Modeling Reionization and its Observational Consequences

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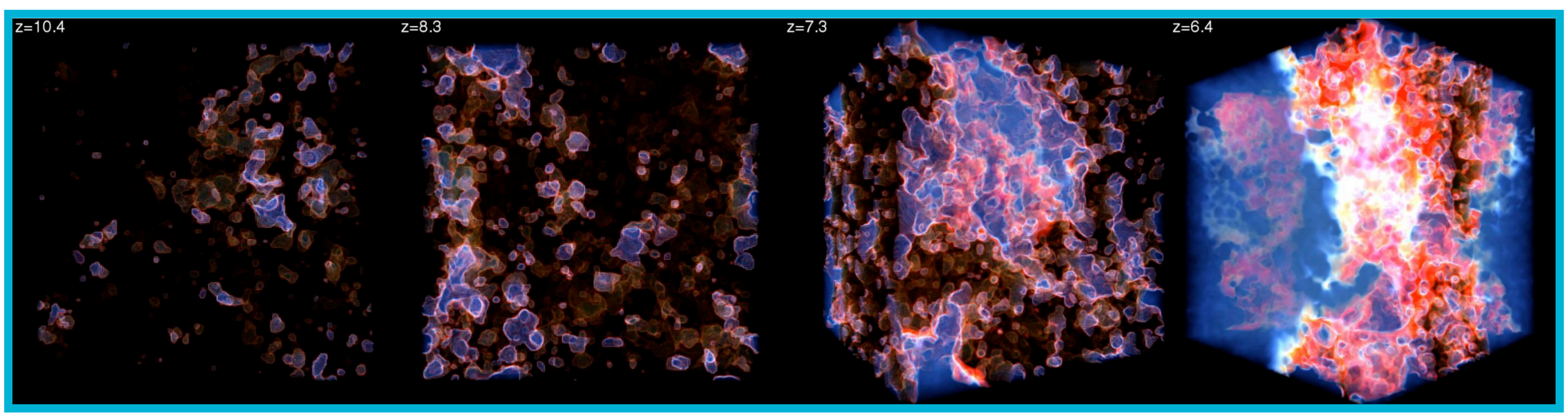


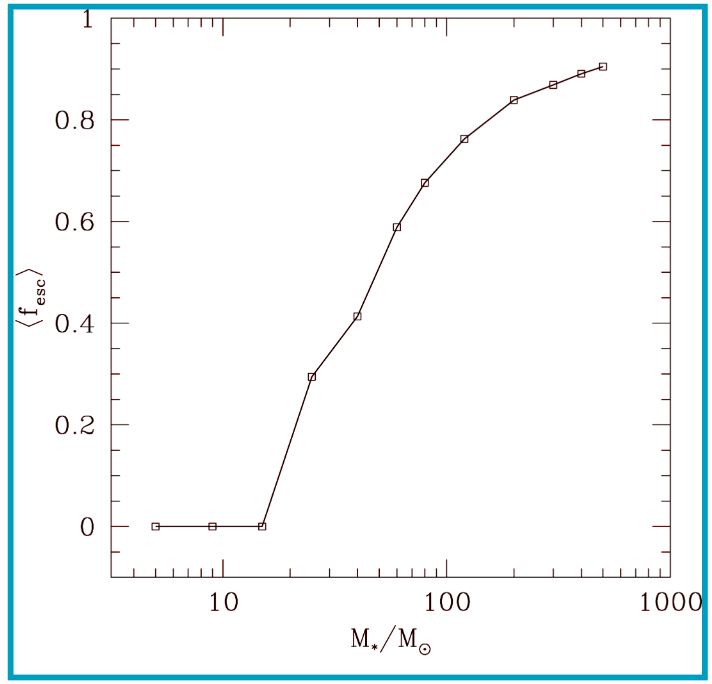
Outline

- Basics of the dominant reionization paradigm
- Important physical effects
- Analytical models
- Numerical simulations of reionization
- 21cm/CMB correlation
- WMAP 3-year results and reionization
- Summary and future directions

Reionization: The Standard Picture

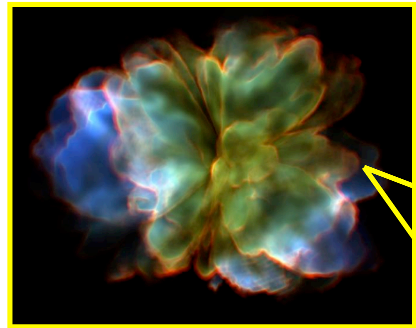
- Reionization driven by ionizing radiation produced by **stars**
- First stars (Pop III) forming in minihalos were likely massive and efficient producers of ionizing radiation at z~10-20
- Pop III stars must have polluted IGM as minihalos merged into larger halos
- Eventually Pop II star forming galaxies dominate as reionization ends at z~6

Alvarez, Bromm & Shapiro (2006)

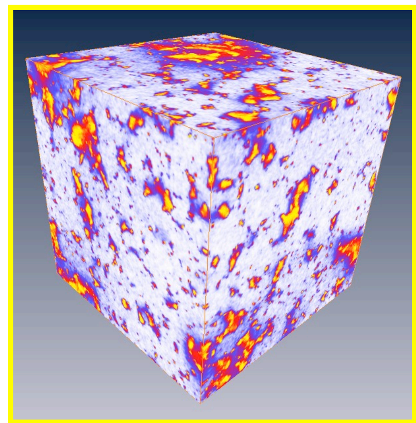


Reionization: The Standard Picture

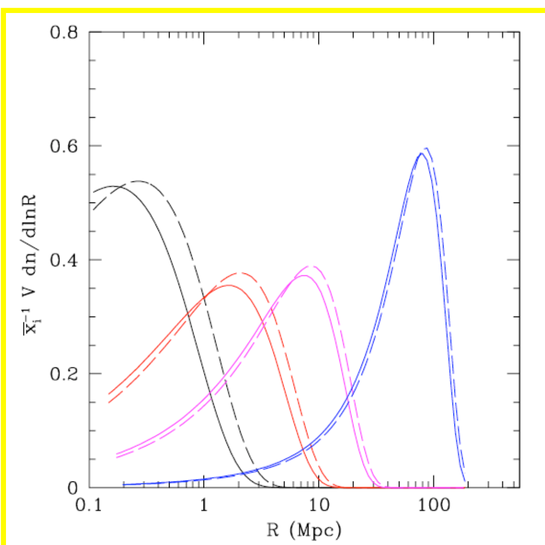
- Vast range of scales are important:
  - First H II regions have radii of order 100 comoving kpc
  - Toward end of reionization, H II regions grow and merge to typical sizes of tens of Mpc
- Approximations must be made!



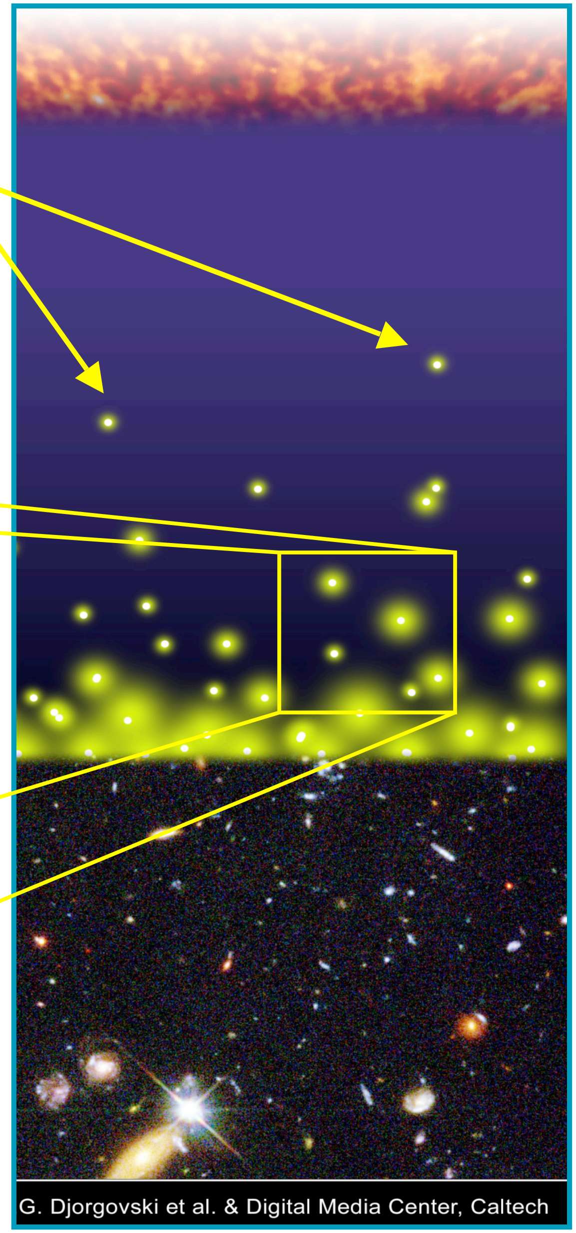
~300 kpc



~150 Mpc



Furlanetto et al. (2007)



Djorgovski et al.

Modeling Reionization

- Initial goal: global **reionization history**
- Ingredients for reionization models:
  - Halo abundance or collapsed fraction
  - Star formation history/IMF/efficiency
  - Escape fraction of ionizing radiation
  - State of the IGM: minihalo abundance, clumping factor, feedback on hydrodynamics, etc…
- More ambitious: **evolving morphology**

Analytical HII Region Size Model

- Furlanetto, Zaldarriaga, & Hernquist (2004) developed powerful analytical model for HII region sizes
- Reionization is "inside out" -- higher density regions reionize first
- Condition for a point to be surrounded by ionized region of mass *m*:

$$\zeta f_{\mathrm{coll}} > 1 \quad f_{\mathrm{coll}}(t) = \mathrm{erfc}\left[\frac{\delta_c(t) - \delta_m}{\sqrt{2\left[\sigma^2_{\mathrm{min}} - \sigma^2(m)\right]}}\right]$$

- This condition results in a scale-dependent "barrier" which is well-approximated as linear funciton of scale-dependent variance

$$B(m, z) \equiv B_0(z) + B_1(z)\sigma^2(m)$$

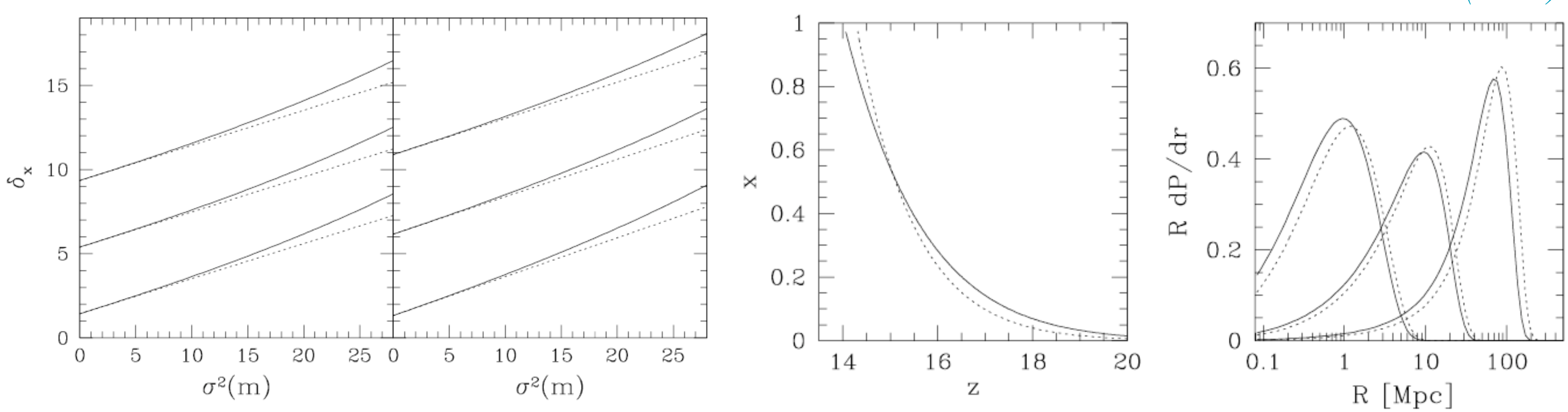
- Allows analytical determination of the "mass function" of bubbles

$$M_b \frac{dn}{dM_b} = \sqrt{\frac{2}{\pi}} \frac{\rho}{M_b} \left|\frac{d\ln\sigma}{d\ln M_b}\right| \frac{B_0}{\sigma(M_b)} \exp\left[-\frac{B^2(M_b, z)}{2\sigma^2(M_b)}\right]$$

Extensions to Model

- Original model assumes a constant ratio of halo mass to ionized mass -- ionization rate proportional to *derivative* of collapsed fraction
- We consider net ionization rate proportional to collapsed fraction (roughly consistent with constant mass to light ratio) -- size distributions are hardly effected due to exponential growth of collapsed fraction
- Other extensions have been made including effect of halo mergers (Cohn & Chang 2006) feedback (Kramer, Haiman, & Oh 2006), recombinations (Furlanetto & Oh 2006), and mass-dependent efficiency (Furlanetto et al. 2006)

Alvarez et al. (2007)

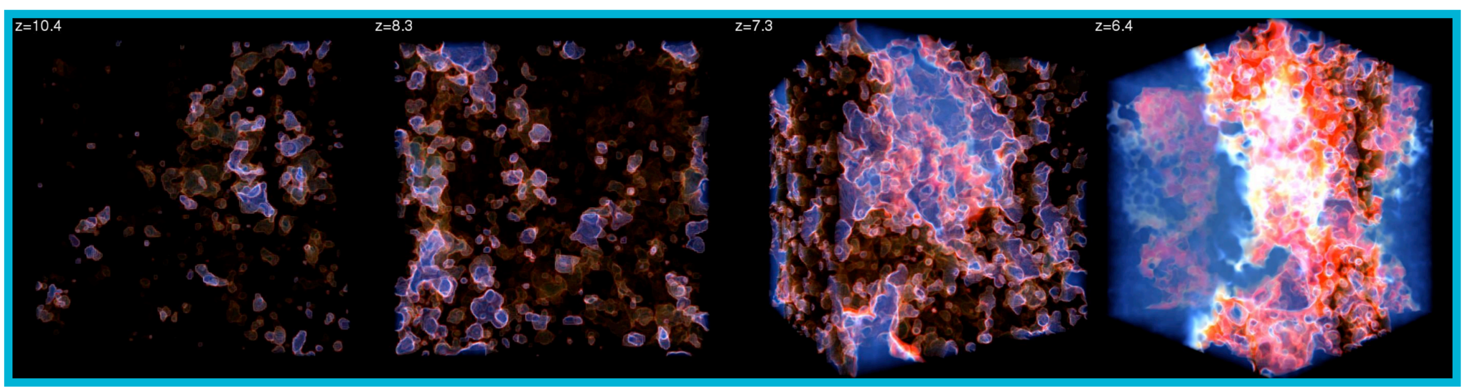


Semi-numerical Reionization

- Zahn et al. (2006) developed a technique for producing 3D evolving ionization field without doing radiative transfer
- Based on Furlanetto, Zaldarriaga, & Hernquist (2004) model
- Only requires linear Gaussian random density field as is usually produced for cosmological N-body simulations
- Smooth around each point and calculate collapsed fraction according to

$$f_{\rm coll}(t) = {\rm erfc}\left[\frac{\delta_c(t) - \delta_m}{\sqrt{2\left[\sigma^2_{\rm min} - \sigma^2(m)\right]}}\right]$$

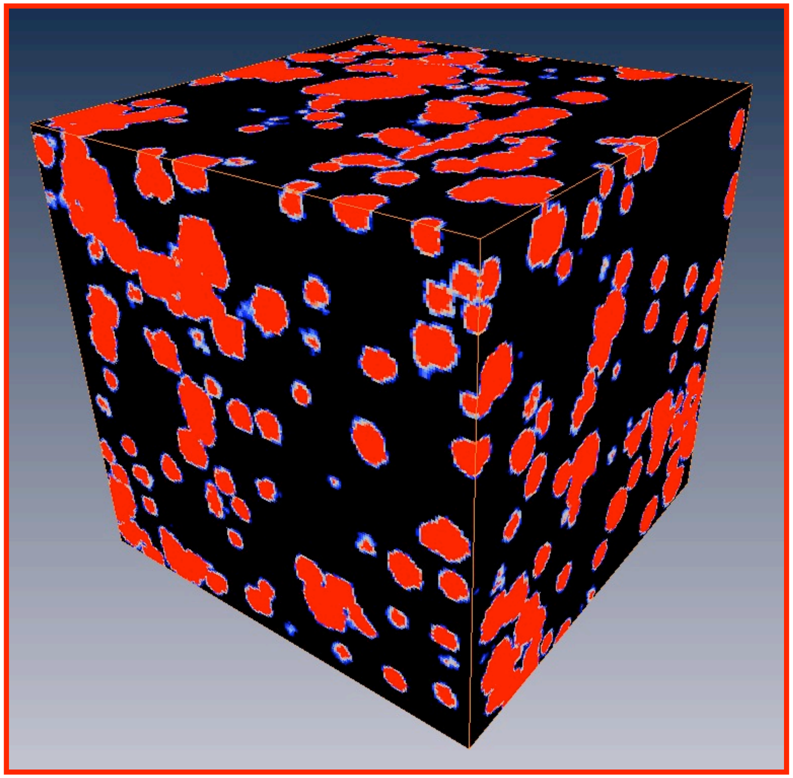
- Point is ionized if $\zeta f_{\rm coll} > 1$ is met for **any** smoothing scale



N-body/Radiative Transfer Simulations: Topology and Scales of Reionization

(Alvarez, Shapiro, Iliev, Mellema & Pen 2007)

- N-body simulations
  - PMFAST (Merz et al. 2005) with $1624^3 \approx 4.3$ billion particles
  - Two different box sizes: 35 and 100 comoving Mpc/h
- Reionization simulations
  - C^2-Ray method (Mellema et al. 2005)
  - Sources placed at the center of DM halos
  - Constant dark matter halo mass-to-light ratio

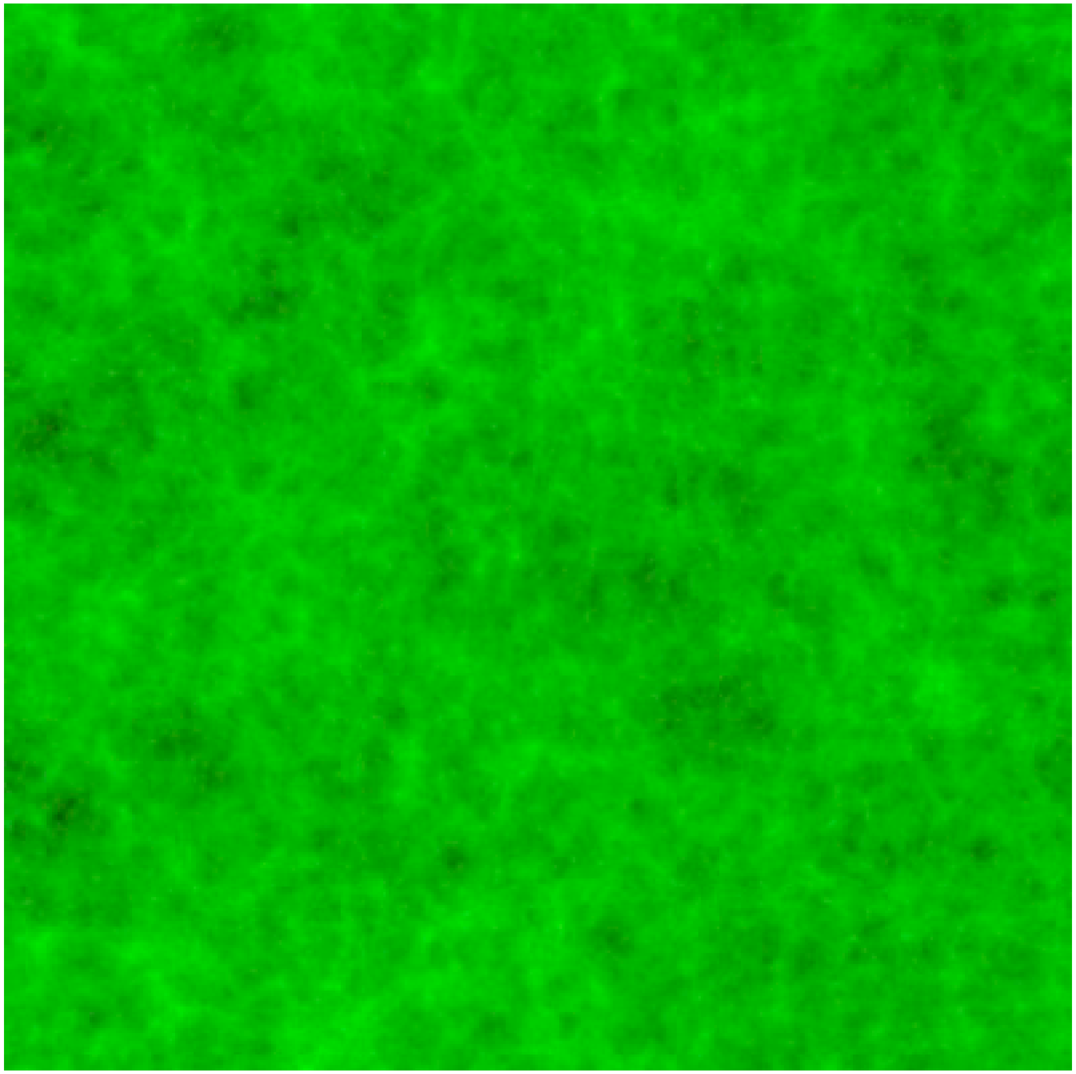


N-body/Radiative Transfer Simulations: Topology and Scales of Reionization

(Alvarez, Shapiro, Iliev, Mellema & Pen 2007)

	f2000	f250	f2000C	f250C	f2000_250	f2000_250S	f250_250S	f2000C_250S	f250C_250S
mesh	203^3	203^3	203^3	203^3	203^3	203^3	203^3	203^3	203^3
box size	100	100	100	100	35	35	35	35	35
$(f_\gamma)_{\rm large}$	2000	250	2000	250	250	250	250	250	250
$(f_\gamma)_{\rm small}$	-	-	-	-	2000	2000	250	2000	250
$C_{\rm subgrid}$	1	1	$C(z)$	$C(z)$	1	1	1	$C(z)$	$C(z)$
$z_{50\%}$	13.6	11.7	12.6	11	16.2	14.5	12.6	13.8	11.6
$z_{\rm overlap}$	11.3	9.3	10.2	8.2	13.5	10.4	9.9	9.1	8.4
$\tau_{\rm es}$	0.145	0.121	0.135	0.107	0.197	0.167	0.138	0.151	0.122

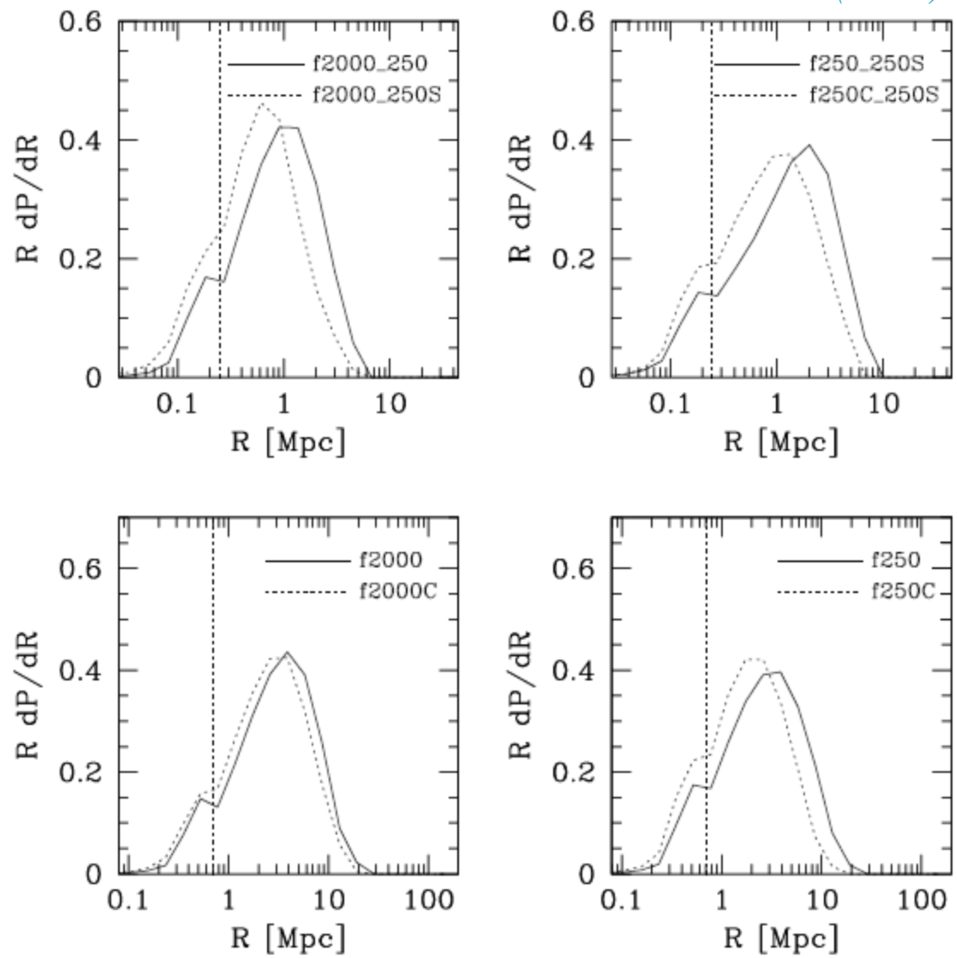
Evolution



Results: Size Distribution

- Used "spherical average" method (Zahn et al. 2006)
- Well-defined peak at Mpc scales
- Comparisons done when each simulation is at "half-ionized" epoch
- Clumping and suppression both shift distributions to smaller scales

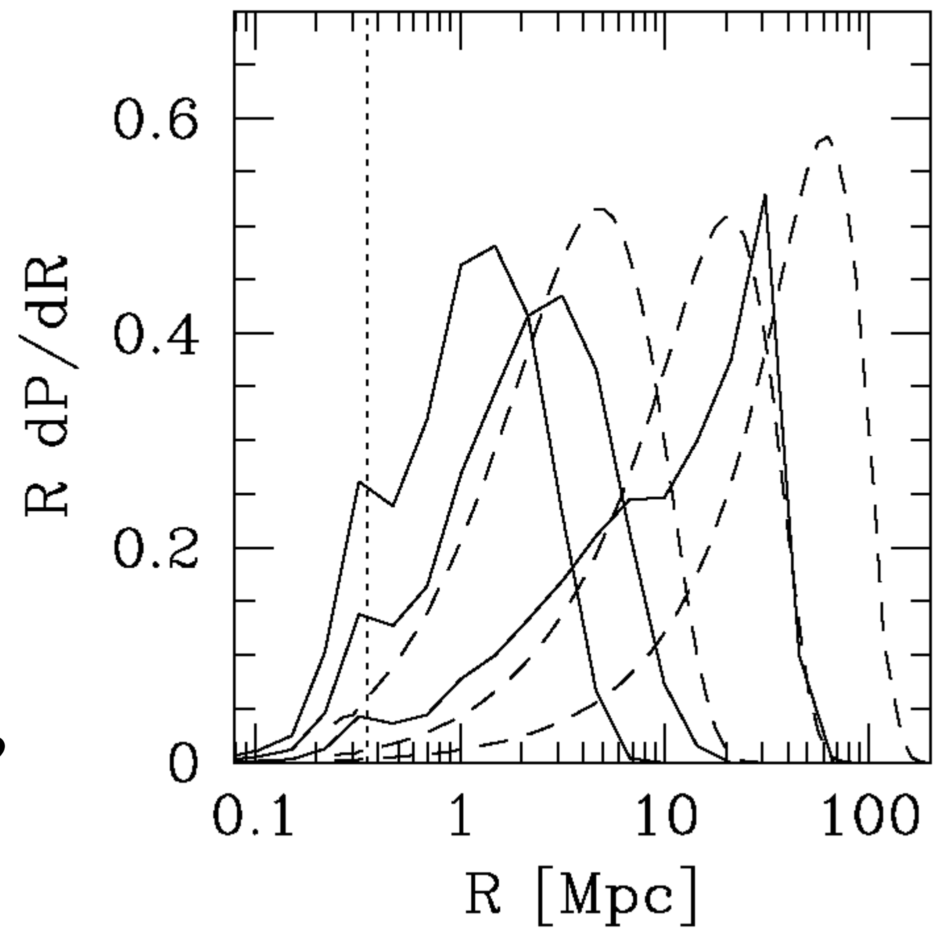
Alvarez et al. (2007)



Results: Size Distribution

- Compared our results to analytical model of Furlanetto et al. (2004)
- Regions get larger and larger as reionization proceeds
- Analytical model predicts larger regions than we see
- Origin of discrepancy not clear… apples to oranges??

Alvarez et al. (2007)



Results: Power Spectrum

Alvarez et al. (2007)

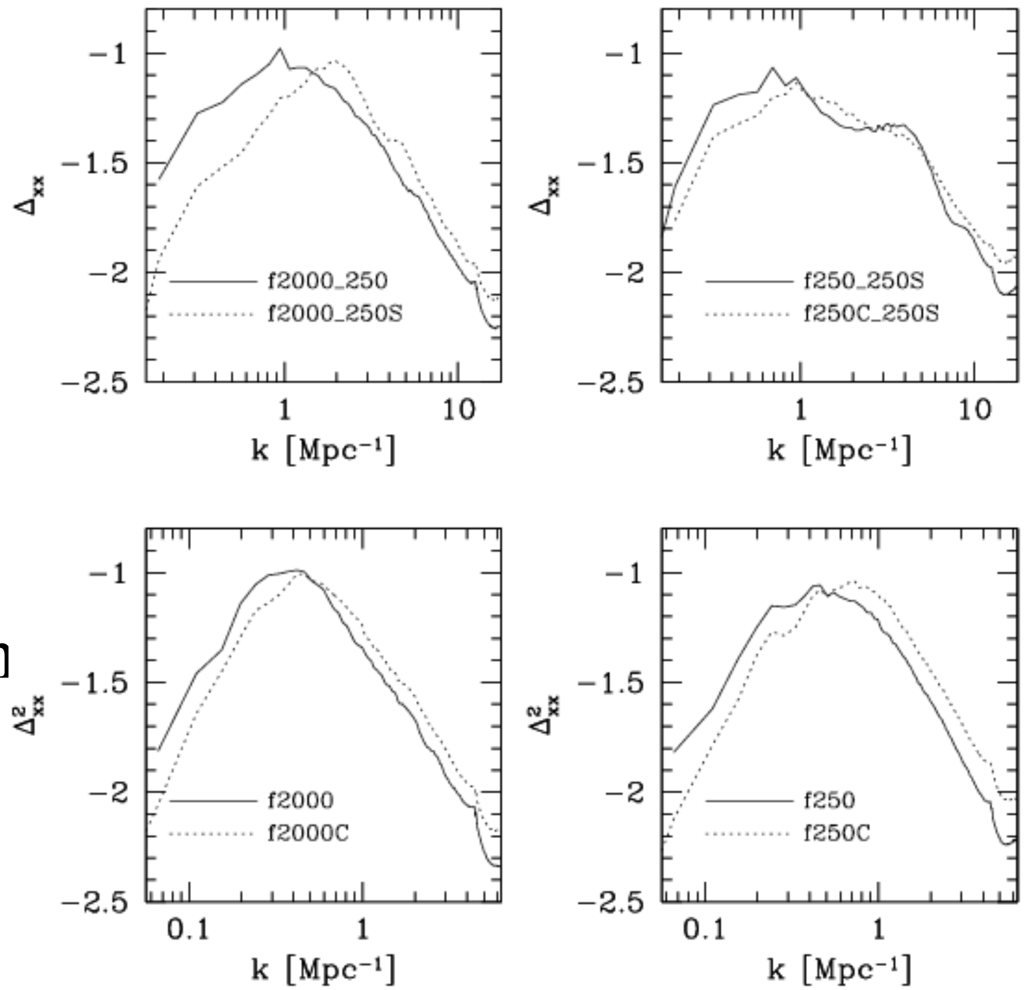
- Power spectrum of ionized fraction:

$$\langle\delta_{x\mathbf{k}}\delta^*_{x\mathbf{k}'}\rangle \equiv \delta^3(\mathbf{k}-\mathbf{k}')(2\pi)^3 P_{xx}(k)$$

- Peak of power spectrum corresponds to peak in size distribution,

  $k \sim 0.3$ - 2 Mpc^{-1}

- Clumping & suppression shift peak to higher k (i.e. smaller scales)



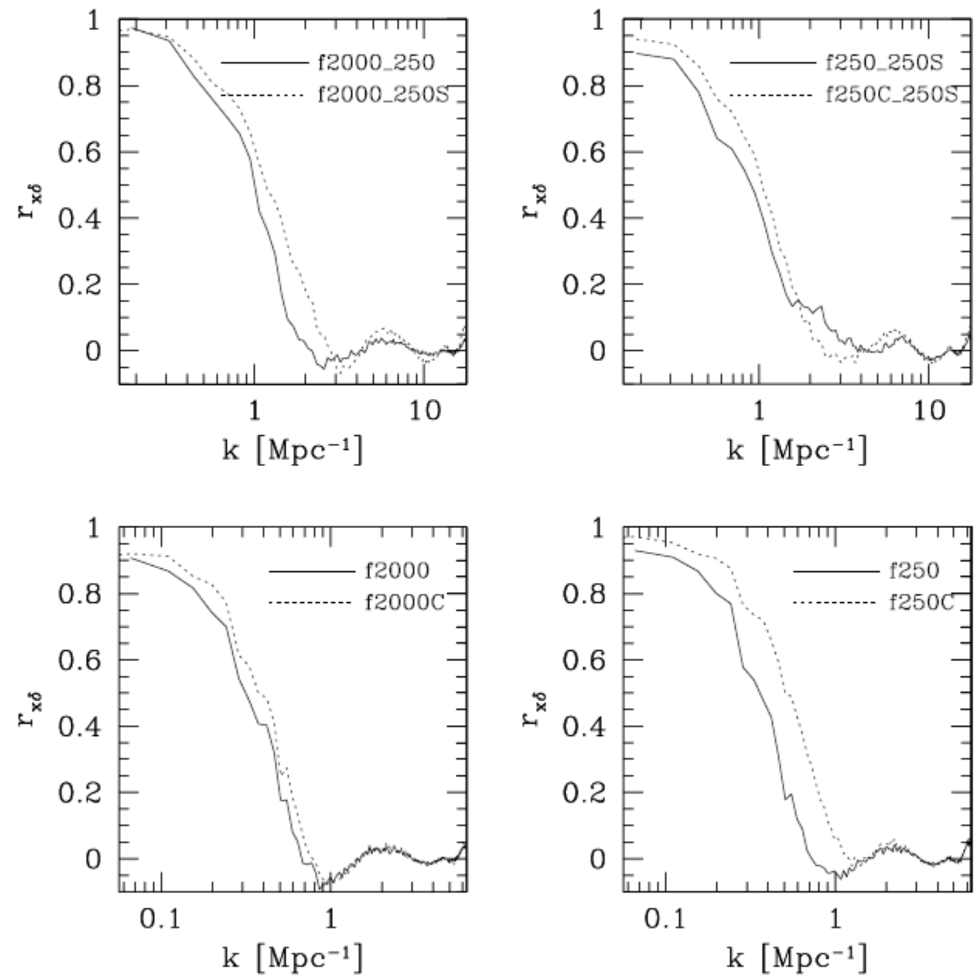
Results: Cross-correlation

Alvarez et al. (2007)

- Cross-correlation coefficient of density and ionization field:

$$r_{x\delta}(k) \equiv \frac{\Delta^2_{x\delta}(k)}{\left[\Delta^2_{xx}(k)\Delta^2_{\delta\delta}(k)\right]^{1/2}}$$

- Well correlated on scales larger than typical bubble size
- Clumping actually increases correlation by shifting bubble scale down

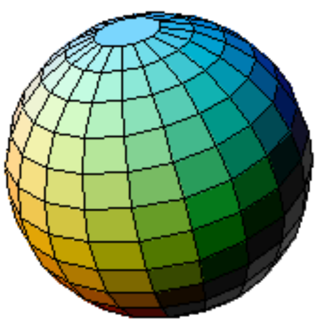


Topology of Reionization: Euler Characteristic

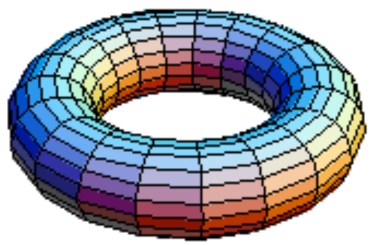
- Fourth Minkowski functional of a surface

$$V_3(f_{\rm th}) \;=\; \frac{1}{4\pi V_{\rm tot}} \int_{\partial F_{\rm th}} d^2S(\mathbf{x}) \frac{1}{R_1 R_2}$$

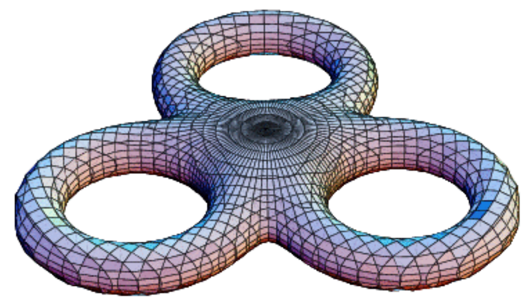
- Equal to the (# parts) - (# tunnels)
- V_3 = 1-genus



$V_3 = 1$



$V_3 = 0$



$V_3 = -2$

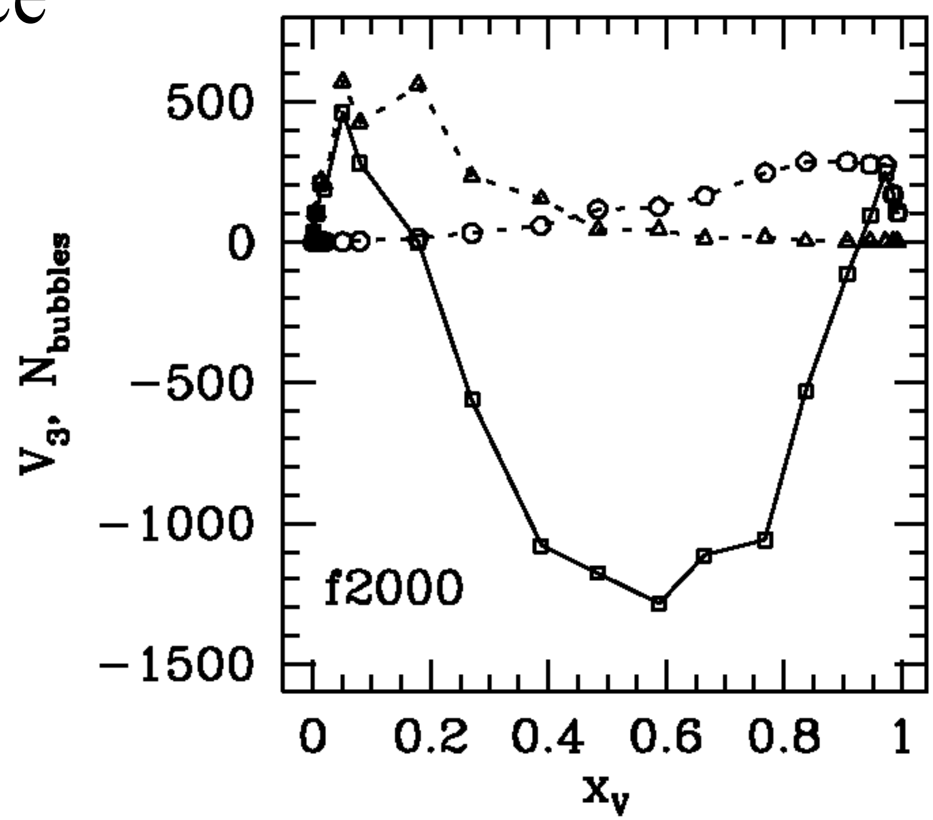
Topology of Reionization: Euler Characteristic

- We calculate V_3 for surface of ionized fraction x=0.5
- Early,

  V_3= # of ***ionized*** regions
- Intermediate time,

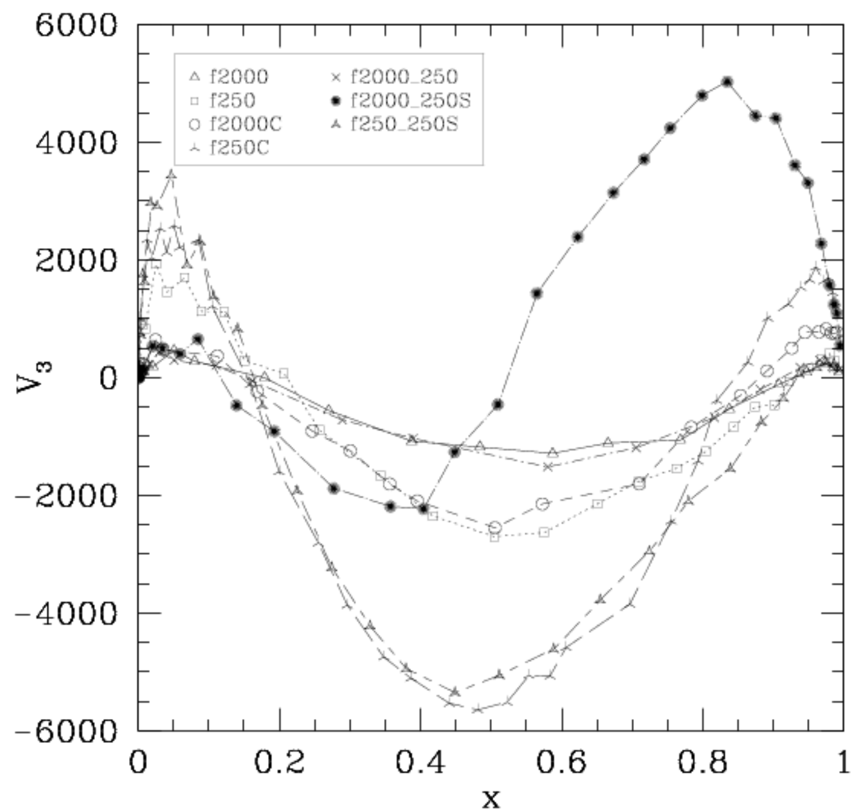
  V_3< 0 as tunnels develop
- Late,

  V_3= # of ***neutral*** regions



Topology of Reionization: Euler Characteristic

- f2000_250S very different from f250_250S
- Suppressed halos form only in neutral regions, “pinch-off” tunnels
- Suppressed halos can remove negative peak if they are very efficient



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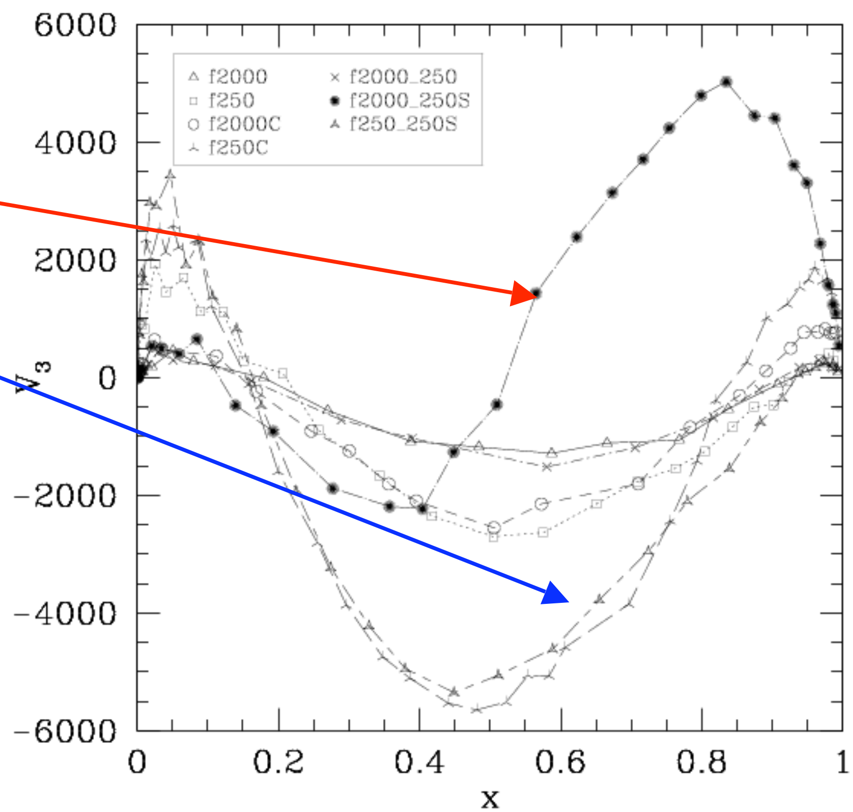
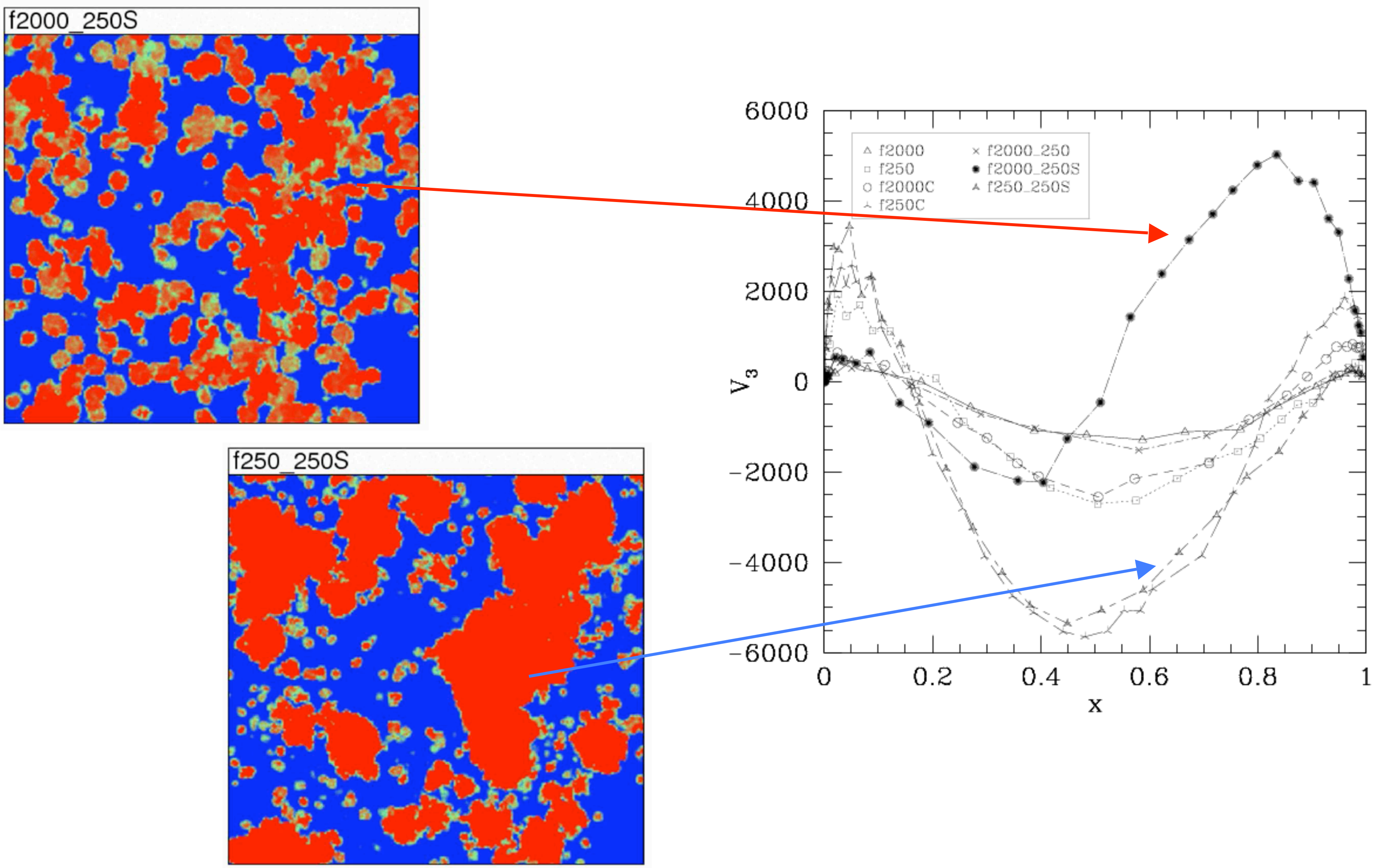


Topology of Reionization: Euler Characteristic

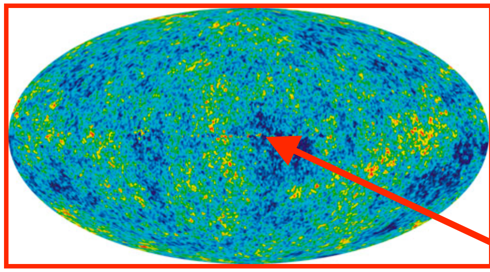


Numerical Simulations: Summary

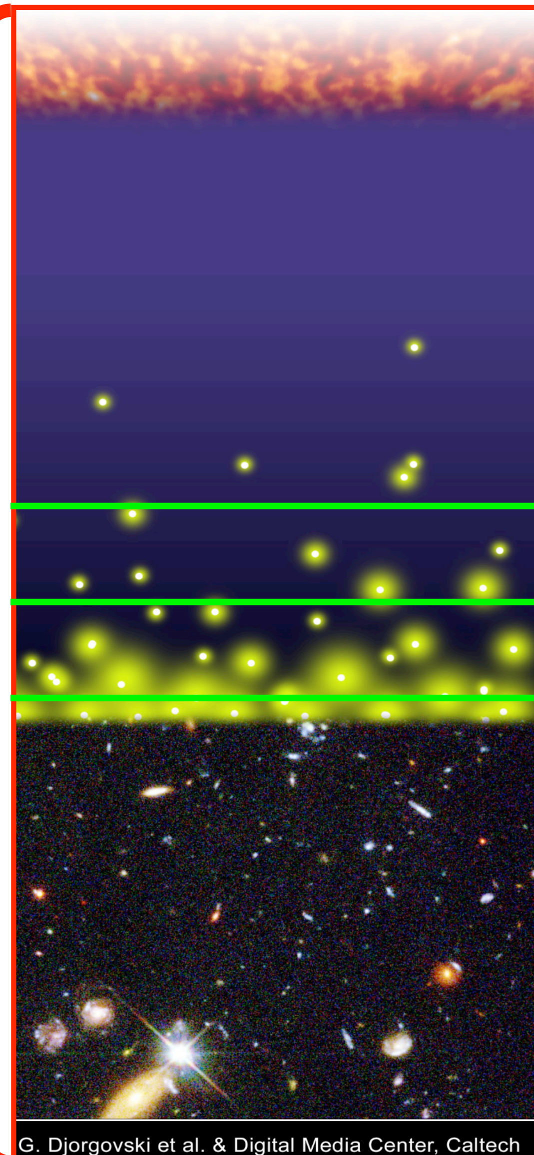
- Clumping and suppression reduce characteristic scale of reionization
- Simulations give smaller H II regions than analytical model of Furlanetto et al. (2004)
- Peak of ionized fraction power spectrum coincides with typical H II region size
- Density and ionized fraction correlated on large scales
- Euler characteristic can discriminate between different reionization scenarios, especially role of source suppression

Reionization & CMB -21cm correlation

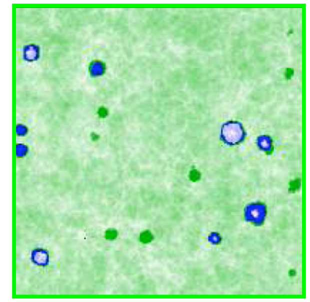
Doppler is a
projected
effect on CMB

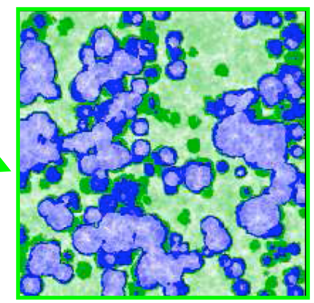


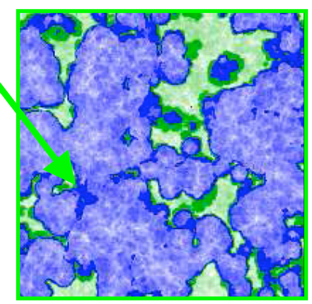
- Doppler effect comes from peculiar velocity along l.o.s.
- 21-cm fluctuations due to density and ionized fraction
- We focus on degree angular scales



21-cm maps result from ***line-emission***







21cm Anisotropy

- To get cross-correlation between 21cm and Doppler, we need expression for spherical harmonic coefficients a_{lm}:

$$a_{lm}^{21}(z) = \int d\hat{\mathbf{n}} T_{21}(\hat{\mathbf{n}}, z) Y_{lm}^{*}(\hat{\mathbf{n}})$$

$$T_{21}(\hat{\mathbf{n}}, z) = T_0(z) \left\{1 - \overline{x}_e(\eta)[1 + \delta_x(\hat{\mathbf{n}}, \eta)]\right\} \left\{1 + \delta_b(\hat{\mathbf{n}}, \eta)\right\}$$

$$a_{lm}^{21}(z) = 4\pi(-i)^l \int \frac{d^3k}{(2\pi)^3} [\overline{x}_H(z)\delta_{b\mathbf{k}} - \overline{x}_e(z)\delta_{x\mathbf{k}}] \alpha_l^{21}(k, z) Y_{lm}^{*}(\mathbf{k})$$

$$\alpha_l^{21}(k, z) \equiv T_0(z) D(z) j_l[k(\eta_0 - \eta)]$$

- To leading order, the anisotropy is dependent on fluctuations in $\boxed{\text{density}}$ and $\boxed{\text{ionized fraction}}$

Doppler Anisotropy

- Doppler arises from integral of velocity field along line of sight

$$T_D(\hat{\mathbf{n}}) = -T_{\rm cmb}\int_0^{\eta_0} d\eta\dot{\tau}e^{-\tau}\hat{\mathbf{n}}\cdot\mathbf{v}_b(\hat{\mathbf{n}},\eta)$$

- Continuity equation ➔ velocity fluctuation proportional to density fluctuation:

$$\mathbf{v}_{b\mathbf{k}} = -i(\mathbf{k}/k^2)\delta_{b\mathbf{k}}\dot{D}$$

- We ignore fluctuations of density (Ostriker-Vishniac) and ionized fraction since they are higher order effects

$$a_{lm}^D = 4\pi(-i)^l\int\frac{d^3k}{(2\pi)^3}\delta_{b\mathbf{k}}\alpha_l^D(k)Y_{lm}^*(\mathbf{k})$$

$$\alpha_l^D(k) \equiv \frac{T_{\rm cmb}}{k^2}\int_0^{\eta_0} d\eta\dot{D}\dot{\tau}e^{-\tau}\frac{\partial}{\partial\eta}j_l[k(\eta_0-\eta)]$$

- To leading order, the Doppler anisotropy is dependent on fluctuations of velocity ⇔ density

Cross-correlation

- Given the coefficients a_{lm} for 21cm and Doppler, the cross-correlation can be found using

$$C_l^{21-D}(z) = \langle a^{21}_{lm}(z)\, a^{D*}_{lm} \rangle$$

$$a^D_{lm} = 4\pi(-i)^l \int \frac{d^3k}{(2\pi)^3} \boxed{\delta_{b\mathbf{k}}}\, \alpha_l^D(k)\, Y^*_{lm}(\mathbf{k})$$

$$a^{21}_{lm}(z) = 4\pi(-i)^l \int \frac{d^3k}{(2\pi)^3} [\overline{x}_H(z)\boxed{\delta_{b\mathbf{k}}} - \overline{x}_e(z)\boxed{\delta_{x\mathbf{k}}}]\, \alpha_l^{21}(k,z)\, Y^*_{lm}(\mathbf{k})$$

$$l^2 C_l^{21-D}(z) \approx -T_{\rm cmb} T_0(z) D(z) \left[\overline{x}_H(z) P_{\delta\delta}\left(\frac{l}{r(z)}\right) - \overline{x}_e(z) P_{x\delta}\left(\frac{l}{r(z)}\right)\right] \frac{\partial}{\partial \eta}(\dot{D}\dot{\tau} e^{-\tau})$$

- Cross-correlation can be found using relation between ionized fraction & density (f=0 ➔ no recombinations; f=1 ➔ Stromgren spheres)

$$\overline{x}_e(z)\boxed{P_{x\delta}(k)} = -\overline{x}_H(z) \ln \overline{x}_H(z) \left[\overline{b}_h(z) - 1 - f\right] \boxed{P_{\delta\delta}(k)}$$

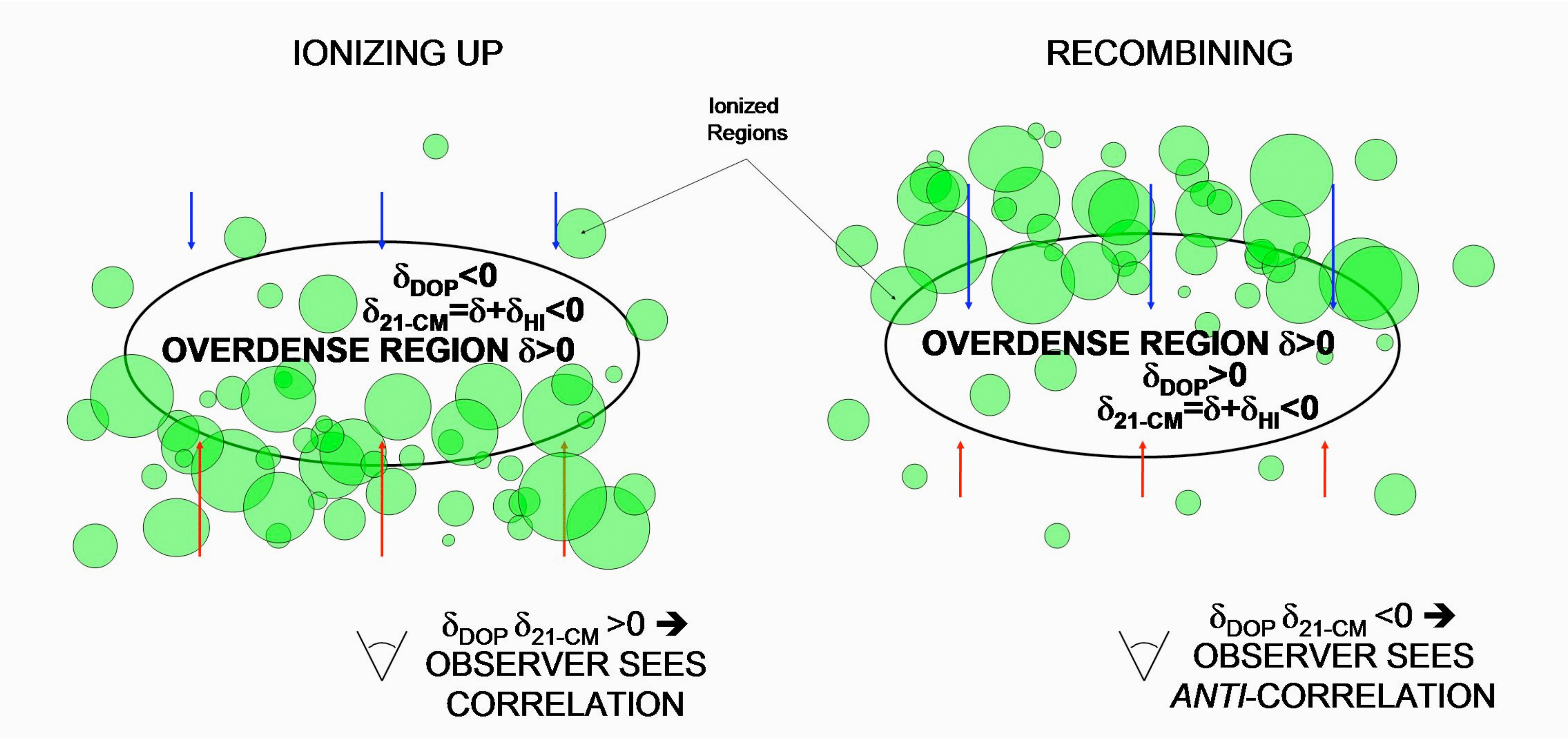
Cross-correlation

- We focus on large scales (l~100; k~0.01 Mpc^{-1}) where patchiness of reionization is averaged over and taken into account by bias factor
- The shape of the correlation traces the linear matter power spectrum at large scales (l~100)

$$\frac{l^2 C_l^{21-D}(z)}{2\pi} \simeq 18.4\ \mu\mathrm{K}^2\ \left[1 + \ln \overline{x}_H(z)\left(\overline{b}_h - f - 1\right)\right] \frac{P_{\delta\delta}[l/r(z), z_N](1+z_N)^2}{10^5\ \mathrm{Mpc}^3}$$
$$\times \left(\frac{\Omega_b h^2}{0.02}\right)^2 \left(\frac{\Omega_m h^2}{0.15}\right)^{1/2} \overline{x}_H(z) \frac{d}{dz}\left[\overline{x}_e(z)(1+z)^{3/2}\right] \left(\frac{1+z}{10}\right)$$

- Because sign of the correlation depends on the derivative of ionized fraction w.r.t. redshift, it can tell us whether universe is reionizing or recombining:
  - Reionization ➔ positive correlation
  - Recombination ➔ negative correlation

Reionization History

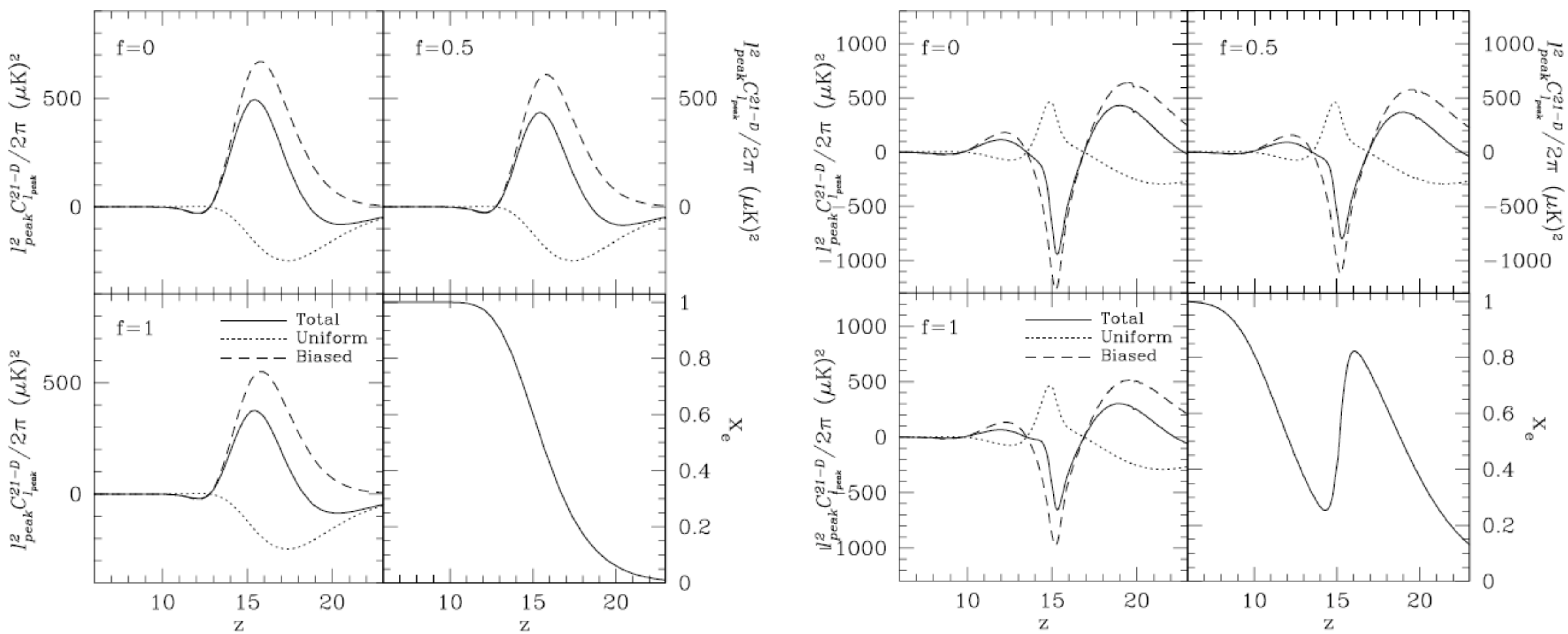


- Reionization ➔ positive correlation
- Recombination ➔ negative correlation

Reionization History

- Cross-correlation peaks when ionized fraction about a half
- Sign and amplitude of correlation constrains derivative of ionized fraction
- Typical signal amplitude ~500 (μK)^2
- Above expected error from Square Kilometer Array for ~1 year of observation ~135 (μK)^2

Alvarez et al. (2006)



WMAP 3-Year Data: Implications for Reionization

Alvarez, Shapiro, Ahn & Iliev 2006, ApJ, *644, L101*

- Optical depth of electron Thomson scattering to last scattering surface, $\boldsymbol{\tau}_{\mathrm{es}}$, from polarization of CMB
  - Constrains ***duration*** of reionization
- First-year WMAP data implied $\boldsymbol{\tau}_{\mathrm{es}}$~0.17 so z_{reion}~17
- Three-year WMAP data implies $\boldsymbol{\tau}_{\mathrm{es}}$~0.09 so z_{reion}~11
- But what does 3-year WMAP data say about the ***ionizing efficiency*** of the sources of reionization?

WMAP 1-yr vs 3-yr

- Most significant changes:
  - Lower optical depth:

$$\boxed{\tau_{es} = 0.17 \longrightarrow 0.09}$$

  - Lower normalization σ_8

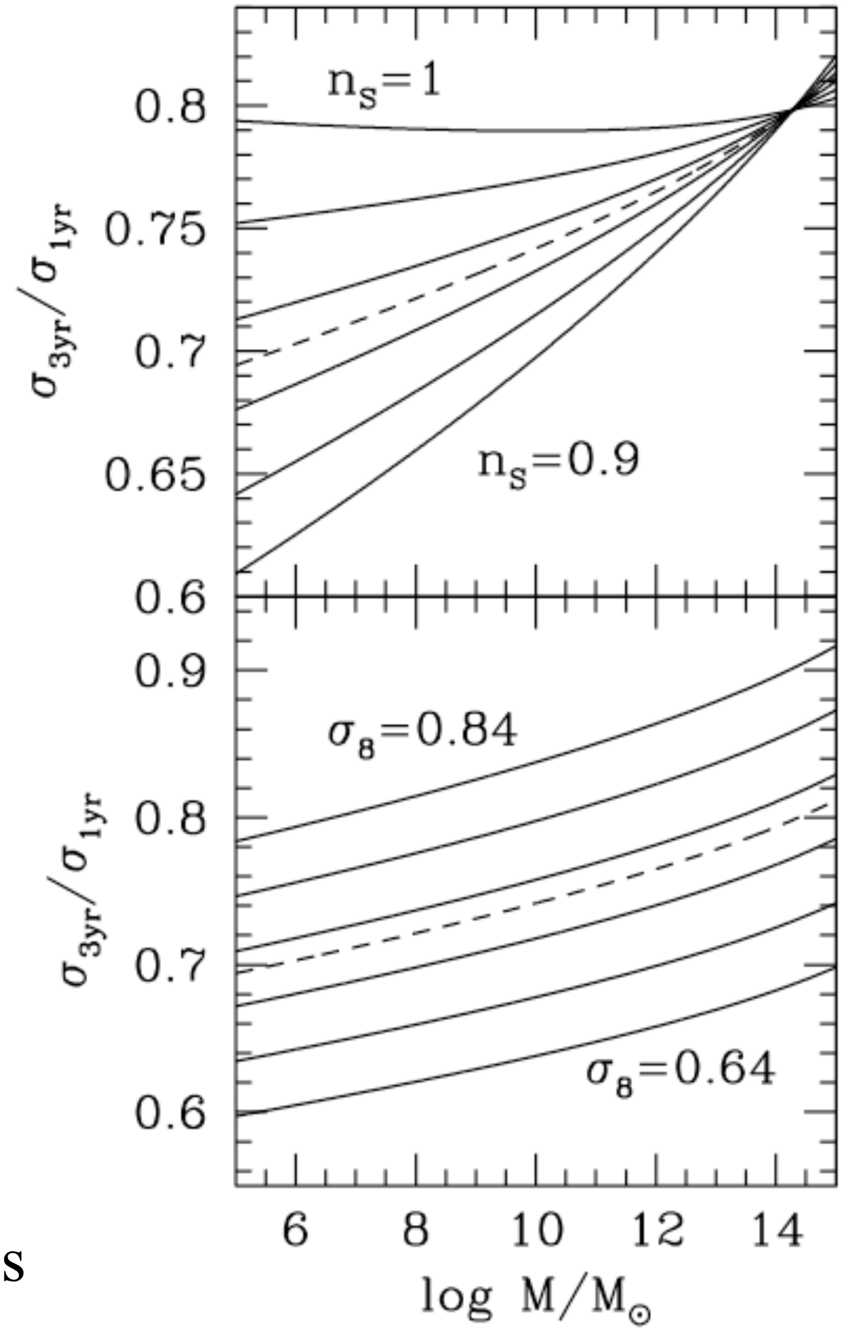
$$\boxed{\sigma_8 = 0.9 \longrightarrow 0.74}$$

  - "tilt" of primordial power spectrum, n_s:

$$P_{\rm prim}(k) \propto k^{n_s}$$

$$\boxed{n_s = 0.99 \longrightarrow 0.95}$$

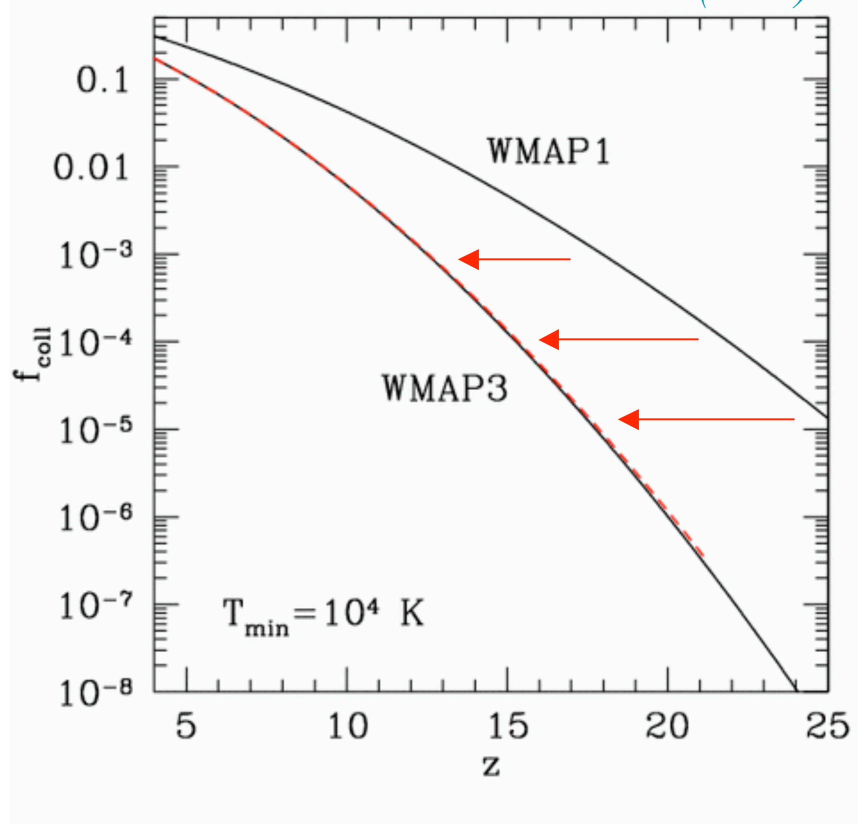
- Tilt affects small scales more
- Normalization affects all scales



WMAP 1-yr vs 3-yr

- Overall effect is much less structure at small scales
- R.M.S. density fluctuation lowered by ~1.4 at low masses
- Density fluctuations grow in proportion to $1/(1+z) \Rightarrow$ structure formation delayed by factor of 1.4 in $(1+z)$

Alvarez et al. (2006)



WMAP 1-yr vs 3-yr

- We use simple reionization model:

$$x_e(z) = \zeta_0 f_{\rm coll}(z)$$

- What happens if we keep the efficiency constant, but use new constraints on power spectrum?
- With same efficiency,

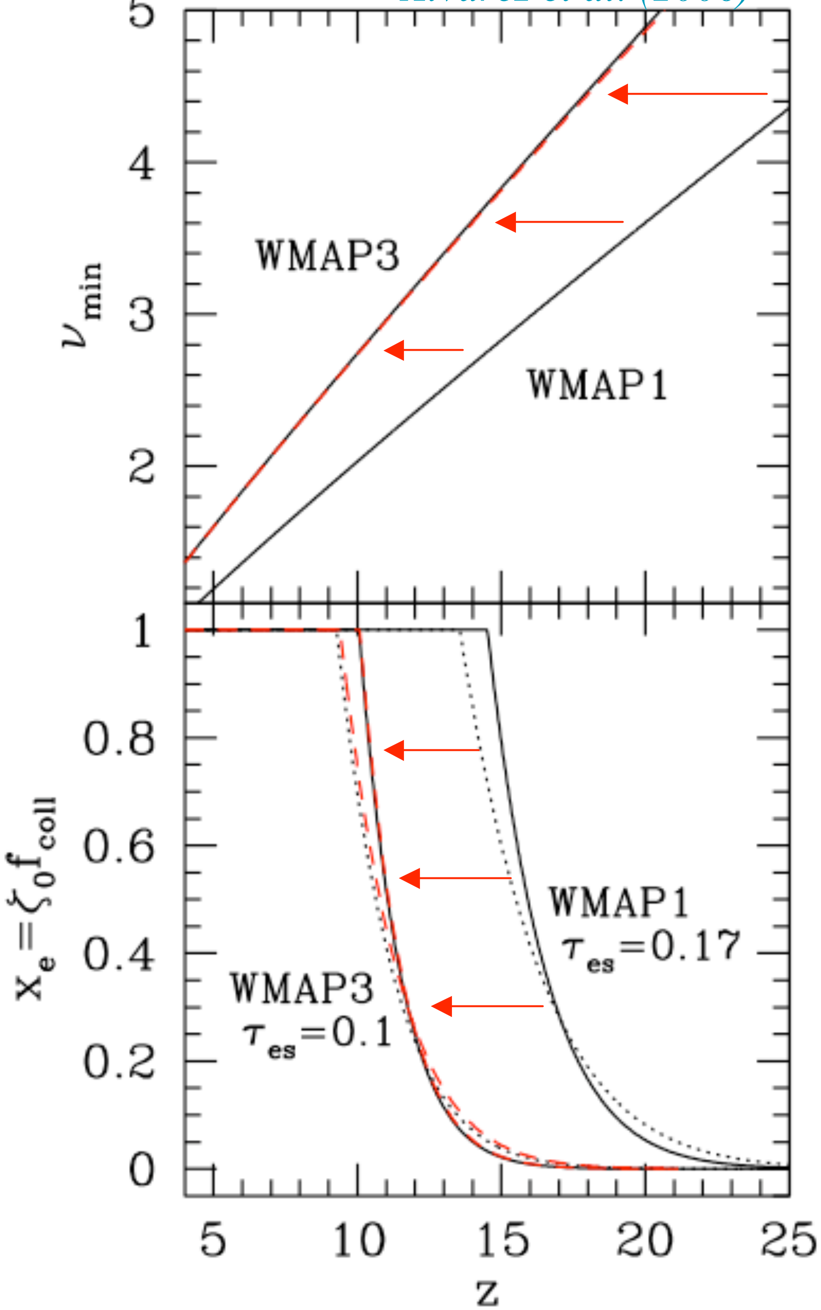
$$\boxed{\tau_{\rm es} = 0.17 \longrightarrow 0.10}$$

- Nearly same change as found by WMAP -- remarkable coincidence!
- Instantaneous reionization at z_r,

$$\tau_{\rm es} \propto (1 + z_r)^{3/2}$$

- Shift of structure formation in scale factor by 1.4 accounts for $1.4^{3/2} \sim 1.7$ change in $\tau_{\rm es}$

Alvarez et al. (2006)



Summary

- "Sub-grid" modeling of reionization a must
- Semi-numerical models for 3D structure of reionization are very useful, can be calibrated against radiative transfer simulations, and incorporated into hydrodynamic simulations
- Large-scale simulations of reionization offer unique insight into topology of reionization -- genus statistics are a useful diagnostic of feedback effects
- Inside-out nature of reionization means that 21cm and CMB are correlated on the largest scales
- New WMAP results taken alone do not substantially reduce demand on ionizing efficiency of collapsed matter