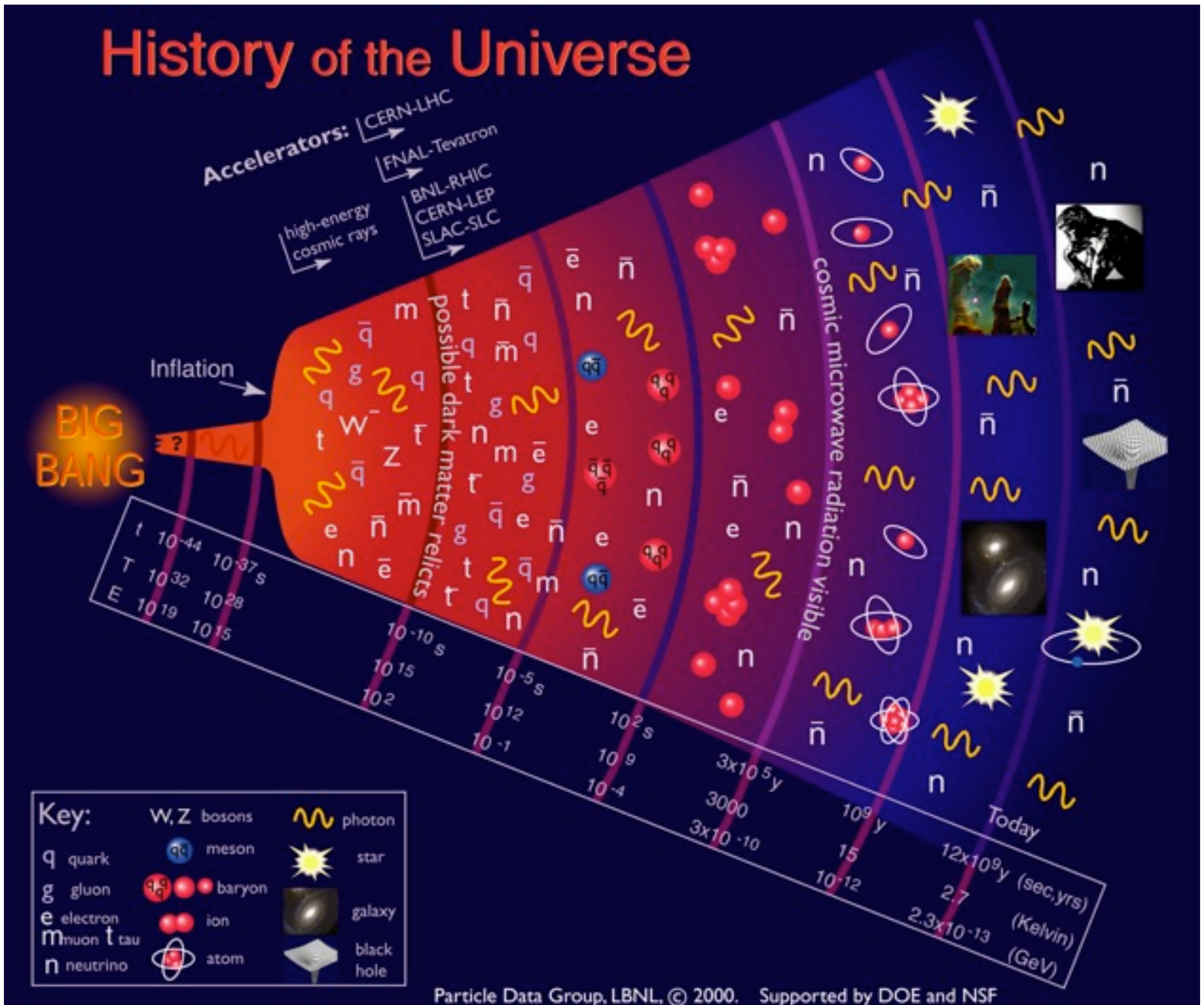
Fast and accurate primordial hydrogen recombination theory

Yacine Ali-Haïmoud
Caltech

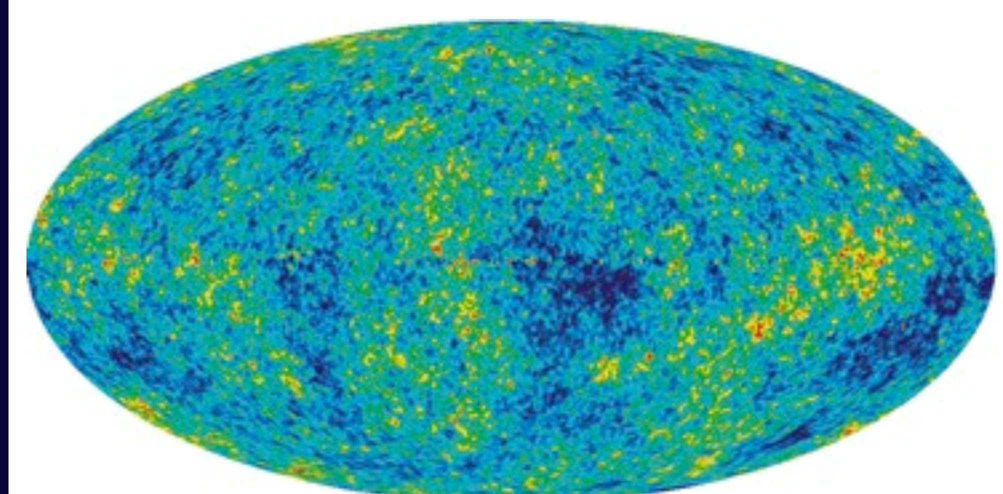
In collaboration with Chris Hirata and Dan Grin

Berkeley Cosmology Seminar, November 16th 2010

Motivations



The CMB: a powerful probe of the geometry, constituents and history of the universe

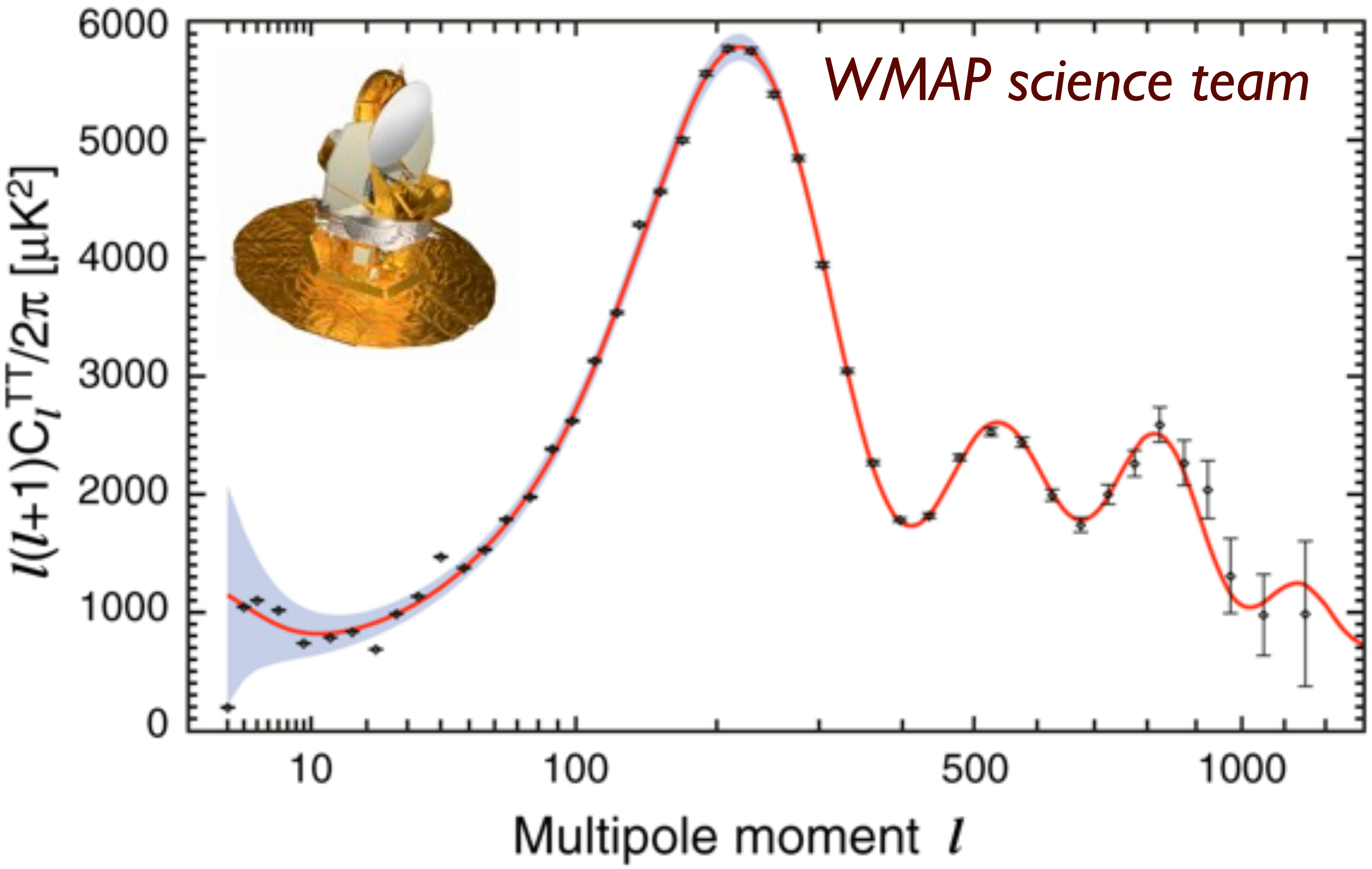


WMAP science team

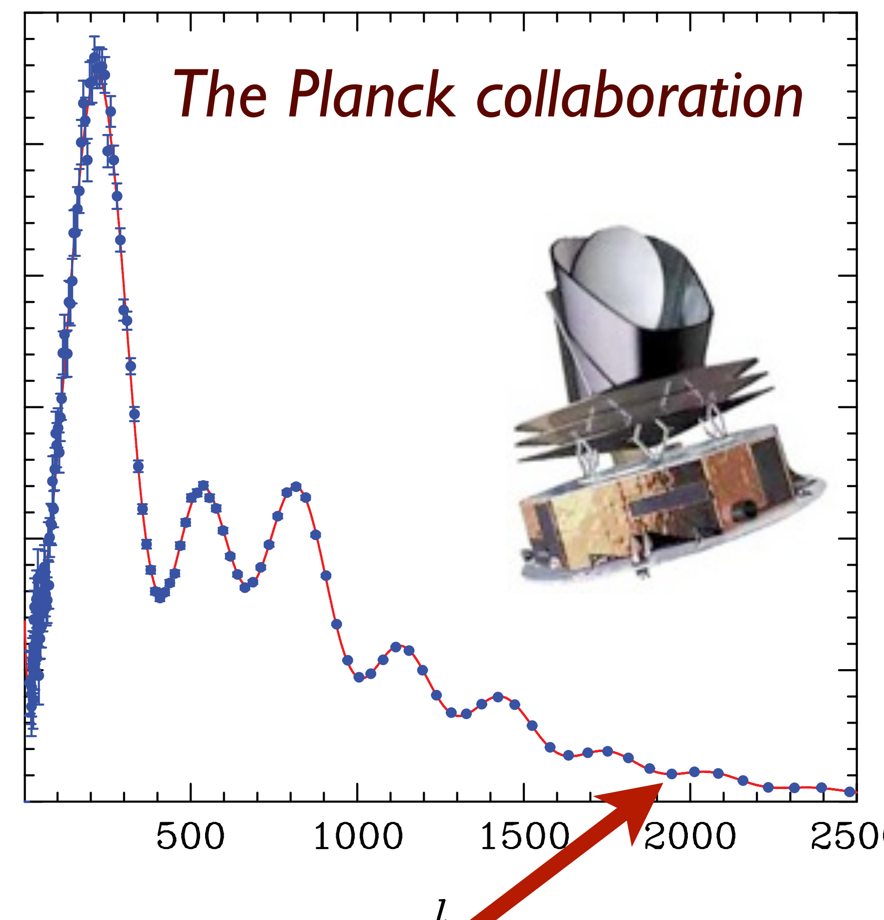
Motivations

Planck: ~3 x resolution, ~5 x sensitivity of WMAP

WMAP seven-year



Planck (simulation)

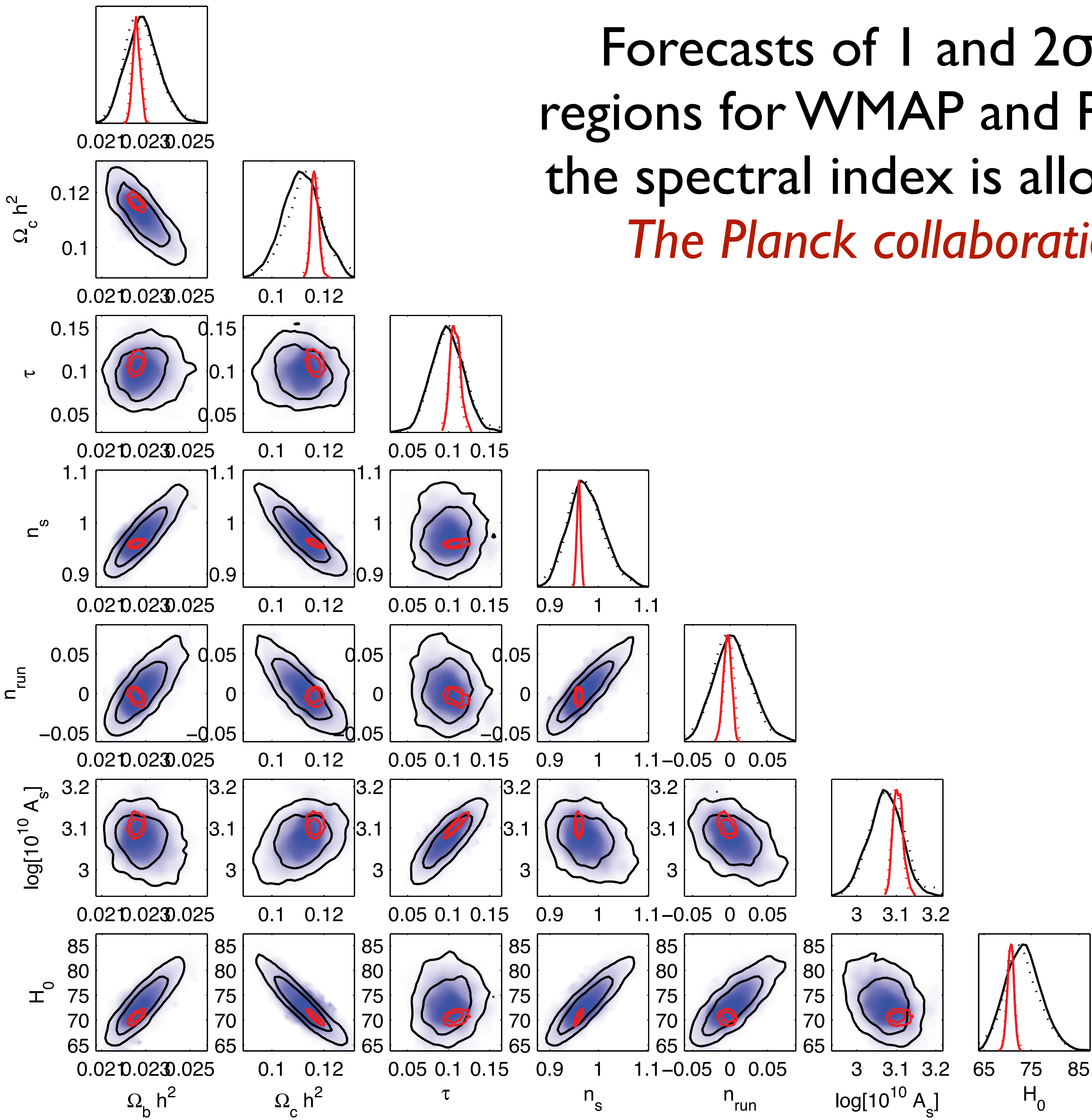


WMAP 7-year: n_s = 0.963 ± 0.014

With *Planck*: n_s = ? ? ? ± 0.0037

Forecasts of 1 and 2σ contour regions for WMAP and Planck when the spectral index is allowed to run

The Planck collaboration 2006



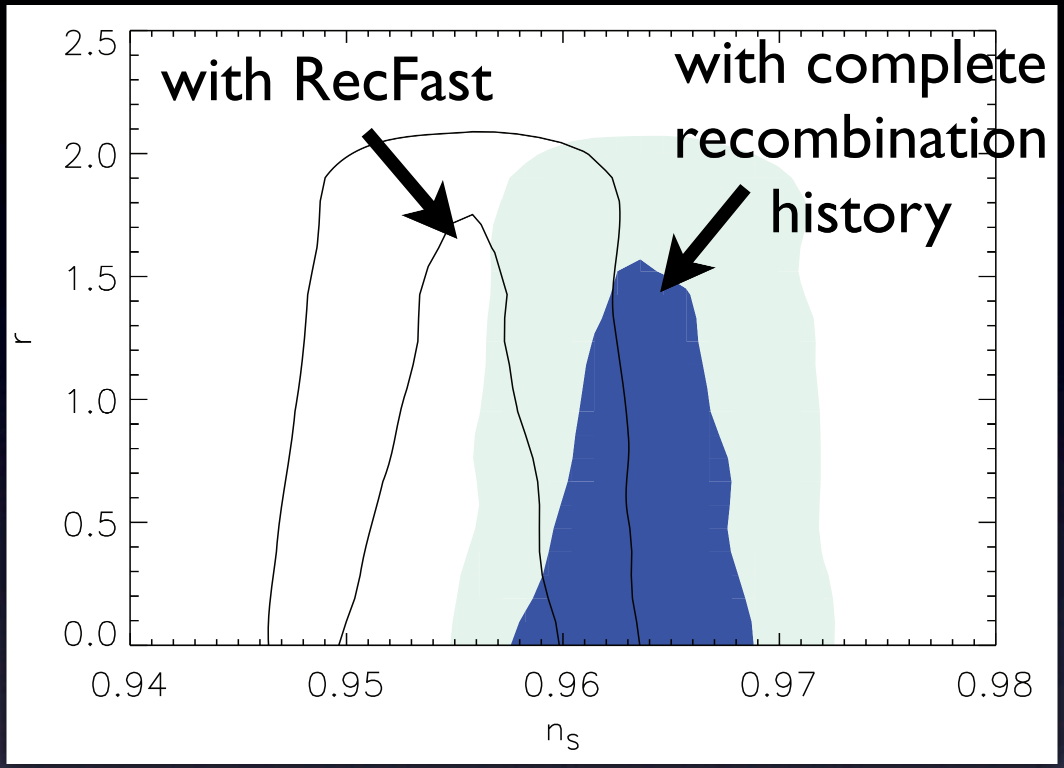
Motivations

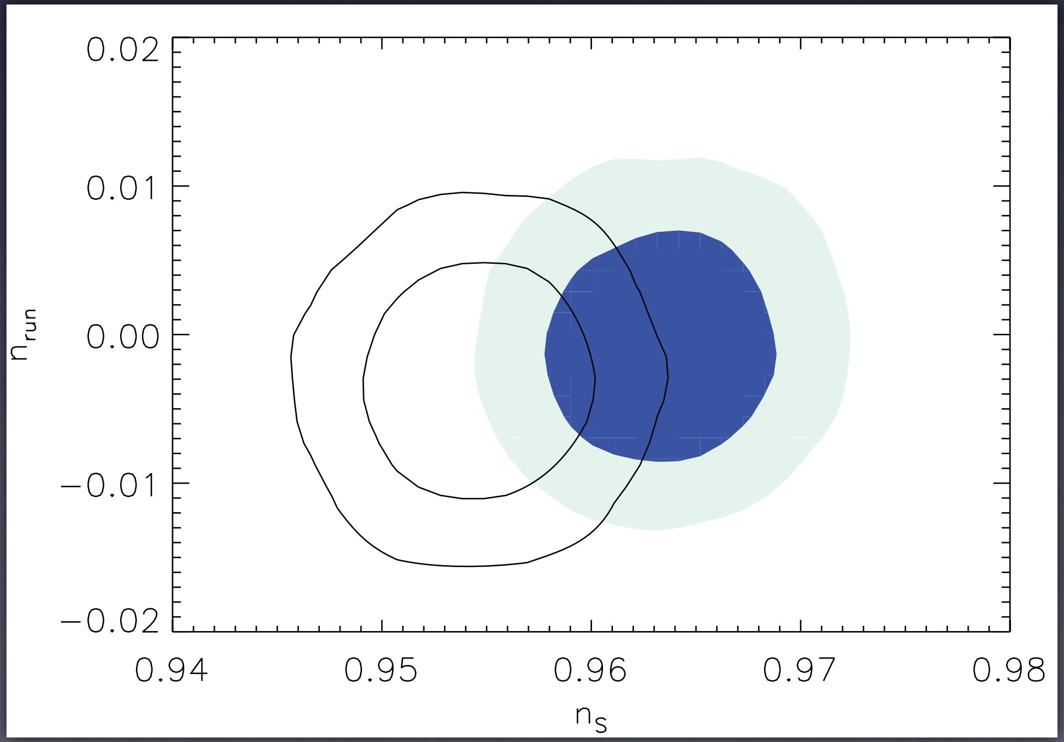
- High-precision data requires a highly accurate theory
- Major uncertainty: recombination (*Hu et al. 1995*)
    - ✦ Position and width of last scattering surface
    - ✦ Silk damping affects high-*l* anisotropy
- Helium recombination: smaller impact (ends at z~1700), but still important.

  See *Switzer & Hirata 2008, Chluba & Sunyaev (2010)...*
- Precision in hydrogen recombination is critical.

Motivations

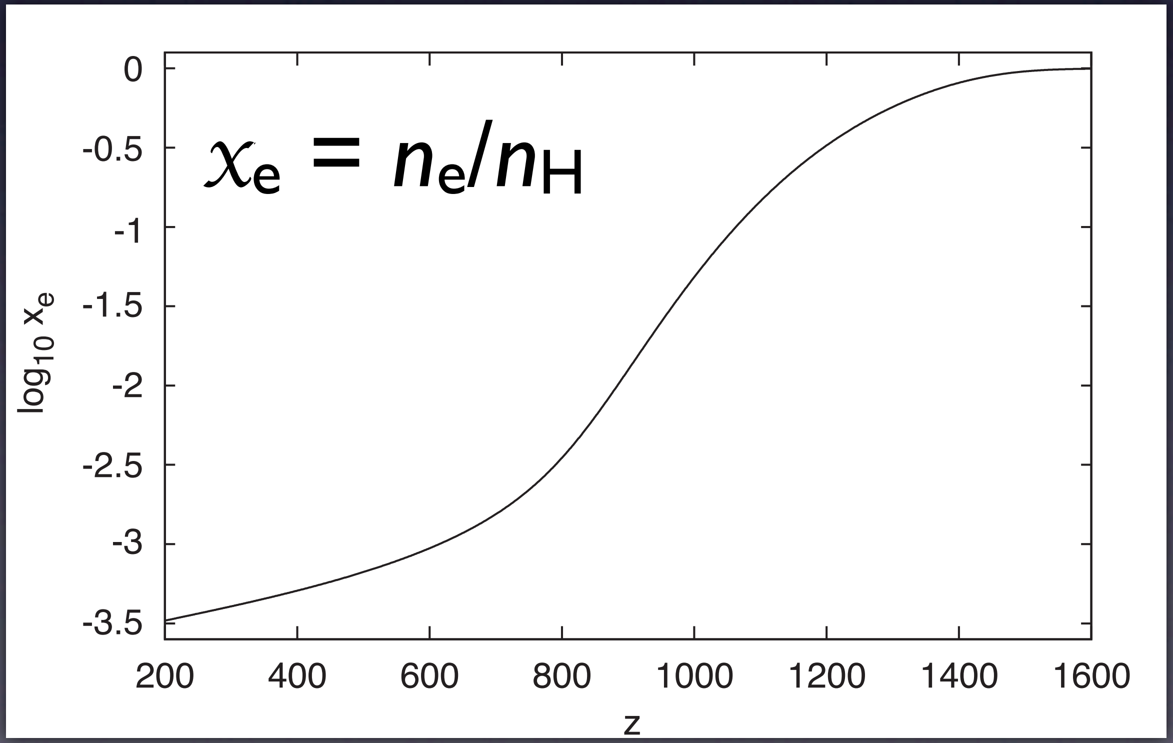
Rubiño-Martín et al. 2010





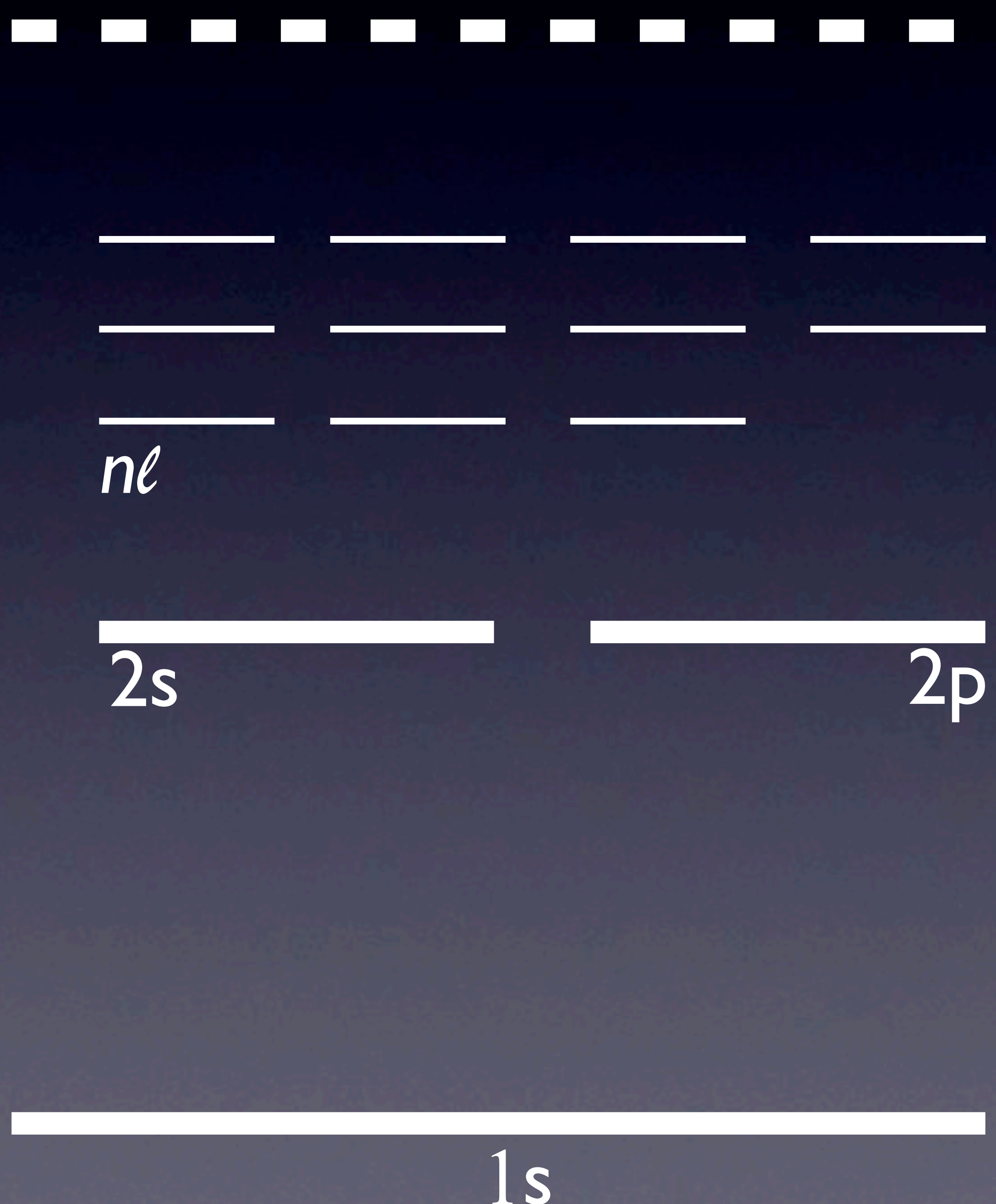
2.5 sigma bias in n_s if using RECFAST.1.4.2 (*Seager et al. 1999*, *Wong et al. 2008*) with Planck data

We need < 0.1% error in $x_e(z)$



The effective three-level atom

Peebles 1968, Zeldovich et al. 1968



- ✦ Assumes excited states in Boltzmann equilibrium with each other

- ✦ Effectively 3 states: 1s, n=2, e$^-$+p

- ✦ At all times $x_2 \ll 1$

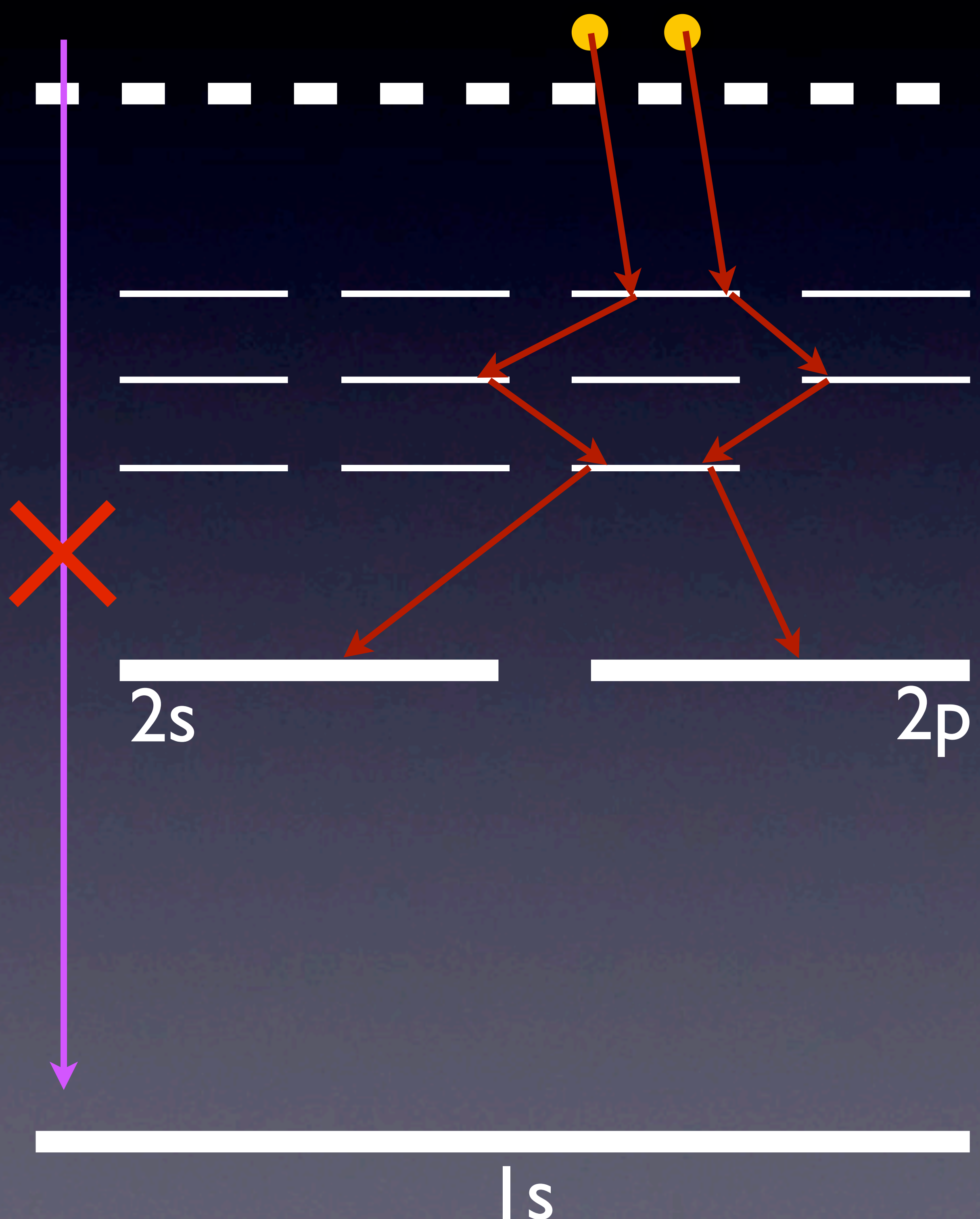
➡ $x_e + x_{1s} = 1$

- ✦ Need $\dot{x}_e(x_e, x_2, z)$

  $\dot{x}_2(x_e, x_2, z)$

The effective three-level atom

Peebles 1968, Zeldovich et al. 1968



✦ Direct recombinations to the ground state are highly inefficient

✦ Recombinations proceed to the excited states, followed by a "cascade" down to n = 2

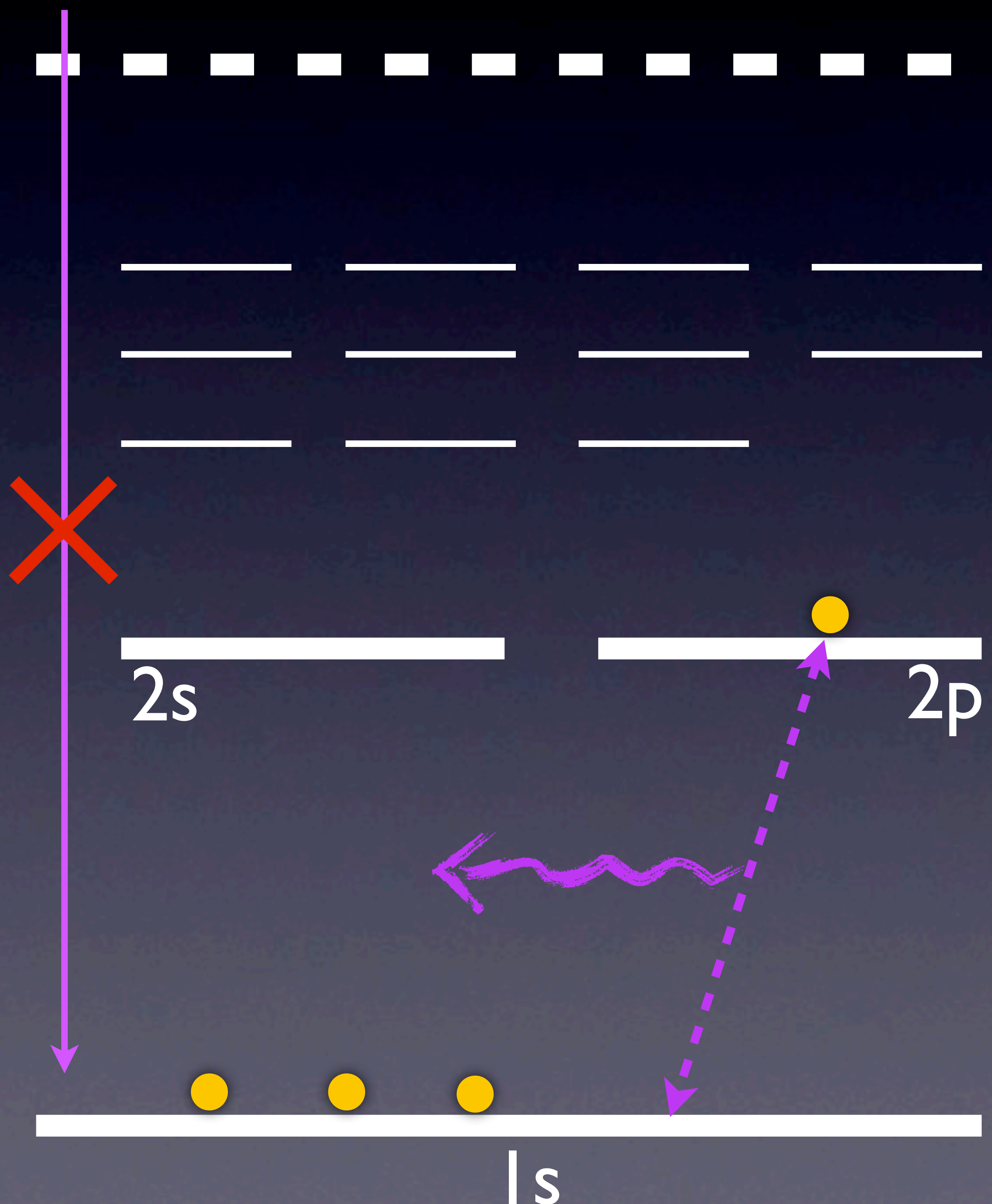
$$\begin{aligned} \dot{x}_e &= -n_{\rm H} x_e^2 \alpha_{\rm B}(T) + x_2 \beta_{\rm B}(T) \\ &= -\dot{x}_2\Big|_{\rm rec} \end{aligned}$$

$$\alpha_B(T) = \sum_{n\geq 2, l} \alpha_{nl}(T)$$

"Case B" recombination

The effective three-level atom

Peebles 1968, Zeldovich et al. 1968



- Decays from 2p are highly suppressed due to re-absorptions
- Hubble expansion allows photons to redshift and escape from resonance
- Sobolev escape probability:

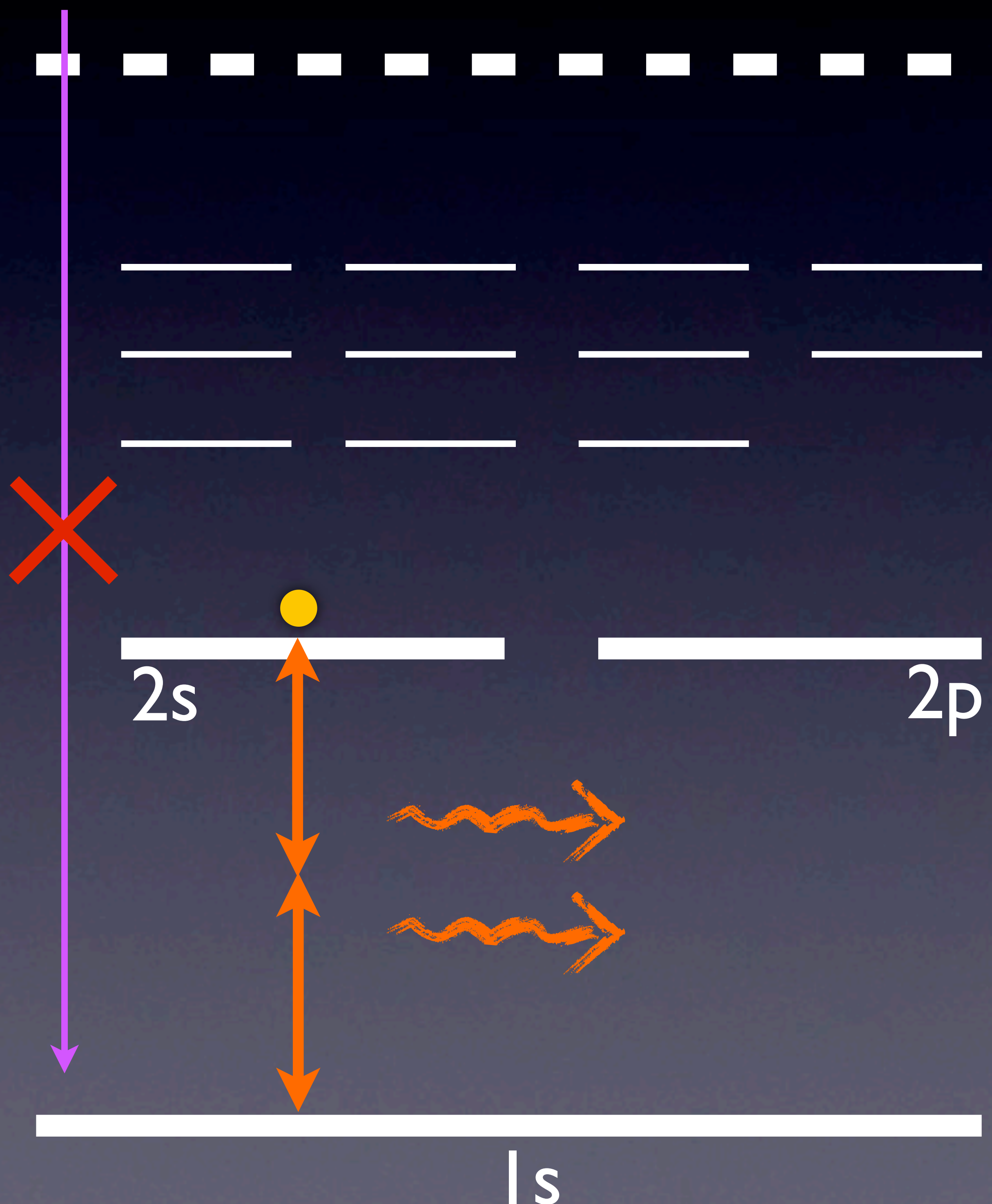
$$P_{\rm esc} = \frac{8\pi H(z)}{n_{\rm H} x_{1s} \lambda^3_{{\rm Ly}\alpha} A_{2p,1s}} \ll 1$$

$$\dot{x}_2\big|_{{\rm Ly}\alpha} = P_{\rm esc} \times A_{2p,1s}\left(-\frac{3}{4}x_2 + 3x_{1s}e^{-E_{21}/T}\right).$$

$$P_{\rm esc} \times A_{2p,1s} \sim 1 - 100 \text{ s}^{-1}$$

The effective three-level atom

Peebles 1968, Zeldovich et al. 1968



✦ The slow two-photon decays from the 2s state are comparable in efficiency to the slow escape of Lyα photons

$$\Lambda_{2s,1s} = 8.22\ \mathrm{s}^{-1}$$
$$\sim P_{\rm esc} \times A_{2p,1s}$$

$$\dot{x}_2\big|_{2\gamma} = \Lambda_{2s,1s}\left(-\frac{1}{4}x_2 + x_{1s}e^{-E_{21}/T}\right).$$

The effective three-level atom

Peebles 1968, Zeldovich et al. 1968

1) $\dot{x}_2 = \dot{x}_2\big|_{\rm rec} + \dot{x}_2\big|_{\rm Ly\alpha} + \dot{x}_2\big|_{2\gamma} \approx 0$

steady-state approximation: atomic rates >> H(z)

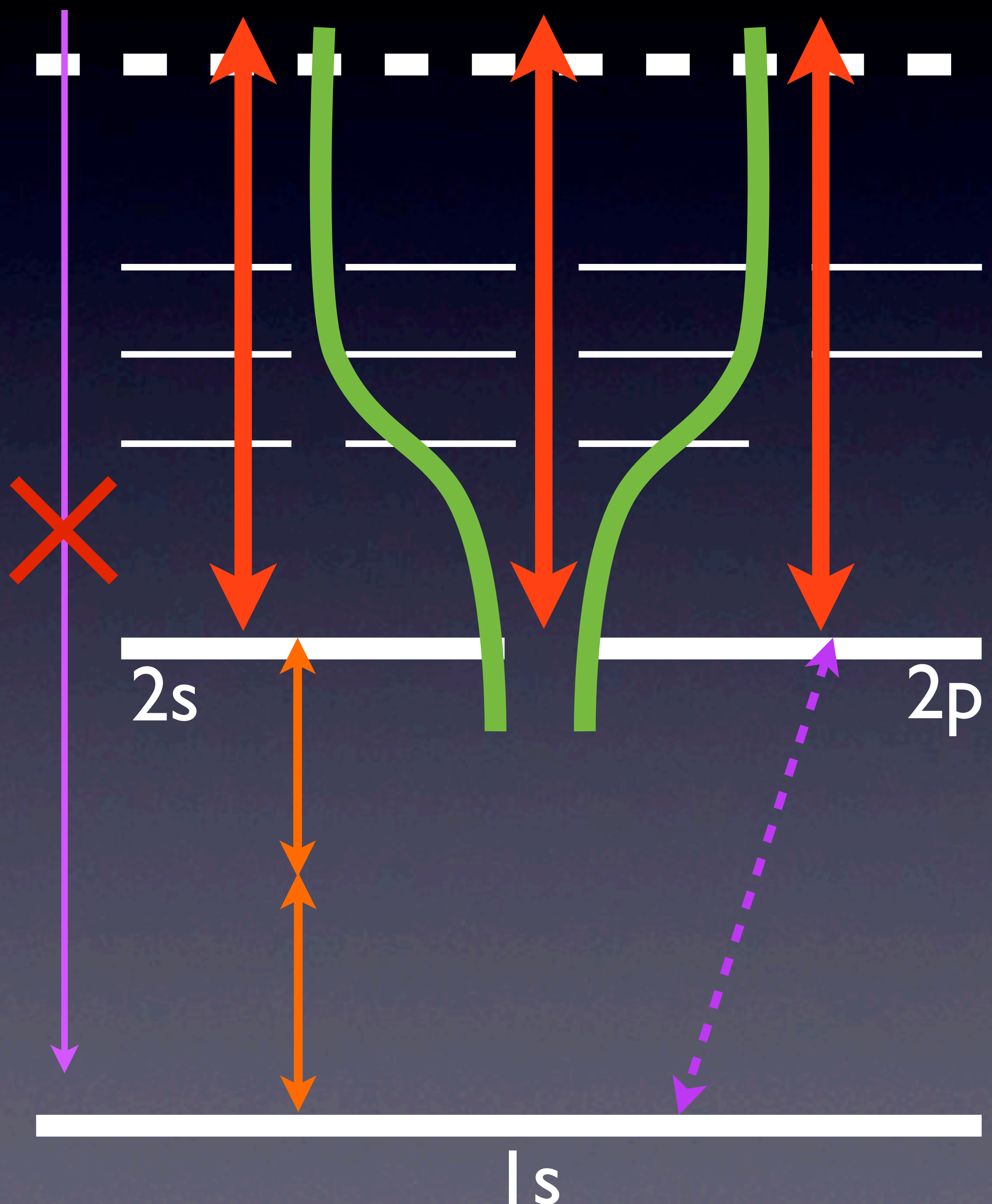
➥ Solve for $x_2(x_e, z; \Omega)$

2) $\dot{x}_e = -n_{\rm H} x_e^2 \alpha_{\rm B}(T) + x_2 \beta_{\rm B}(T)$

➥Obtain $x_e(z; \Omega)$

- Simple, yet very insightful!
- Not very accurate

Early times (z > 800-900)



✦ Intense radiation field
➡ Excited atoms are much more likely to be photoionized than decay to 1s
➡ Recombination dynamics governed by the slow decay rate to 1s

"n=2 bottleneck"

Requires accurate 2s↔1s and 2p↔1s rates

↪ Radiative transfer

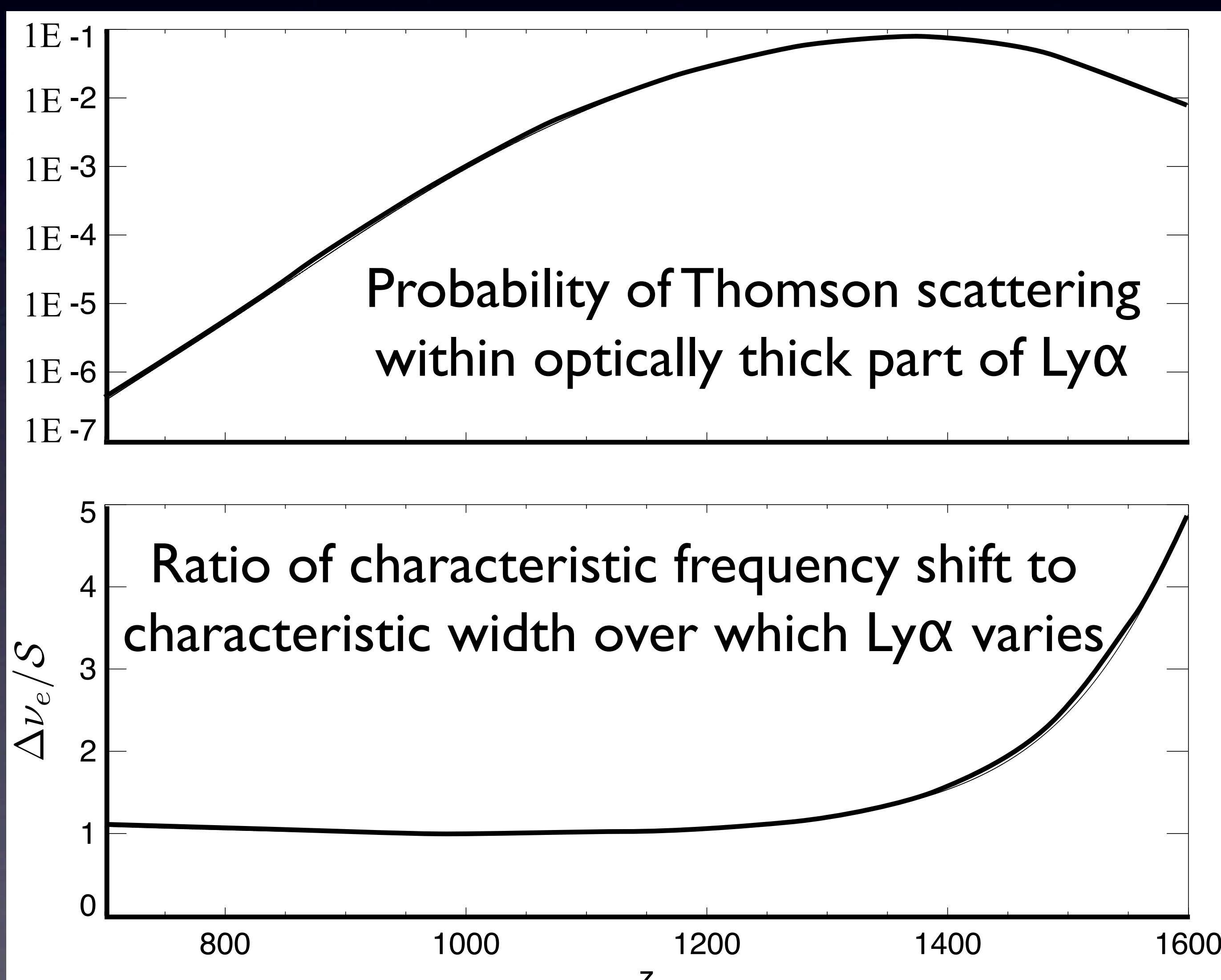
Important radiative transfer effects

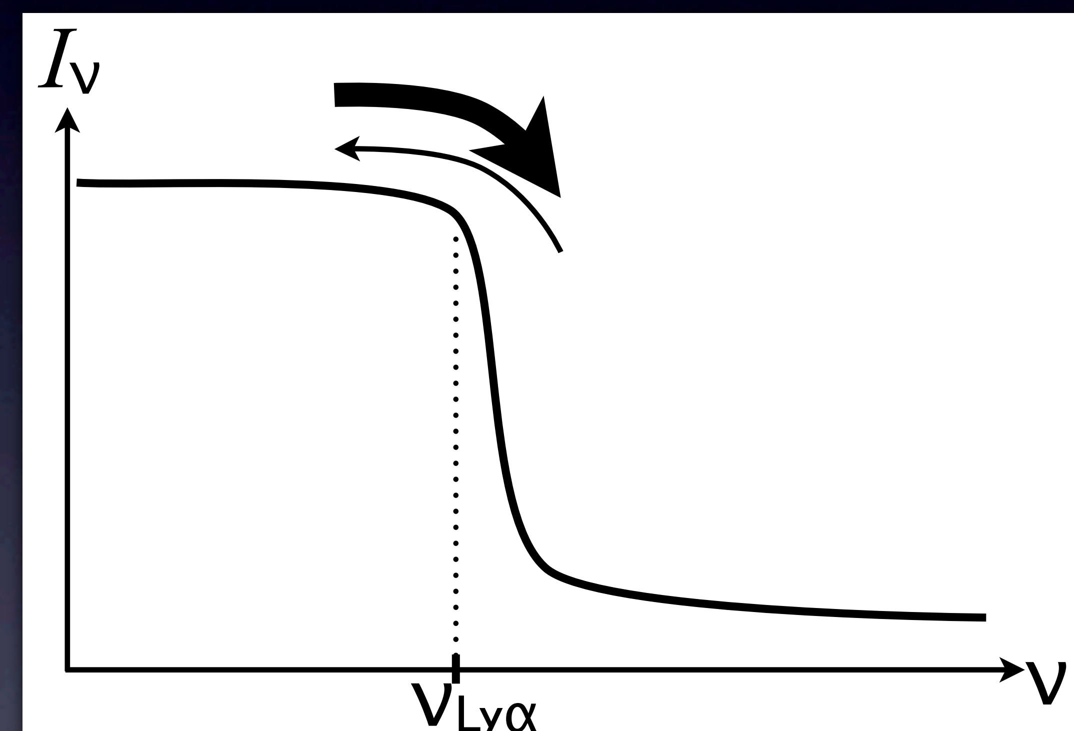
- 2s↔1s: include stimulated decays (*Chluba & Sunyaev 2006*) and non-thermal absorptions (*Kholupenko & Ivanchik 2006*)
- Feedback between Lyman lines
- Sobolev approximation breaks down for Lyα decays:
  - ✦ Time-dependent effects (*Chluba & Sunyaev 2009*)
  - ✦ Absorption profile ≠ emission profile
- Two-photon decays from *ns*, *nd* (n > 2) (*Dubrovich & Grachev 2005, Chluba & Sunyaev 2008, Hirata 2008*)
- Frequency diffusion in Lyα (*Hirata & Forbes 2009, Chluba & Sunyaev 2009*)

Other radiative transfer effects

Ali-Haïmoud, Grin & Hirata 2010 (arXiv:1009.4697)

- *Thomson scattering in Lyα*





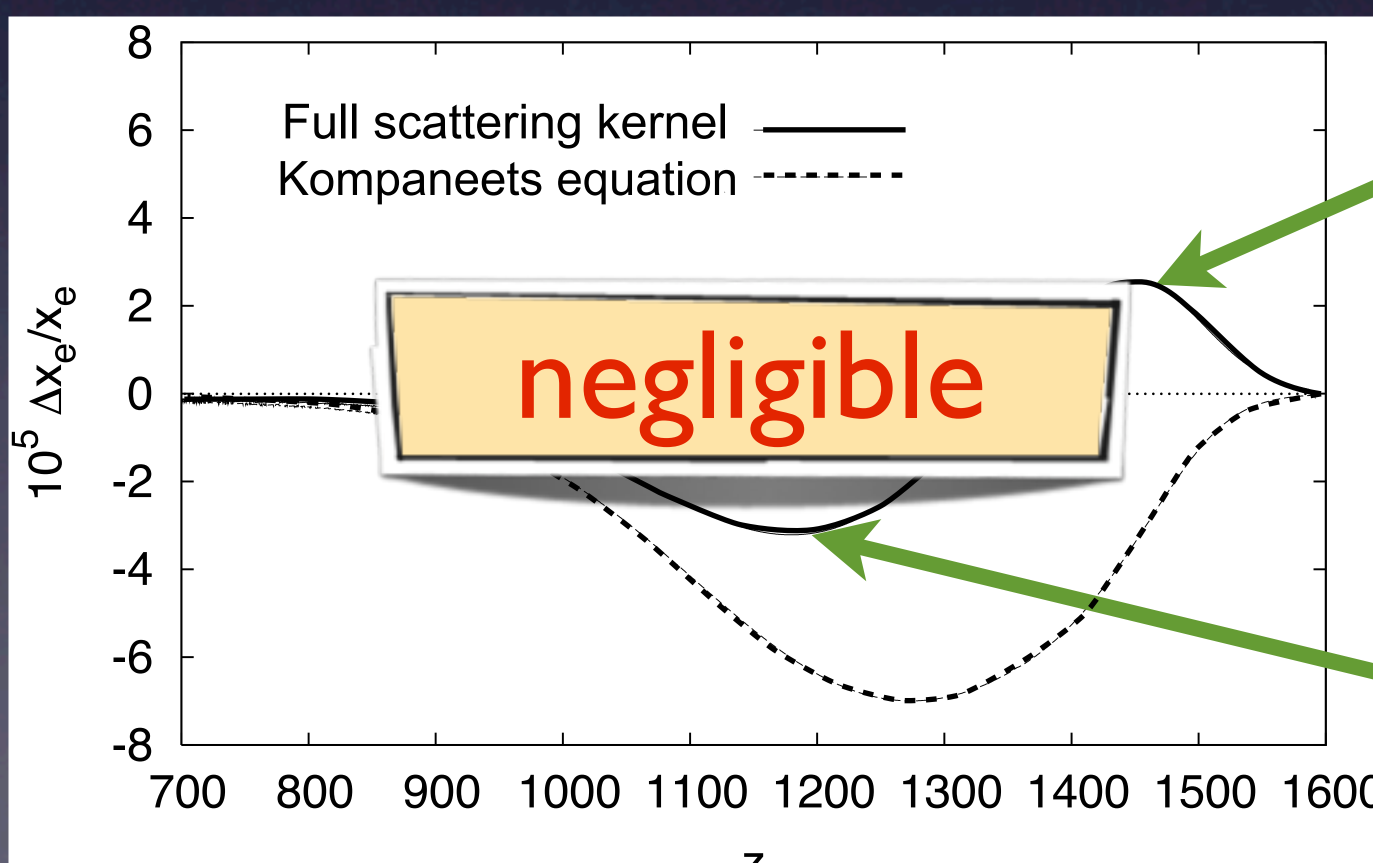
Kompaneets equation not valid in this context

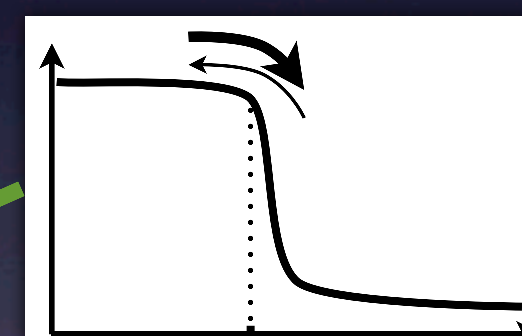
Other radiative transfer effects

- *Thomson scattering in Lyα*

Thomson collision term with full scattering kernel:

$$\dot{\mathcal{N}}_\nu\Big|_{\rm T} = n_e \sigma_{\rm T} c \left[-\mathcal{N}_\nu + \int \mathcal{N}_{\nu'} R_{\rm T}(\nu', \nu) {\rm d}\nu\right]$$



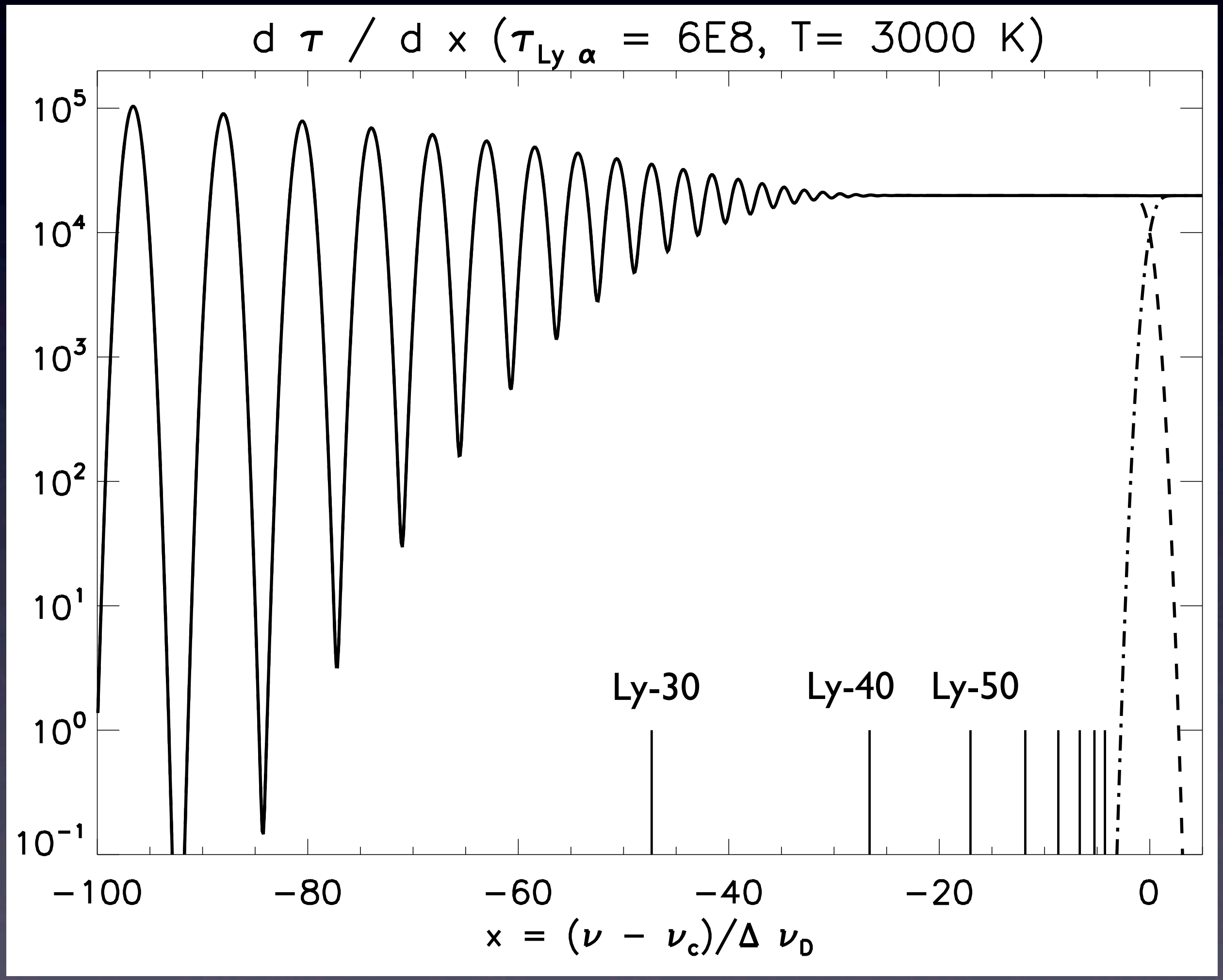


Systematic recoil

$$\frac{\langle \Delta\nu \rangle}{\nu} = \frac{4kT_m - h\nu}{m_e c^2}$$

Other radiative transfer effects

- Overlap of the high lying Lyman lines

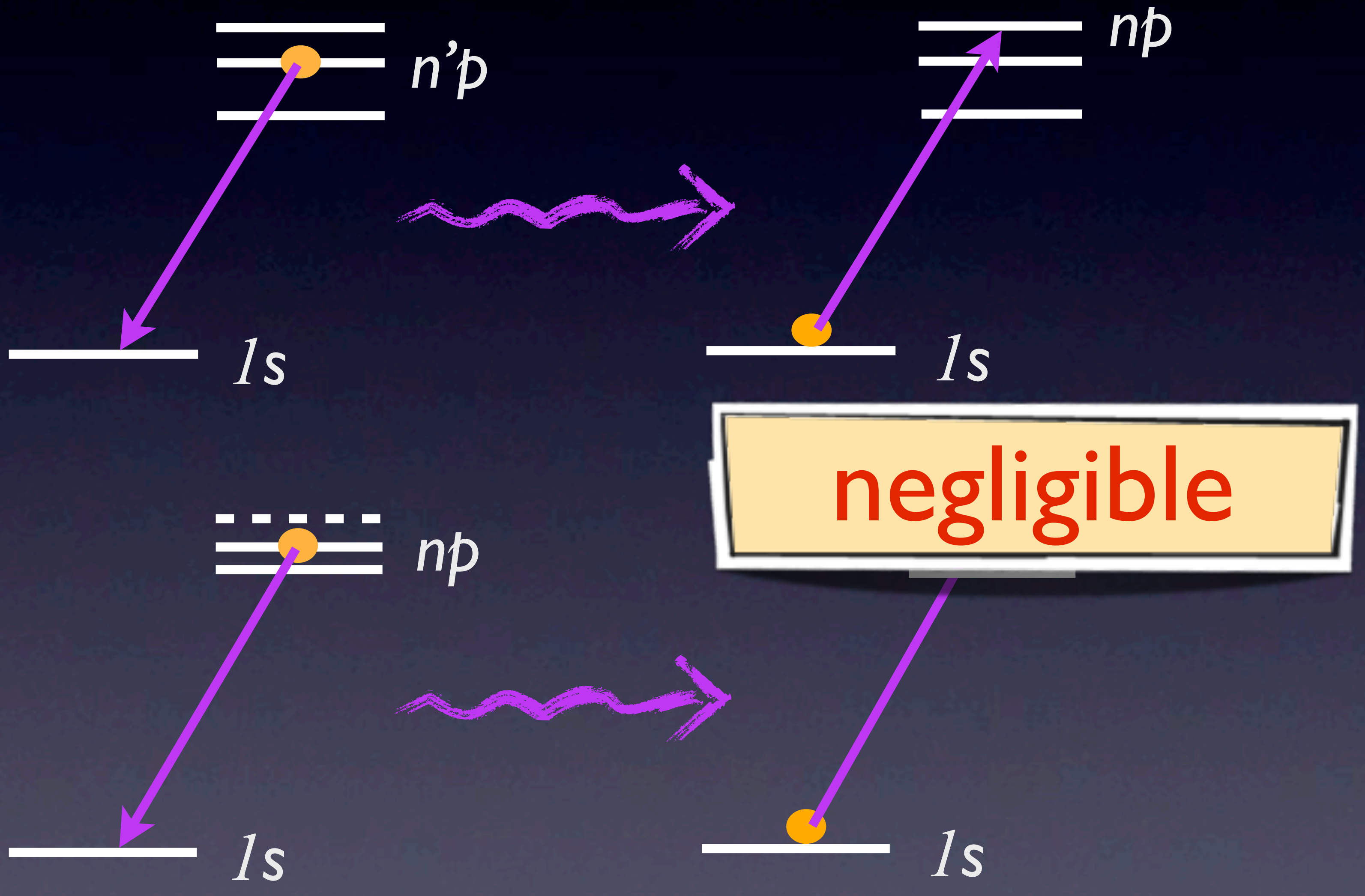


- For n > 40, Ly-n and Ly-(n+1) are within 1 Doppler width of each other

- For n > 200, Ly-n is within 1 Doppler width of Ly-continuum.

Other radiative transfer effects

- Overlap of the high lying Lyman lines



Overlap-induced transitions:

$$R_{n'p,np}(T_m)$$

Overlap-induced photoionizations and recombinations:

$$\beta_{np}(T_m), \alpha_{np}(T_m)$$

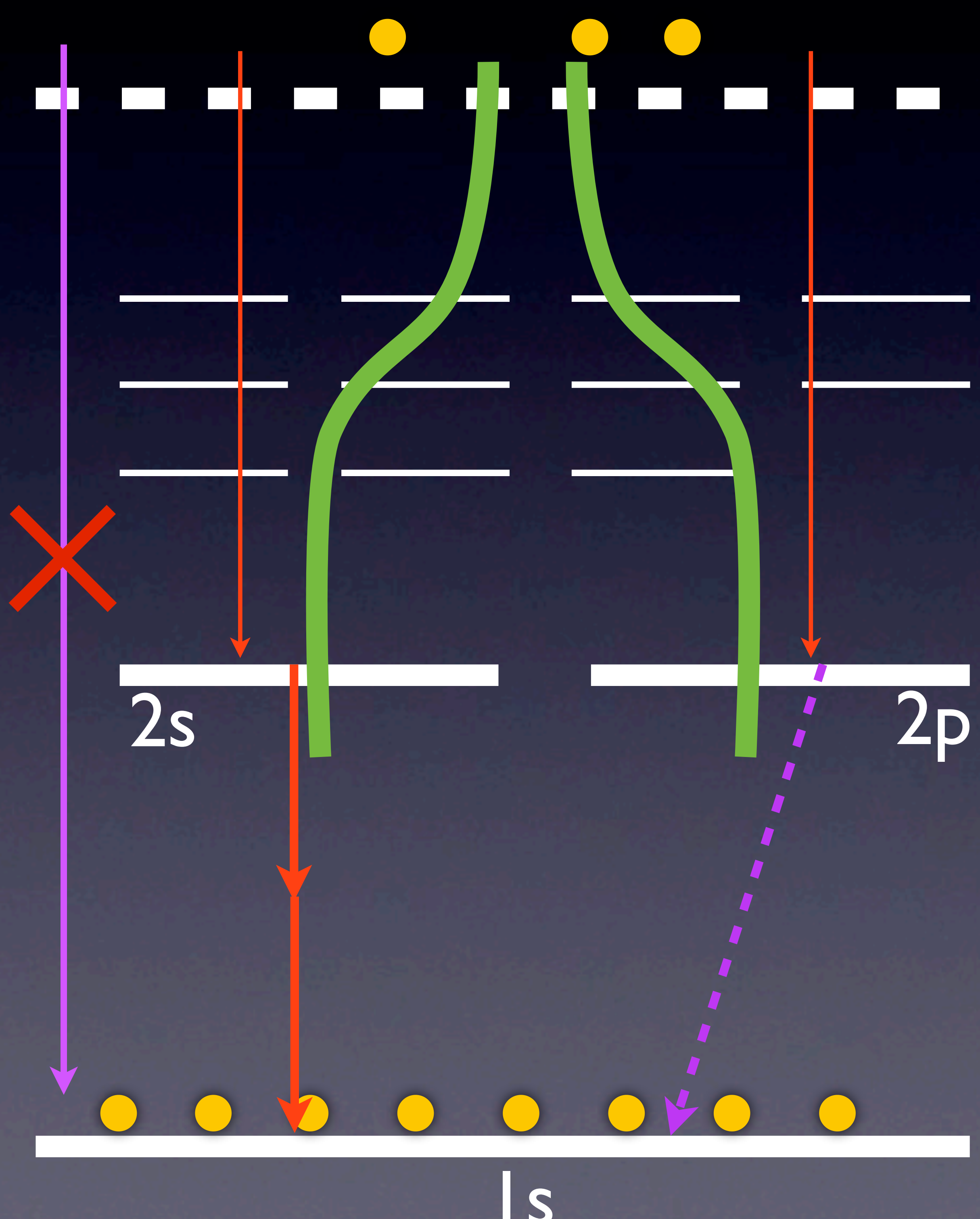
Implemented in a multi-level atom code (to be described in a few minutes)

Other radiative transfer effects

- Interaction of hydrogen and deuterium Ly-α lines
- Masers
- Quadrupole transitions *ns/nd ↔ 1s (Grin & Hirata 2010)*
- .....

negligible

Late times (z < 800-900)



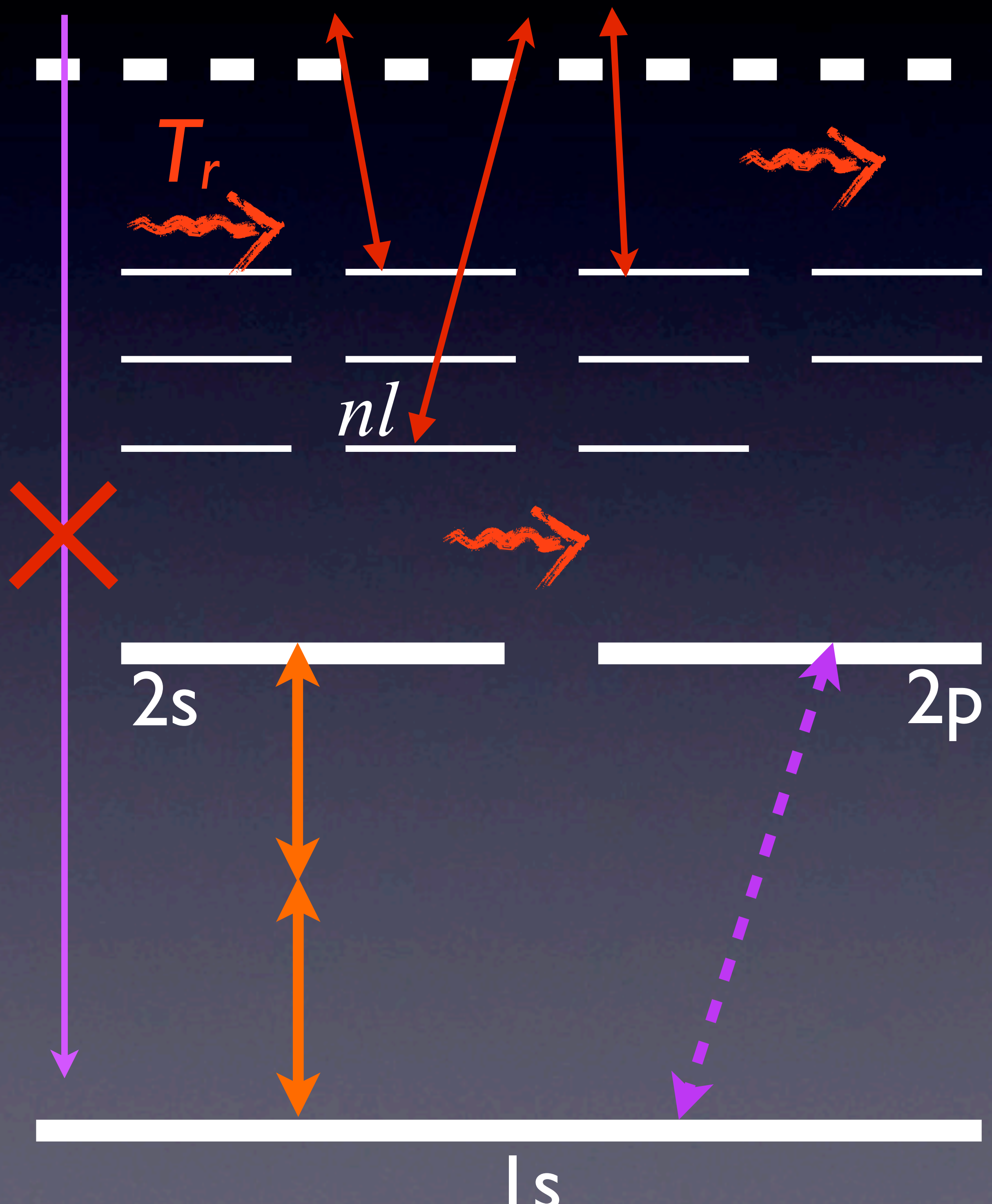
✦ Few free electrons and protons
➡ low rate of recombinations

$$\dot{x}_e \approx -n_{\rm H} x_e^2 \alpha_B(T)$$

✦ Low temperature
➡ excited states are out of Boltzmann equilibrium

Require accurate effective recombination rate and accounting for out-of-equilibrium effects

The multi-level atom (MLA)



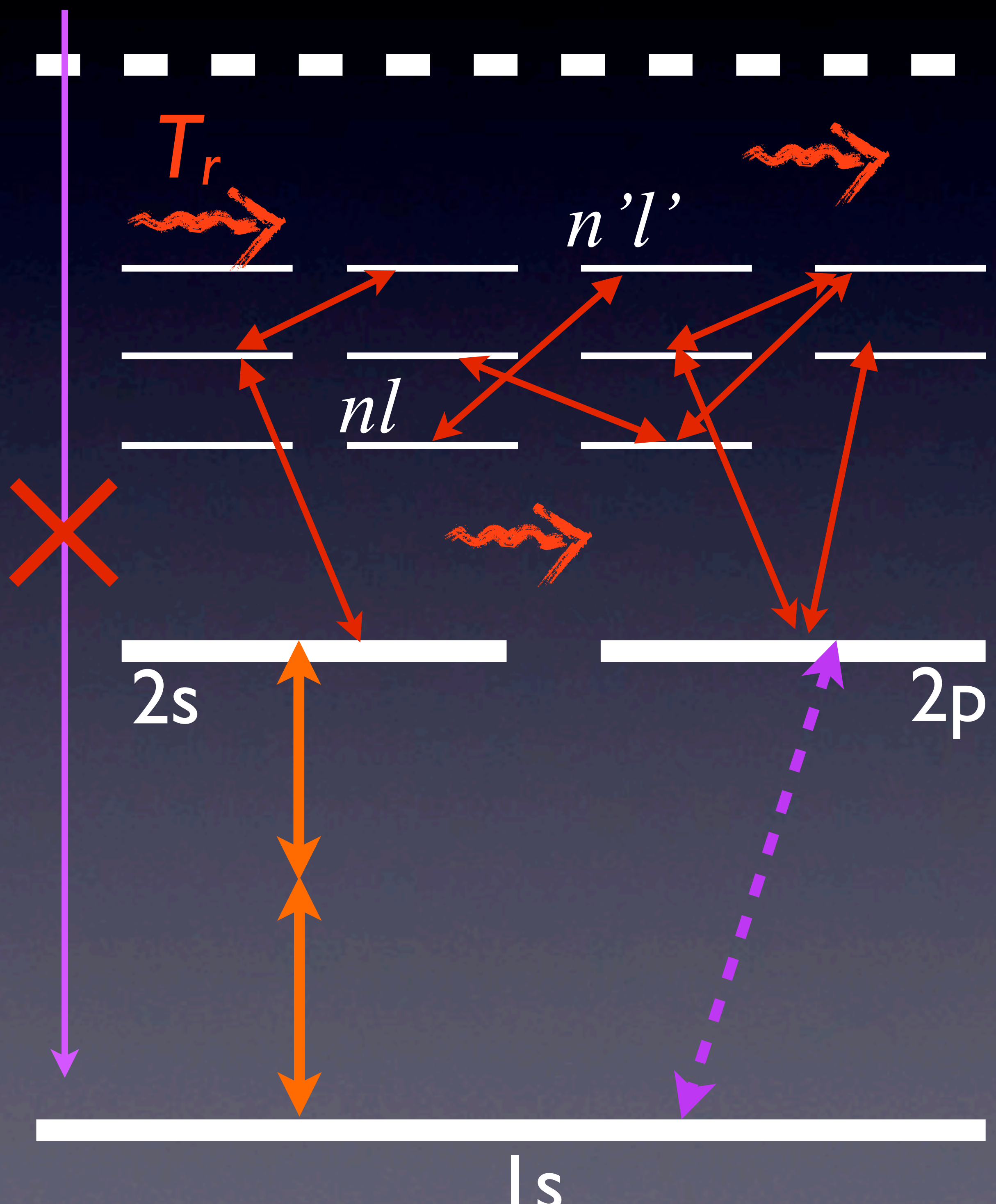
- Bound-free transitions:
- Recombination coefficient to nl (including stimulated by blackbody photons):

$$\alpha_{nl}(T_m, T_r)$$

- Rate of photoionization by blackbody photons from nl:

$$\beta_{nl}(T_r)$$

The multi-level atom (MLA)



✦ Bound-bound transitions:

✦ Transition rate from nl to $n'l'$:

$$R_{nl,n'l'}(T_r)$$

(absorption of blackbody photons if n < n', emission stimulated by blackbody photons if n > n')

The standard MLA method

- Follow populations of all excited state, x_{nl} , x_{2s} , x_{2p}:

$$0 \; \dot{x}_{nl} = n_{\rm H} x_e^2 \alpha_{nl} - x_{nl}\beta_{nl} + \sum_{n'l'} x_{n'l'} R_{n'l',nl} - \sum_{n'l'} x_{nl} R_{nl,n'l'}$$

$$0 \; \dot{x}_{2s} = n_{\rm H} x_e^2 \alpha_{2s} - x_{2s}\beta_{2s} + \sum_{n'l'} x_{n'l'} R_{n'l',2s} - \sum_{n'l'} x_{2s} R_{2s,n'l'}$$
$$+ x_{1s}\tilde{R}_{1s,2s} - x_{2s}\tilde{R}_{2s,1s}$$

- Solve for the populations of the excited states in the ***steady-state approximation***

- Invert linear system, obtain $x_{nl}\,(x_e, z)$, $x_{2s}\,(x_e,\ z)$, $x_{2p}\,(x_e,\ z)$

- Evolve x_e: $\dot{x}_e \approx -\dot{x}_{1s} = x_{1s}\tilde{R}_{1s,2s} - x_{2s}\tilde{R}_{2s,1s} + x_{1s}\tilde{R}_{1s,2p} - x_{2p}\tilde{R}_{2p,1s}$

- ***Iterate at each timestep***

The standard MLA method

- Seager et al. 1999, 2000: MLA up to $n_{\rm max}$ = 300, assuming statistical equilibrium of angular momentum substates $$x_{nl} = (2l+1)/n^2 \times x_n$$
  Results fitted with a fudged effective three-level atom:
  $$\alpha_B(\text{used}) = 1.14 \times \alpha_{\rm B}$$
- At late times, ℓ-substates fall out of equilibrium (Chluba et al. 2006)
- Grin & Hirata 2010, Chluba et al. 2010: need $n_{\rm max}$ > 100 for Planck. ➡ Need to follow $n_{\rm max}(n_{\rm max}+1)/2$ > 5000 states!

The standard MLA method

- Fastest codes take hours to days for a single run.

Too slow for inclusion in Markov Chains for cosmological parameter estimation

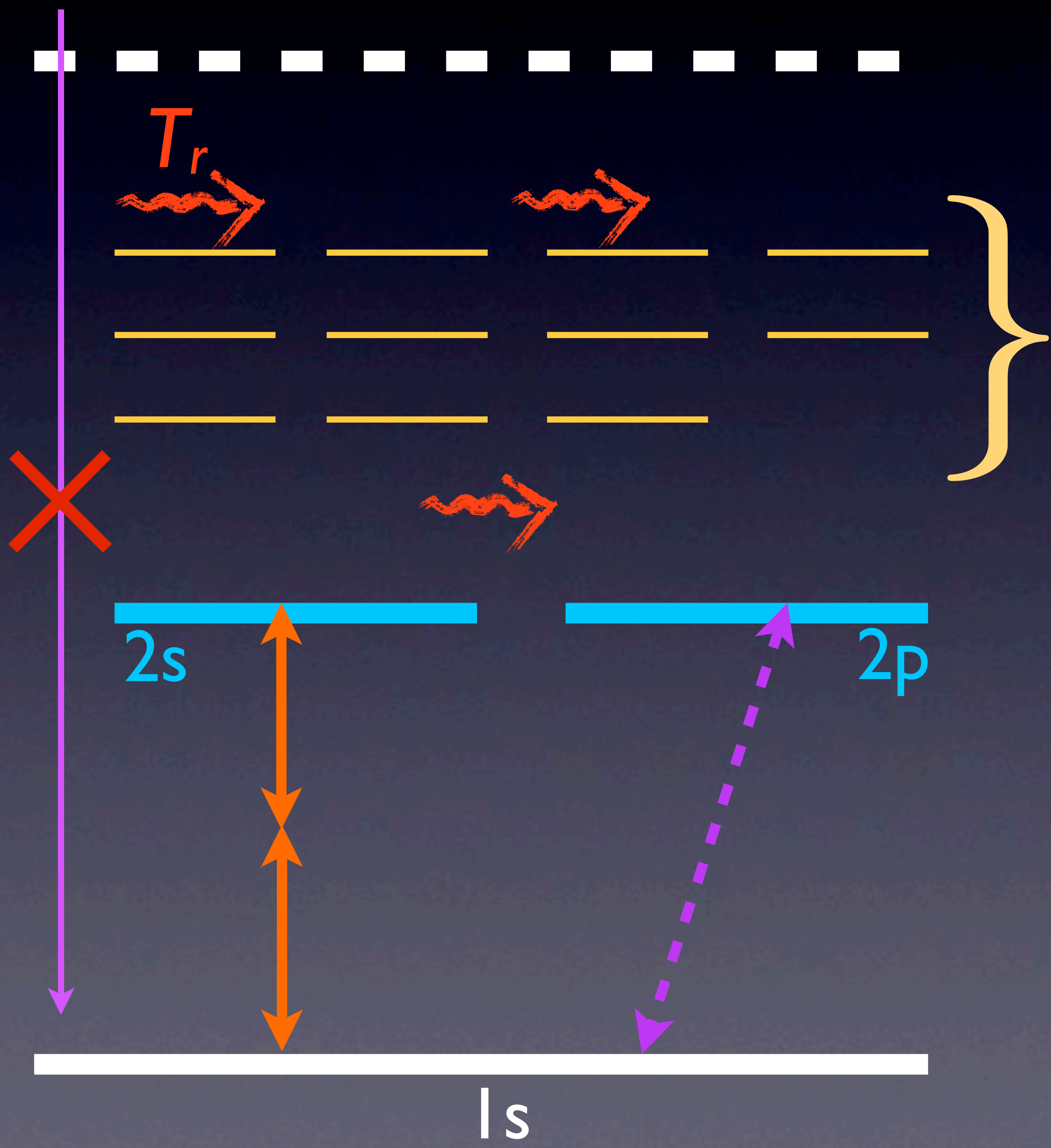
- Suggested solutions:
- More fudge factors (Wong & Scott 2007)
- Multidimensional interpolation for

$x_e(z\,; T_0, \Omega_b h^2, \Omega_m h^2, \Omega_\Lambda h^2, H_0, Y_{He}, \ldots)$ (Fendt et al 2009)

- Work in principle, but in fact not needed.

The effective MLA approach

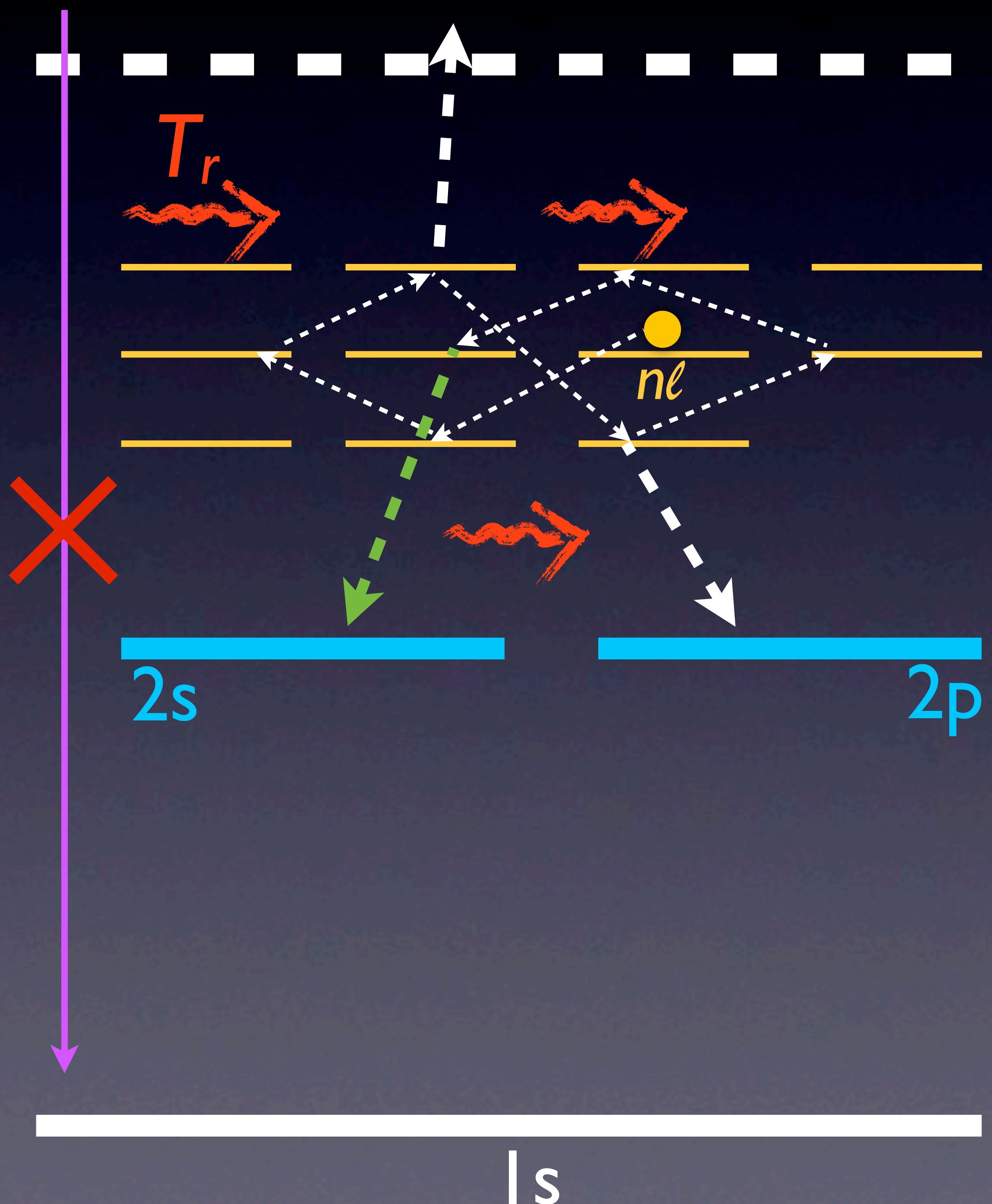
Ali-Haïmoud & Hirata, PRD, 2010 (arXiv:1006.1355)



✦ "interior" states: not directly connected to 1s
✦ Very fast *optically thin* transitions. Depend only on T_r

✦ "interface" states" radiatively connected to 1s
✦ Transition rates may be complicated functions of x_e, z, cosmological parameters Ω

The effective MLA approach



✦ Instead of computing $x_{nl}(x_e, z, \Omega)$, consider the probabilities:

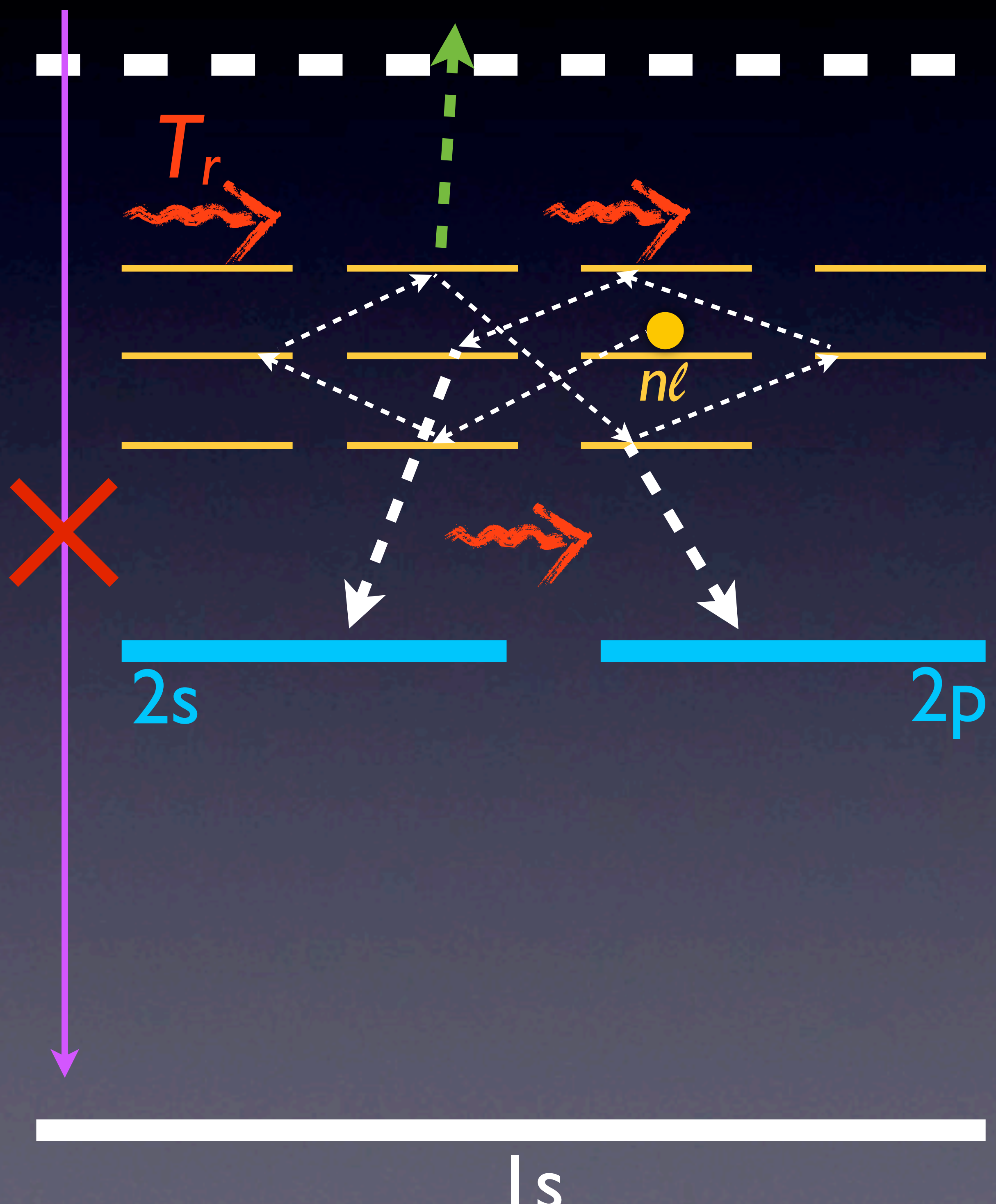
$$P(nl \dashrightarrow 2s), P(nl \dashrightarrow 2p), P(nl \dashrightarrow e^- p)$$

$$P(nl \dashrightarrow 2s) = \frac{R_{nl,2s}}{\Gamma_{nl}} + \sum_{n'l'} \frac{R_{nl,n'l'}}{\Gamma_{nl}} P(n'l' \dashrightarrow 2s)$$

$$\Gamma_{nl} \equiv R_{nl,2s} + \sum_{n'l'} R_{nl,n'l'} + \beta_{nl}$$

Depend only on T_r

The effective MLA approach



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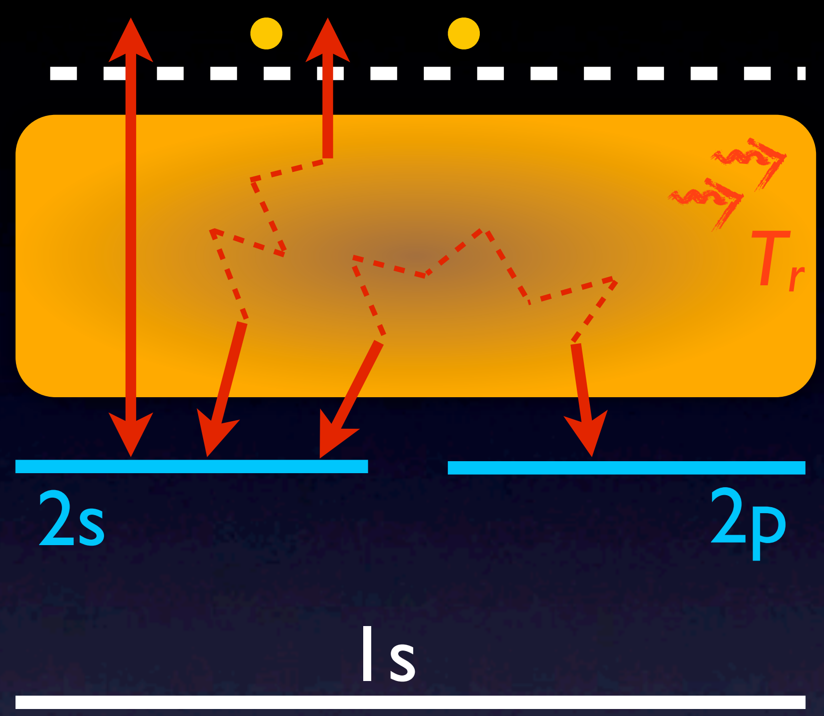
$$\Gamma_{nl} \equiv R_{nl,2s} + \sum_{n'l'} R_{nl,n'l'} + \beta_{nl}$$

Depend only on T_r

The effective MLA approach



✦ Effective recombination coefficient to the 2s state:

$$\mathcal{A}_{2s}(T_m, T_r) \equiv \alpha_{2s}(T_m, T_r) + \sum_{nl} \alpha_{nl}(T_m, T_r) P(nl \dashrightarrow 2s)$$

✦ Effective photoionization rate from the 2s state:

$$\mathcal{B}_{2s}(T_r) \equiv \beta_{2s}(T_r) + \sum_{nl} R_{2s,nl}(T_r) P(nl \dashrightarrow e^- p)$$

✦ Effective transfer rate from 2s to 2p:

$$\mathcal{R}_{2s,2p}(T_r) \equiv \sum_{nl} R_{2s,nl}(T_r) P(nl \dashrightarrow 2p)$$

The effective MLA approach

- Tabulate $\mathcal{A}_{2s}(T_m, T_r), \mathcal{A}_{2p}(T_m, T_r), \mathcal{R}_{2s,2p}(T_r)$

- Effective FOUR-level atom 1s, 2s, 2p, e$^-$+p (can be extended to include Lyβ decays...)

$$
\begin{aligned}
0\ \cancel{\dot{x}_{2s}} &= n_{\rm H} x_e^2 \mathcal{A}_{2s} - x_{2s}\mathcal{B}_{2s} + x_{2p}\mathcal{R}_{2p,2s} - x_{2s}\mathcal{R}_{2s,2p} \\
&+ x_{1s}\tilde{R}_{1s,2s} - x_{2s}\tilde{R}_{2s,1s}
\end{aligned}
$$

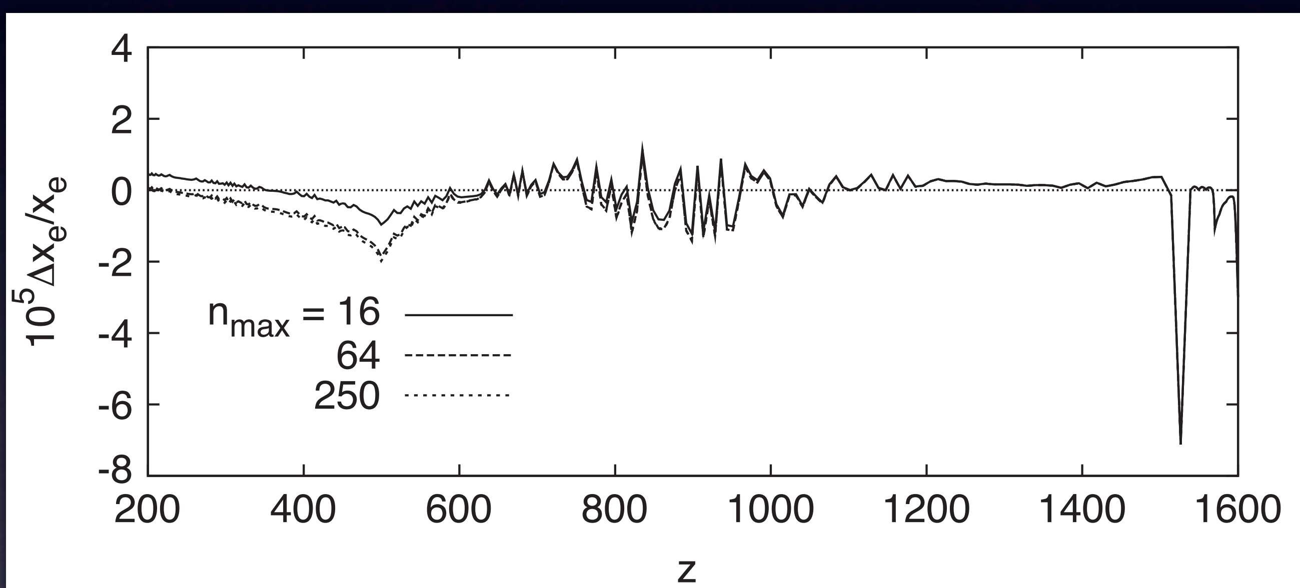
$$
\begin{aligned}
0\ \cancel{\dot{x}_{2p}} &= n_{\rm H} x_e^2 \mathcal{A}_{2p} - x_{2p}\mathcal{B}_{2p} + x_{2s}\mathcal{R}_{2s,2p} - x_{2p}\mathcal{R}_{2p,2s} \\
&+ x_{1s}\tilde{R}_{1s,2p} - x_{2p}\tilde{R}_{2p,1s}
\end{aligned}
$$

$$\dot{x}_e = -n_{\rm H} x_e^2 \mathcal{A}_{2s} + x_{2s}\mathcal{B}_{2s} - n_{\rm H} x_e^2 \mathcal{A}_{2p} + x_{2p}\mathcal{B}_{2p}$$

The effective MLA approach

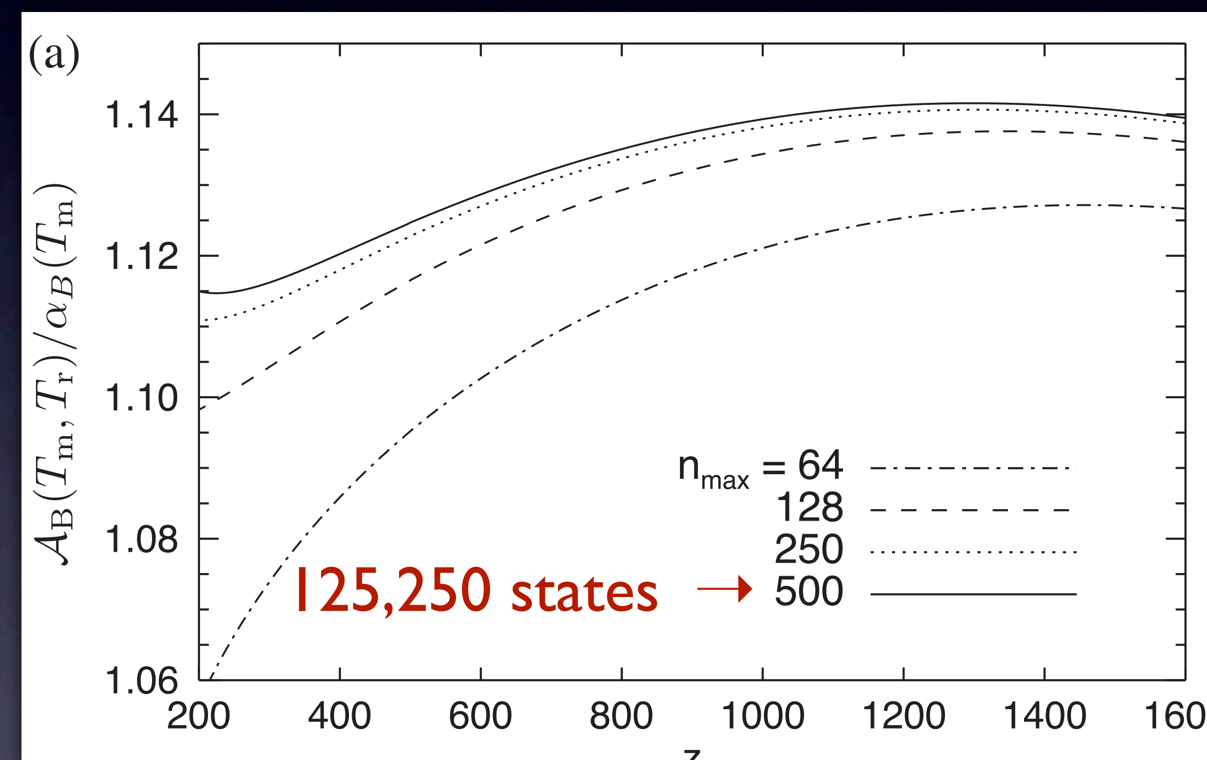
- Exactly equivalent to the standard MLA method.

Proof involves $\vec{x}\cdot\mathrm{M}\cdot\vec{y} = \vec{y}\cdot\mathrm{M}^{\mathrm{T}}\cdot\vec{x}$



Fractional difference with Dan Grin's standard MLA code

0.08 sec instead of 1 week!



The "exact fudge factor"

$$\mathcal{A}_{\rm B} \equiv \mathcal{A}_{2s} + \mathcal{A}_{2p}$$

The effective MLA approach

This is just Peebles' three-level atom with a twist (Peebles 1968, Zeldovich et al. 1968):

Peebles: $$\alpha_B(T_m) = \sum_{n\geq 2,l} \alpha_{nl}(T_m, T_r = 0)$$

EMLA: $$\mathcal{A}_{2s}(Tm, Tr), \mathcal{A}_{2p}(Tm, Tr)$$

$$\mathcal{A}_B(T_m, T_r) = \sum_{n\geq 2,l} \alpha_{nl}(T_m, T_r) P(nl \dashrightarrow 2)$$

accounts for stimulated recombinations

$\neq 1$

Advertising time

HyRec

A code for primordial hydrogen and helium recombination including radiative transfer

Ali-Haïmoud & Hirata, arXiv:1011.3758

- Contains all the effects mentioned before + helium corrections (*Switzer & Hirata 2008*)
- Original “non-perturbative” solution of radiative transfer
- Aside from collisions, accuracy: a few times 10^{-3} for helium, a few times 10^{-4} for hydrogen
- Computes a recombination history in ~2 seconds
- Also recently released: J. Chluba’s code (ongoing detailed comparison)

Conclusions

- To fully take advantage of Planck and other upcoming high-precision CMB experiments, an accurate recombination history is required
- We are starting to believe that all major radiative transfer effects have now been addressed
- The "high-n" MLA problem now solved
- Future work: accounting for collisions. Accurate rates are required. Effective rates will then all depend on n_e, T_m, T_r
- HyRec now available!
- Refs: arXiv:1006.1355, arXiv:1009.4697, arXiv:1011.3758

Thank you!