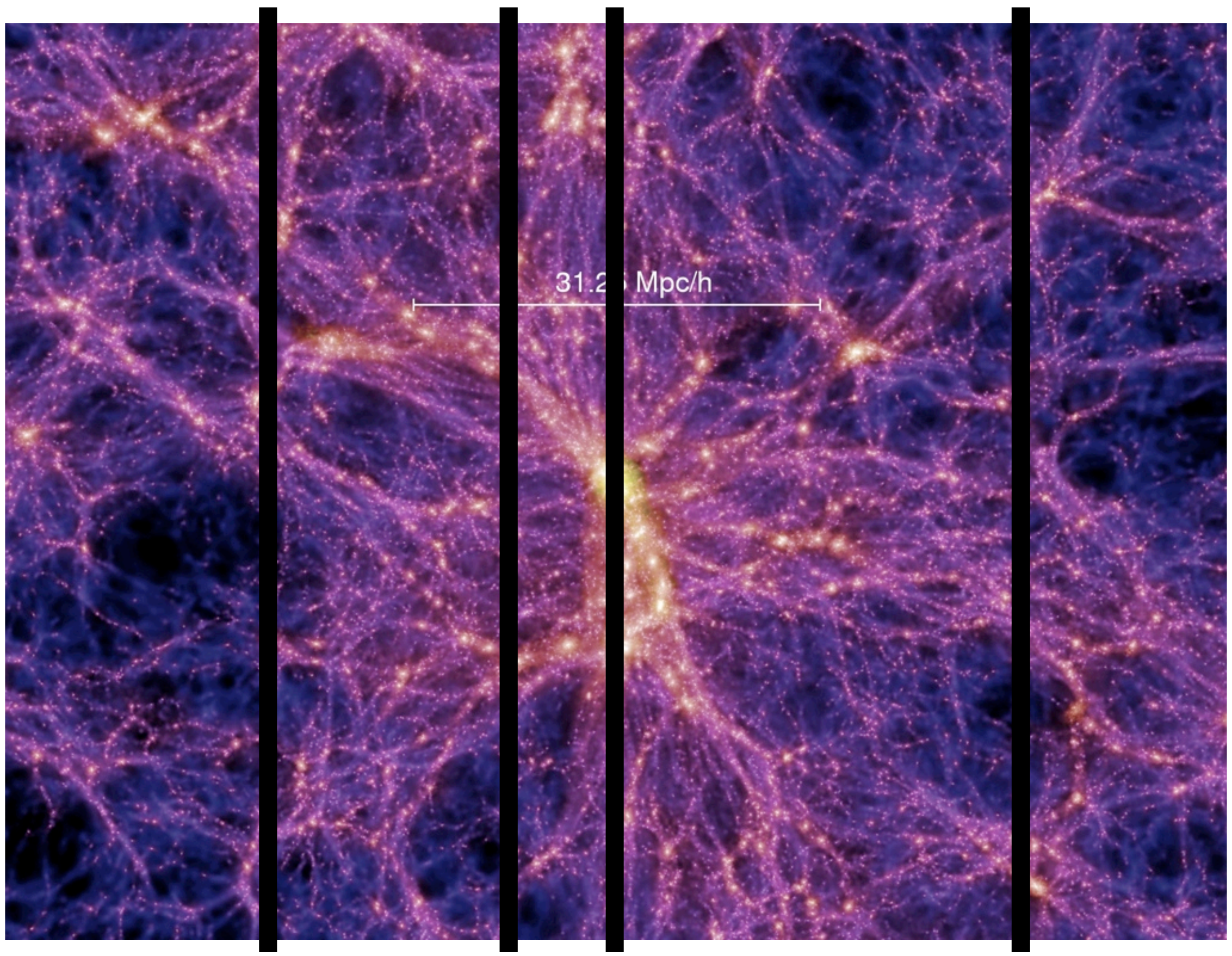
Flux Power Spectrum



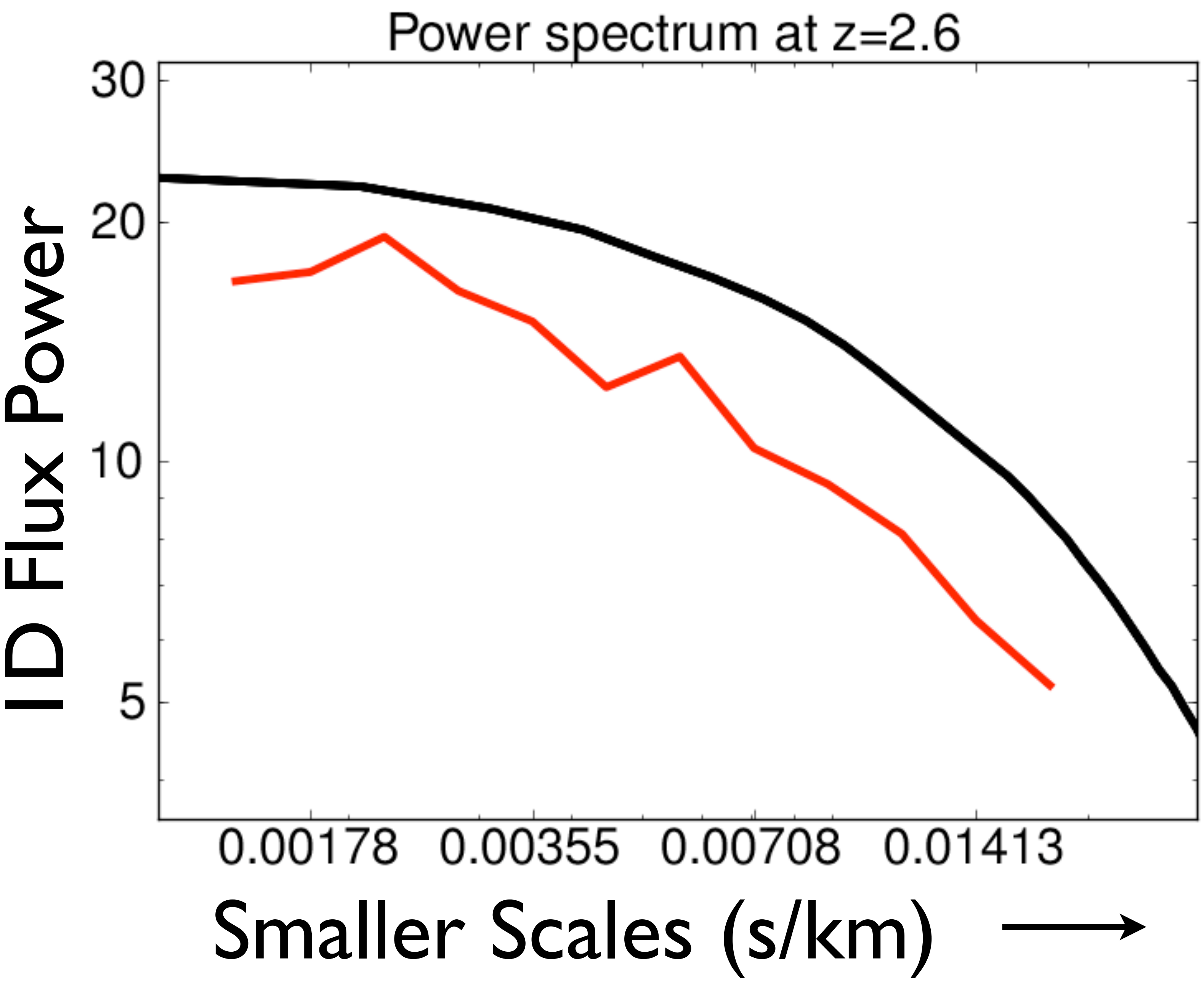
Simulated Spectra



- Draw skewers through density field
- Calculate absorption along skewers
- Average of two-point statistics

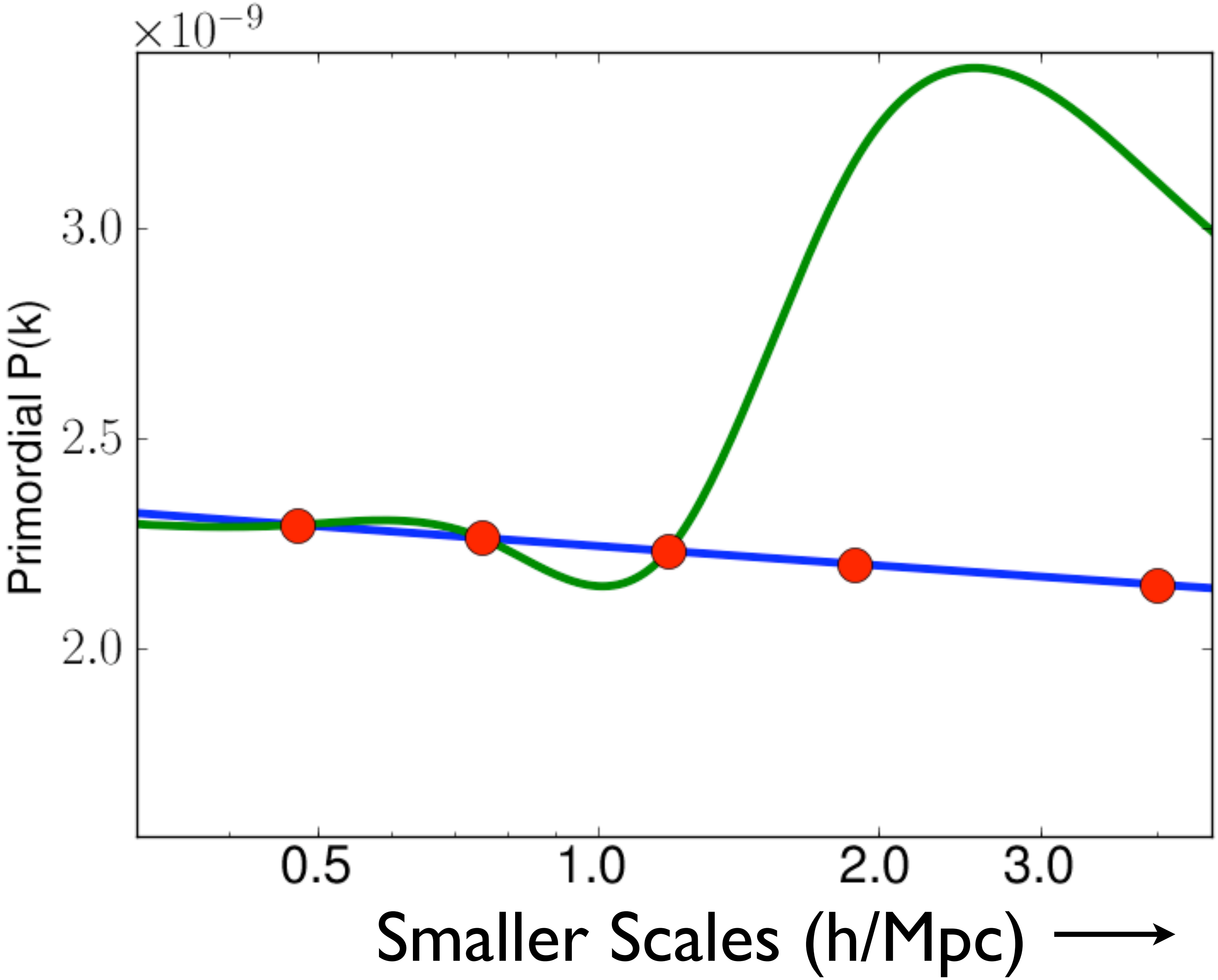
Image: Millennium Simulation

Flux Power Spectrum

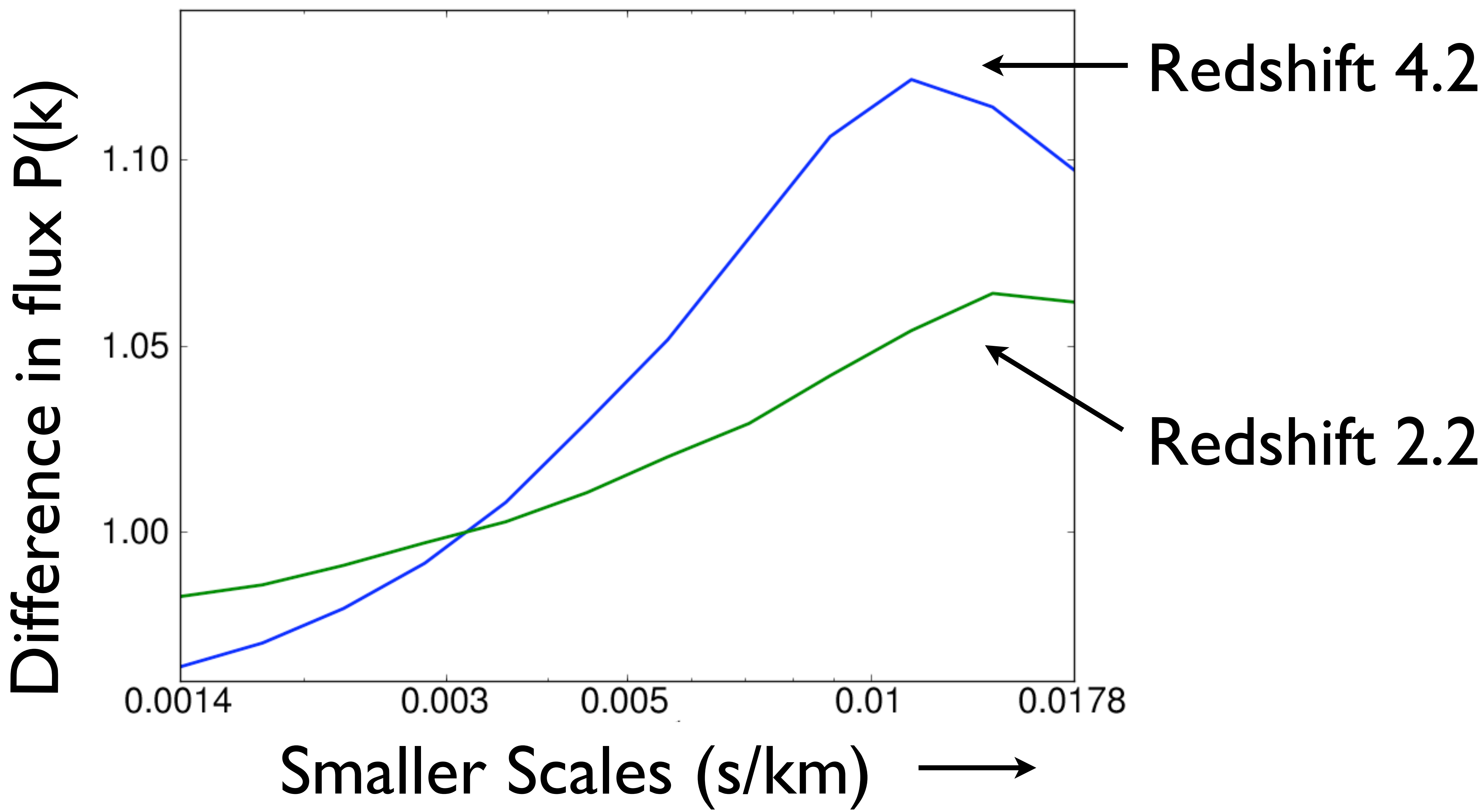


What is effect of the initial power spectrum?

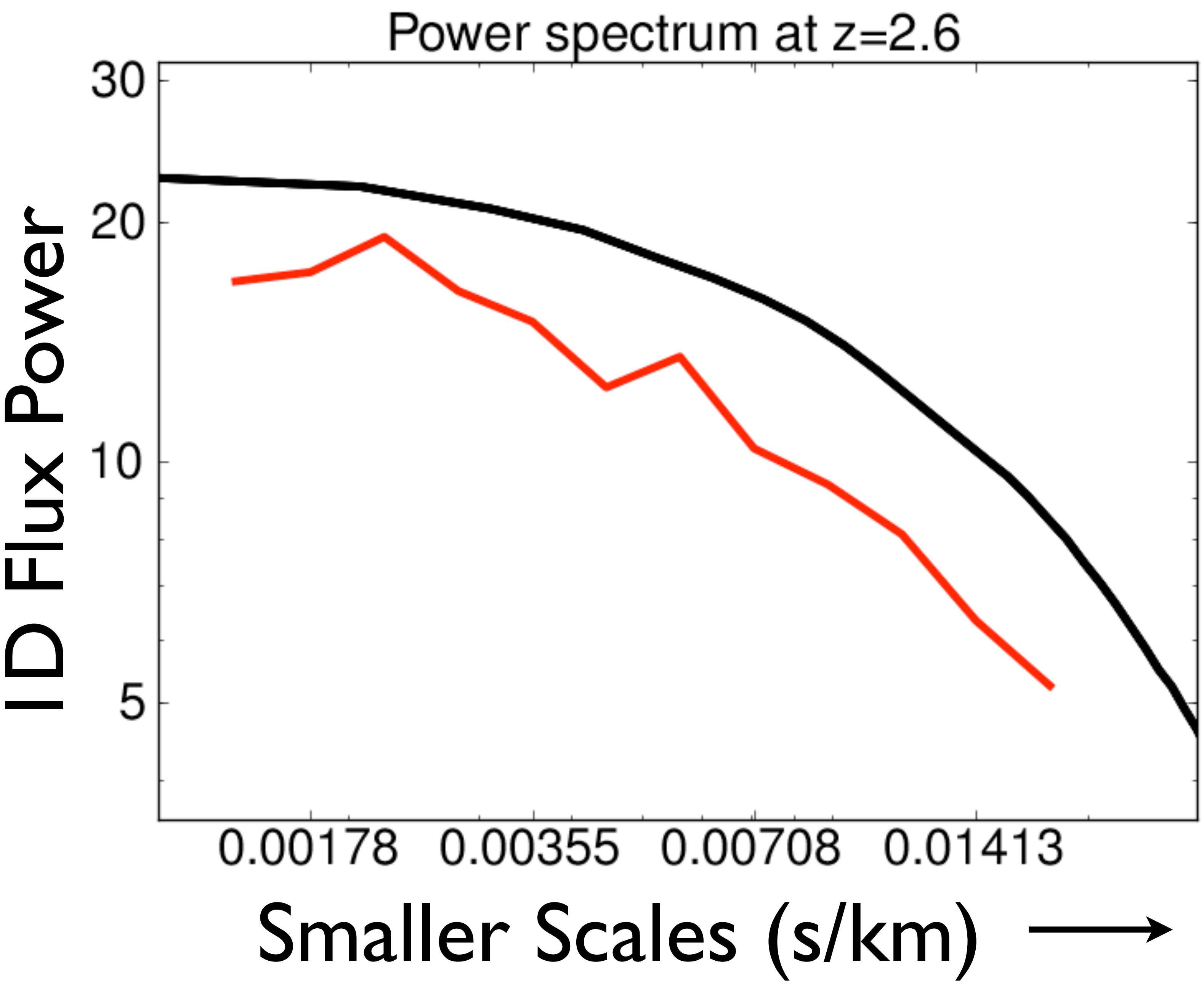
Flux Power Spectrum



Flux Power Spectrum



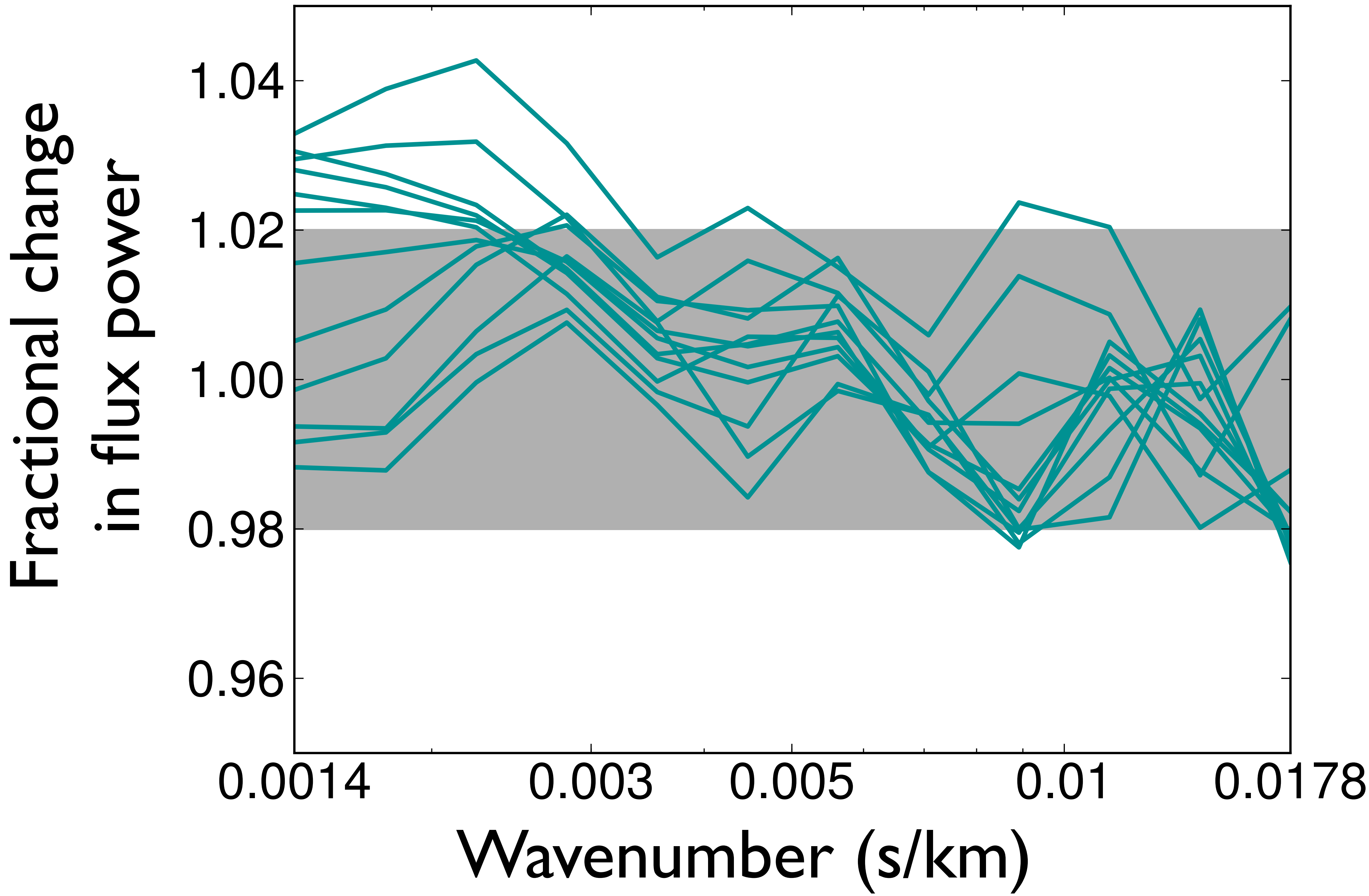
Flux Power Spectrum



What are the effects of numerics?

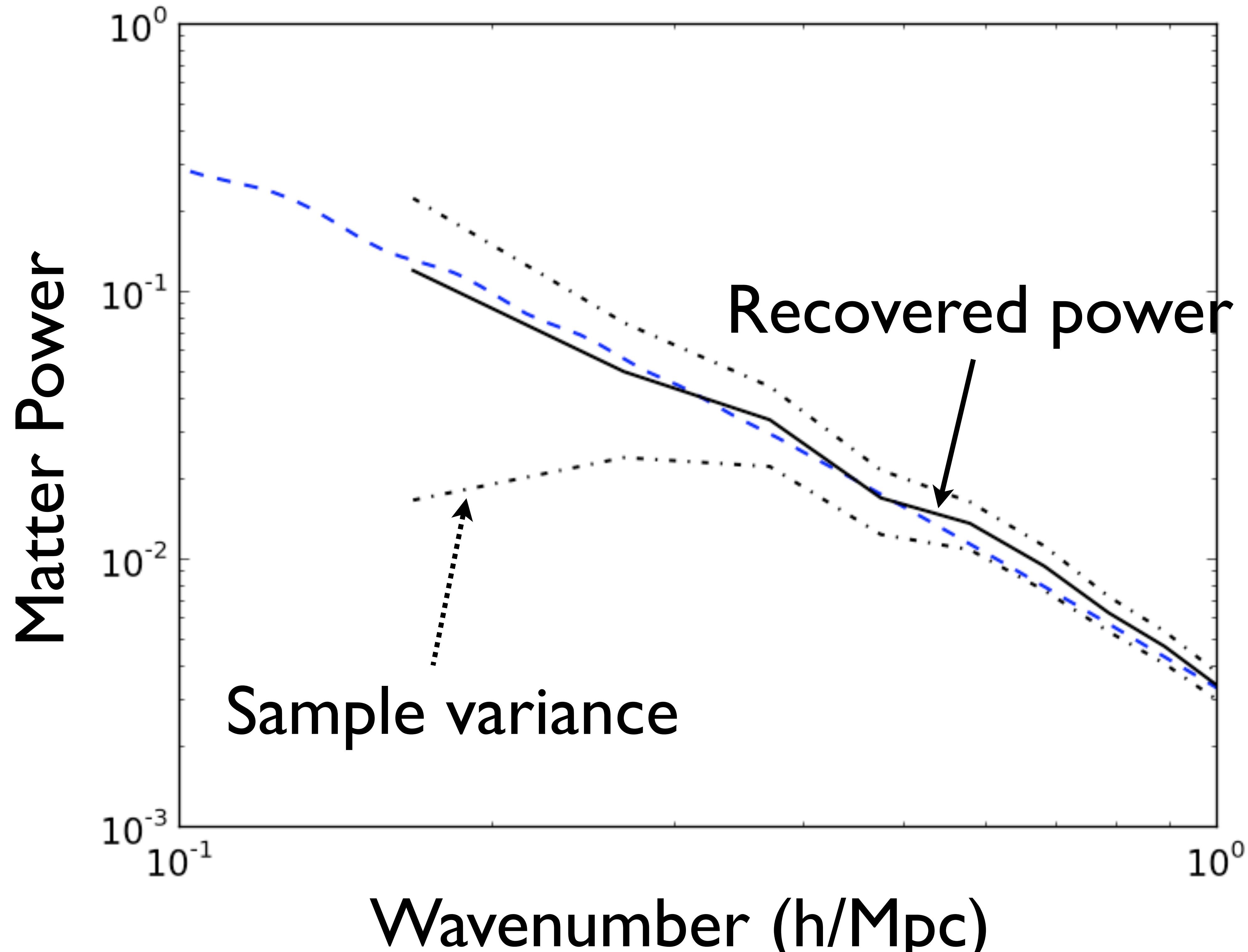
Are our simulations resolved?

Box Size

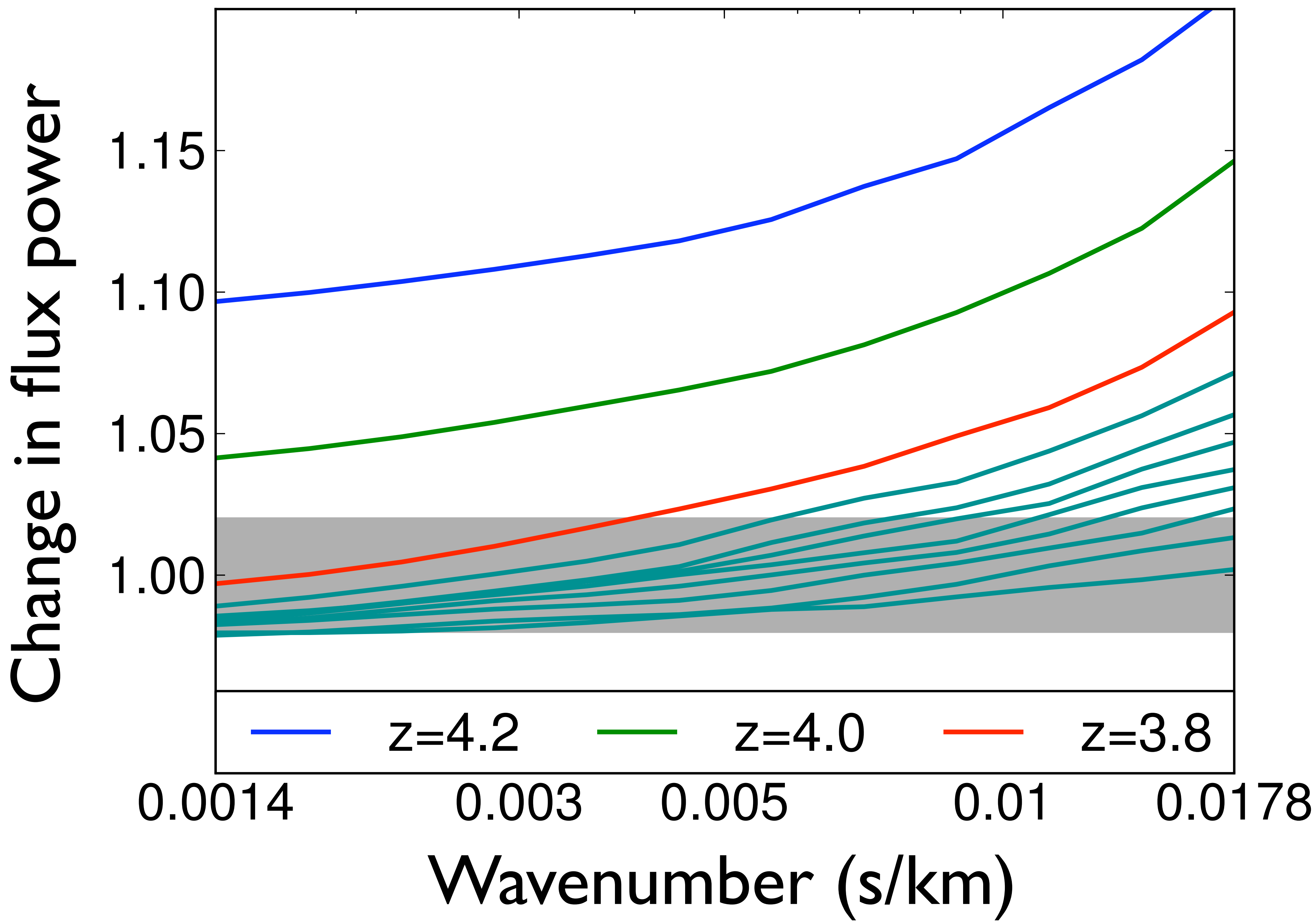


1.04
1.02
1.00
0.98
0.96
Fractional change in flux power
0.0014
0.003
0.005
0.01
0.0178
Wavenumber (s/km)

Box correction



Resolution



Change in flux power
1.15
1.10
1.05
1.00
z=4.2
z=4.0
z=3.8
0.0014
0.003
0.005
0.01
0.0178
Wavenumber (s/km)

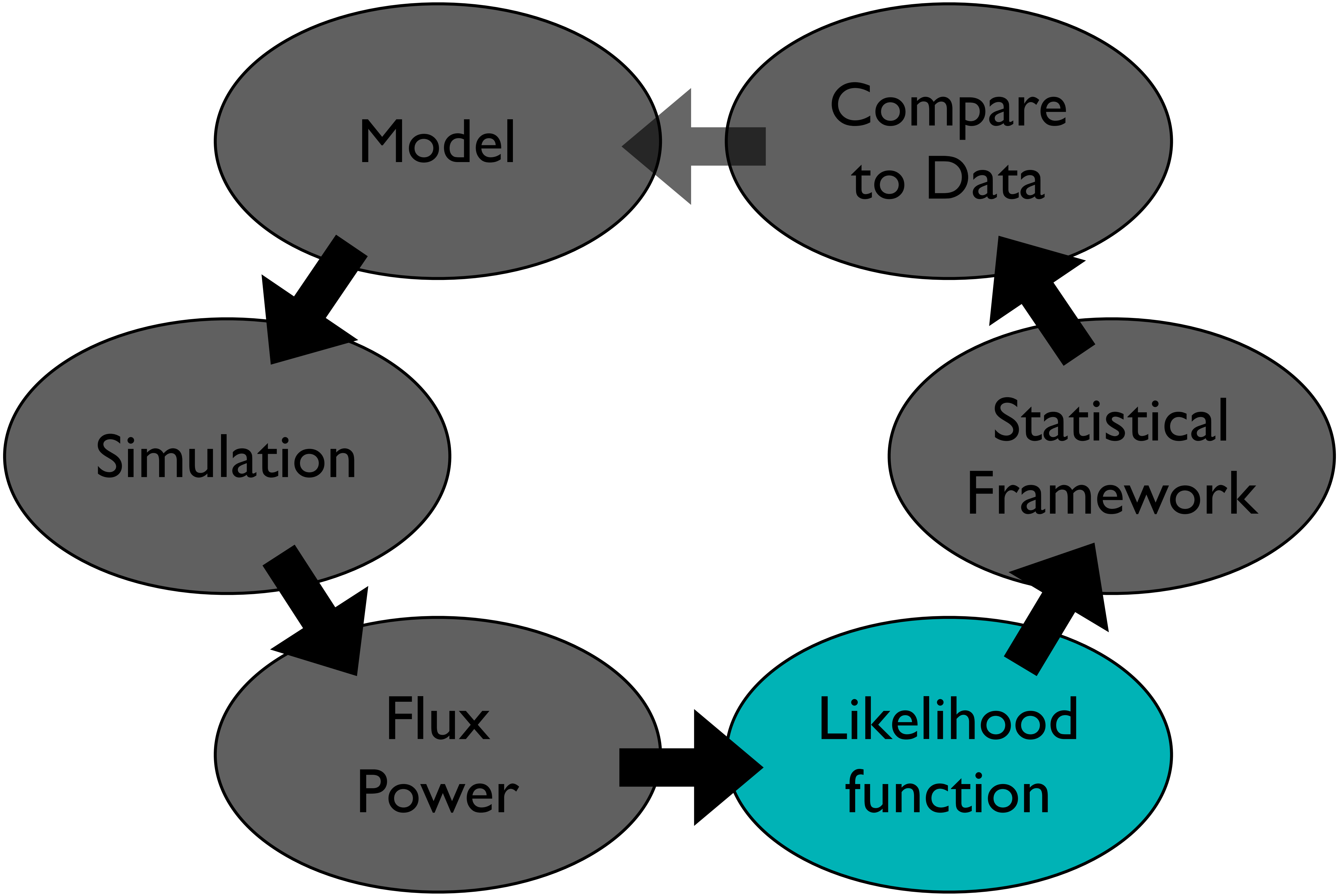
Resolution

Low Redshift

- Absorption from near mean density

High Redshift

- Absorption from voids
- SPH allocates particles to high density regions
- Voids are poorly resolved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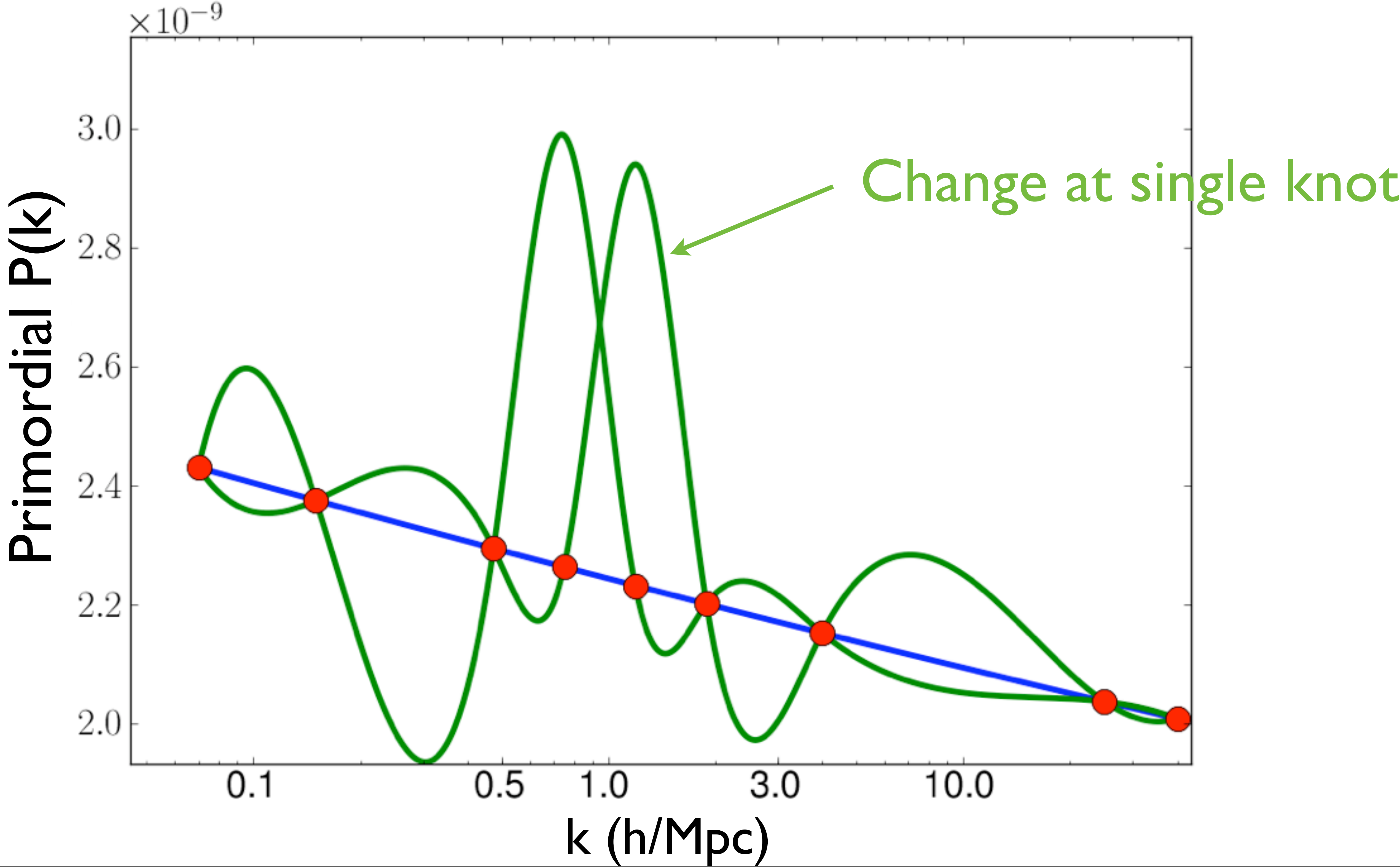
Likelihood Construction

- Vary one parameter at a time.
- Fit change in flux power with a polynomial

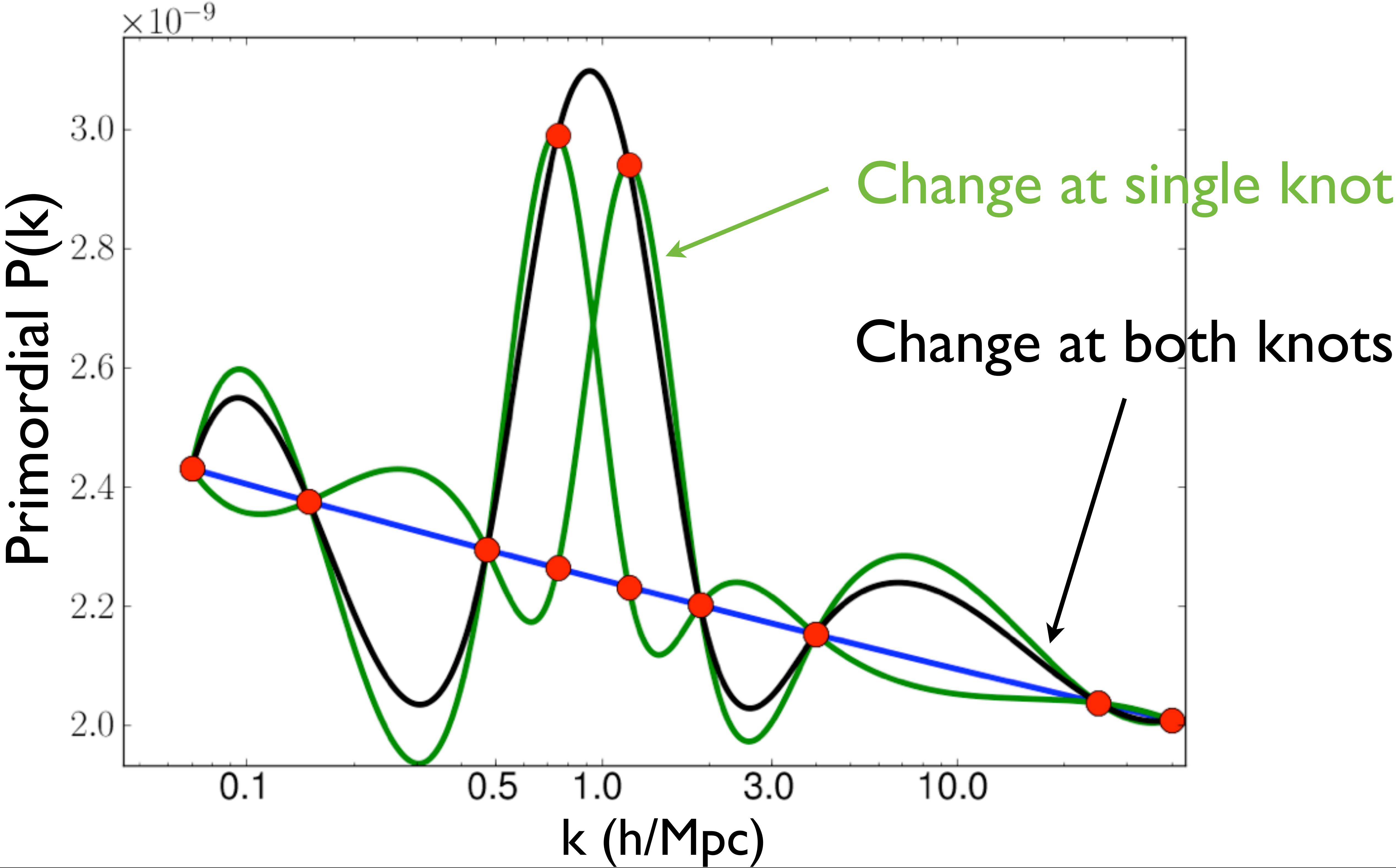
$$\delta P_{\mathrm{F}}(p_i) = \Sigma_i \left(a\delta p_i^2 + b\delta p_i\right)$$

- Check accuracy with jack-knifing.

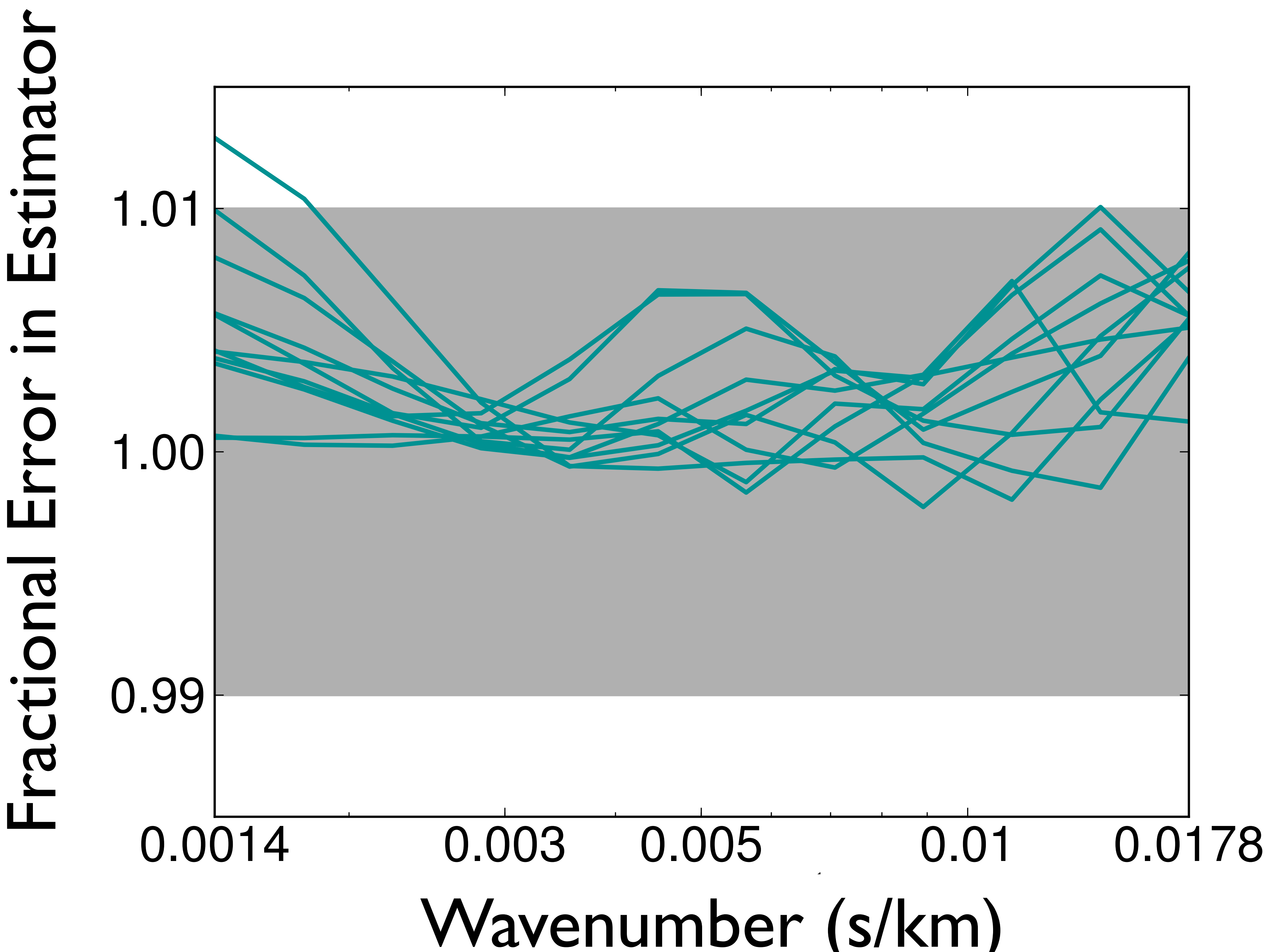
Estimator


$\times 10^{-9}$
Primordial P(k)
3.0
2.8
2.6
2.4
2.2
2.0
Change at single knot
0.1
0.5
1.0
3.0
10.0
k (h/Mpc)

Estimator

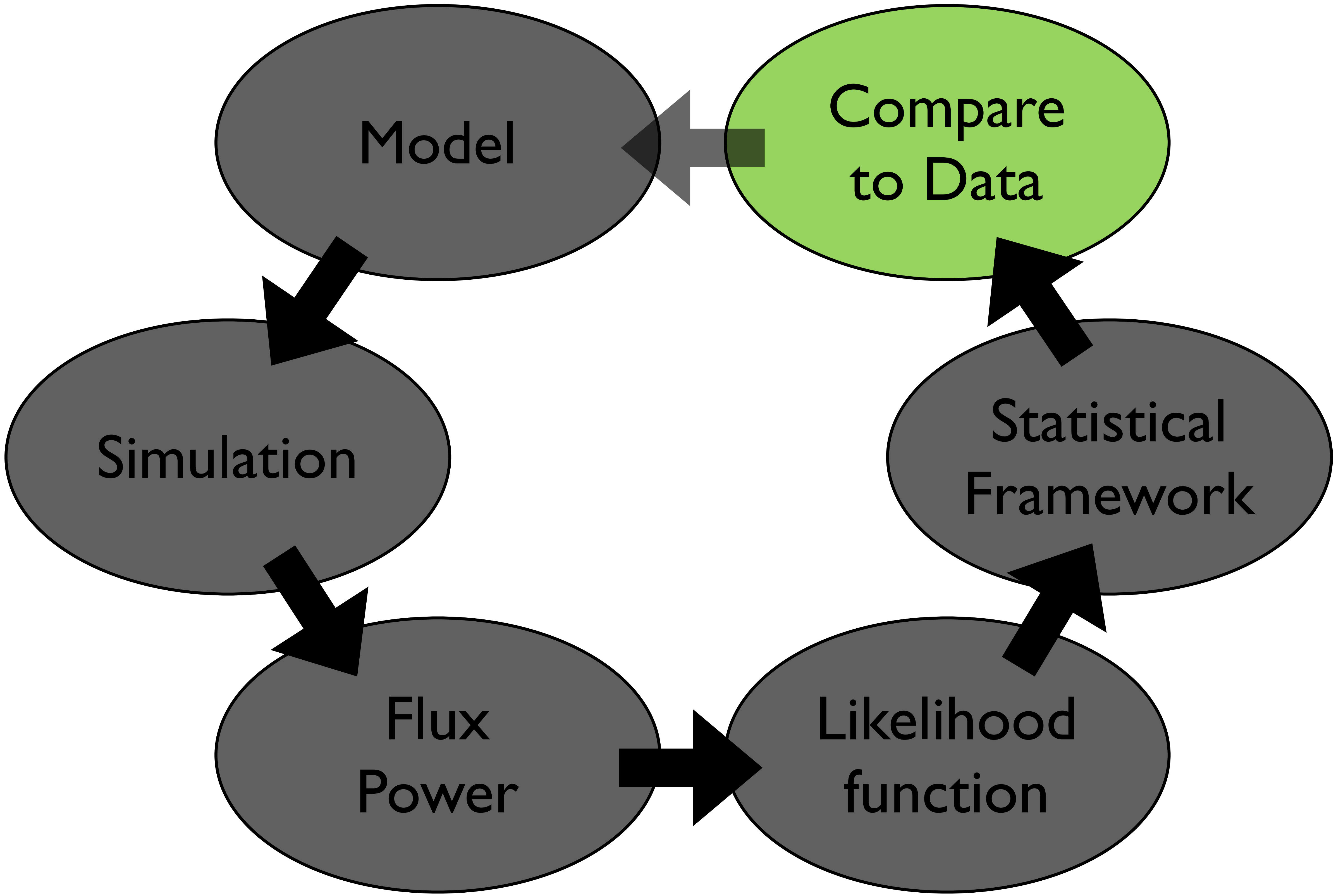

$\times 10^{-9}$
3.0
2.8
2.6
2.4
2.2
2.0
Primordial P(k)
0.1
0.5
1.0
3.0
10.0
k (h/Mpc)
Change at single knot
Change at both knots

Estimator Error



Likelihood Construction

- Marginalise over thermal parameters:
  - Temperature
  - Temperature-density relation
  - Mean optical depth, aka ionising radiation density
- Correct for resolution and box effects, damping wings, SiIII, ...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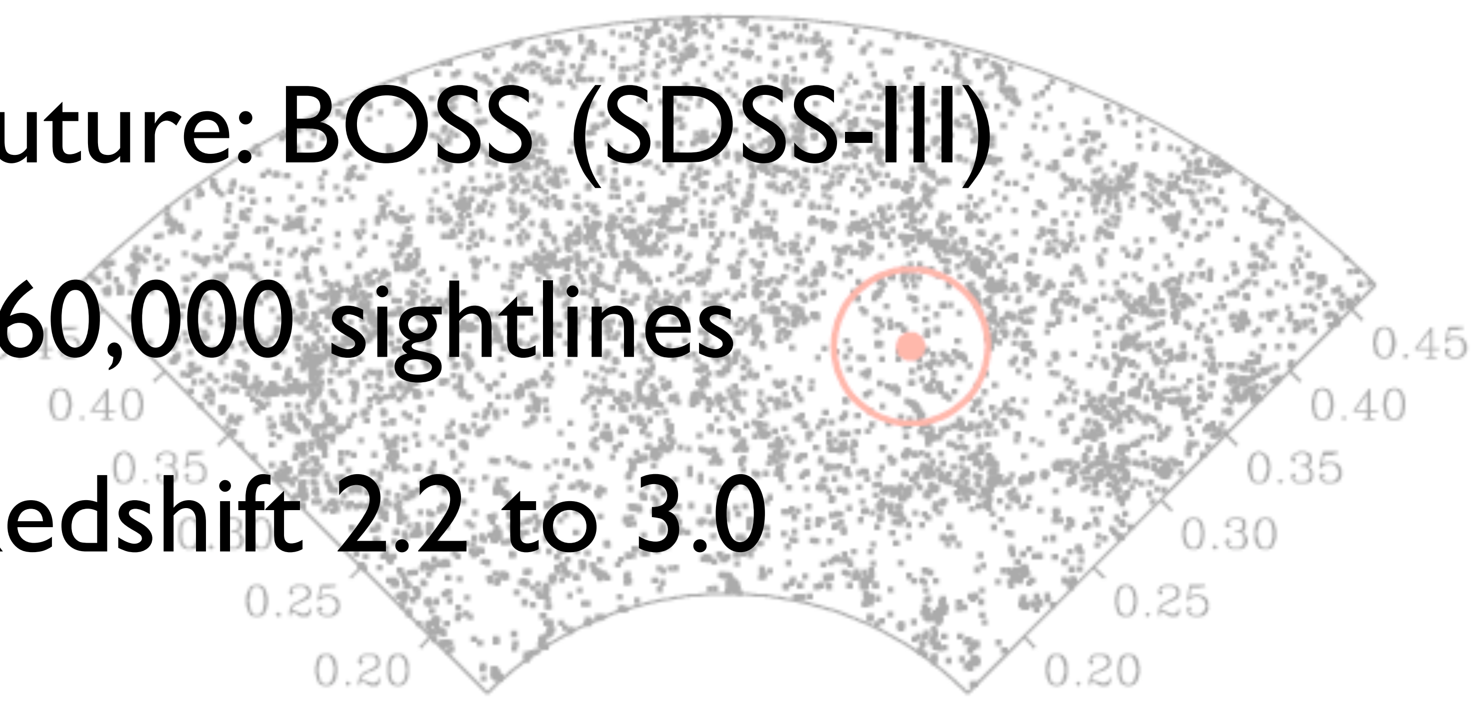
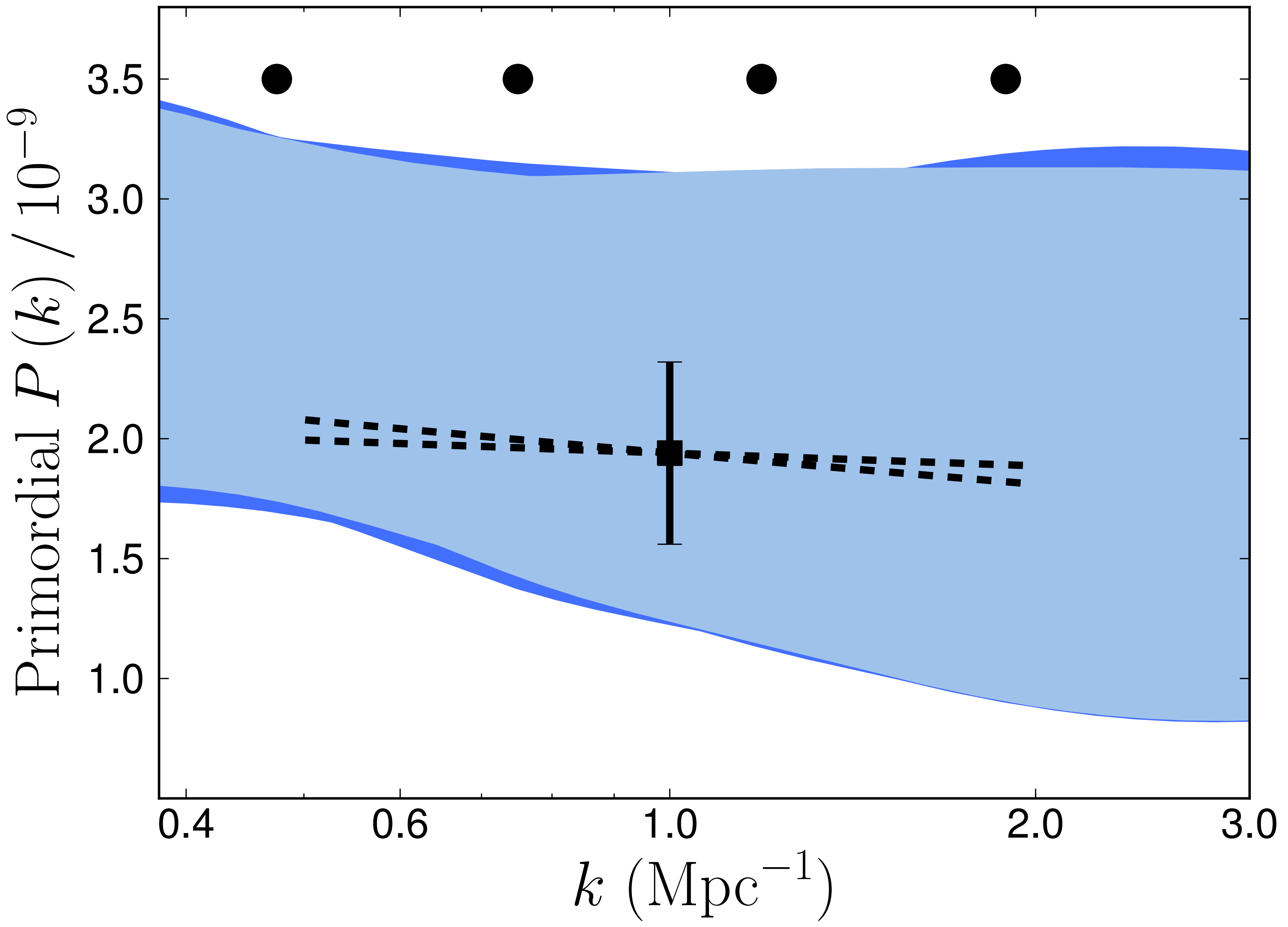
Data Comparison

- Current data: SDSS quasar flux power spectrum from McDonald et al 2005.
- ~3000 quasar sightlines
- Redshift 2.2 to 4.2

- Future: BOSS (SDSS-III)
- 160,000 sightlines
- Redshift 2.2 to 3.0



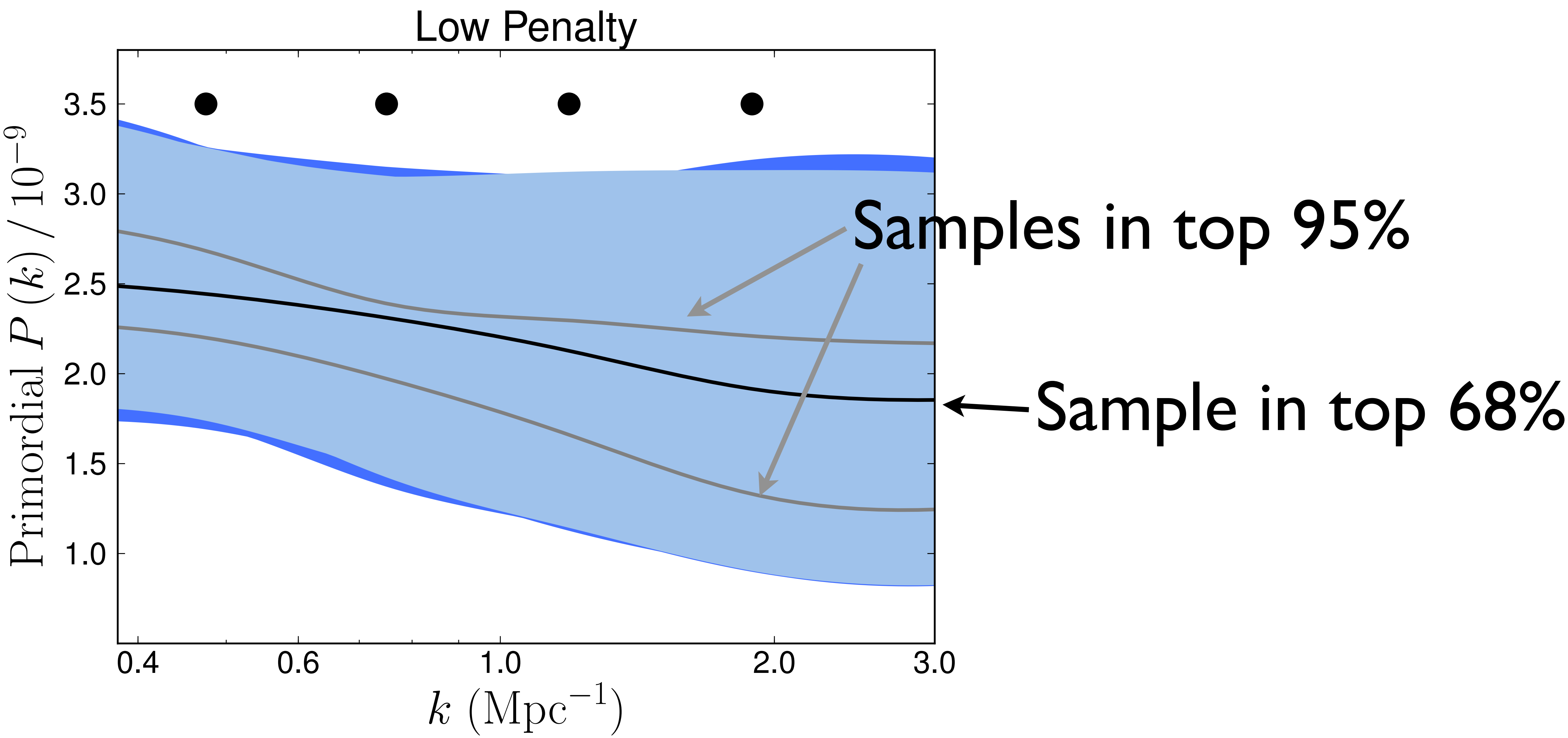
Results



- "Envelope" of splines with likelihood in top 95%.

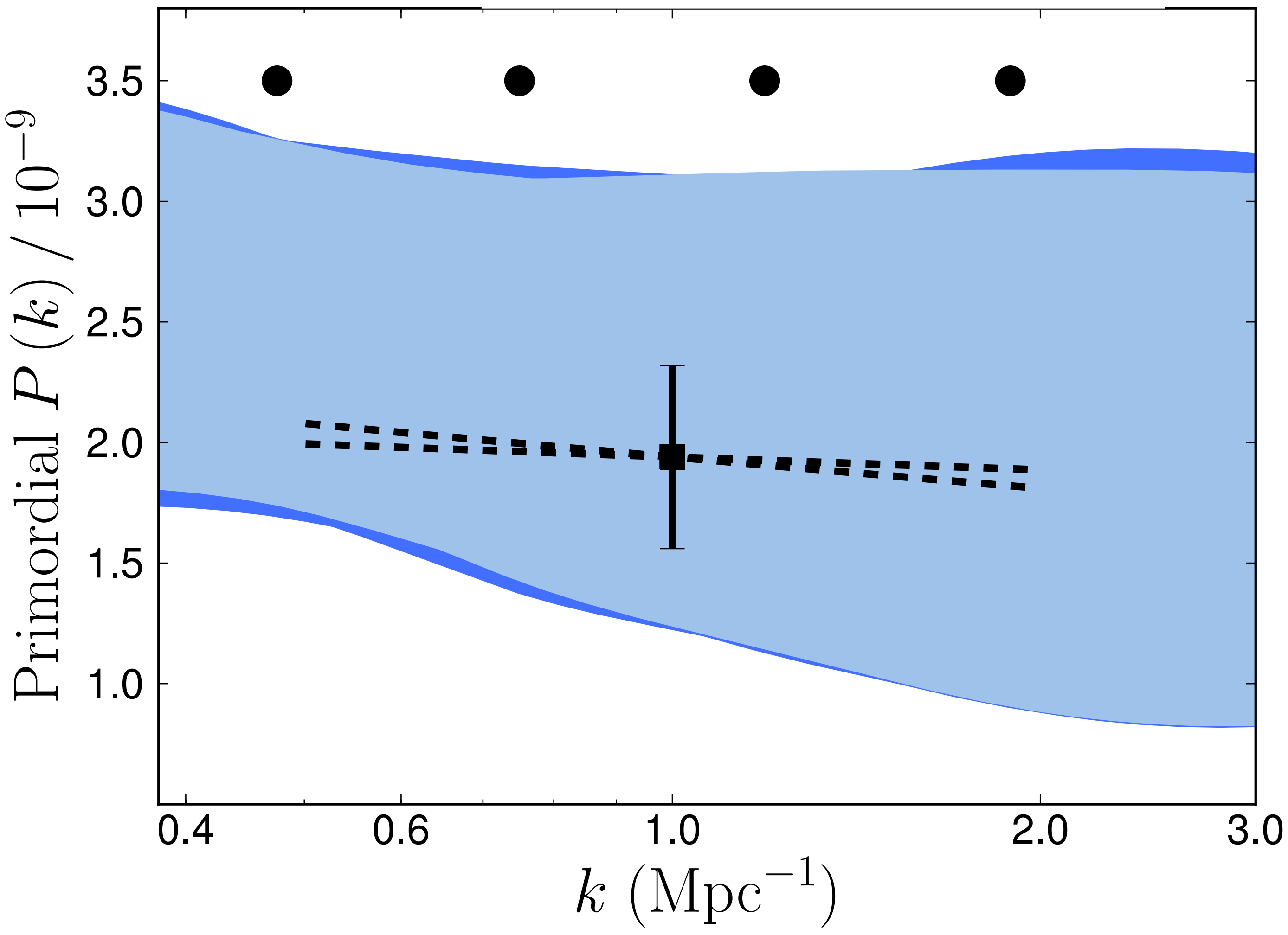
Bird et al (2010), arxiv:1010.1519

Results



- 68% and 95% have similar envelopes; lower likelihood splines have more features.

Results

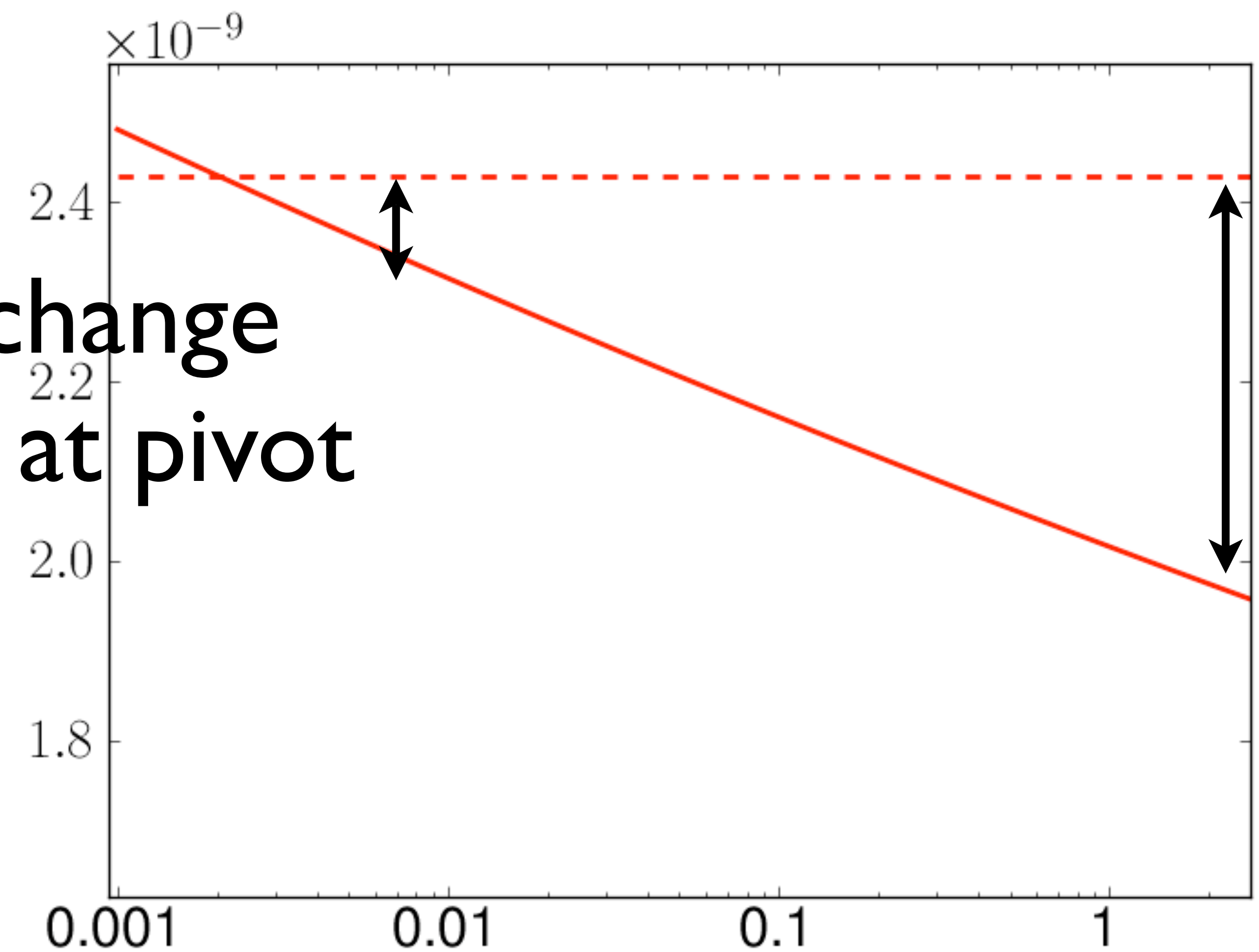


- Error bar shows constraints from parameter estimation
- Driven by prior assumption of power law form

A sufficiently high penalty reproduces the previous results.

Results

- Constraints on $n_s = 1 + \dfrac{d \ln P}{d \ln k}$: 0.2 - 1.2
- Constraints from parameter estimation helped by long baseline



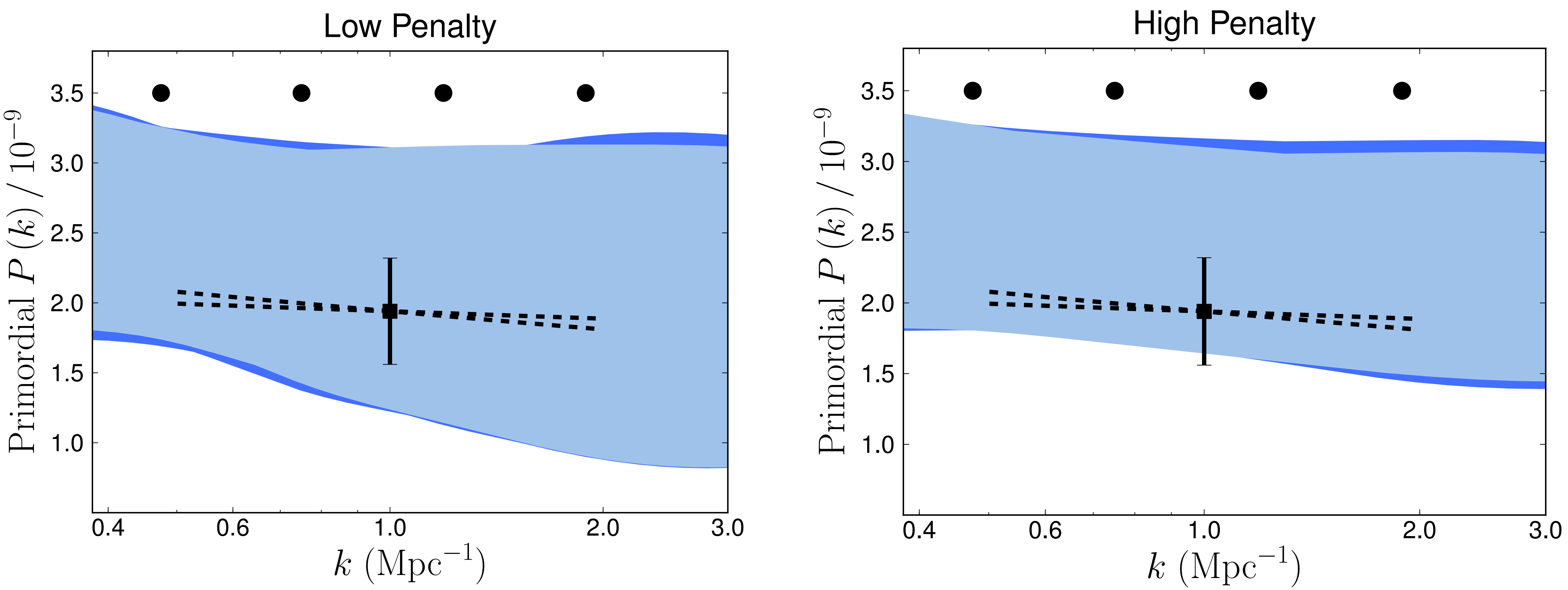
Small change in slope at pivot

Big change by Lyman-α scales

$$n_s = 0.97$$

$$P(k) = A_s \left(\frac{k}{k_0} \right)^{n_s - 1}$$

Results

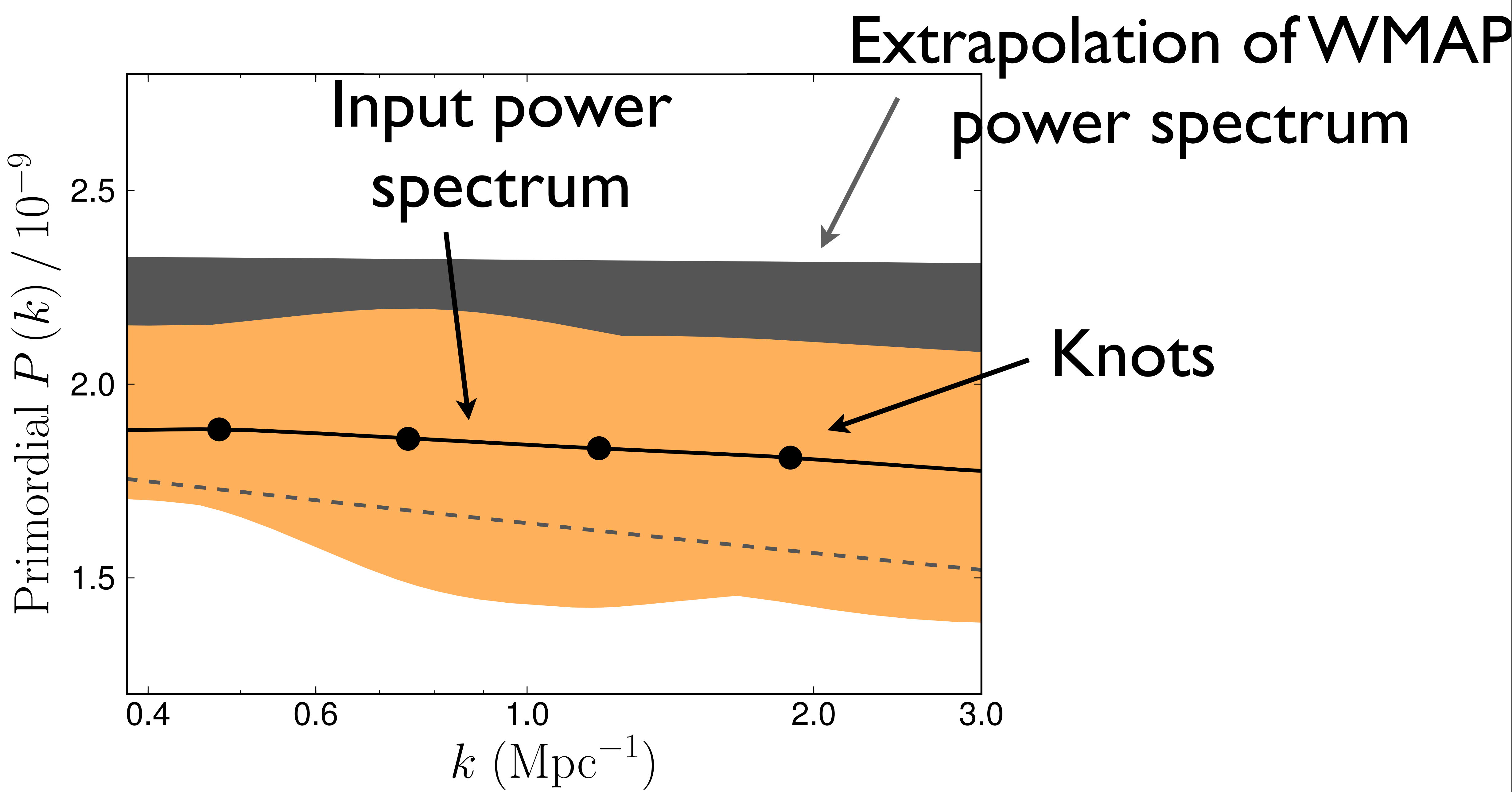


- CV score constant with penalty
- Cannot distinguish between above plots.

BOSS simulation

- Simulated flux power spectrum with theoretically motivated parameters
- Simulate BOSS covariance matrix by dividing SDSS-II covariance matrix by 80.
- Add Gaussian noise to simulated flux power spectrum
- Add SiIII, resolution...

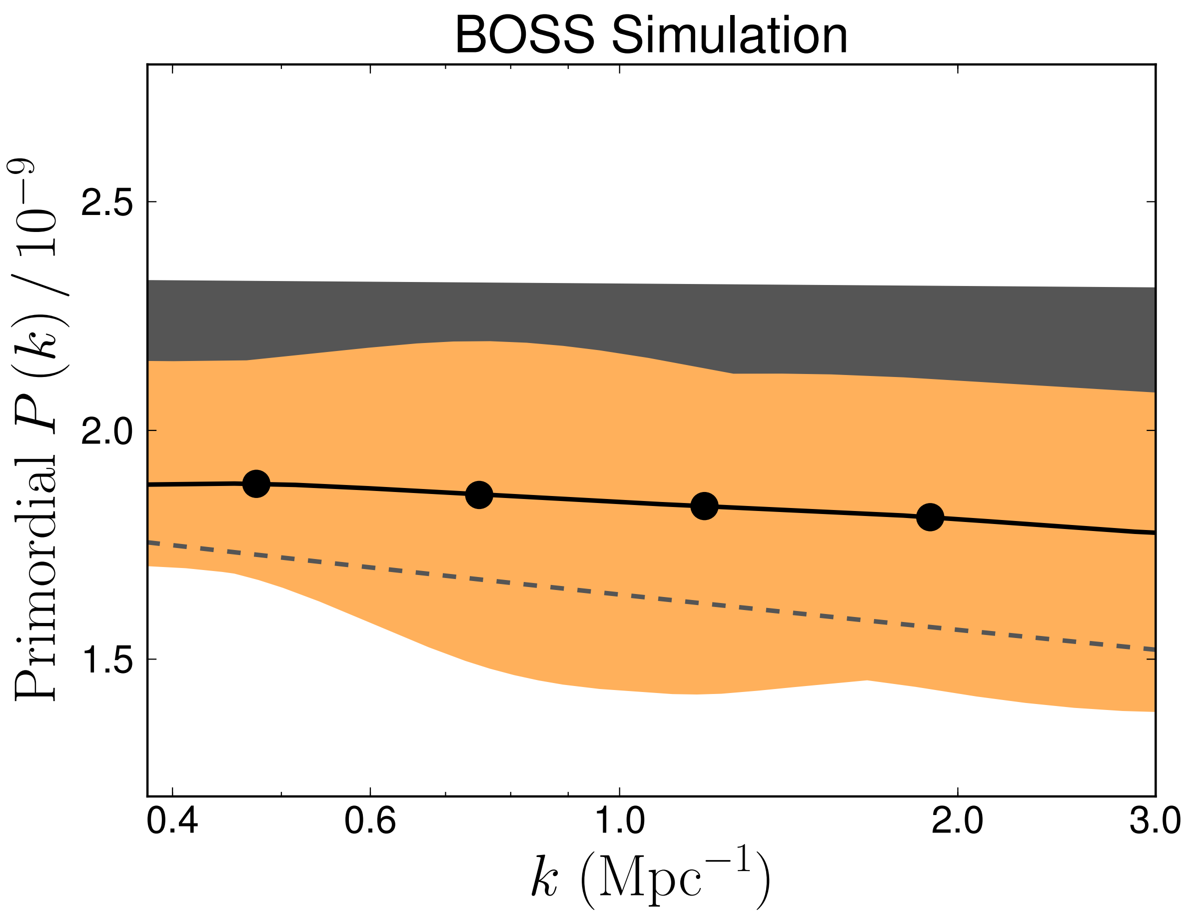
Results: BOSS Simulation



Comparable error bars to the CMB!

Bird et al (2010), arxiv:1010.1519

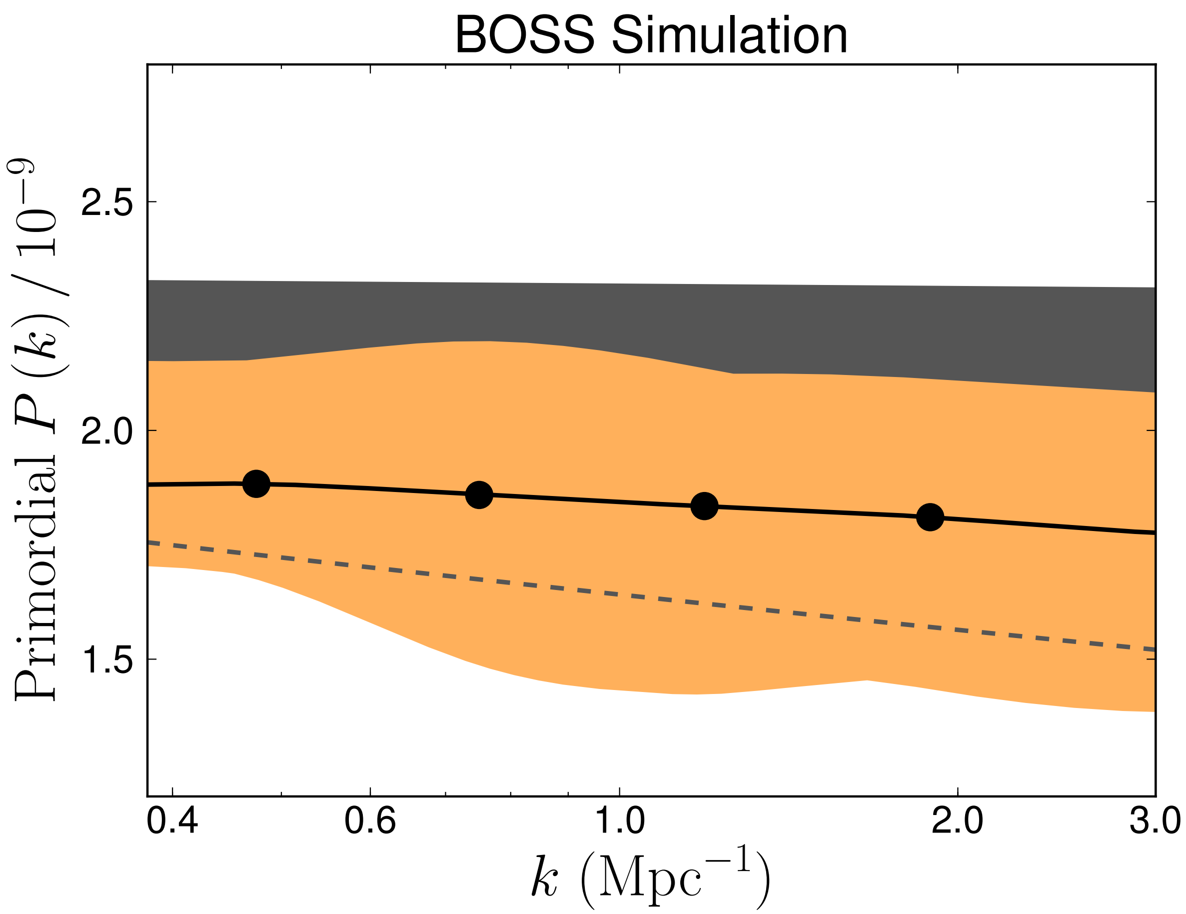
Results: BOSS



- Added near-future priors on thermal parameters
- No significant change

- BOSS already constrains some thermal parameters better than the priors.
- Degeneracies already broken

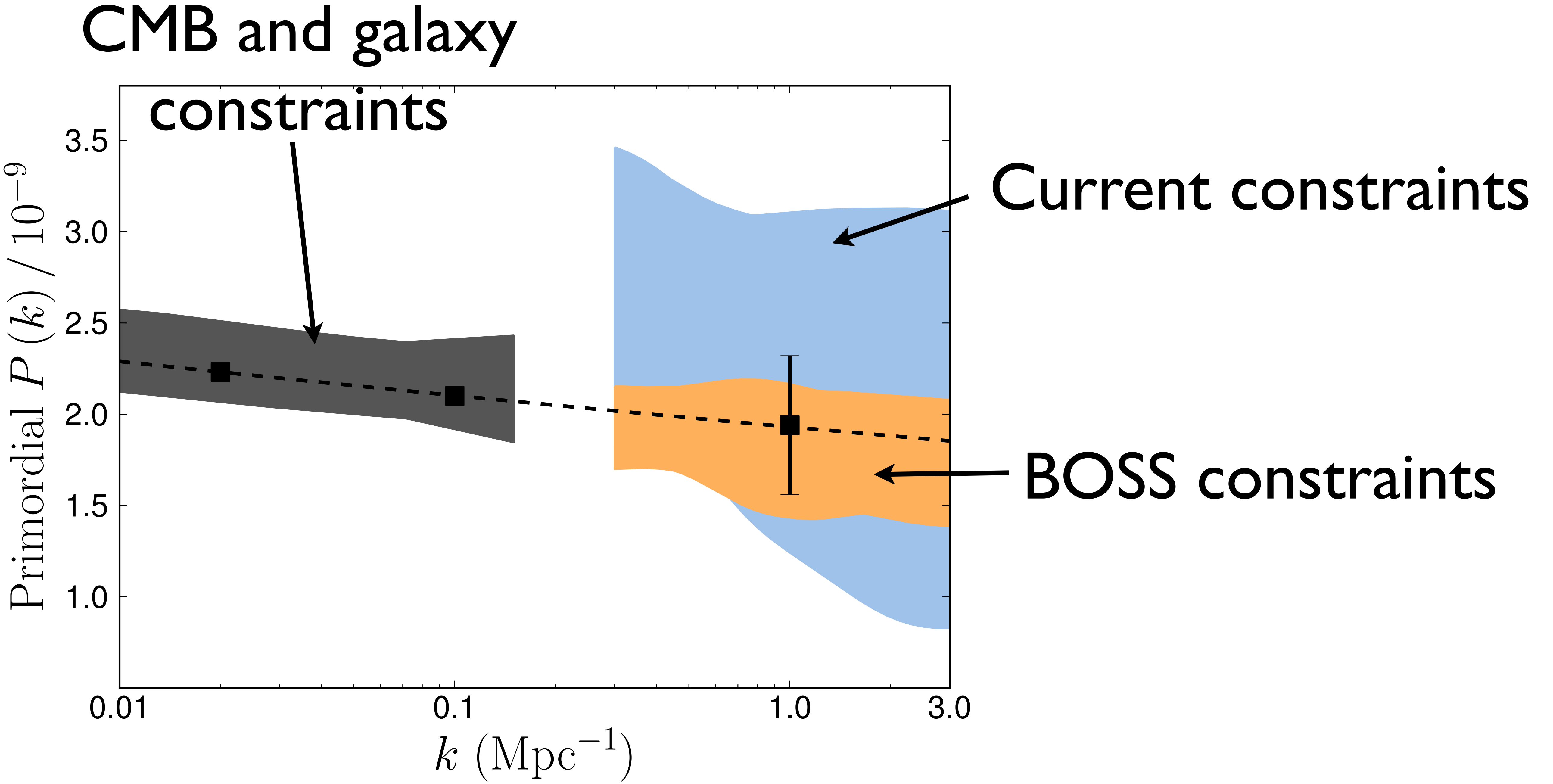
Results: BOSS



- Reproduce earlier results with SDSS covariance matrix
- CV score again constant with penalty
- Fixing thermal params finds preferred prior

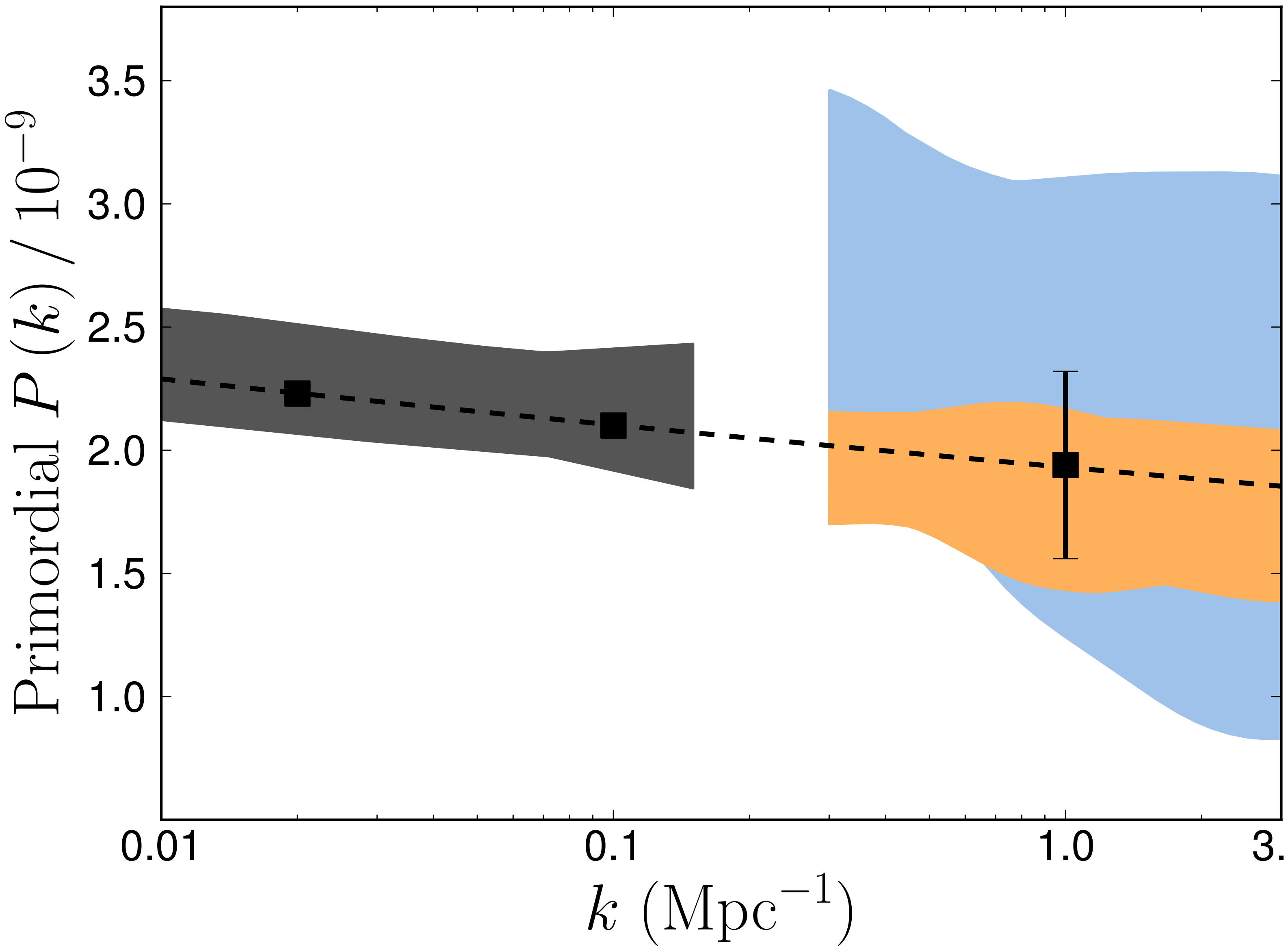
No preferred prior for current data due to systematic and statistical error.

Conclusions



Bird et al (2010), arxiv:1010.1519

Conclusions



Ultimate goal: combine Lyman-α with large-scale datasets