Dark Matter Halo Mergers & Smoluchowski's Equation

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Overview

- Press-Schechter theory and extensions
  - Progenitor mass functions
  - Merger trees
  - Pr
- Sm
  - W
  - S

- Goal is full-featured merger tree factory
  - Masses
  - Concentrations
  - Spins
  - Shapes
  - Dynamical Evolution

- Simple analytic forms
  - Fits
  - Mass function evolution

What is Press-Schechter?

- An analytical model for the distribution of dark matter halo masses in hierarchical Universes
- Based on the statistics of peaks in Gaussian fields
- First derived by Press & Schechter (1974), extended in early 90's by Lacey & Cole, BCEK, Bower etc.
- Predicts halo mass function:

$$n(M;t)=\sqrt{\frac{2}{\pi}}\frac{\rho_b}{M^2}\frac{\delta_c(t)}{\sigma(M)}\left|\frac{d\ln\sigma}{d\ln M}\right|\exp\left(\frac{-\delta_c^2(t)}{2\sigma^2(M)}\right)$$

- Rate of change of mass function easily determined

$$\dot{n}(M;t)=n(M;t)\left|\frac{\dot{\delta}_c}{\delta_c}\right|\left(\frac{\delta_c^2(t)}{\sigma^2(M)}-1\right)$$

Extended Press-Schechter (ePS)

- Prediction for halo merger rate: Γ = $n(M_1)n(M_2)Q_{ePS}(M_1,M_2;t)$

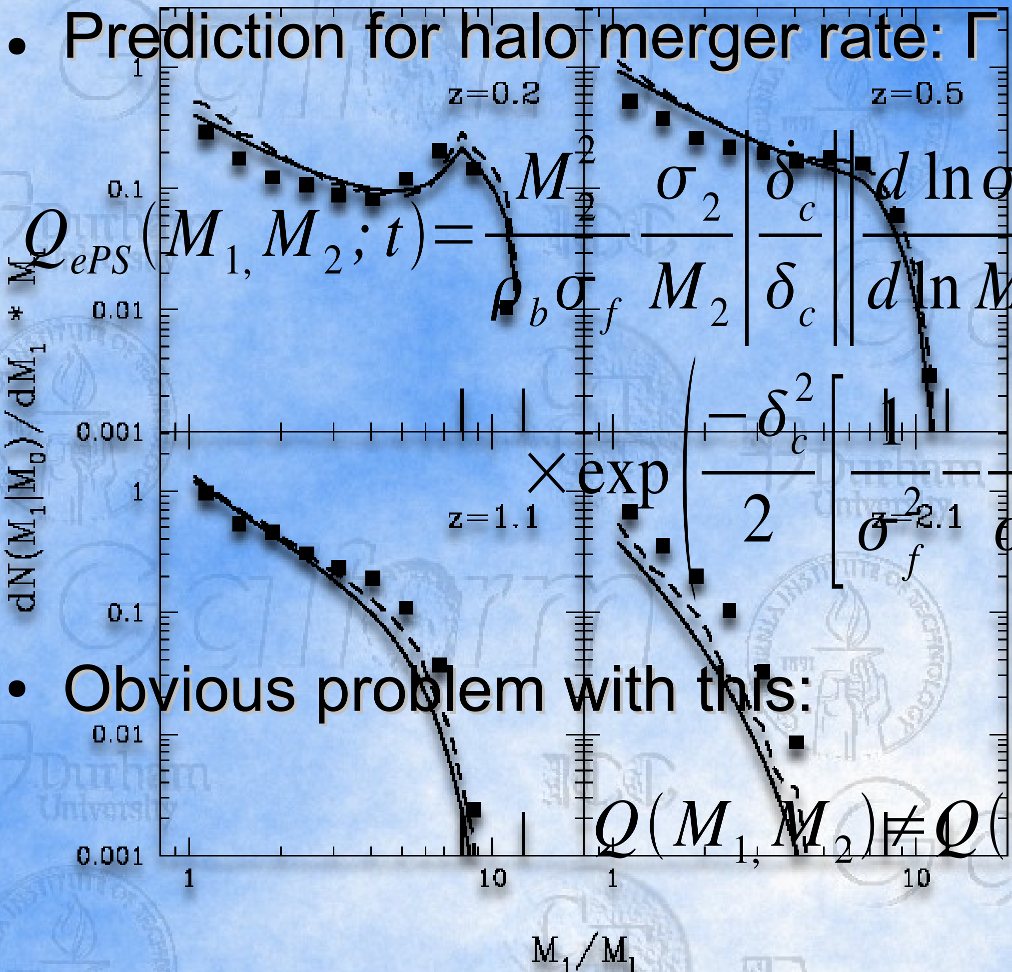
$$Q_{ePS}(M_1,M_2;t)=\frac{M_2^2}{\rho_b\sigma_f}\frac{\sigma_2}{M_2}\left|\frac{\dot{\delta}_c}{\delta_c}\right|\left|\frac{d\ln\sigma_f}{d\ln M_f}\right|\left|\frac{d\ln\sigma_2}{d\ln M_2}\right|^{-1}\frac{1}{(1-\sigma_f^2/\sigma_1^2)^{3/2}}$$

$$\times\exp\left(\frac{-\delta_c^2}{2}\left[\frac{1}{\sigma_f^2}-\frac{1}{\sigma_2^2}-\frac{1}{\sigma_1^2}\right]\right)$$

- Obvious problem with this:

$$Q(M_1,M_2)\neq Q(M_2,M_1)$$

- Theory can be manipulated to predict progenitor mass functions
- Agree reasonably well with N-body simulations
- Leads to predictions for merger rates of halos...



Somerville et al. (2001)

Merger Trees

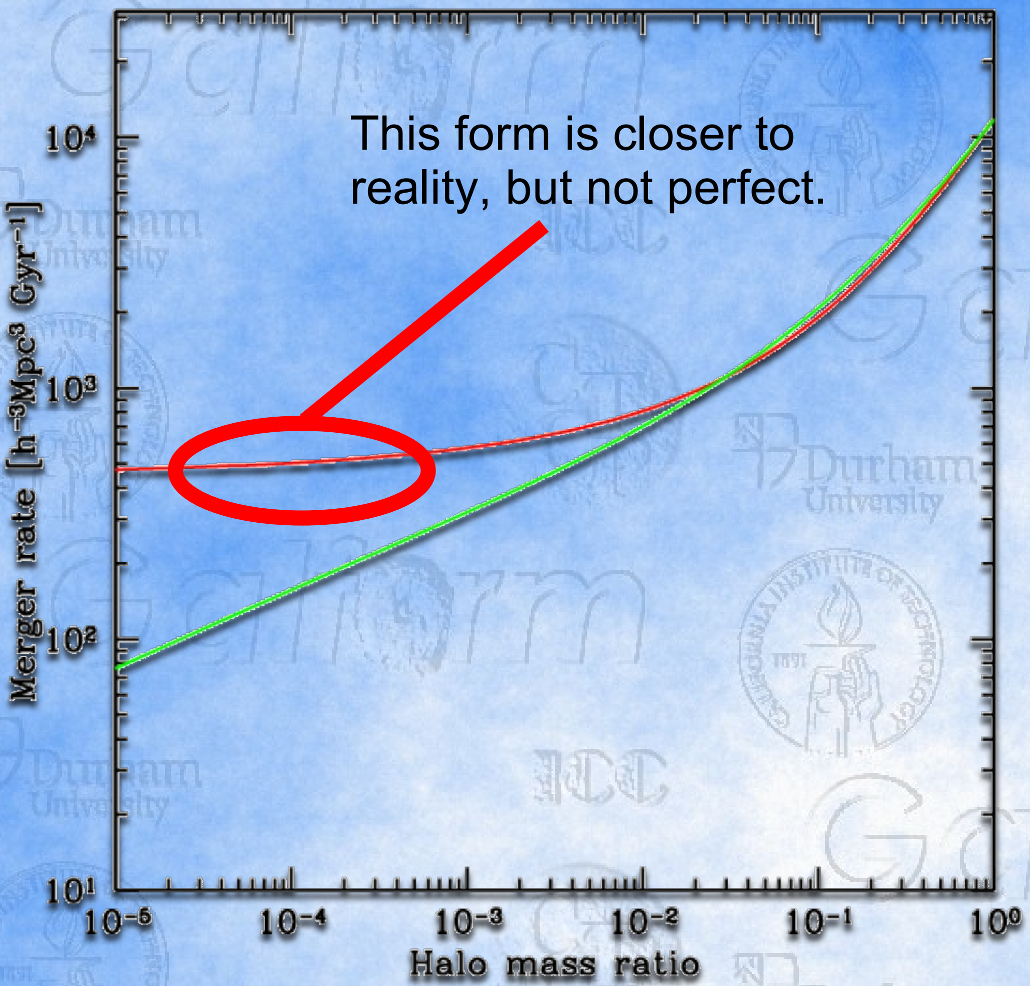
- Halos form through merging of sub-units
- Process of merging described by merger trees
- Extended Press-Schechter can be used to construct these statistically

Uses of Press-Schechter

- Press-Schechter (and in particular ePS) is used extensively in studies of cosmology and galaxy formation
  - Galaxy evolution
  - Galaxy morphology
  - AGN activity
  - Lyman-break galaxies
  - Abundance of binary SMBHs...
  - ....and resulting event rate for LISA
  - Formation of the first stars
  - Substructure in Galactic halos
  - Reionization of the Universe
  - Halo angular momenta and concentrations
  - Particle acceleration in clusters
  - Formation redshifts of clusters

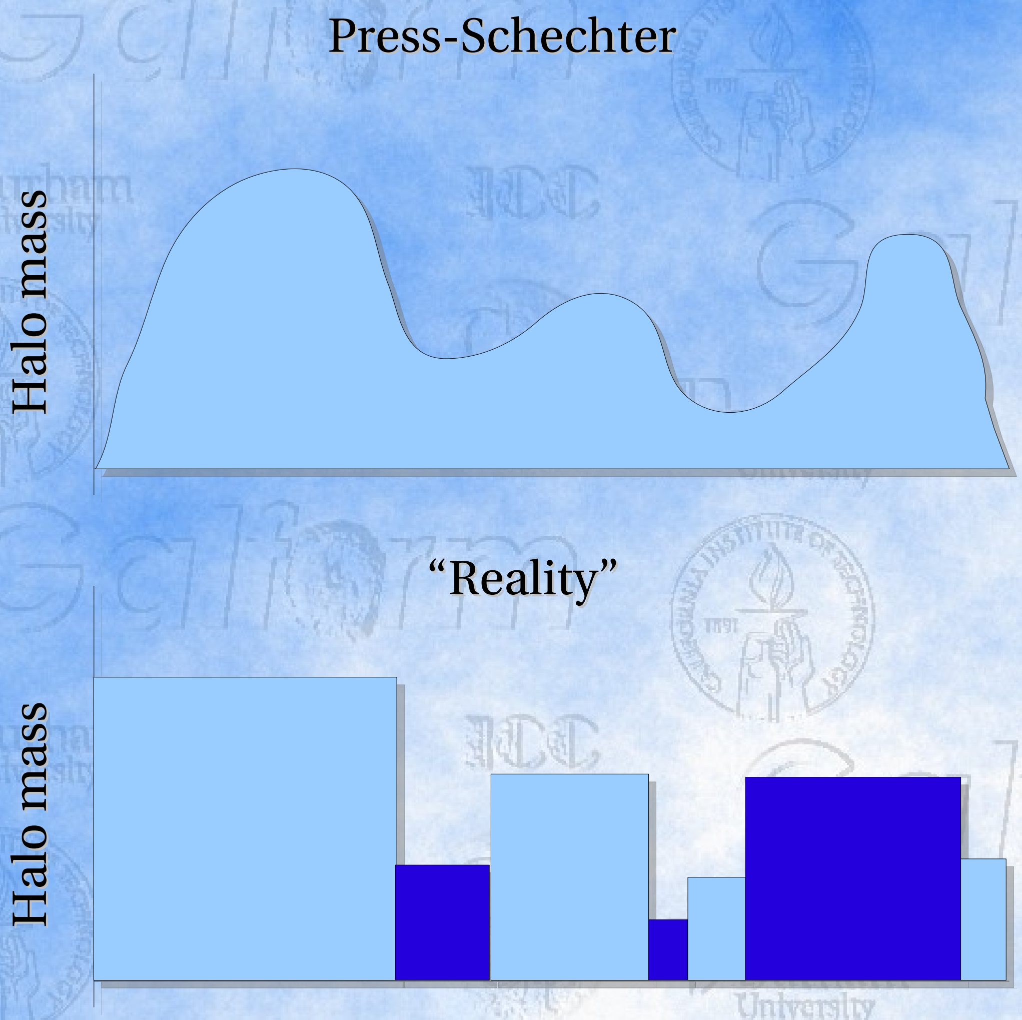
Out of 686,000 refereed articles in the ADS database, Lacey & Cole (1993) is the 142nd most cited

The Problem with ePS



- Problem is that ePS predicts *two different* merger rates!
- Rates are similar for equal mass mergers, but very different for larger mass ratios
- Can affect many calculations using ePS

Why is There a Problem?



- Press-Schechter doesn't deal with discrete halos
- Halo mass is a continuously varying function of position
- So, it doesn't really incorporate halo mergers at all
- (Also, filtering is not spatially localized)

Smoluchowski's Coagulation Equation

- Smoluc… …overning coag…
- Form… …archical form…
- Hal…
- …al…
- Net r…

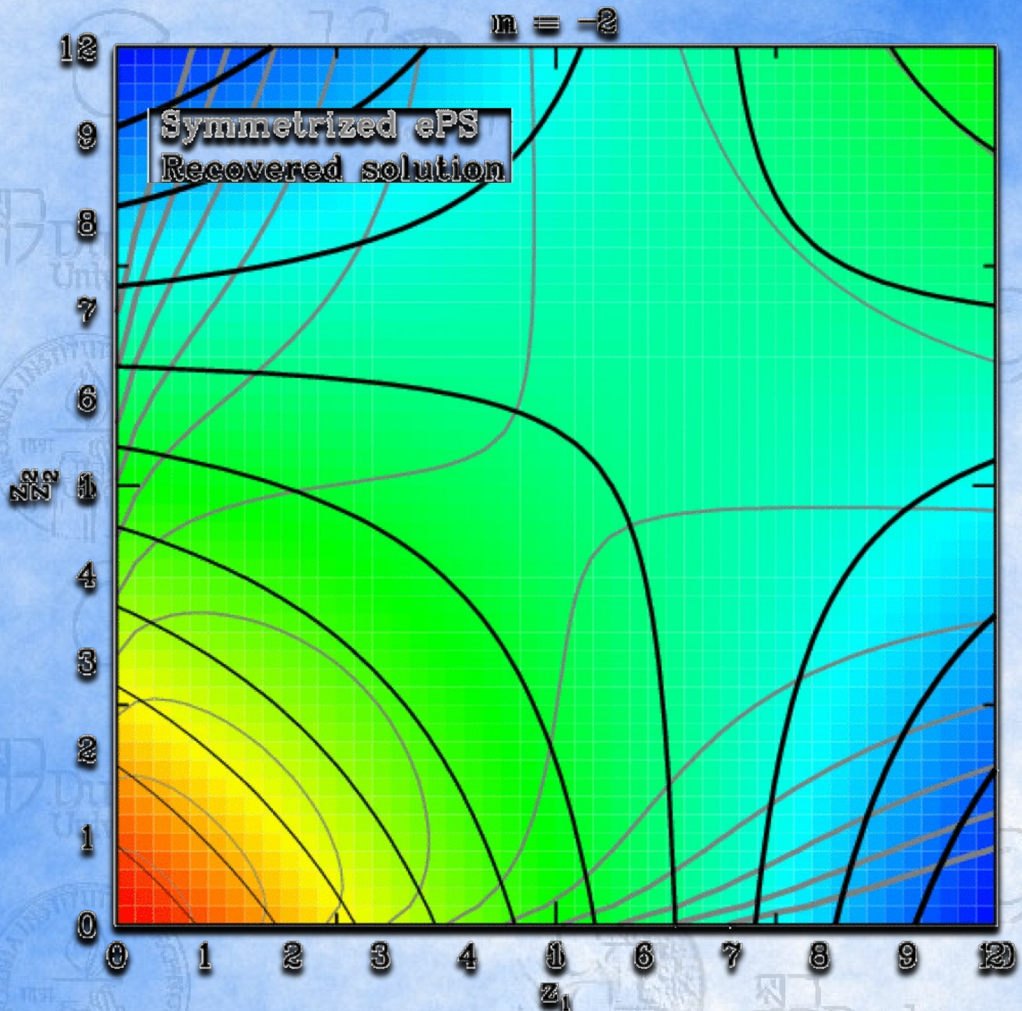
Caveats

- Assumes mass is conserved in mergers
  - *Not true in N-body simulations*
- Assumes binary mergers
  - *May be true......*
- Ignores fragmentation
  - *Happens in N-body simulations*
    - Shoot-through
    - Ejection

$$\ldots\, dM' \qquad dt \qquad 2$$

$$-\int_0^\infty Q(M, M')\, n(M)\, n(M')\, dM'$$

Results: Merger Rate Functions



$$P(k) \propto k^n$$

- Merger rate functions are symmetric and smooth
- Similar form to symmetrized ePS merger rates
- Can be parameterized by a simple fitting formula

CDM Power Spectra..............

- Apply same techniques to CDM power spectra
- Used same regularization conditions
- Improved solver with improved dynamic range
- Can find solutions to Smoluchowski's equation......
- ....but do not agree with N-body simulations!
- The smoothest solution is not the correct one
- Tried other regularization conditions, but no real success

Modified ePS Merger Rates

- Recently proposed empirical modifications to ePS
- Tuned to match N-body progenitor mass functions
- Parkinson, Cole & Helly (2007) method:

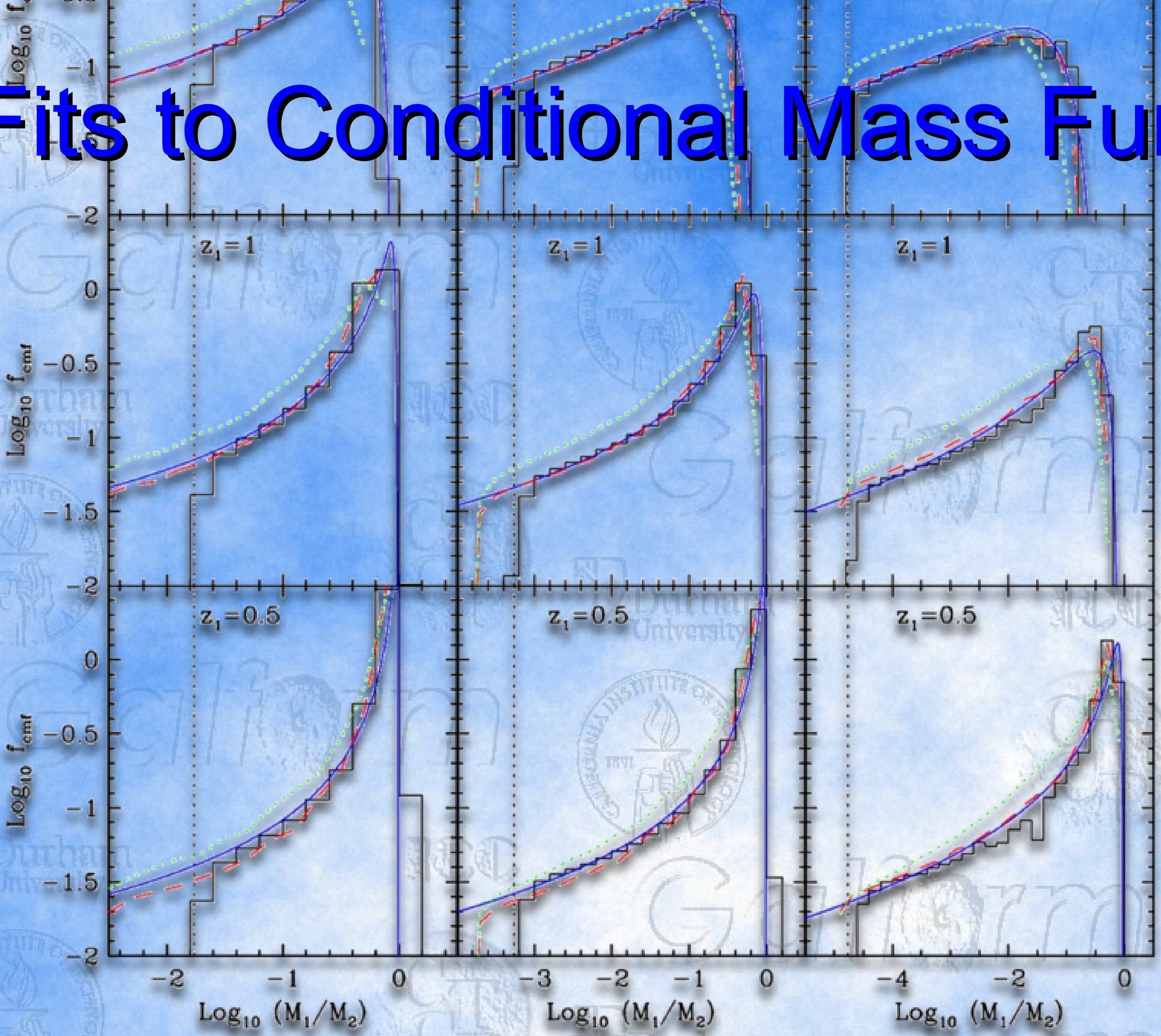
$$\frac{dN}{dM_1} \rightarrow \frac{dN}{dM_1} G\left(\frac{\sigma_1}{\sigma_f}, \frac{\delta_f}{\sigma_f}\right)$$

Progenitor mass function: Number of progenitors of mass M_1 for final halo of mass M_f

$$G\left(\frac{\sigma_1}{\sigma_f}, \frac{\delta_f}{\sigma_f}\right) = G_0 \left(\frac{\sigma_1}{\sigma_f}\right)^{\gamma_1} \left(\frac{\delta_f}{\sigma_f}\right)^{\gamma_2}$$

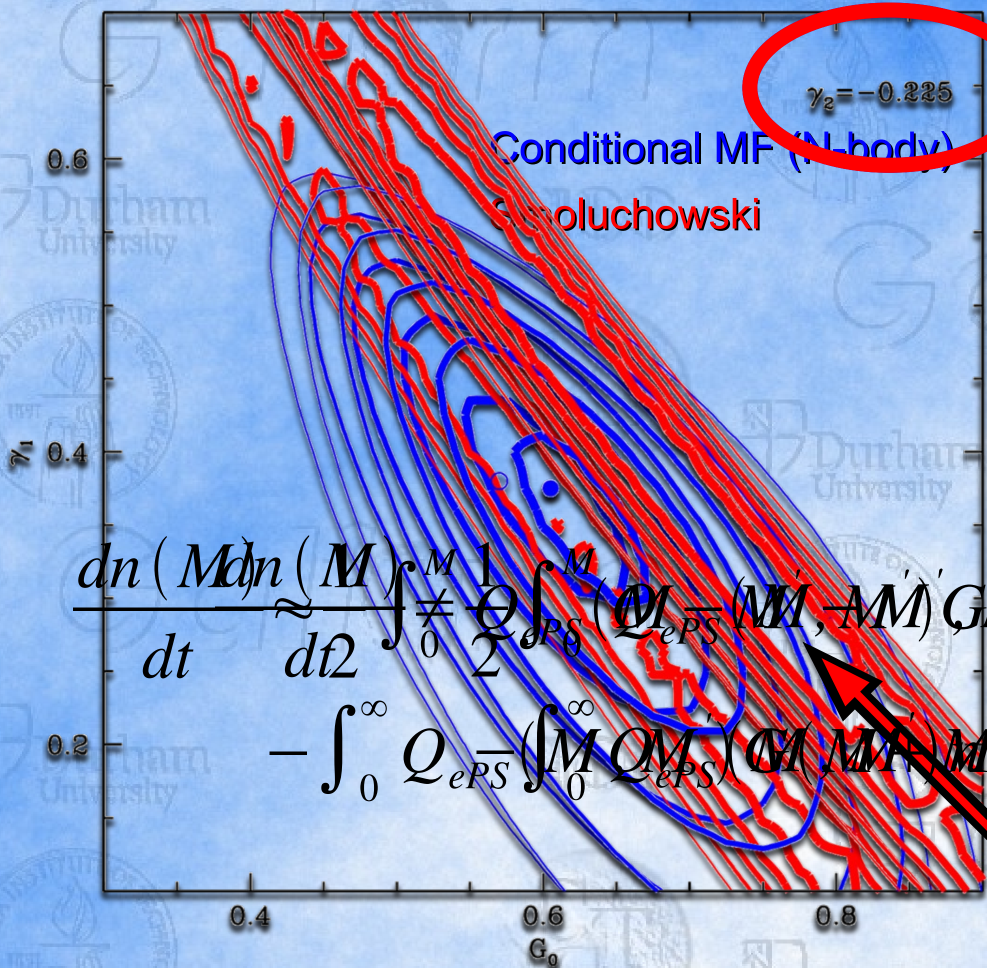
- In Smoluchowski terms, just multiplies the merger kernel

Fits to Conditional Mass Functions



$G_0 = 0.57$
$\gamma_1 = 0.38$
$\gamma_2 = -0.01$

Constraints from Smoluchowski



Best fit to N-body conditional mass functions

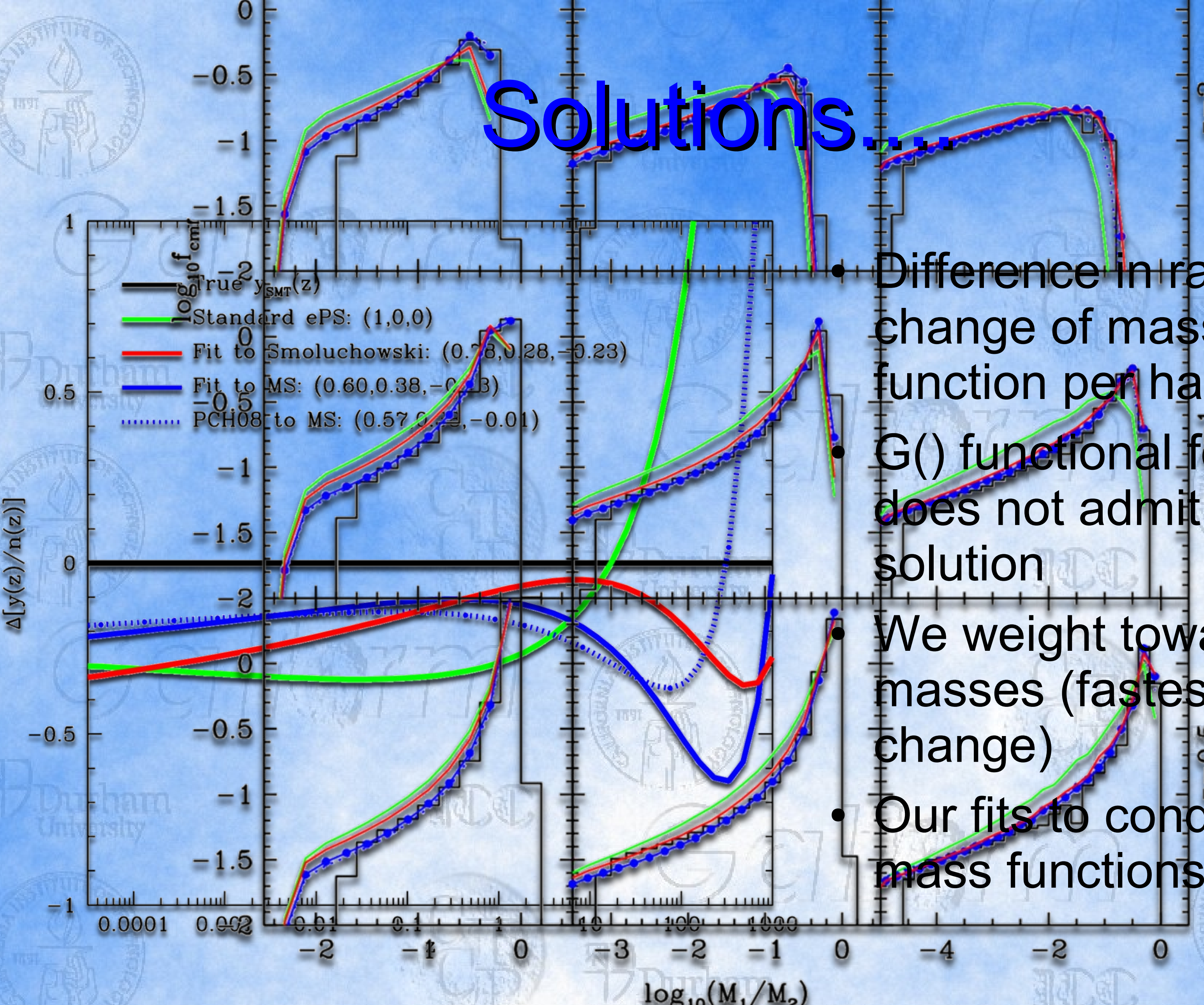
Best solution to Smoluchowski equation

- Additional constraint on 3 parameters from Smoluchowski equation
- Slice through parameter space at fixed γ_2
- Highly degenerate
- Consistent constraints from N-body and Smoluchowski

$$\frac{dn(M)}{dt} = \frac{1}{2}\int_0^M Q_{ePS}(M', M-M')\, n(M')\, n(M-M')\, dM' - \int_0^\infty Q_{ePS}(M, M')\, n(M)\, n(M')\, dM'$$

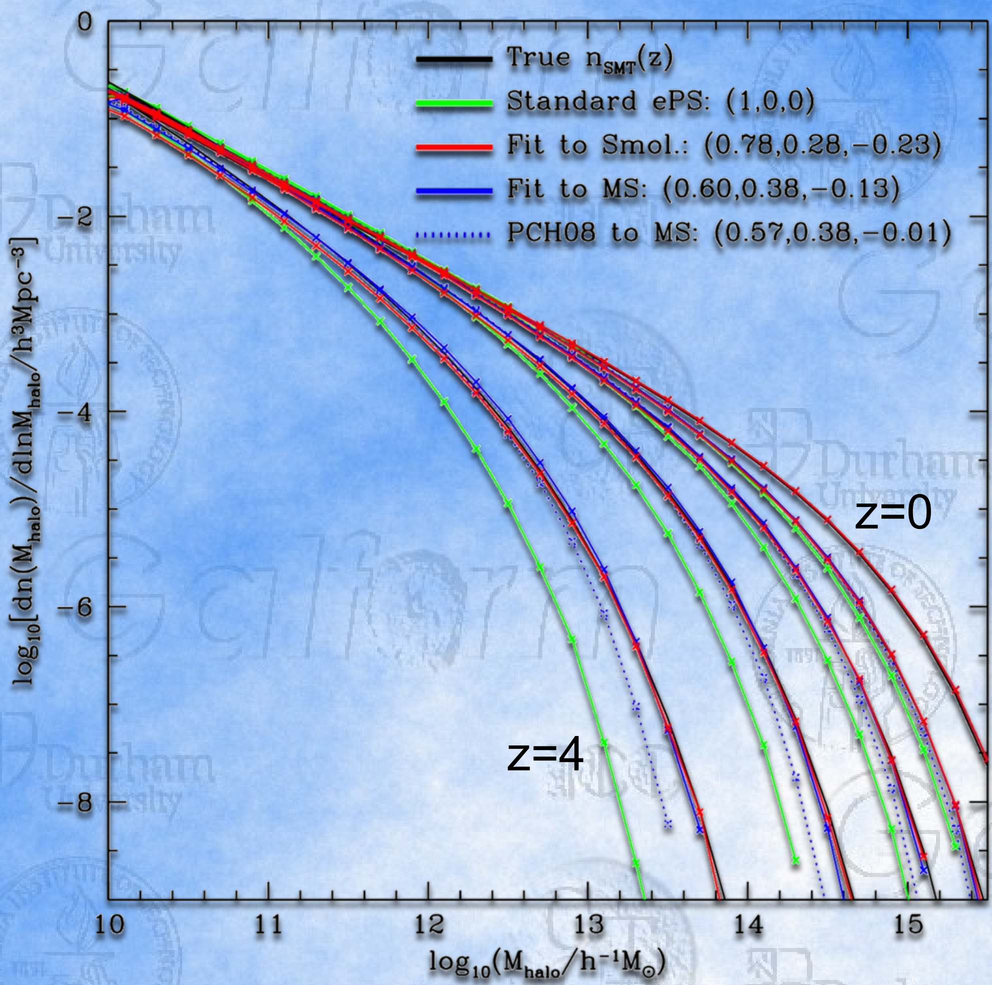
Fit to N-body *mass function*, but **not** N-body *conditional mass functions*.

Solutions....



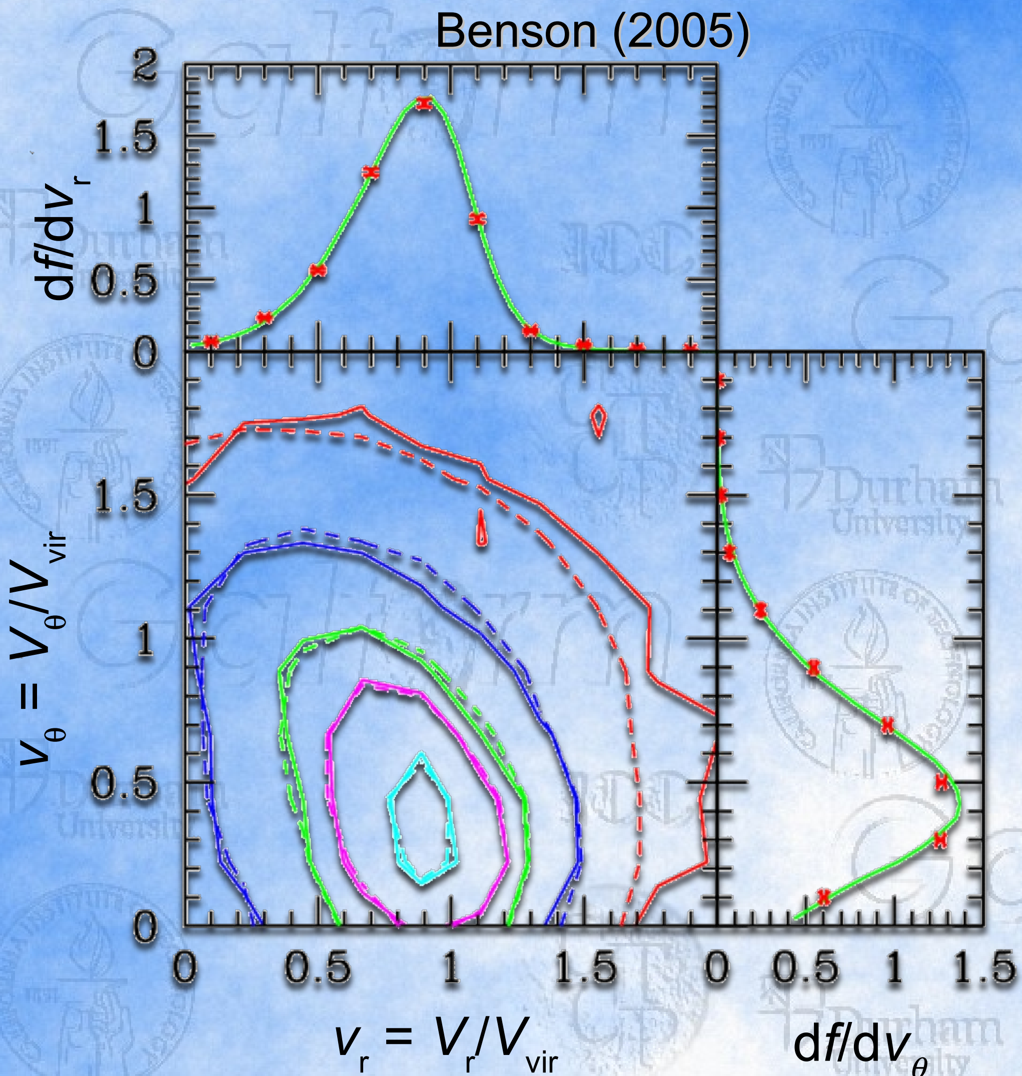
- Difference in rate of change of mass function per halo
- G() functional form does not admit an exact solution
- We weight towards high masses (fastest change)
- Our fits to conditional mass functions...

Constraints from Smoluchowski



- Evolve Sheth-Tormen mass function over large redshift intervals
- New fits work extremely well, even for very low abundance halos
- Other functional forms for G() could work better still

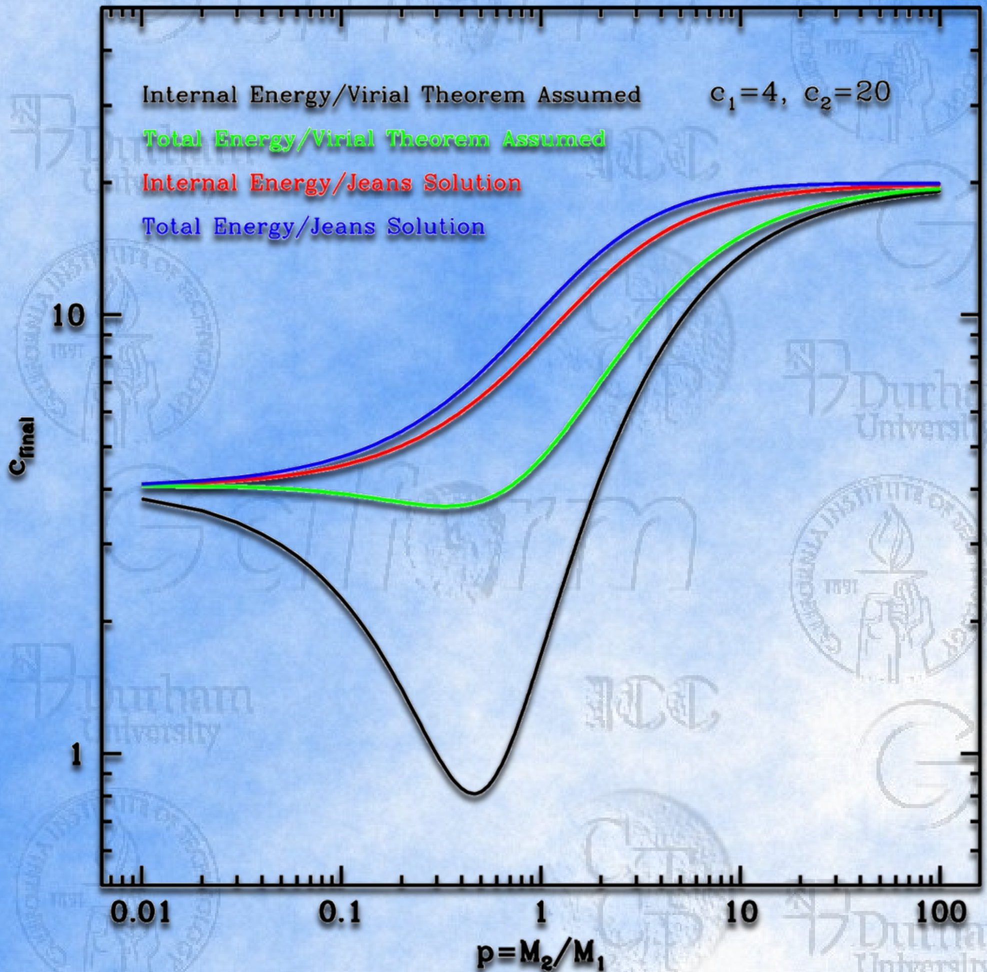
Merger Tree Factory: Orbits



- What are orbital properties of just merged halos?
- How to find them
  - *N-body simulation*
  - *Find all the halos*
  - *Find those about to merge*
- Distribution of orbital velocities
- Measured to very good precision

Halo Mass-Concentration

Dan Grin & AJB



- Halo concentrations well measured
  - *Millennium Simulation (Neto et al., arXiv:0706.2919)*
  - *C(M,z)*
- Scatter larger than current analytic models predict (<30%)
- Can we rectify this by considering complete merging history?
- Currently testing this hypothesis using Millennium Simulation data

Summary

- Merger trees ubiquitous in structure/galaxy formation work
  - *Need to be accurate*
  - *N-body trees good, but still very limited*
- Extended Press-Schechter doesn't work
  - *Inconsistencies*
  - *Inaccurate*
- Coagulation equation
  - *Can solve directly, but what regularization to use?*
  - *Using simple analytic forms a better/easier approach*
- High-accuracy merger trees possible
  - *Can construct trees back to z≥4 maintaining mass function*