Simulating Massive Neutrinos

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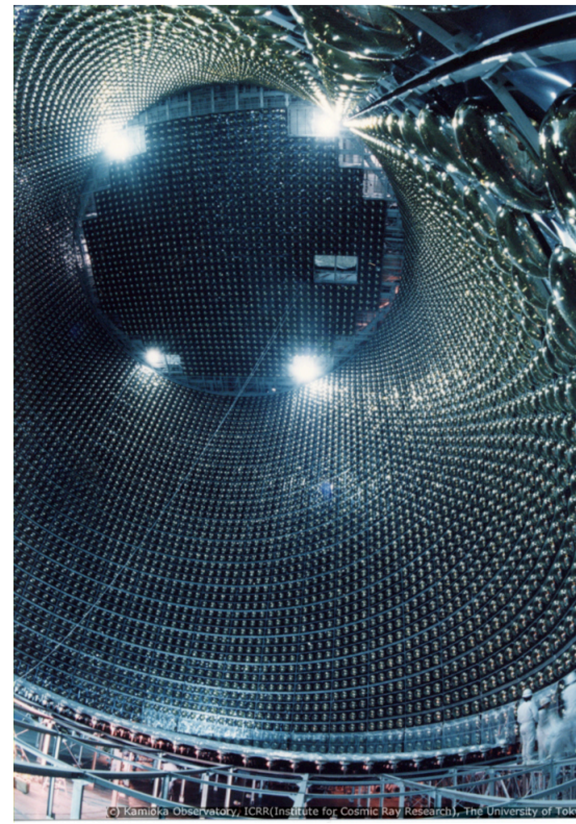
arXiv:1209.0461
https://github.com/sbird/fs-neutrino

Massive Neutrinos

- Super Kamiokande found neutrino oscillations in 1999
- Measures mass splitting between particles
- Neutrinos have mass



Current Lower Limits

- Neutrino oscillations:

$$\Delta m^2_{21} = (7.59 \pm 0.21) \times 10^{-5}\mathrm{eV}^2$$

$$\Delta m^2_{32} = (2.43 \pm 0.13) \times 10^{-3}\mathrm{eV}^2$$

(Particle Data Group)

- At least two massive flavours
- Total mass: $M_\nu > 0.05$ eV

Upper Limits

- Oscillations don't measure total mass
- Tritium beta decay $m_{\nu e} < 2.2$ eV
- Future (KATRIN) $m_{\nu e} < 0.2$ eV
- Challenging as $m_{\nu e} << m_{e,}\ m_{p}$

Gap between 0.05 and 0.6 eV

Massive Neutrinos

Last standard model particles without known mass

"All science is either physics or stamp collecting"

– Ernest Rutherford

The first stamp

The last stamp?

Cosmic Neutrino Background

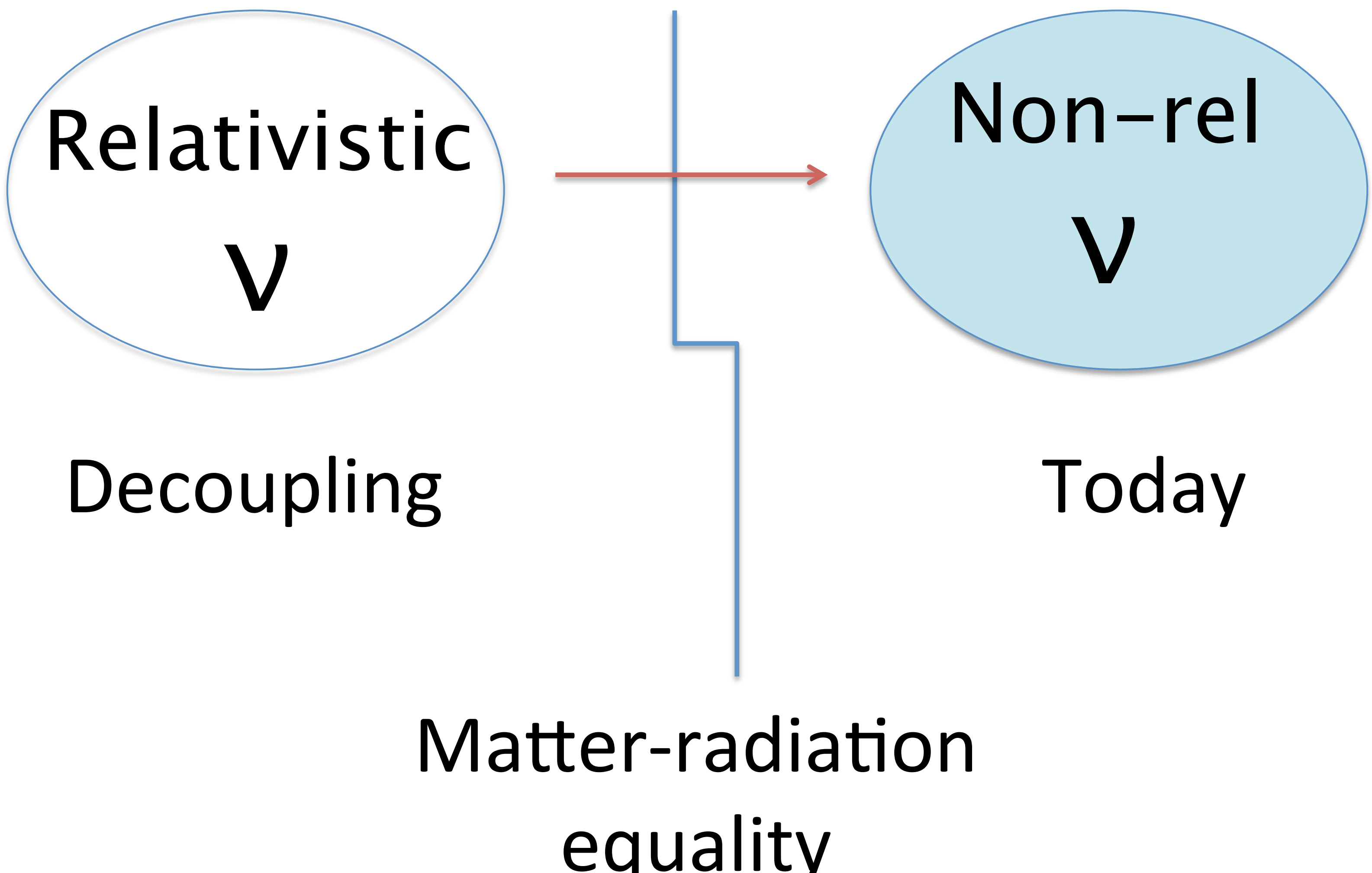
Produced at nucleosynthesis

$$\Omega_\nu = \frac{M_\nu}{93.14h^2}$$

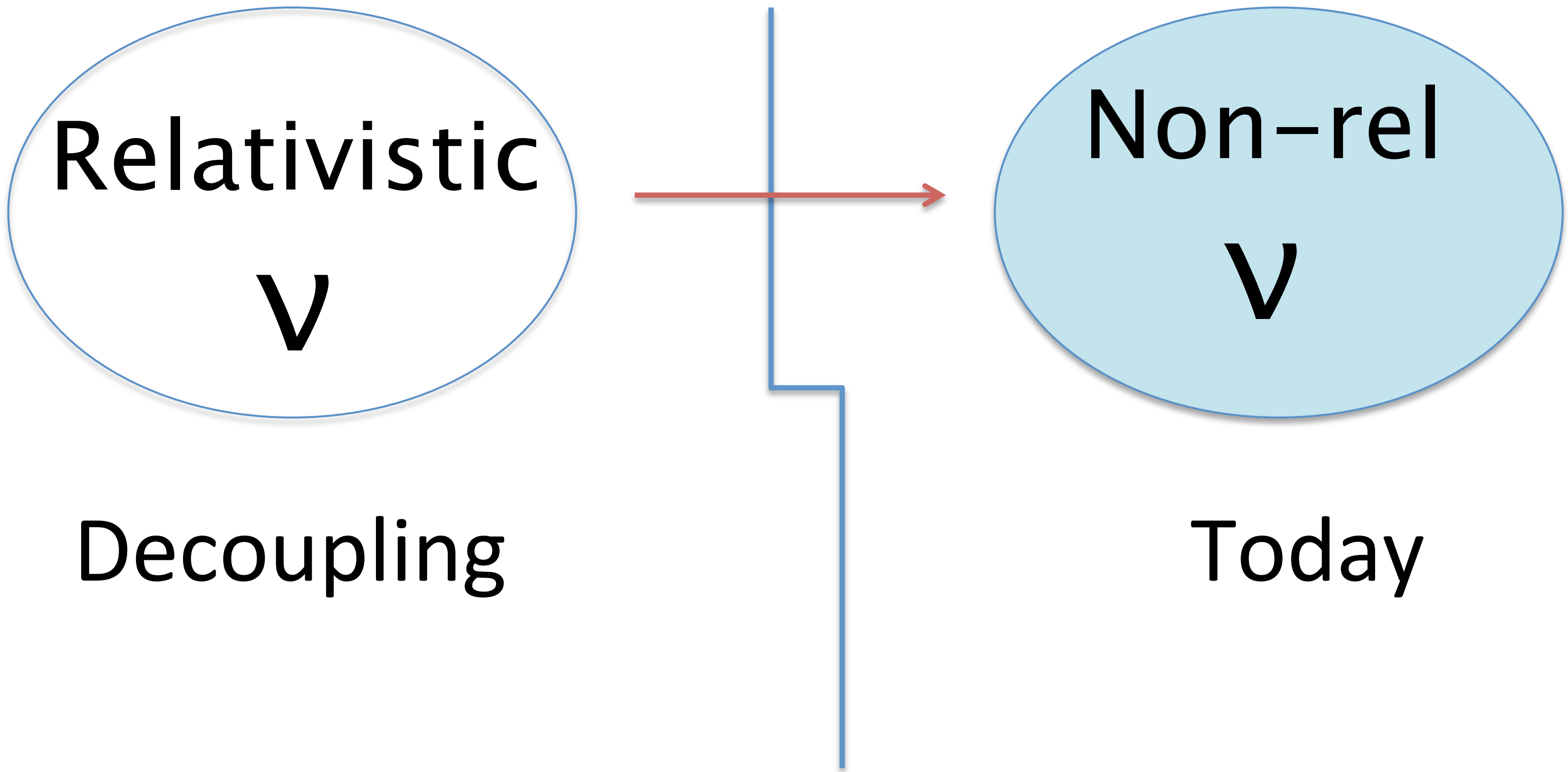
Simplest upper bound by imposing

$$\Omega_\nu \leq \Omega_0$$

Background Effect

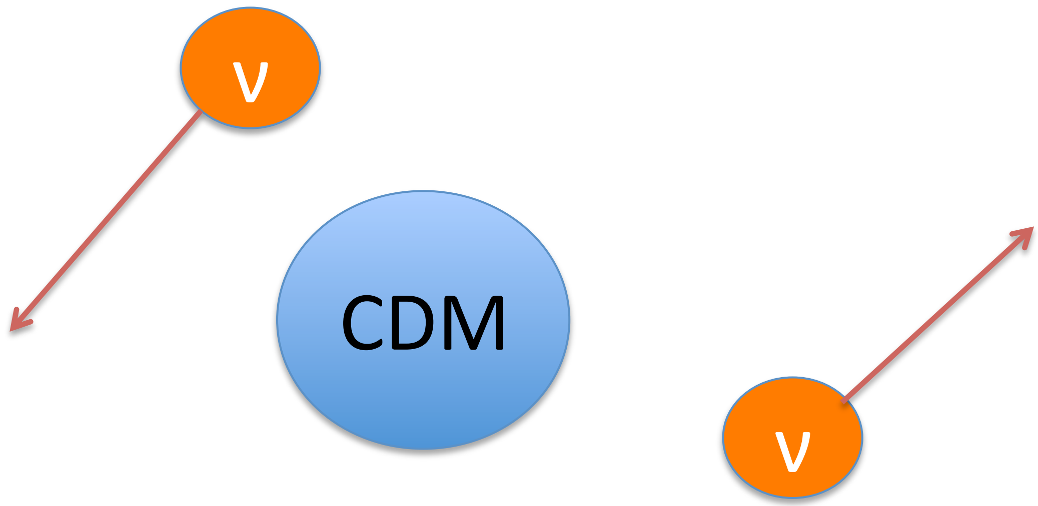


Background Effect



$$M_{\nu} < 1.3\, eV$$ (WMAP)

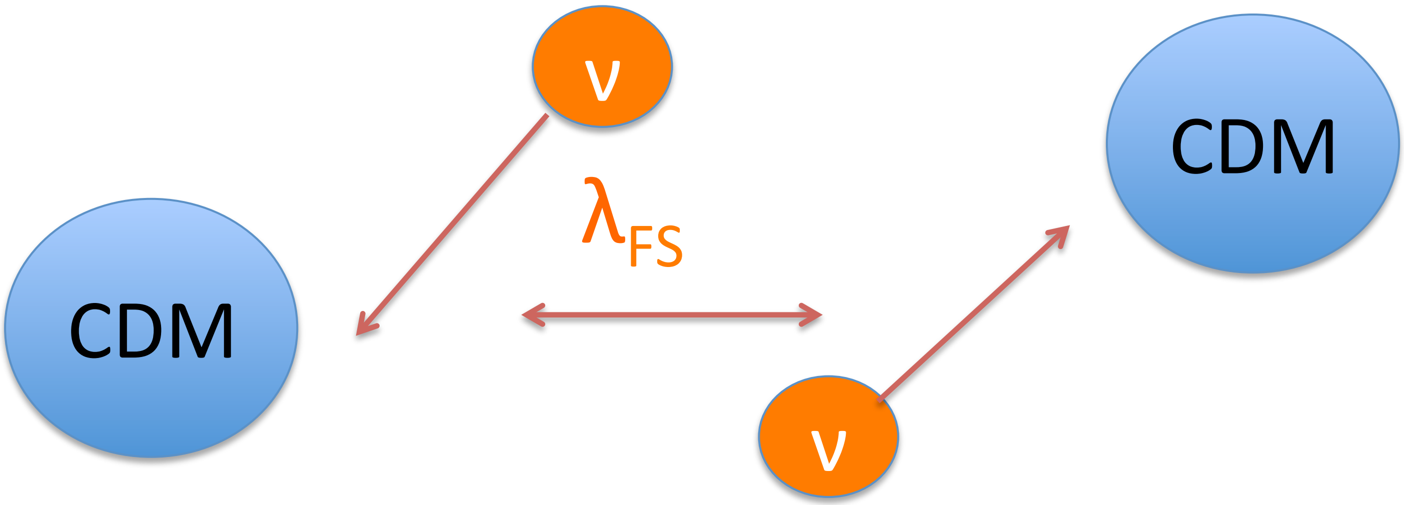
Perturbations



Massive neutrinos are

hot (later warm) dark matter

Perturbations



Thermal pressure prevents clustering below free-streaming length

Suppresses matter power spectrum

Linear Growth

Much of the effect from growth function

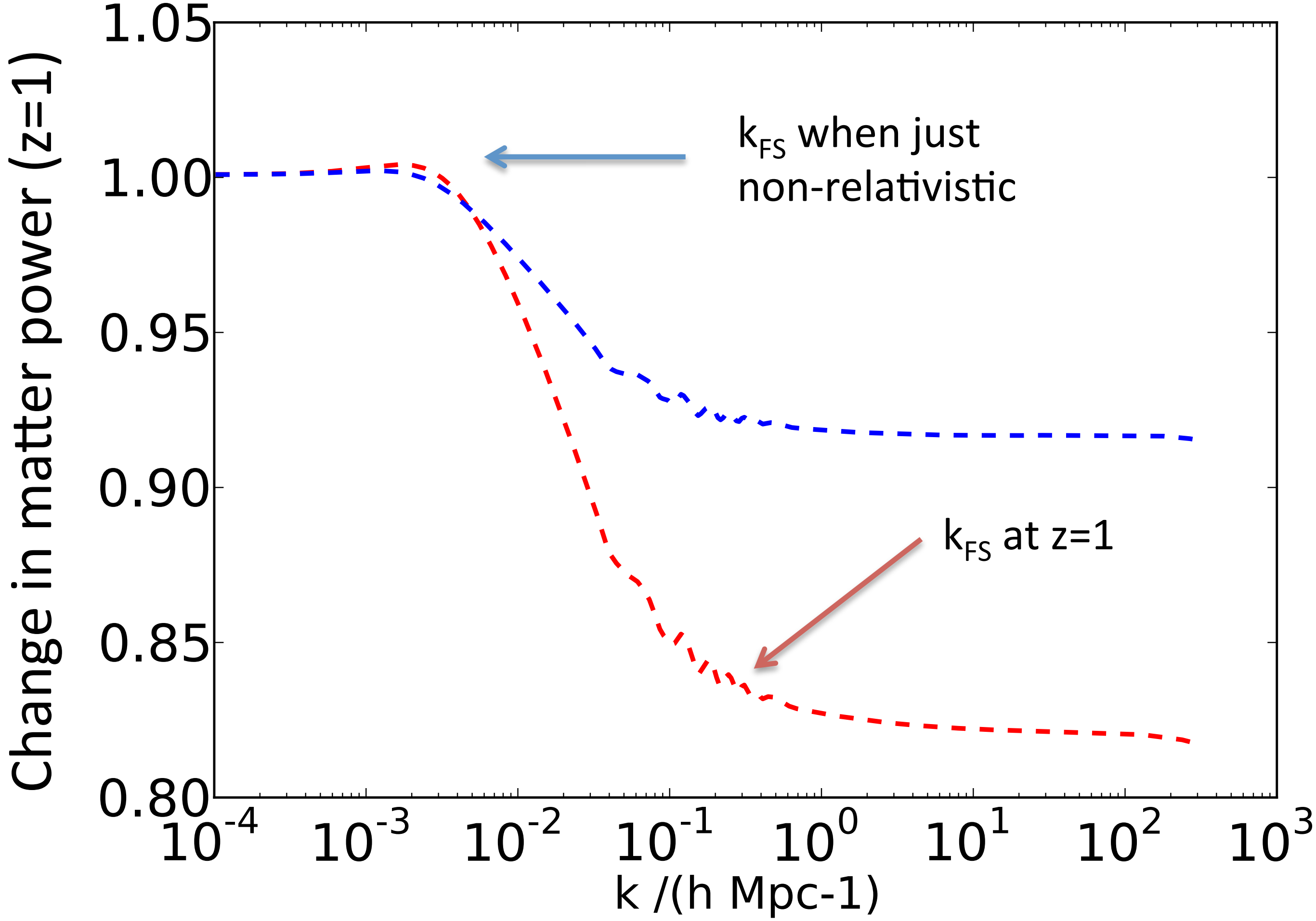
Background expansion and large scales:

$$\Omega_{\rm M} \sim \Omega_{\rm CDM} + \Omega_\nu$$

Small scales:

$$\Omega_{\rm M} \sim \Omega_{\rm CDM}$$

Linear theory: Power Spectrum Step



Proportional to:

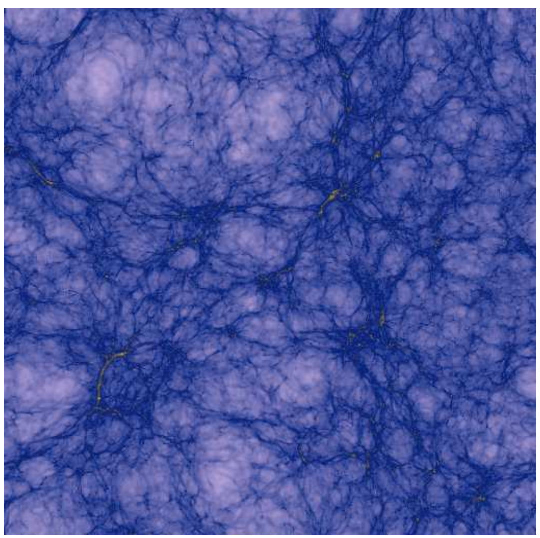
$$f_\nu = \frac{\Omega_\nu}{\Omega_M}$$

Free-streaming scale k_{FS}

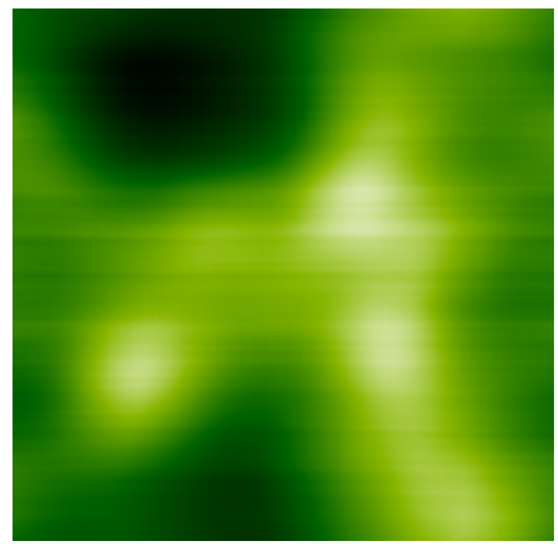
Time-dependent prefactor

Simulating Neutrinos as Particles

Neutrinos are fast-moving dark matter:
Add an extra particle species



Cold Dark Matter



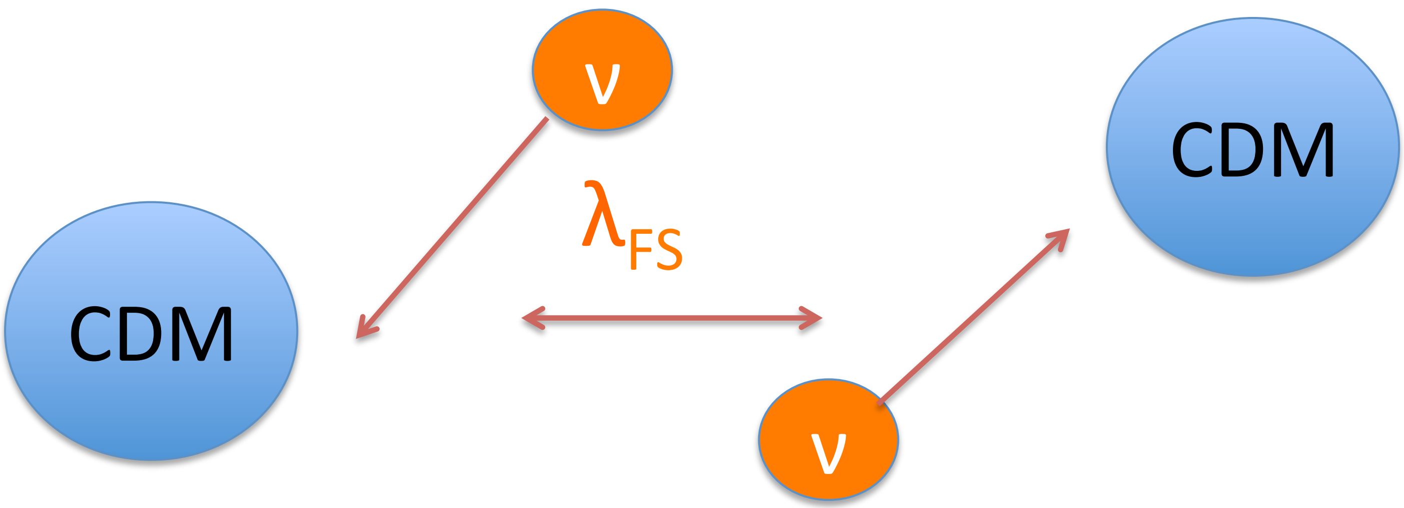
Neutrinos
(hot dark matter)

(Viel 2010)

Particle Neutrinos

Mixed DM model –

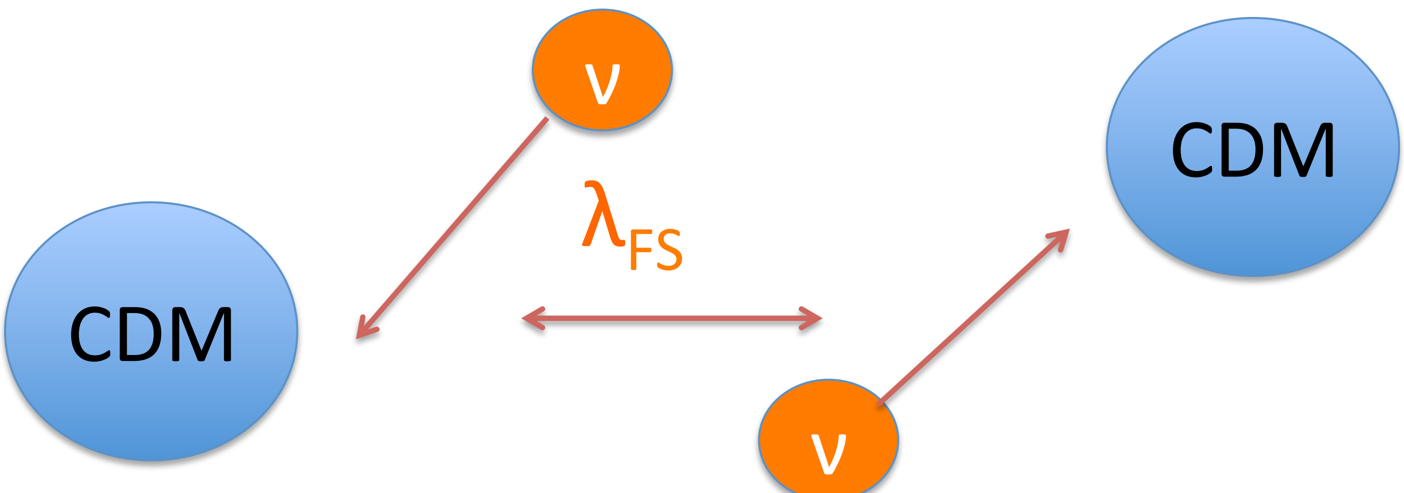
- Neutrinos: light particles, large initial velocity
- CDM: heavy particles, low initial velocity



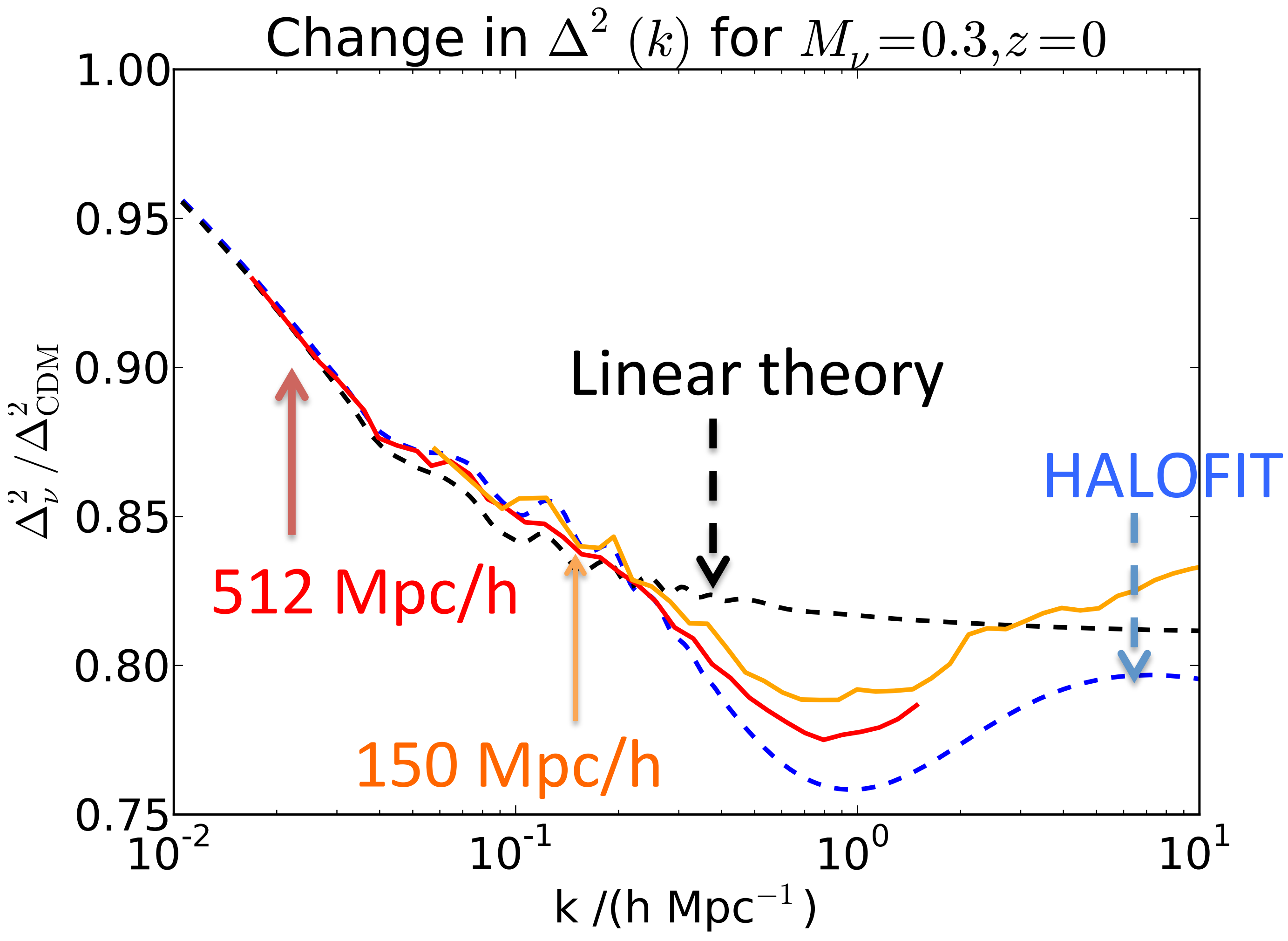
Particle Neutrinos

Mixed DM model –

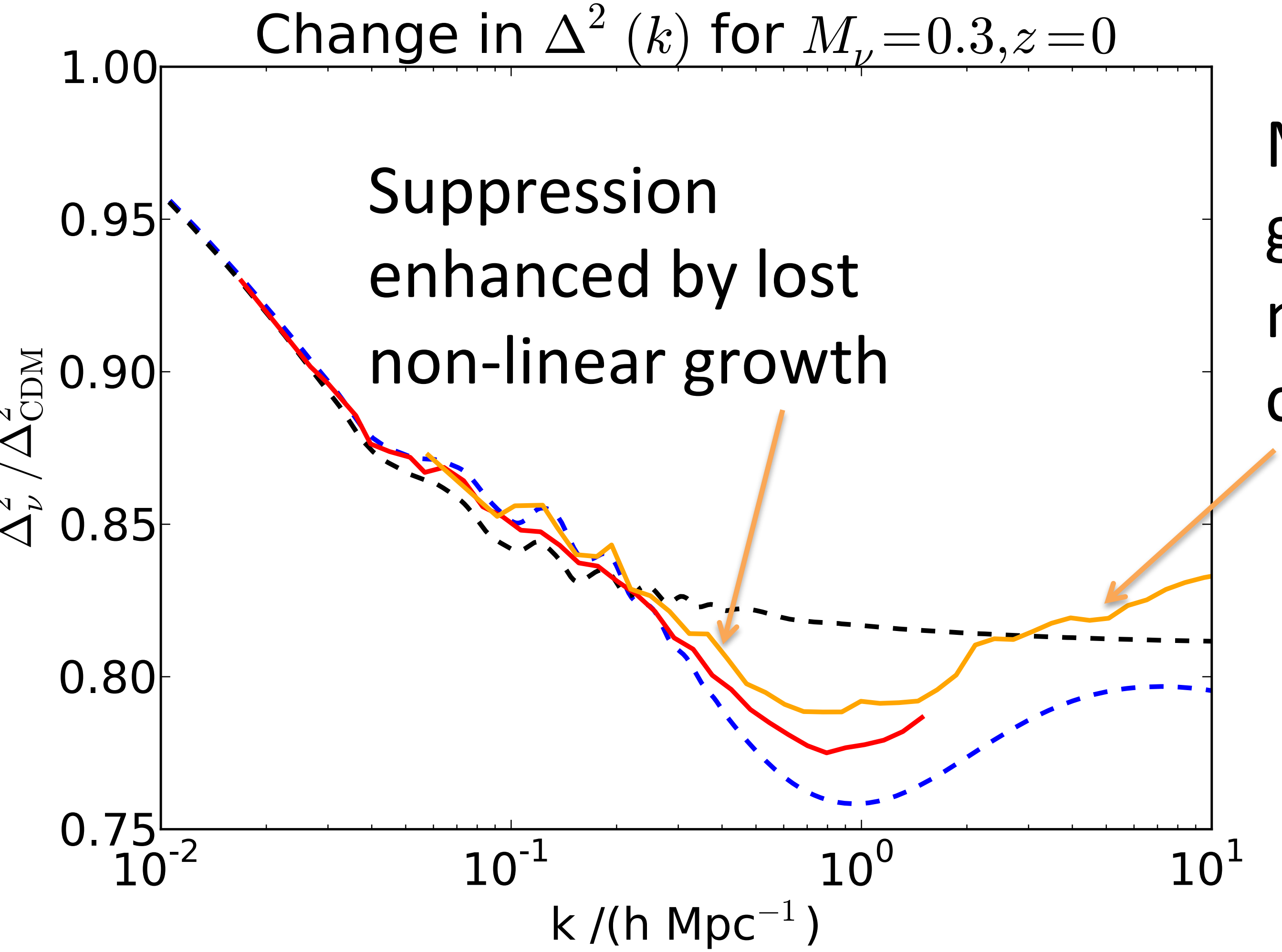
- Large thermal velocity numerically tricky
- Doubles memory consumption



Non-Linear Neutrino Clustering



Non-Linear Neutrino Clustering



More non-linear growth wipes out memory of initial conditions

Maximal Suppression

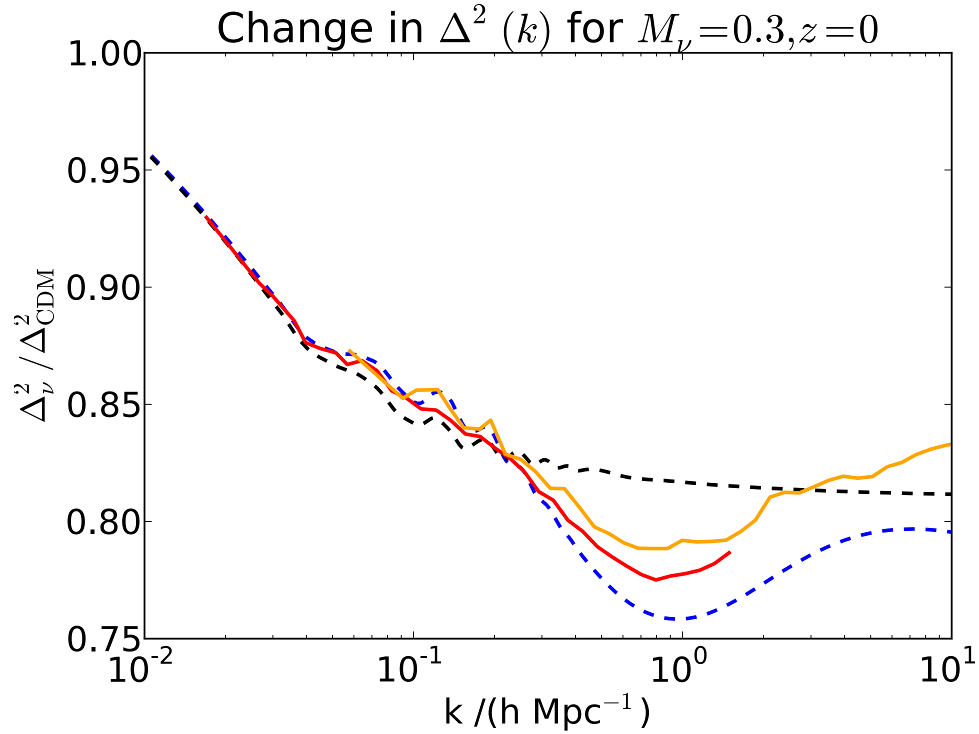
- Linear theory:

$$\frac{\delta P_L}{P_L} \sim -8 f_\nu$$

- Non-linear result:

$$\frac{\delta P_{NL}}{P_{NL}} \sim -10.5 f_\nu$$

Non-Linear Neutrino Clustering



Good news for linear theory forecasts:

- constraints are **tighter**

Bad news for halofit forecasts:

- constraints are **weaker**

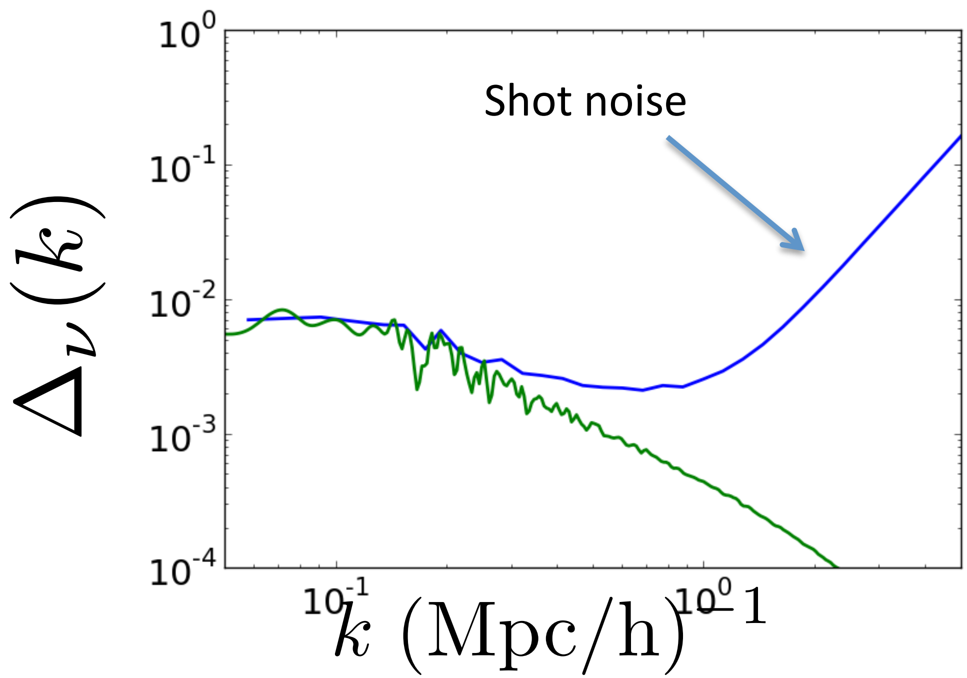
Particle Neutrinos

- Some problems with this method
  - Shot noise
  - Early-time relativistic effects
  - No neutrino hierarchy

- Hard and expensive to perform

Shot Noise

- Neutrinos cluster very weakly, so 1/N shot noise can dominate at high z or low mass
- Can mitigate with more particles



Early-Time Relativistic Effects

Particles assume:

$$\Omega_\nu(a) = \Omega_\nu^0 a^{-3}$$

But at early times:

$$\Omega_\nu(a) = \Omega_\nu^0 a^{-4}$$

Cosmologically interesting masses have

$$\Omega_\nu(a = 0.01) > \Omega_\nu^0 a^{-3}$$

Picking Initial Redshift

Avoiding transients → EARLY

Non-relativistic matter density → LATE

- Initial redshift 24 for $M_\nu = 0.15$ eV
- Up to redshift 99 for $M_\nu = 0.6$ eV

Neutrino Hierarchy

- Neutrinos are three species with different masses
- Matters for M < 0.15
- Can include 2-3 particle species

Particle Neutrinos

- Some problems with this method
  - Shot noise
  - Early-time relativistic effects
  - Neglects neutrino hierarchy

- These problems are easy analytically
- Look for analytic method

Linear Neutrinos, Non-Linear CDM (Brandbyge & Hannestad)

- Neutrinos free-stream on non-linear scales
- Assume neutrino power is as in linear theory

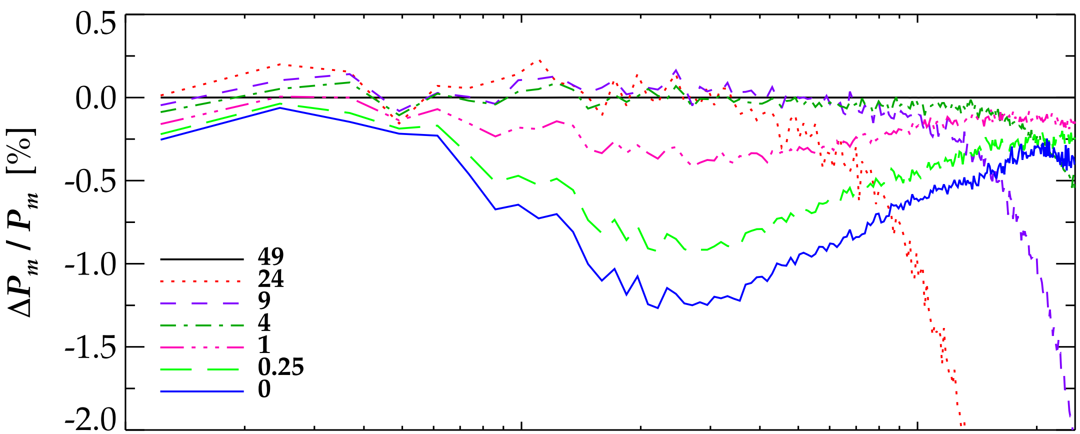
$$P_{NL}^2(k) = f_{CDM} P_{NL,CDM}^2 + f_\nu P_{L,\nu}^2$$

From N-body simulation

From CAMB

Linear Neutrinos, Non-Linear CDM (Brandbyge & Hannestad 2009)

- How good is this approximation?
- Fine for M= 0.15
- 1% error in P(k) for M = 0.6:



Linear Neutrinos, Non-Linear CDM (Brandbyge & Hannestad)

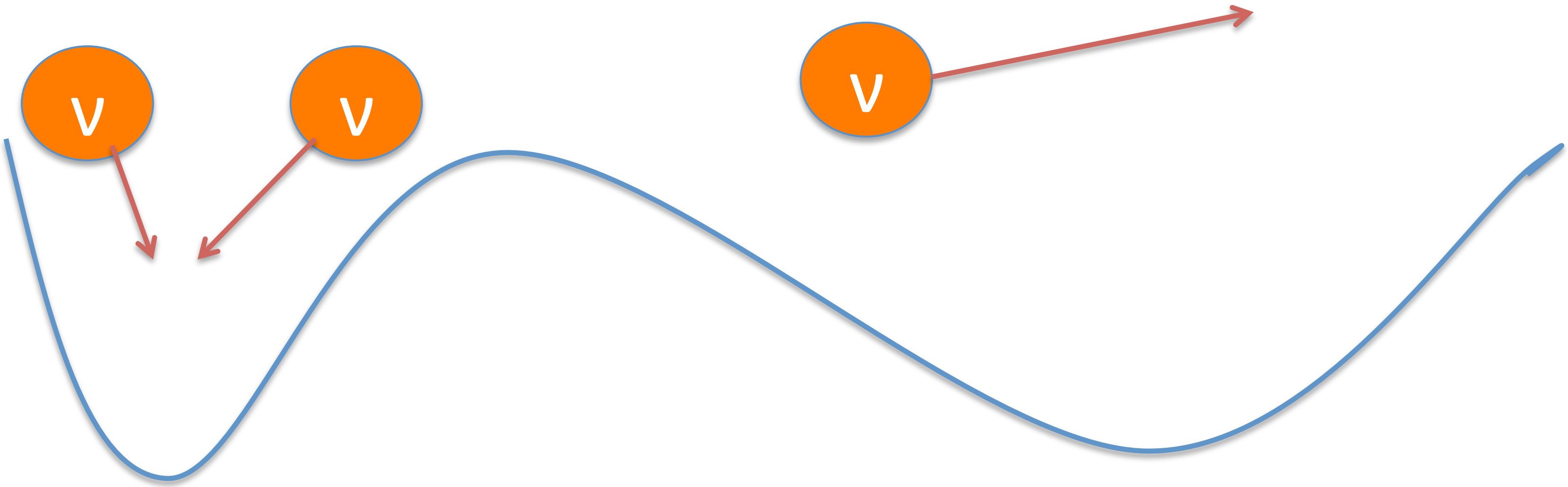
- Free to compute

- Avoids shot noise
- Manifestly correct in linear regime

- Neglects effects of non-linear DM growth on neutrino distribution

Why?

CDM clustering sources neutrino clustering



Deeper wells due to non-linear DM growth:
Neutrinos cluster more than linear theory

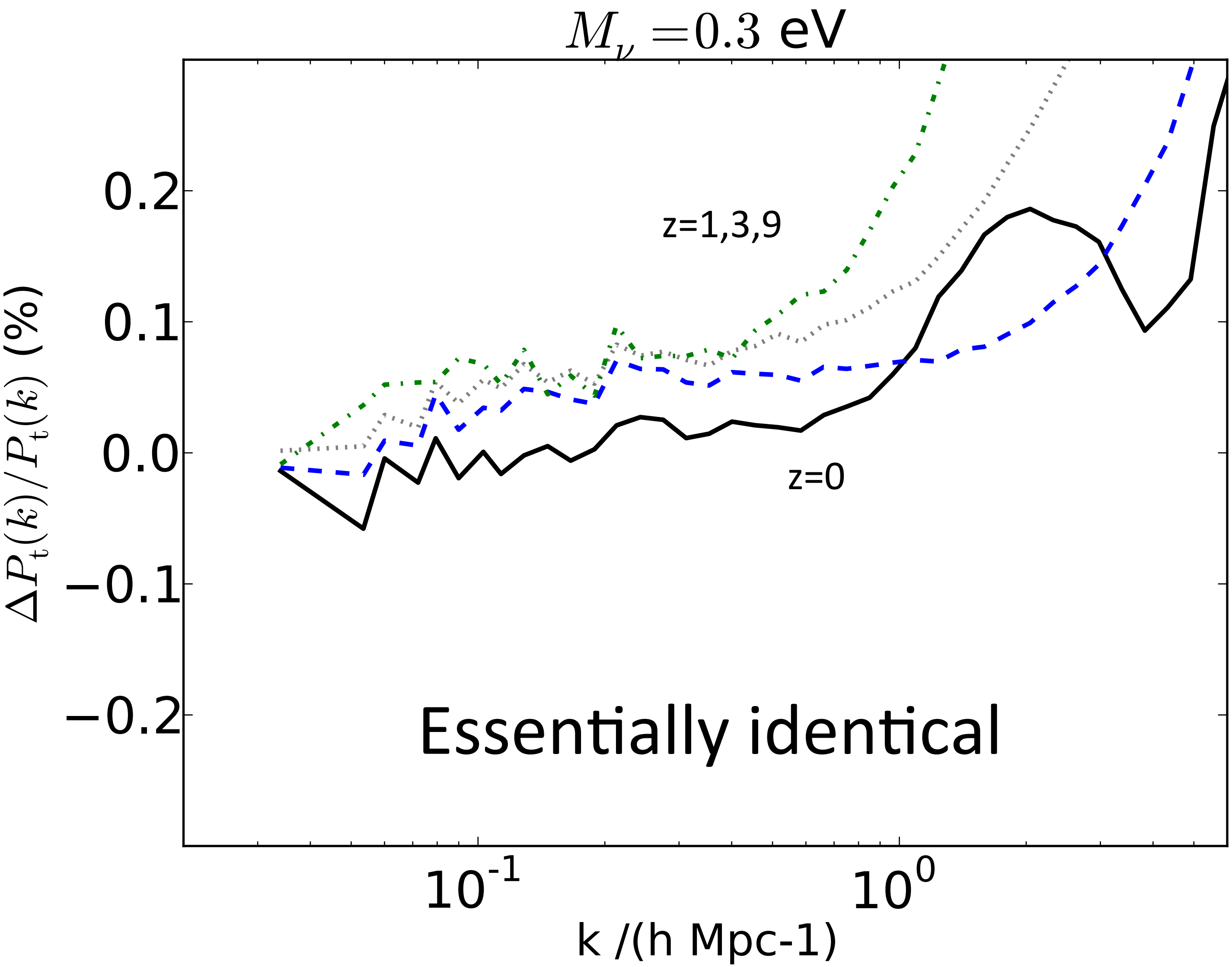
Our Improvement

- Calculate CDM power spectrum every timestep

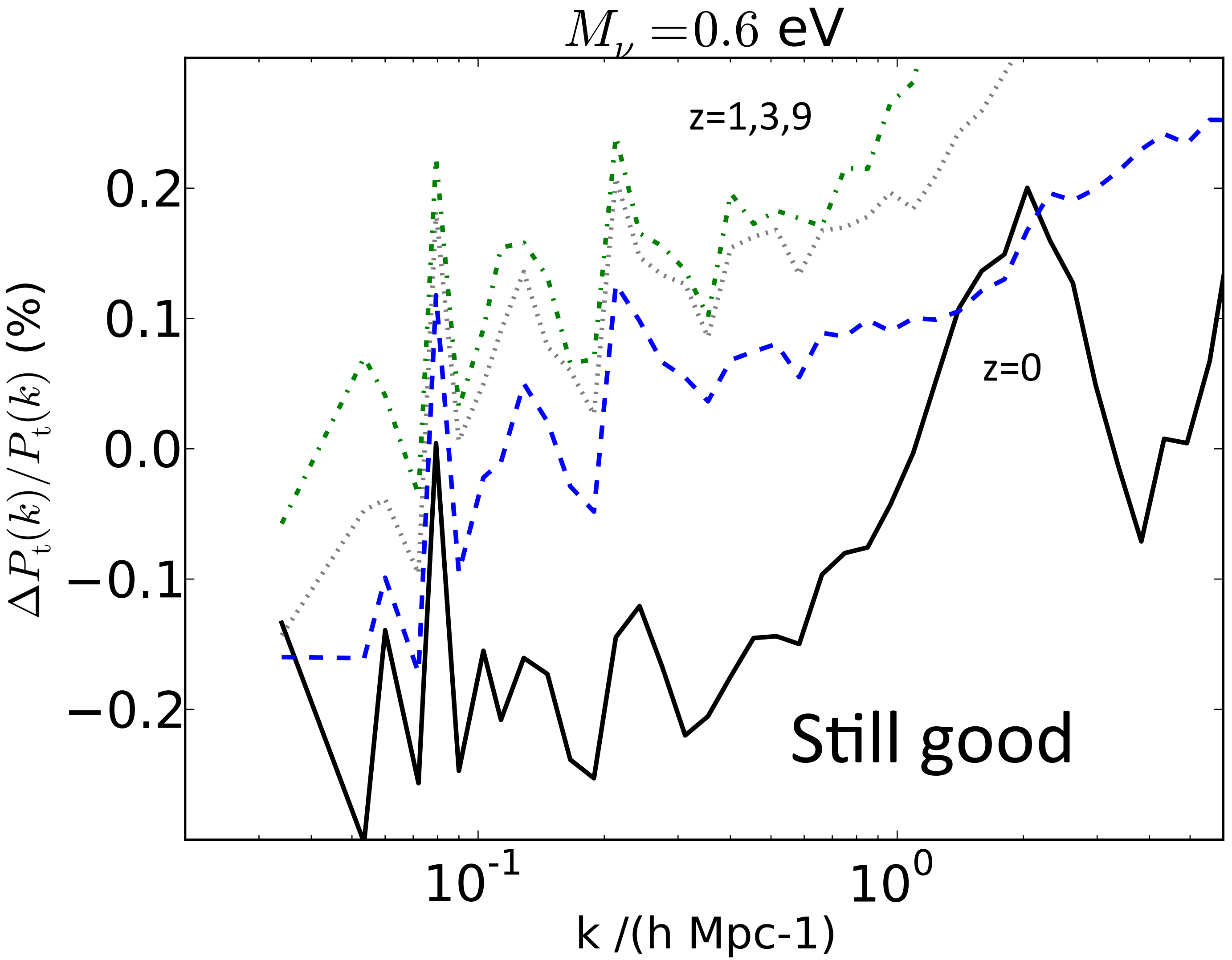
- Calculate using linear theory neutrino power spectrum sourced by the non-linear CDM potential

- Add this to the CDM

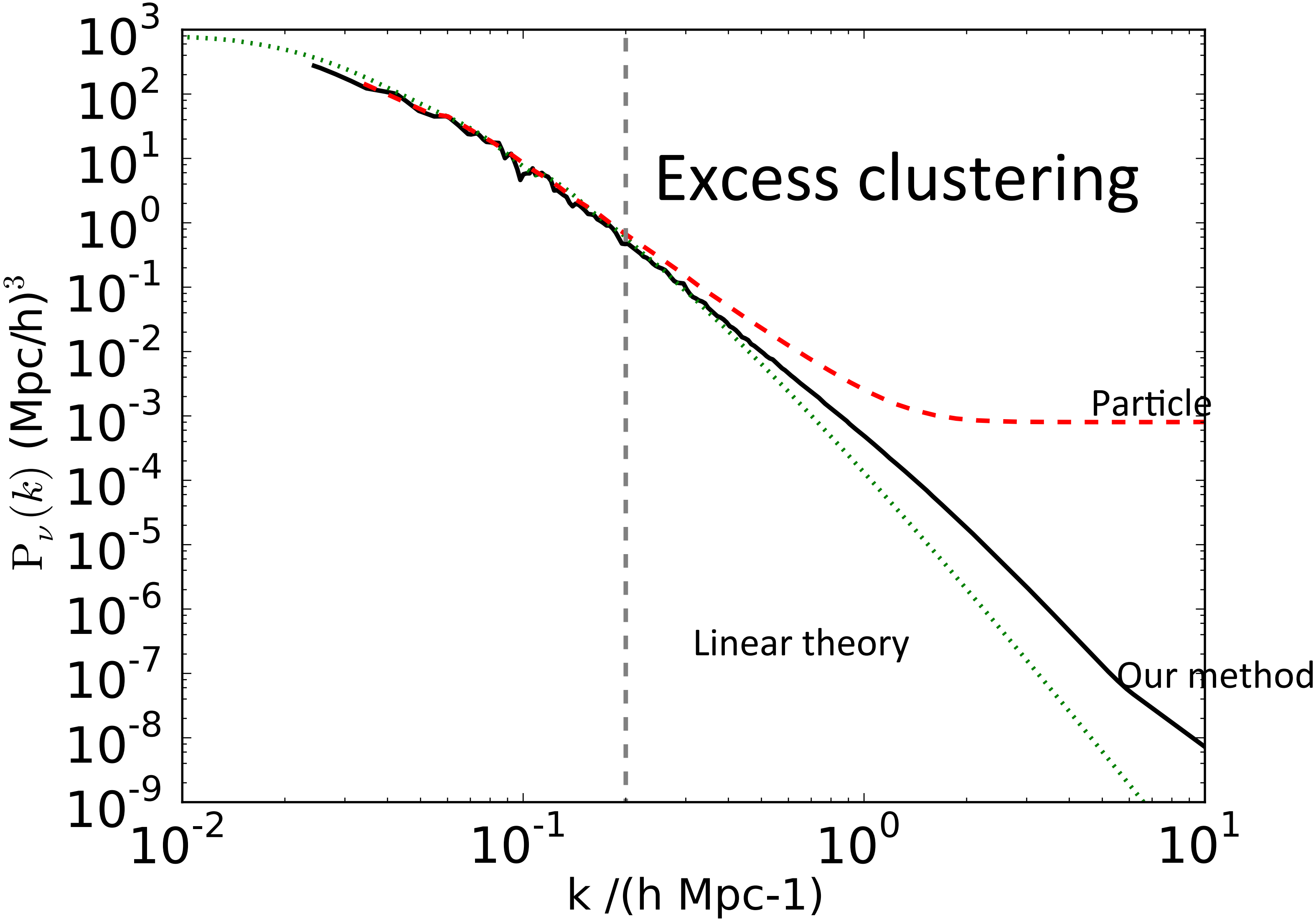
Particle vs. Fourier-Space



Particle vs. Fourier-Space



Neutrino power at z=0



Neutrino power spectrum

- Neutrinos in the slow tail of the velocity distribution cluster nonlinearly at $z < 0.5$
- Extra clustering in regions where CDM is clustering vastly more
- Low matter density
- Doesn't measurably affect CDM

Conclusion

- Improved method accurate in non-linear regime
- Free – same cost as simulating CDM
- Easy to add extra physics, eg, neutrino hierarchy.

PUBLIC CODE

https://github.com/sbird/fs-neutrino