Abundance of halos and voids in the excursion set approach

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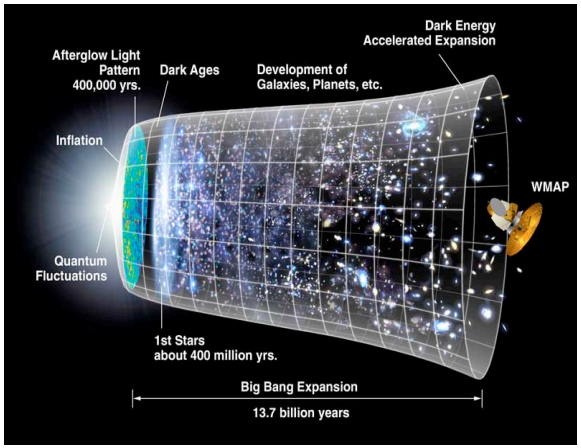
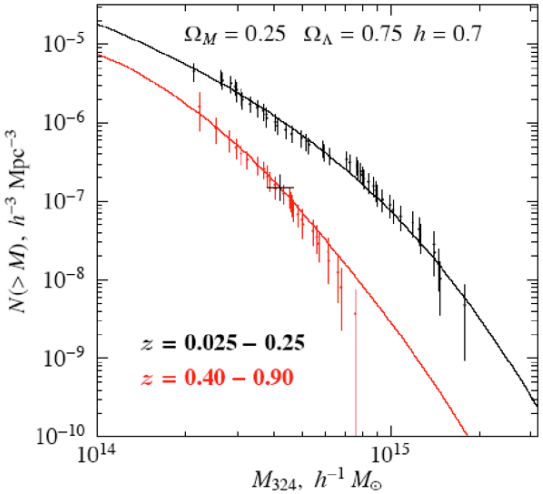
Cosmic Structures & Dark Matter

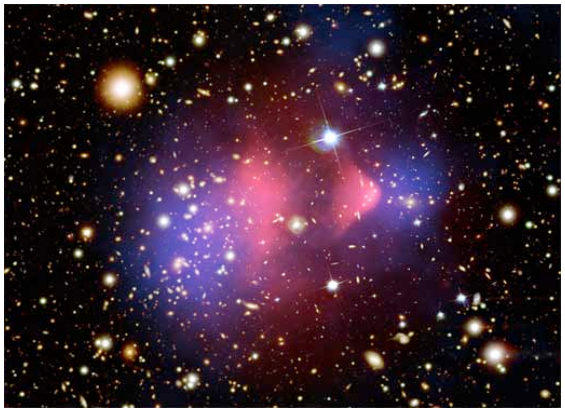
Hierarchical scenario:
- Initial density perturbation = seed for LSS
- Virialized structure=DM halos
- Building block of the cosmic structure



Dark Matter Halos mass function:
- Statistics of primordial fluctuation
- Non-linear gravitational processes
- Evolution in time sensitive to underline cosmology



Measurement of the halo mass function from ROSAT (Vikhlinin et al. 2007



X-Ray: NASA/CXC/CfA/M.Markevitch et al.; Optical: NASA/STScI; Magellan/U.Arizona/D.Clowe et al.; Lensing Map: NASA/STScI; ESO WFI; Magellan/U.Arizona/ D.Clowe et al

OUTLINES

Excursion set Theory

- Analytical predictions for Marvovian walks
- Corrections for realistic mass definitions

A Drifting Diffusing Barrier

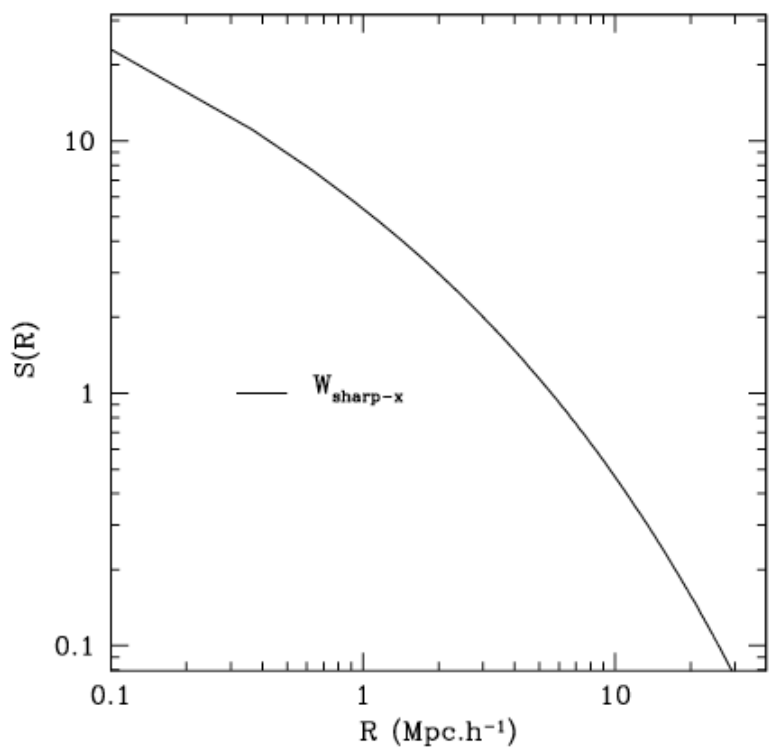
- Comparison with N-body simulations
- A self-consistent theory
- Voids abundance

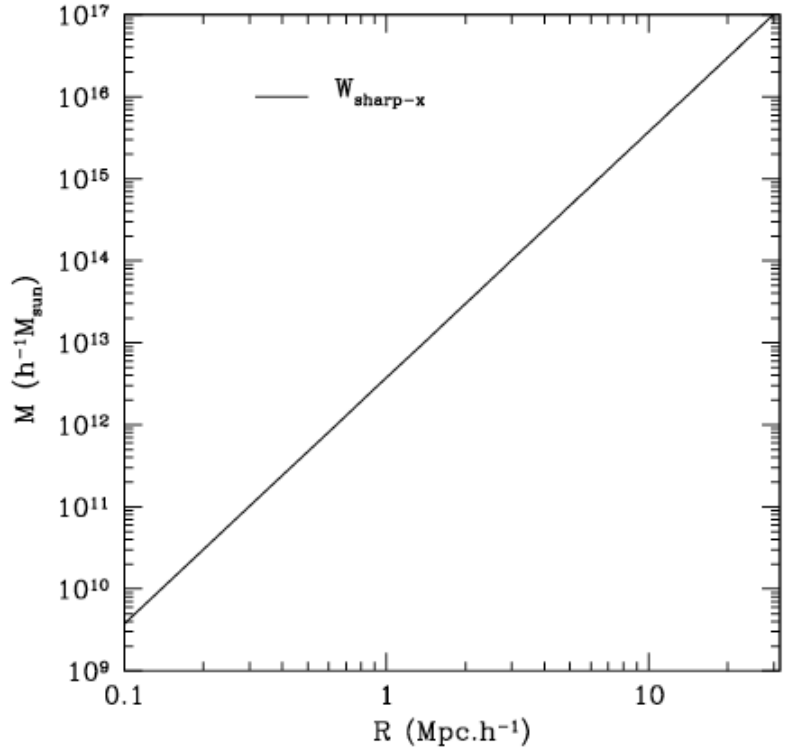
Conclusions & Perspectives

- Sensitivity of collapse parameters to non standard cosmology
- Application to the halo model

Variance Smoothing Radius & Mass

$$S = \langle \delta^2(R) \rangle \equiv \sigma^2(R[M]) = \frac{1}{2\pi^2} \int dk\ k^2 P(k) \tilde{W}^2[k, R(M)]$$



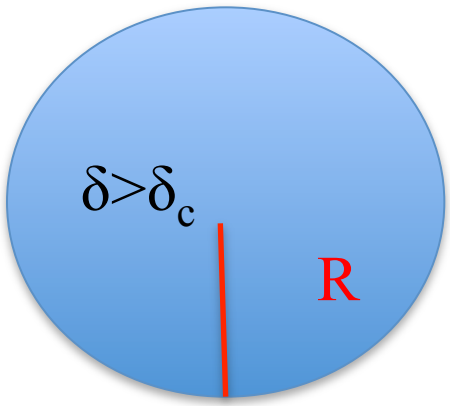


$$M = V(S)\bar{\rho}$$

Press-Schechter approach (70's)

Idea:

- DM halo mass M(R)= region where linear density filed is above some threshold (e.g. δ_c)



$M=\rho V(R)$

Definition:

- The Probability density of finding a value δ on scale R(S) = $\Pi_{PS}(\delta,R(S))$

- The fraction of collapsed volume:

$$F_{PS}(S) = \int_{\delta_c}^{\infty} \Pi_{PS}(\delta, S)\, d\delta$$

$$\frac{dn(M)}{dM} = \frac{M}{\bar{\rho}} \frac{dF}{dM} \left|\frac{dM}{dS}\right|,$$

$$f(\sigma^2) = 2\sigma^2 \frac{dF(\sigma^2)}{d\sigma^2}$$

Excursion Set Theory

Bond et al (90's)

Additional constrain:

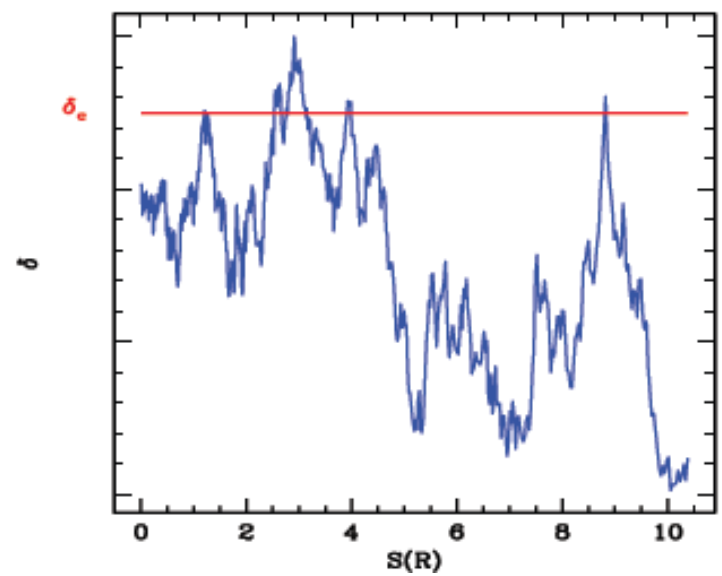
the scale which verified the collapse condition to be the larger one (id: absorbing boundary)

$$\delta(\mathbf{x}, R) = \frac{1}{(2\pi)^3} \int d^3k\, \tilde{W}(k, R)\tilde{\delta}(\mathbf{k})e^{-i\mathbf{k}\,\mathbf{x}}$$

Gaussian initial statistics + sk filter:

$$\tilde{W}(k, R) = \Theta(k_f - k) \text{ with } k_f = 1/R \propto S$$

$$\frac{\partial \delta}{\partial S} = \eta(S) \qquad \langle \delta(S_1)\delta(S_2)\rangle = min(S_1, S_2)$$



- No memory of previous steps:

Exact analytical solution for spherical collapse and linear barrier

The Mass definition problem

Mass contained in the smooth field:

$$< M(R) >= \bar{\rho} V \text{ with } V = \int W(R) d^3 R$$

Sharp-k filter:

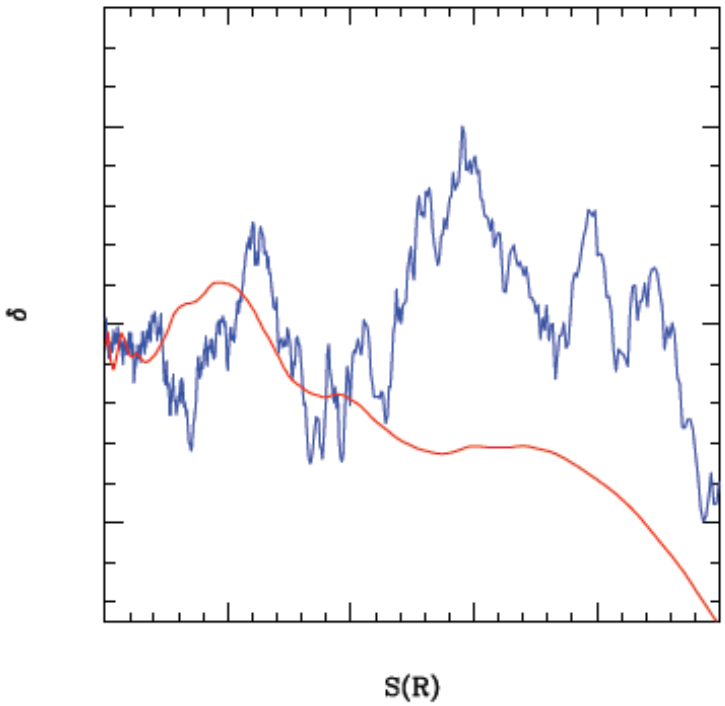
$$V(R)_{sharp-k} = 6\pi^2 R^3 - 12\pi R^3 \int_0^\infty \cos x \, dx$$

- Undefined volume unless you regularize it (e.g. Lacey & Cole)
- Incoherent with observational and N-body simulations mass definition

Non-Markovian Random Walks

Sharp-x filter:

$V=V_{sphere}$ but $$< \delta(S_1)\delta(S_2) >= \int \frac{d^3k}{(2\pi)^3} \tilde{W}(k,R_1)\tilde{W}(k,R_2)P(k)$$



Correlation between steps
→ no analytical solution

Perturbative Approach

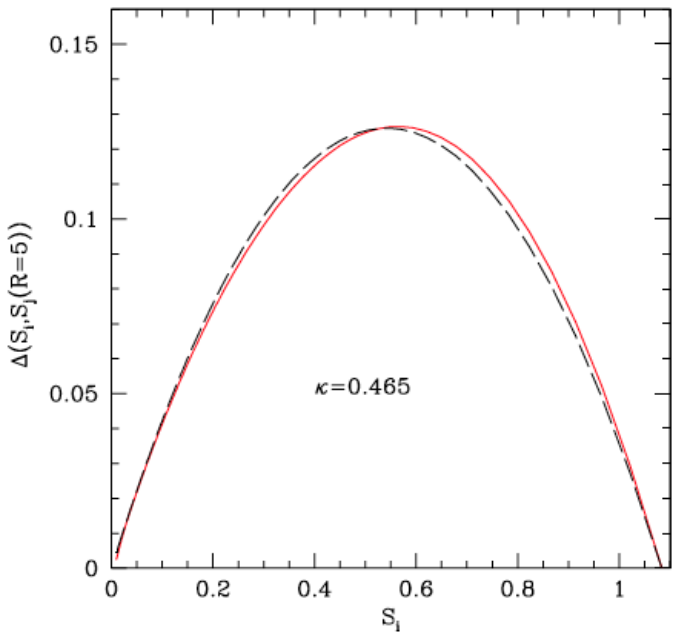
Maggiore & Riotto (MR1 2009)

In the mass range of interest we can approximate the 2-point correlator of the sx filter by the sk filter

$$\langle \delta(S_1)\delta(S_2)\rangle - \min(S_1, S_2) = \Delta(S_1, S_2)$$

For a ΛCDM model :

$$\Delta(S_1, S_2) \simeq \kappa \frac{S_1(S_2 - S_1)}{S_2}$$



Path Integral Formalism (MR1)

Procedure:

discretize $S_i = i\,\varepsilon$ trajectory = $\{\delta_1 \ldots \delta_n\}$

$$\Pi_\varepsilon(\delta_0, \delta_n, S_n) = \int_{-\infty}^{\delta_c} d\delta_1 \ldots \int_{-\infty}^{\delta_c} d\delta_{n-1} \int \mathcal{D}\lambda \; e^{i\sum_{i=1}^{n} \lambda_i \delta_i} e^{Z} \qquad Z \equiv \langle e^{-i\sum_{i=1}^{n} \lambda_i \delta(S_i)} \rangle$$

Expand around the Markovian solution:

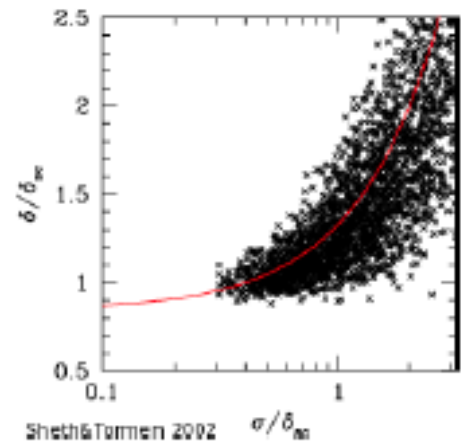
$$\Pi_\varepsilon(\delta_0, \delta_n, S_n) = \int_{-\infty}^{\delta_c} d\delta_1 \ldots \int_{-\infty}^{\delta_c} d\delta_{n-1} \int \mathcal{D}\lambda \; e^{i\sum_{i=1}^{n} \lambda_i \delta_i} e^{-\frac{1}{2}\sum_{i,j=1}^{n} [\min(S_i,S_j) + \Delta(S_i,S_j)]\lambda_i \lambda_j}$$

$$f(\sigma) = (1-\kappa)\left(\frac{2}{\pi}\right)^{1/2} \frac{\delta_c}{\sigma}\, e^{-\delta_c^2/(2\sigma^2)} + \frac{\kappa}{\sqrt{2\pi}} \frac{\delta_c}{\sigma} \Gamma\left(0, \frac{\delta_c^2}{2\sigma^2}\right),$$

Is it enough to match the N-body halo mass function ?

A Drifting Diffusive Barrier?

Ellispoidal collapse distribution:
Bond & Meyers (1996); Sheth Mo & Tormen (2001)
$<B(S)>=\delta_c+\beta\, S^{\gamma}$ Other moments are negligible ?



Analysis from simulations:
"Critical overdensity is sensitive to the initial displacement of a given mass element from the perturbation center, thus changing how the collapse barriers depend on auxiliary properties of the tidal field." (Ludlow &Porciani 2011)

Assumption:

Take a simple Gaussian distribution to model the barrier

$$< B(S) > \equiv \bar{B}(S) = \delta_c + \beta S$$

$$< B(S)B(S') >_c = D_B \min(\mathrm{S}, \mathrm{S}')$$

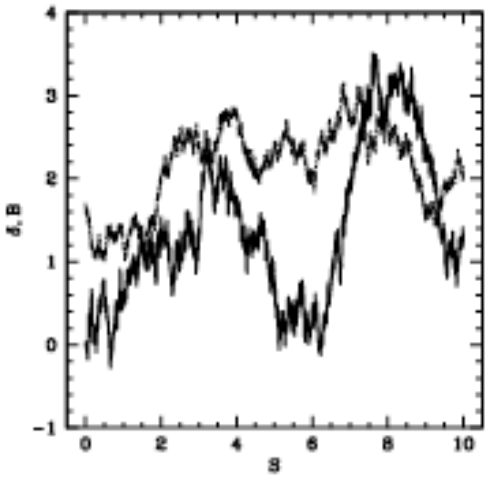
$$\Pi_B(B_0, B, S) = \frac{e^{-\frac{(B-\bar{B})^2}{2D_B S}}}{\sqrt{2\pi D_B S}}$$

Implementation of the Barrier in the Excursion Set framework

PS. Corasaniti & IA (arxiv 1012.3448/1107.1251)

Exact Markovian solution:

$$f_0(\sigma) = \sqrt{\frac{2}{\pi}}\frac{\delta_c}{\sigma}e^{-\frac{a}{2\sigma^2}(\delta_c+\beta\sigma^2)^2}$$



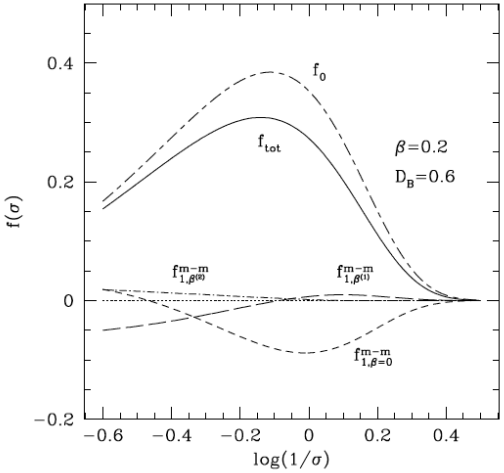
Non-Markovian corrections:

$$f(\sigma) = f_0(\sigma) + f_{\kappa=1}(\sigma)$$

$$f^{m-m}_{1,\beta=0}(\sigma) = -a\,\kappa\sqrt{\frac{2}{\pi}}\frac{Y_0\sqrt{a}}{\sigma}\left(e^{-\frac{aY_0^2}{2\sigma^2}} - \frac{1}{2}\Gamma\left[0, \frac{aY_0^2}{2\sigma^2}\right]\right),$$

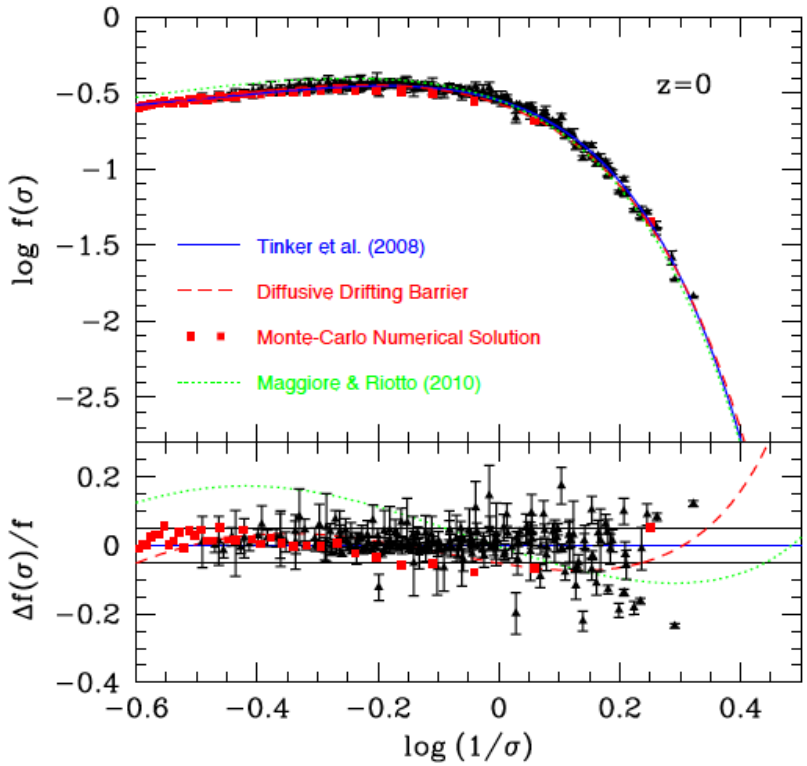
$$f^{m-m}_{1,\beta^{(1)}}(\sigma) = -a\,Y_0\,\beta\left(a\,\kappa\,\mathrm{Erfc}\left[Y_0\sqrt{\frac{a}{2\sigma^2}}\right] + f^{m-m}_{1,\beta=0}(\sigma)\right),$$

$$f^{m-m}_{1,\beta^{(2)}}(\sigma) = -\frac{a}{2}\beta^2\sigma^2 f^{m-m}_{1,\beta=0}(\sigma) - aY_0\beta f^{m-m}_{1,\beta=1}(\sigma)$$



Comparison with N-body simulations

Agreement within 5% N-body simulation uncertainties of Tinker et al. 2008 with only 2 free physical parameters !



- β =0.057 deviation rate from spherical collapse

- D_B=0.3 scatter amplitude of the collapse

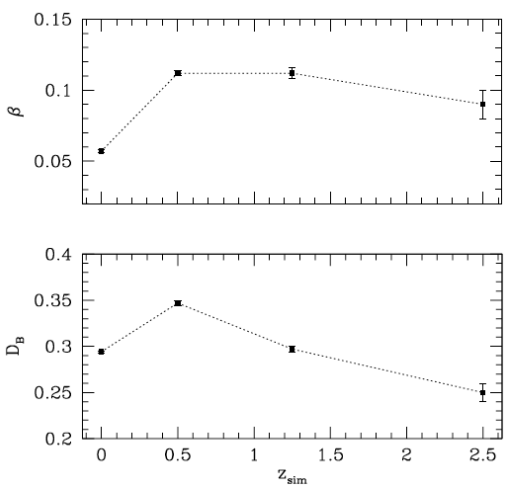
$$f_{Tinker}(\sigma) = A\left[\left(\frac{\sigma}{b}\right)^{-a} + 1\right] e^{-c/\sigma^2}$$

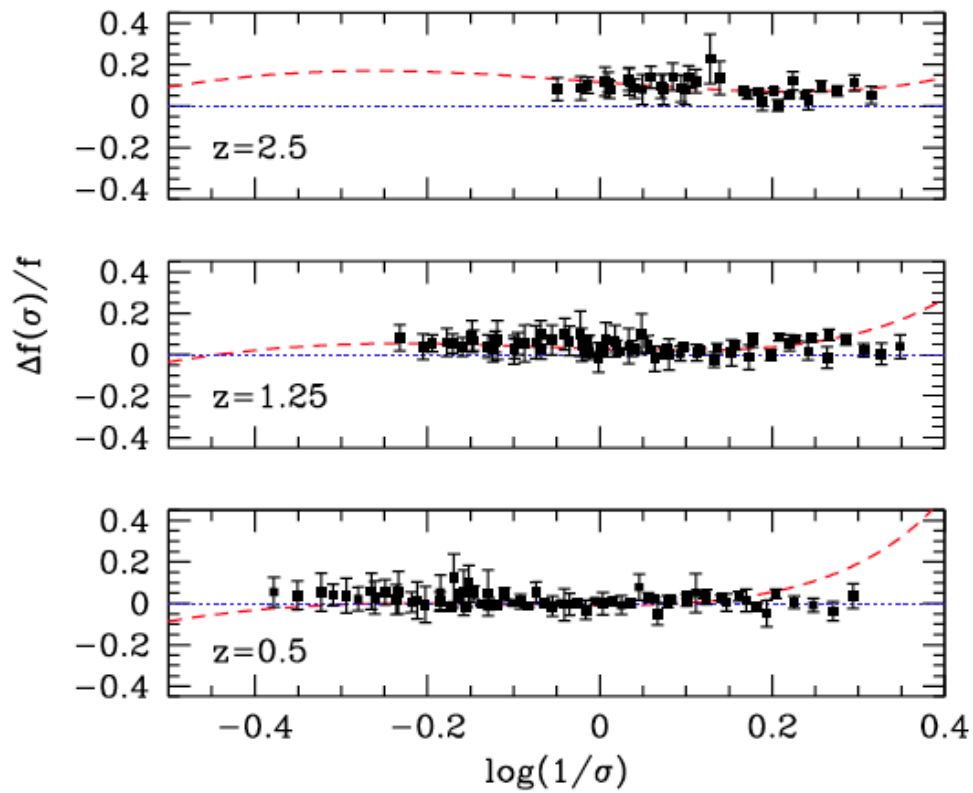
Comparison at different redshift

Evolution of Tinker parameters:

$$A(z) = A_0(1+z)^{-0.14},$$
$$a(z) = a_0(1+z)^{-0.06},$$
$$b(z) = b_0(1+z)^{-\alpha},$$
$$\log \alpha(\Delta) = -\left[\frac{0.75}{\log(\Delta/75)}\right]^{1.2}$$

Evolution of DDB parameters:

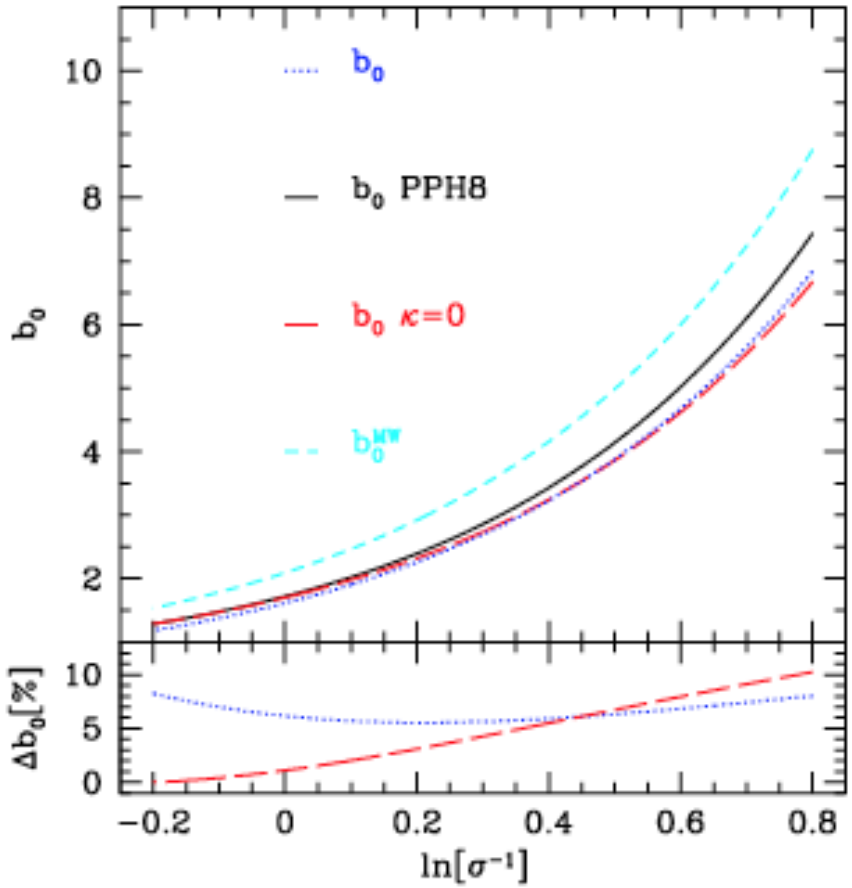




Red: DDB model

Blue: Tinker et al. fitting formula

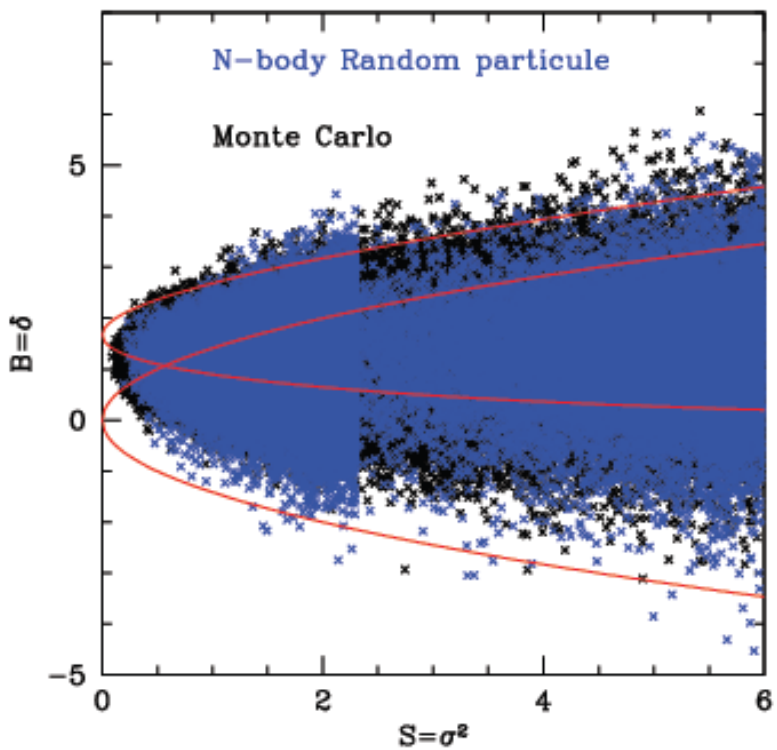
Linear Halo bias

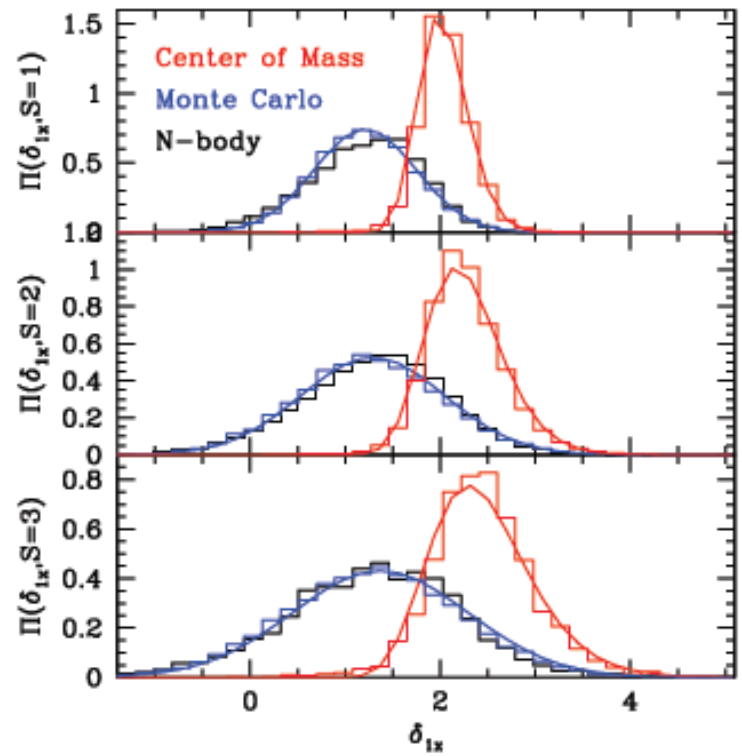


Can we predict β,D_B from N-body simulation?

IA, Y. Rasera, R. Sheth & PS Corasaniti (arxiv 1212.1166)

DEUS N-body simulation





MC analysis for (D_B ,β) best fit

Distribution of B=δ compared to the one from the MC

Test the excursion set and predict barrier parameters at the same time!

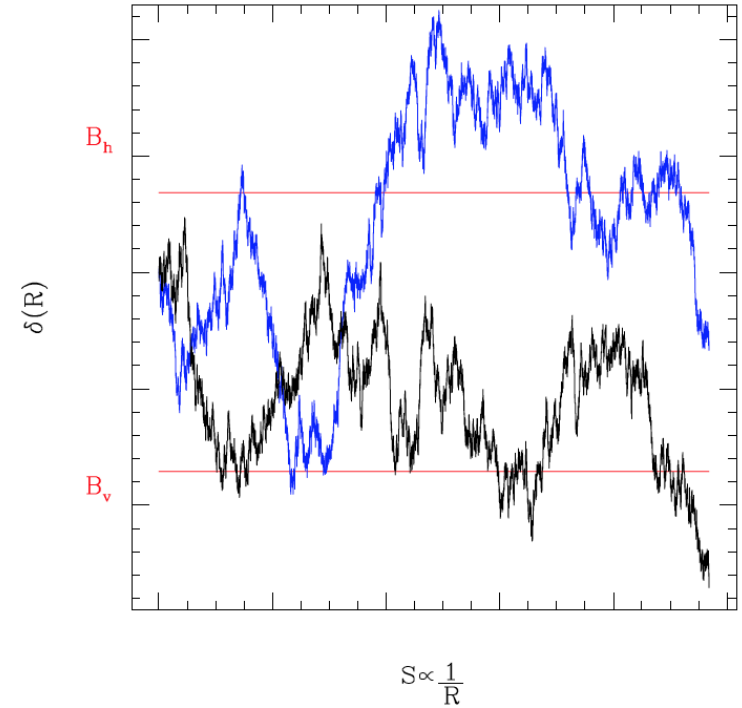
Voids hierarchy & Analytical treatment

Sheth & Van de Weygaert: (SVdW 04)

- Initial underdense regions = voids

$$V\frac{\mathrm{d}n}{\mathrm{d}\ln R} = f(\sigma)\left|\frac{\mathrm{d}\ln\sigma}{\mathrm{d}\ln R}\right|$$

- Spherical threshold Bv=-2.7
- Boundary conditions
  void-in-void; void-in-cloud; cloud-in-void



Shortcoming:

- Spherical evolution threshold?
- Inconsistent with spherical volume (sk filter)

Testing spherical evolution for modeling void abundances

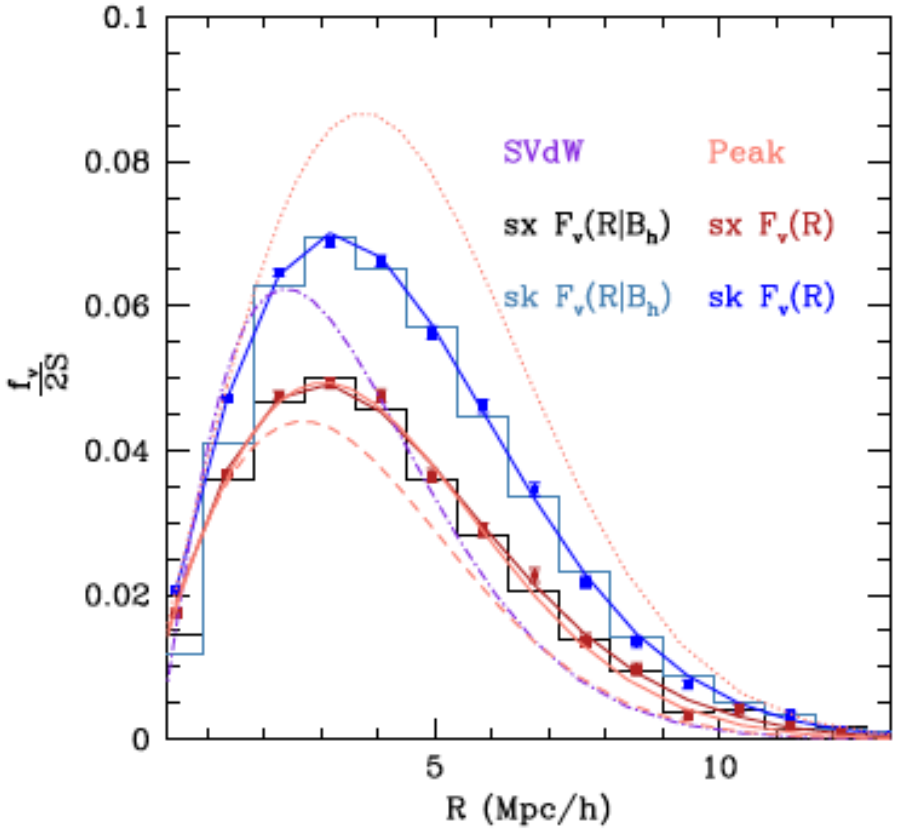
IA, M. Neyrinck & A. Paranjape (arxiv 1309.3799)

Pertinence of the void-in-cloud?

- Small scales effect
- Negligible for sx !

Applications to realistic model of barriers

- Path integral approach to compute SX corrections
- Peak predictions



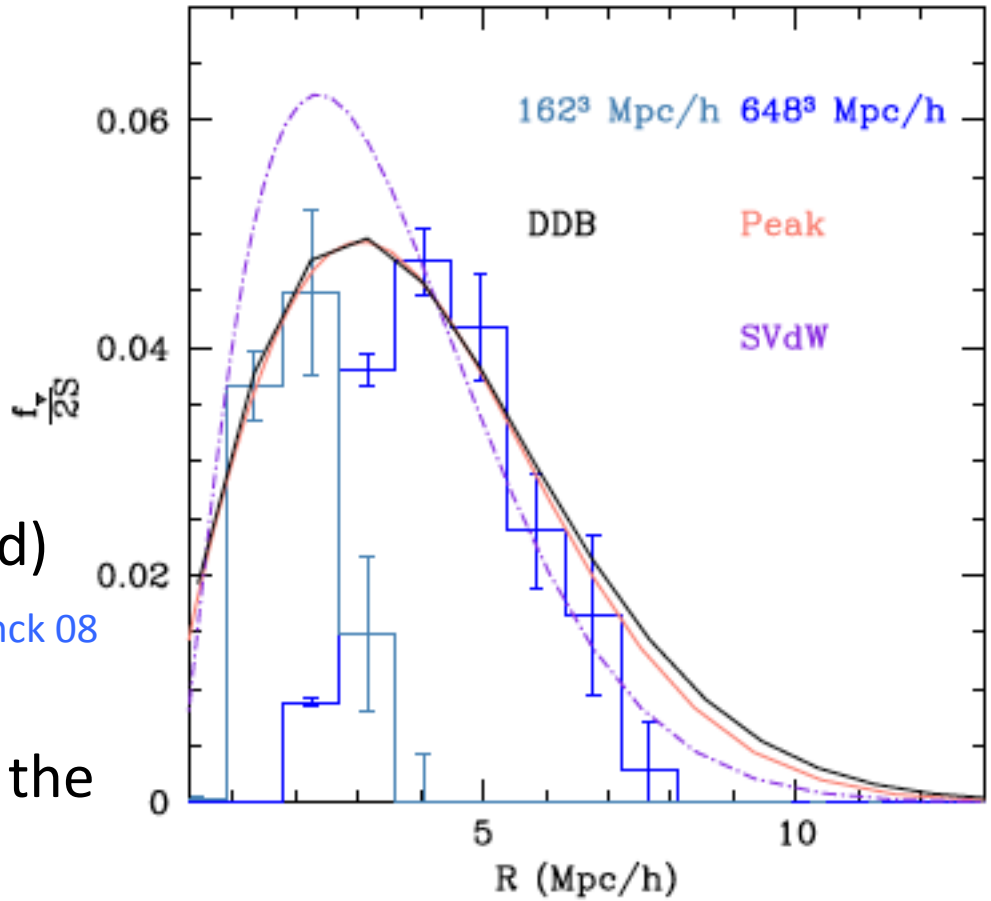
Peaks and Excursion Set predictions with DDB can be very similar!!

Comparison with N-body simulations

Spherical voids Vs Zobov:
- find density minima using watershed transform

Post-processing:
- Remove subvoid (void-in-void) using a statistical criteria Neyrinck 08

- Density minima threshold of the core void



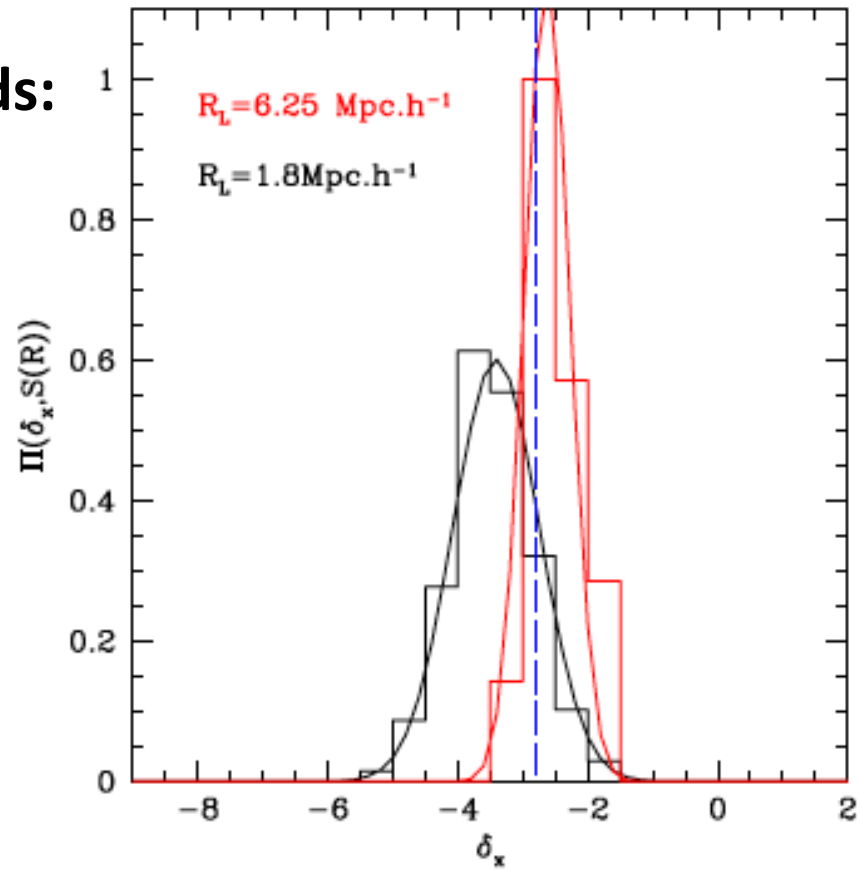
DEUSS N-body simulations (obspm-LUTh)
ZOBOV void finder (Neyrinck 08)

Consistency of the spherical evolution

Critical underdensity leading to voids:
- Scale dependent
- Scatter around the mean

simple Bv=-2.7 is not enough !

Assumptions:
- Spherical shape for voids
- $R_L = R_E/1.7$ (spherical evolution)



Conclusion & Percpectives

- **DDB reproduces N-body simulations precisely and consistent with EST**

- **Extension to non-LCDM cosmology ?**

  - PNG **(IA & P.S. Corasaniti arxiv:1207.4796 & 1109.3196)**
  - Modify gravity: f(R) **(M. Kopp, S. Appleby, IA & J. Weller et al. arxiv:1306.3233)**

- **Measurement of D_B and β can be predict from CI!**

  - Sensitivity to different cosmology ?

Next steps:

- **Halo model for a DDB: modify gravity…**
- **Halo Merger Trees and galaxy formation**
- **Voids defined by biased tracers**

…

Conditional Halo Mass function

Motivations

Merging tree, halo bias

Main equation:

$$P(\delta_n, S_n|\delta_m, S_m) = \frac{\int_{-\infty}^{\delta_c} d\delta_1 ... \hat{d\delta_m} ... \int_{-\infty}^{\delta_c} d\delta_{n-1} W(\delta_0, ...\delta_n, S_n)}{\int_{-\infty}^{\delta_c} d\delta_1 ... d\delta_{m-1} W(\delta_0, ...\delta_m, S_m)}$$

Ma et al. 2010

- Well normalized (Bayes theorem)
- Does not work for diffusive barrier (unless $S_m \rightarrow 0$)

Conditional sk for generic scale (Sm)

Marginalisation over the Barrier:

$$P(\delta_n, S_n|\delta_m, S_m) = \frac{\int dB_1...dB_{n-1} W_B(B_0, ...B_n) \int_{-\infty}^{B_1} d\delta_1...\hat{d\delta_m}... \int_{-\infty}^{B_{n-1}} d\delta_{n-1} W(\delta_0, ...\delta_n, S_n)}{\int dB_1...dB_{m-1} W_B(B_0, ...B_m) \int_{-\infty}^{\delta_c} d\delta_1...d\delta_{m-1} W(\delta_0, ...\delta_m, S_m)}$$

Extended conditional in case of a diffusive barrier:

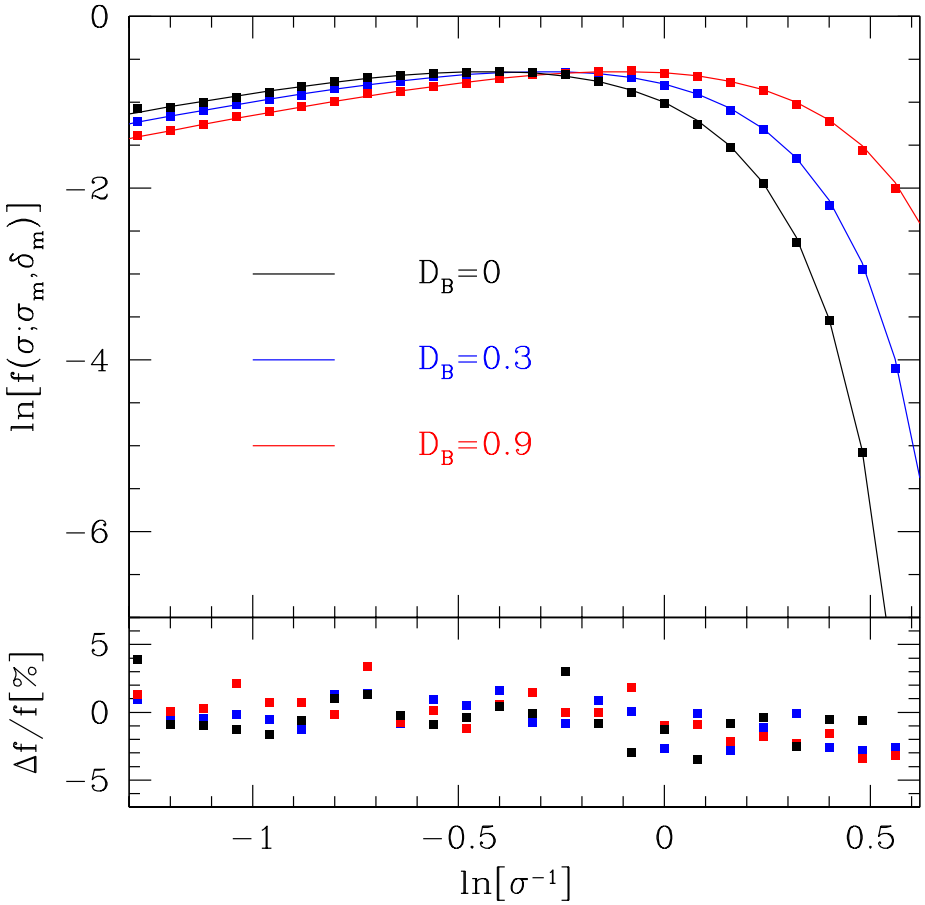
$$\mathcal{F}(S|\delta_m, S_m) = \sqrt{\frac{1}{2\pi}} \frac{(\delta_c - \delta_m)(1 + D_B)}{[(1 + D_B)S - Sm]^{3/2}} e^{-\frac{(\delta_c - \delta_m)^2}{2(S(1+D_B) - S_m)}}$$

Special case:

D_B=0 →Bond et al. ✓

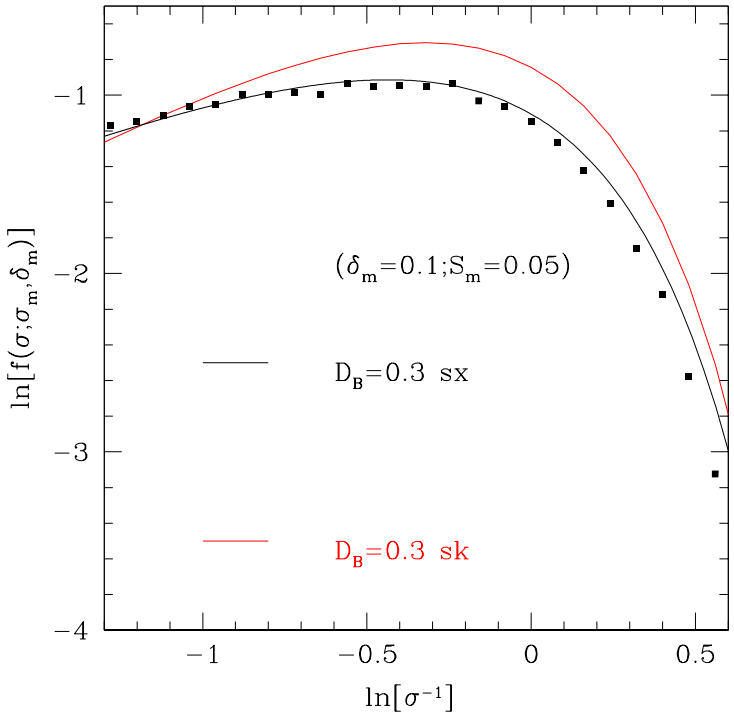
S_m=0→ DDB ✓

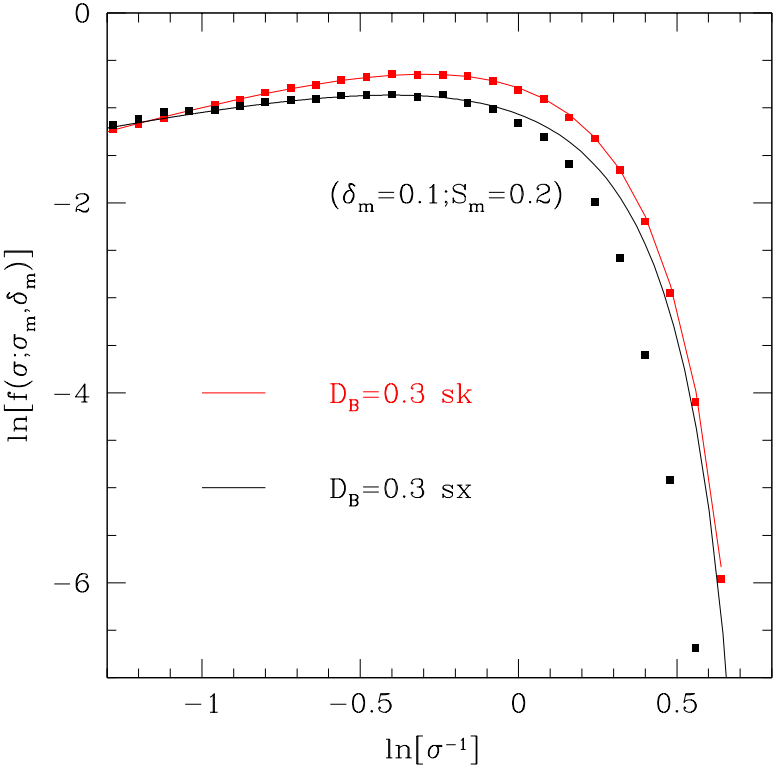
Comparison with MC solution



Work in progress…

sharp-x filter





Work in Progress…