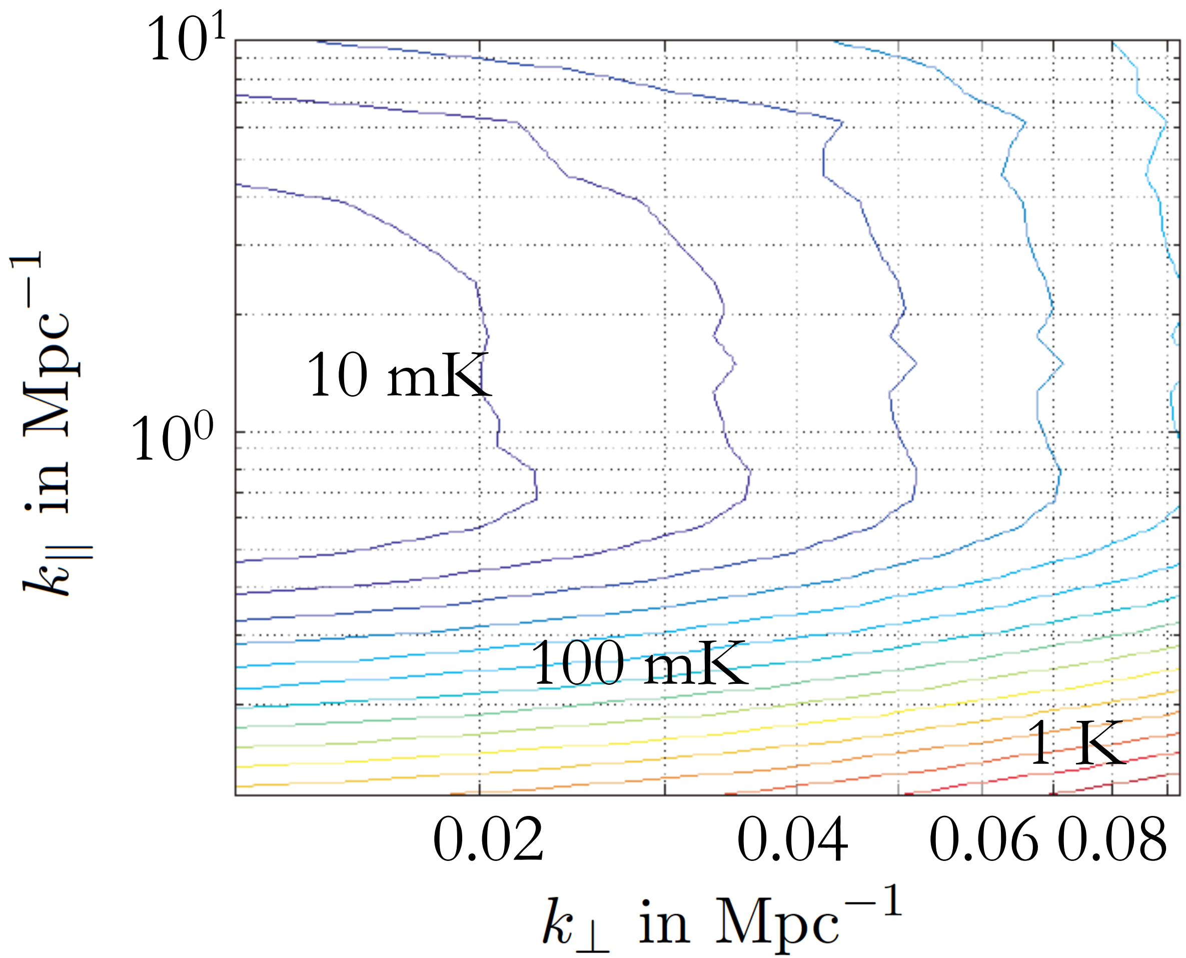
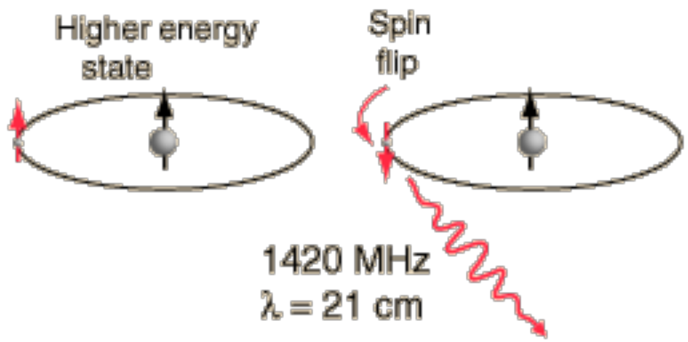
From Theoretical Promise to Observational Reality: Calibration and Foreground Subtraction in 21cm Tomography



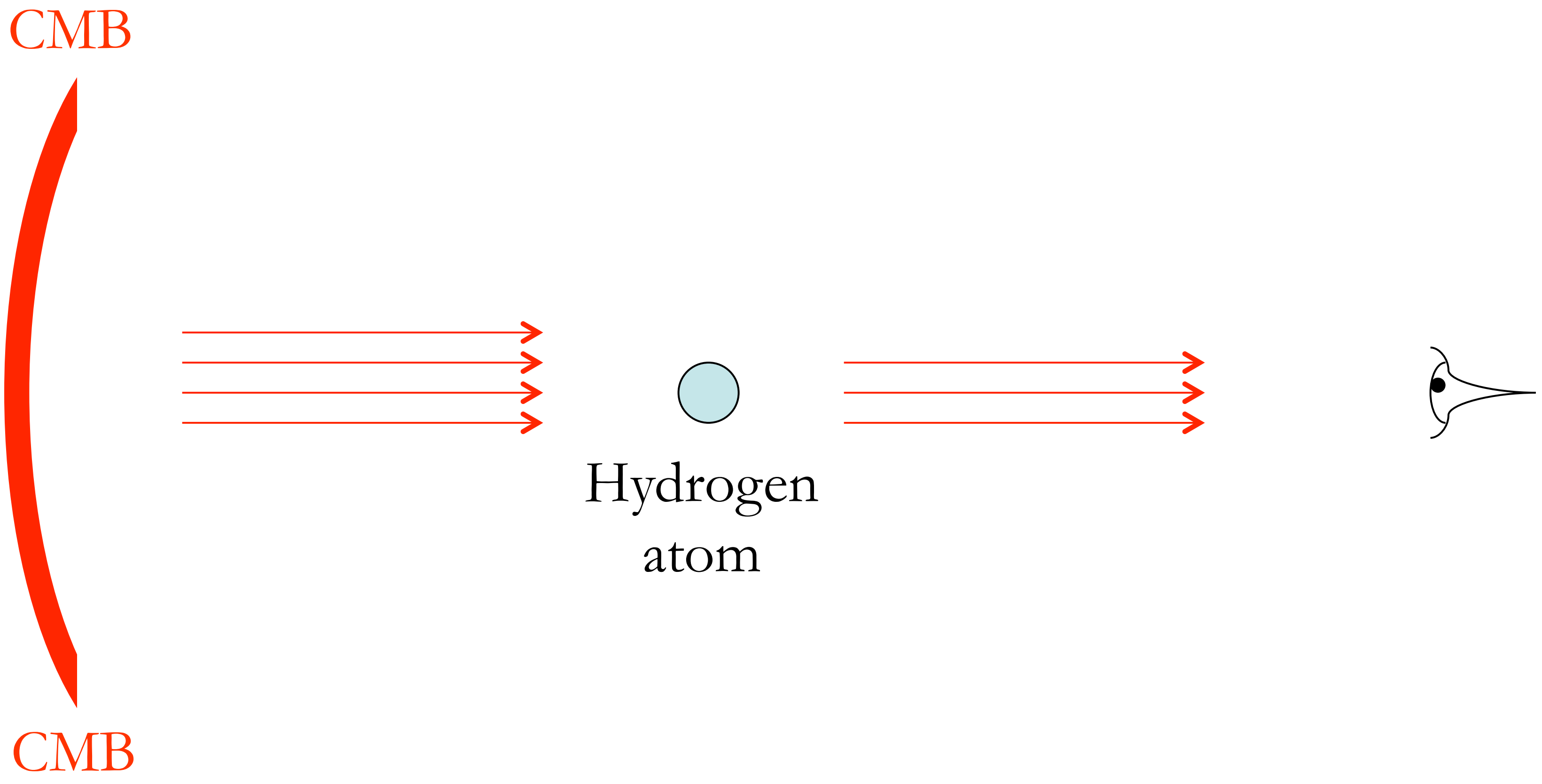
Adrian Liu, MIT

The Vision for 21cm Tomography

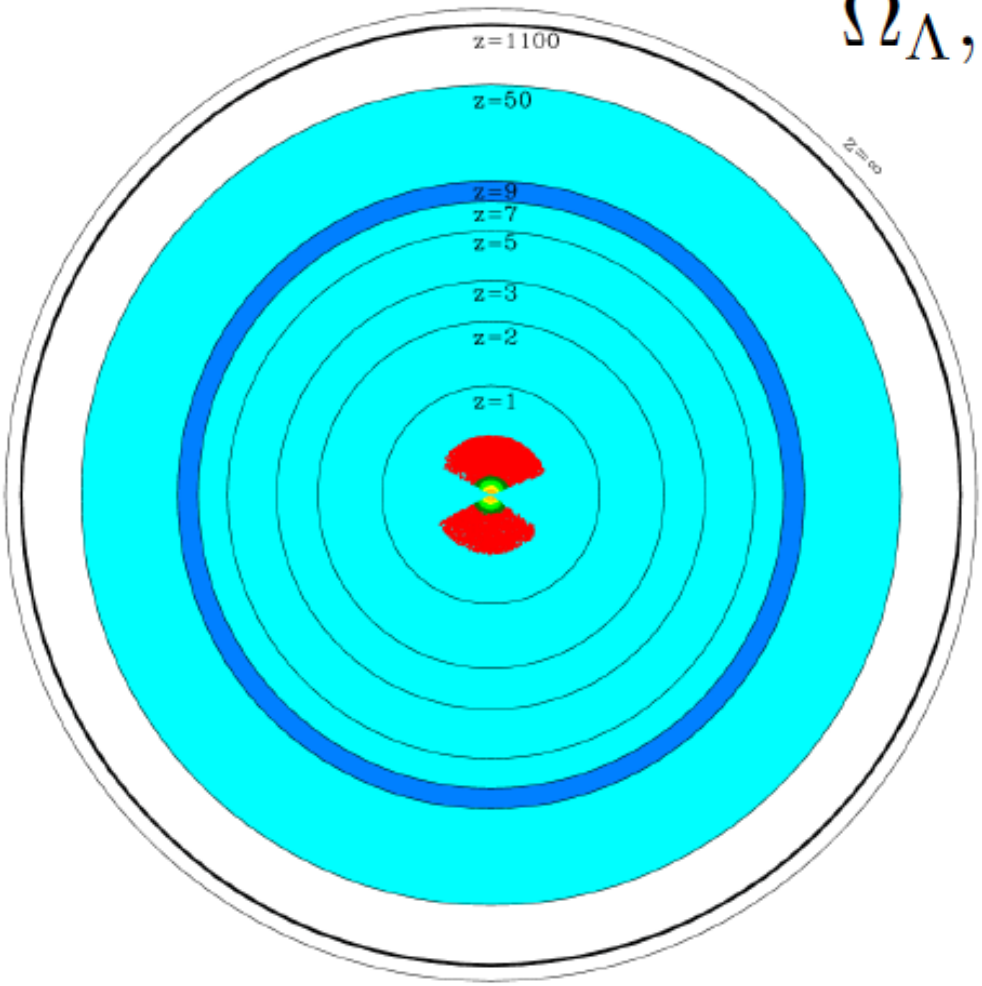
Vision



Vision



Vision



$$\Omega_\Lambda, \Omega_m, \Omega_b, n_s, A_s, \tau, \overline{x}_H,$$
$$\Omega_k, m_\nu, \alpha, \kappa, w, f_{NL}$$

E.g. Spatial curvature:

WMAP+SDSS: $\Delta\Omega_{tot}$= 0.01

Planck: $\Delta\Omega_{tot}$= 0.003

21cm: $\Delta\Omega_{tot}$=0.0002

Mao, Tegmark, McQuinn, Zahn, Zaldarriaga 2008

Vision

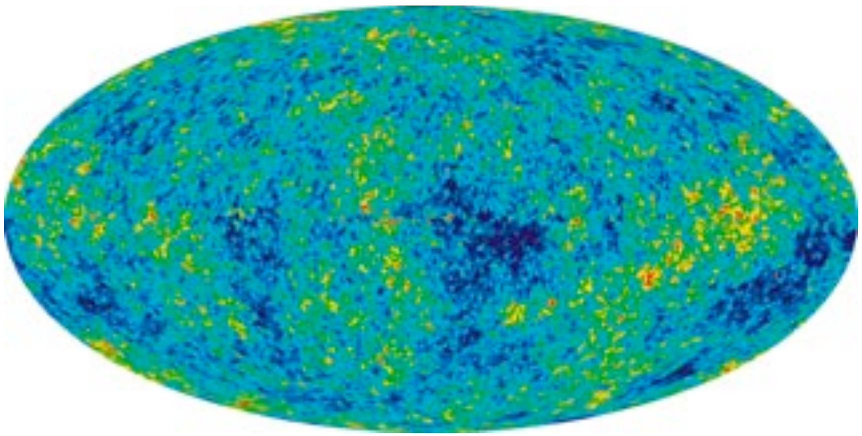


Image credit: WMAP team

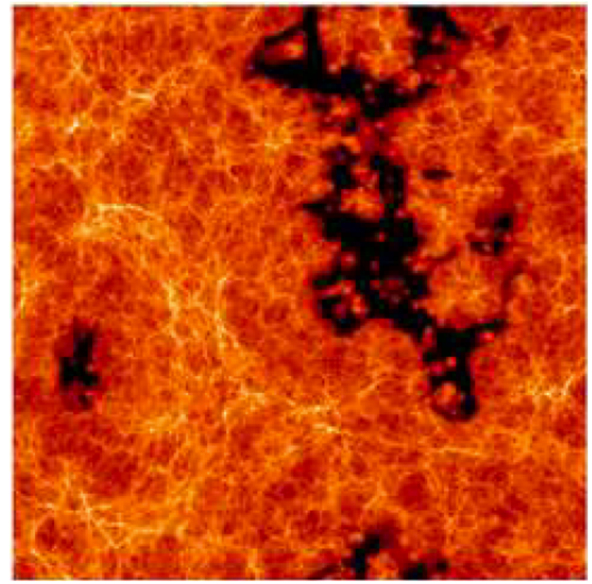


Image credit: Trac & Cen 2007

C_l

P(k) and much more!

Vision

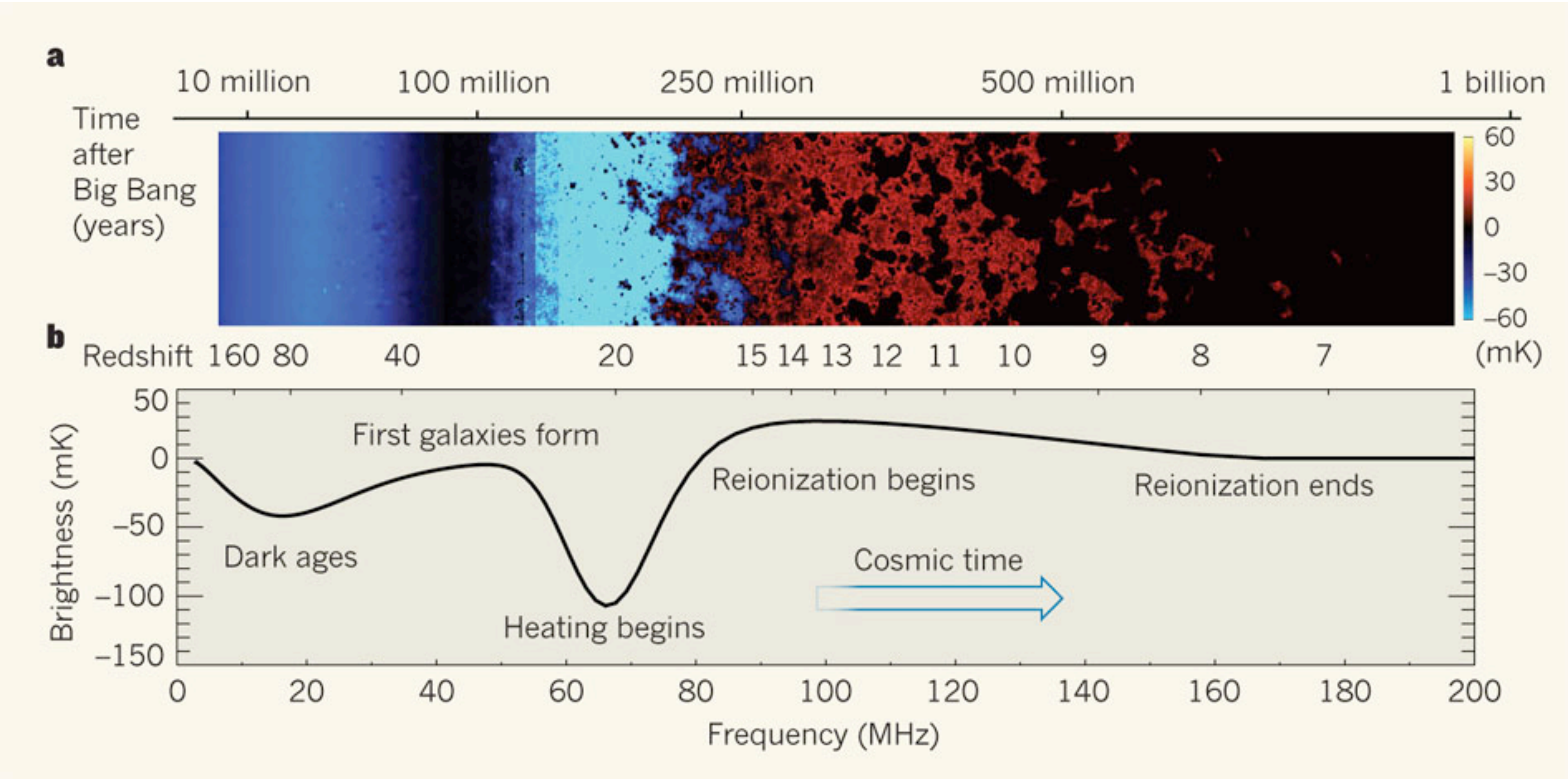


Image credit: Pritchard & Loeb 2010

The Problem

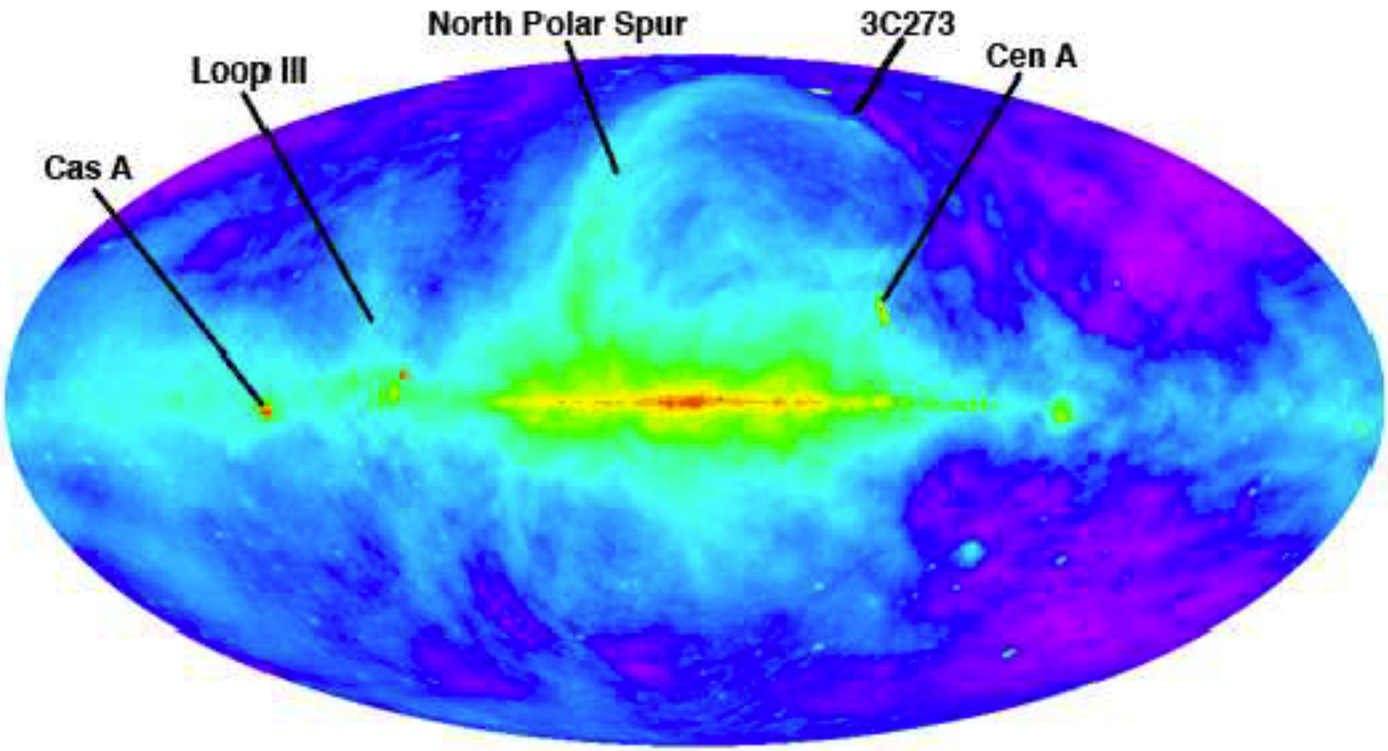
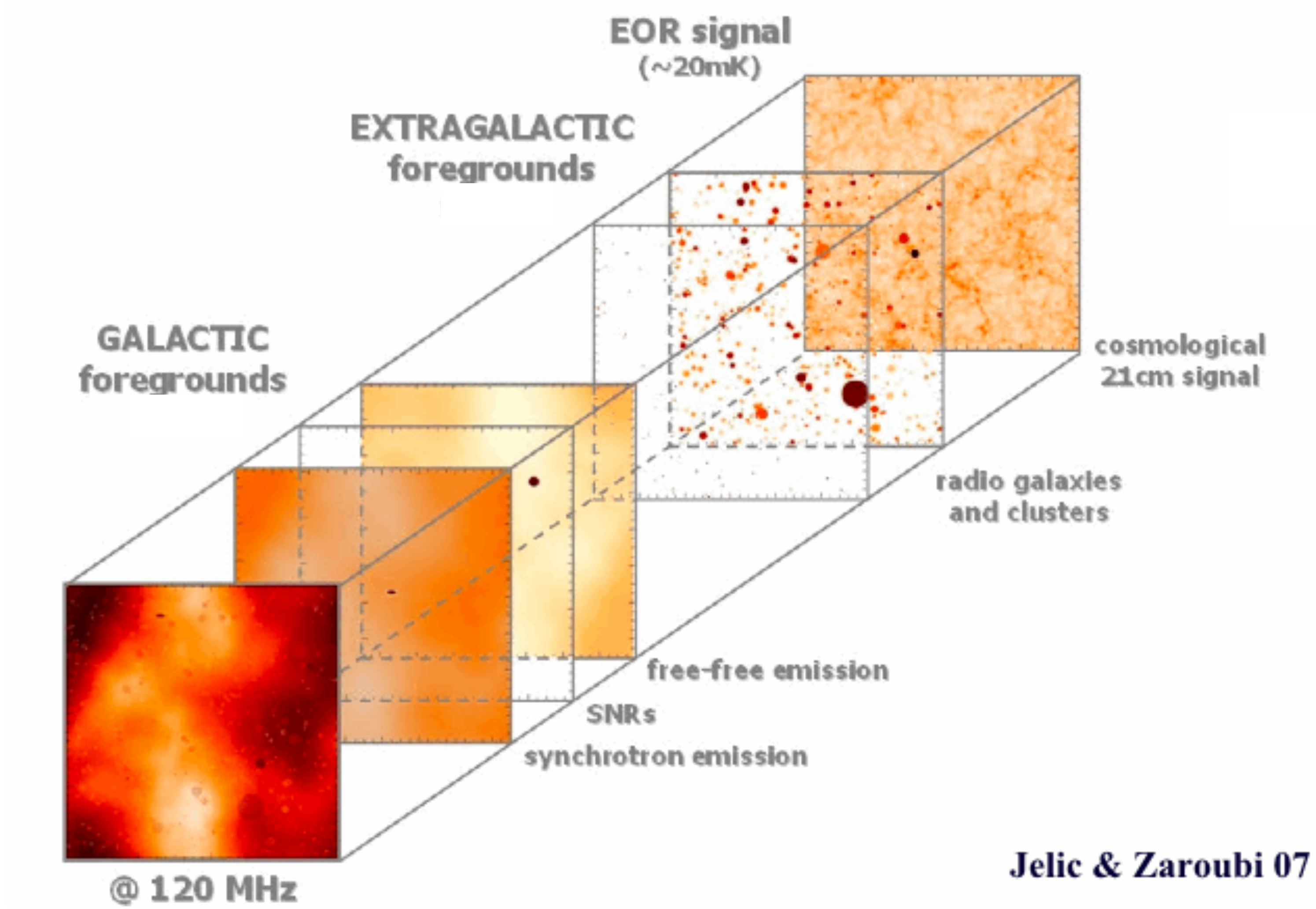


Image credit: de Oliveira-Costa et. al. 2008


EOR signal
(~20mK)
EXTRAGALACTIC
foregrounds
GALACTIC
foregrounds
cosmological
21cm signal
radio galaxies
and clusters
free-free emission
SNRs
synchrotron emission
@ 120 MHz
Jelic & Zaroubi 07

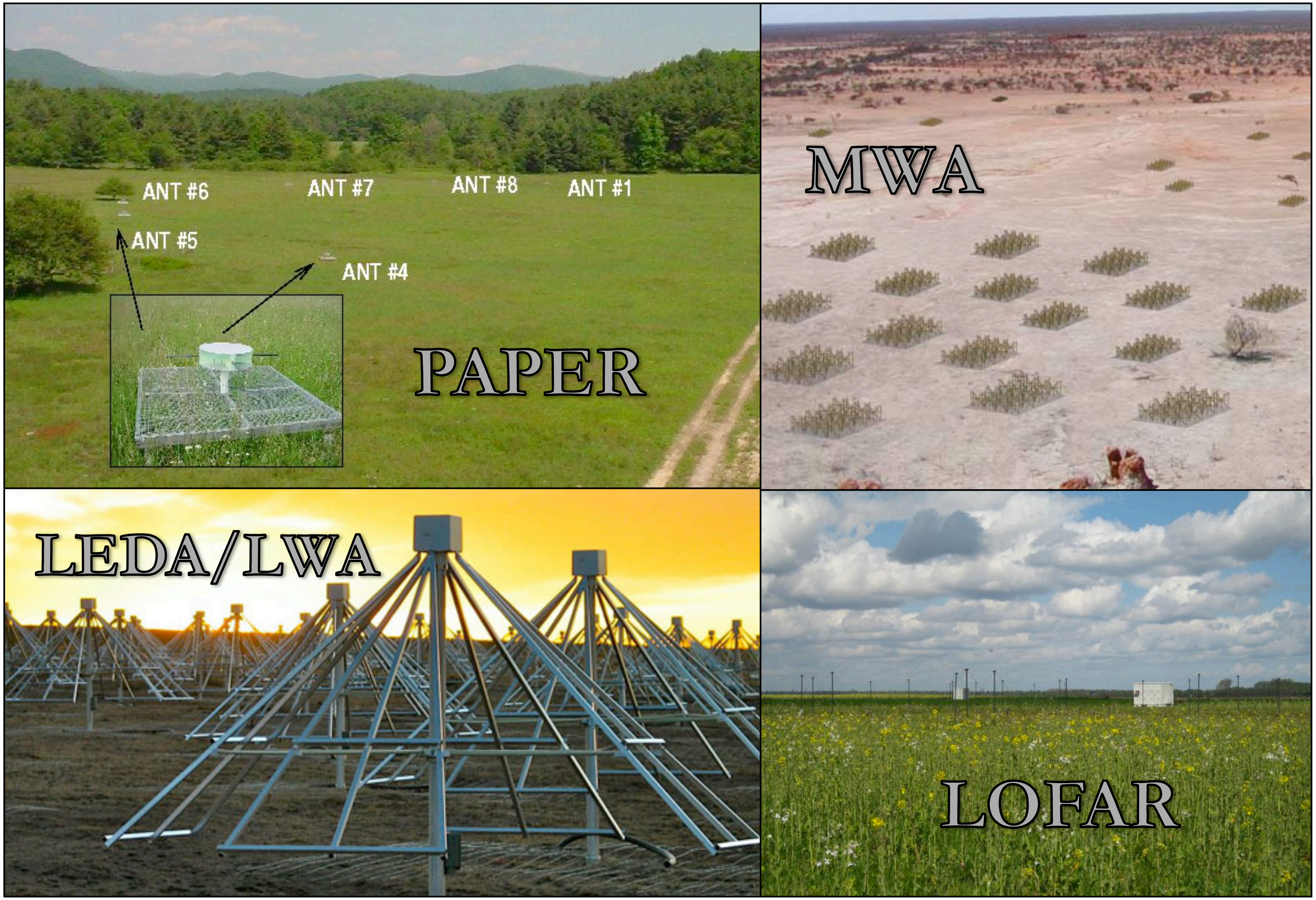
Outline

- Precision Calibration for Precision Cosmology
  - What makes calibration a new problem in 21cm tomography?
  - Why redundant calibration? What are some of its subtleties?
  - How does redundant calibration relate to traditional algorithms?
- Precision Subtraction for Precision Cosmology
  - What are some "traditional" proposals for 21cm foreground subtraction?
  - Can we do better?

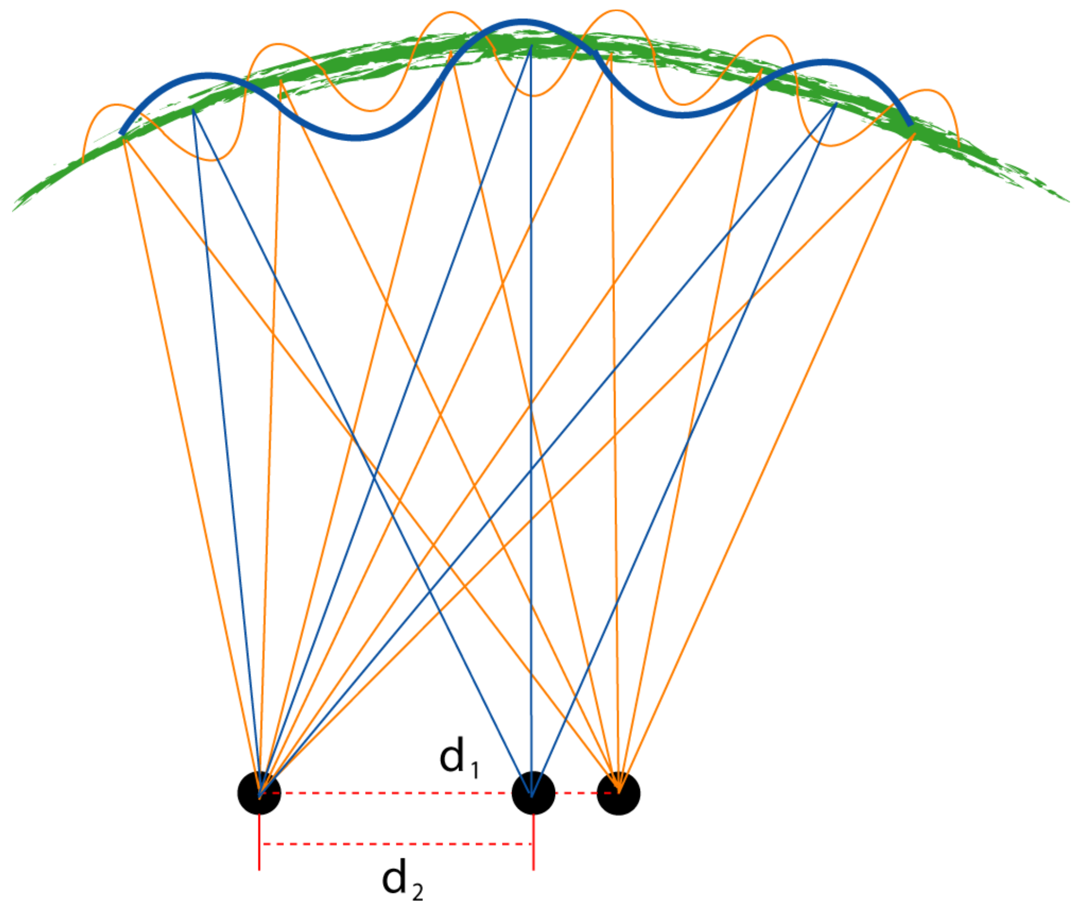
Precision Calibration for Precision Cosmology

A. Liu, M. Tegmark, S. Morrison, A. Lutomirski, M. Zaldarriaga,
MNRAS **408**, 1029, Oct. 2010

21cm tomography requires interferometer arrays



In principle, each baseline probes a Fourier mode of the sky, but…



In principle, each baseline probes a Fourier mode of the sky, but…

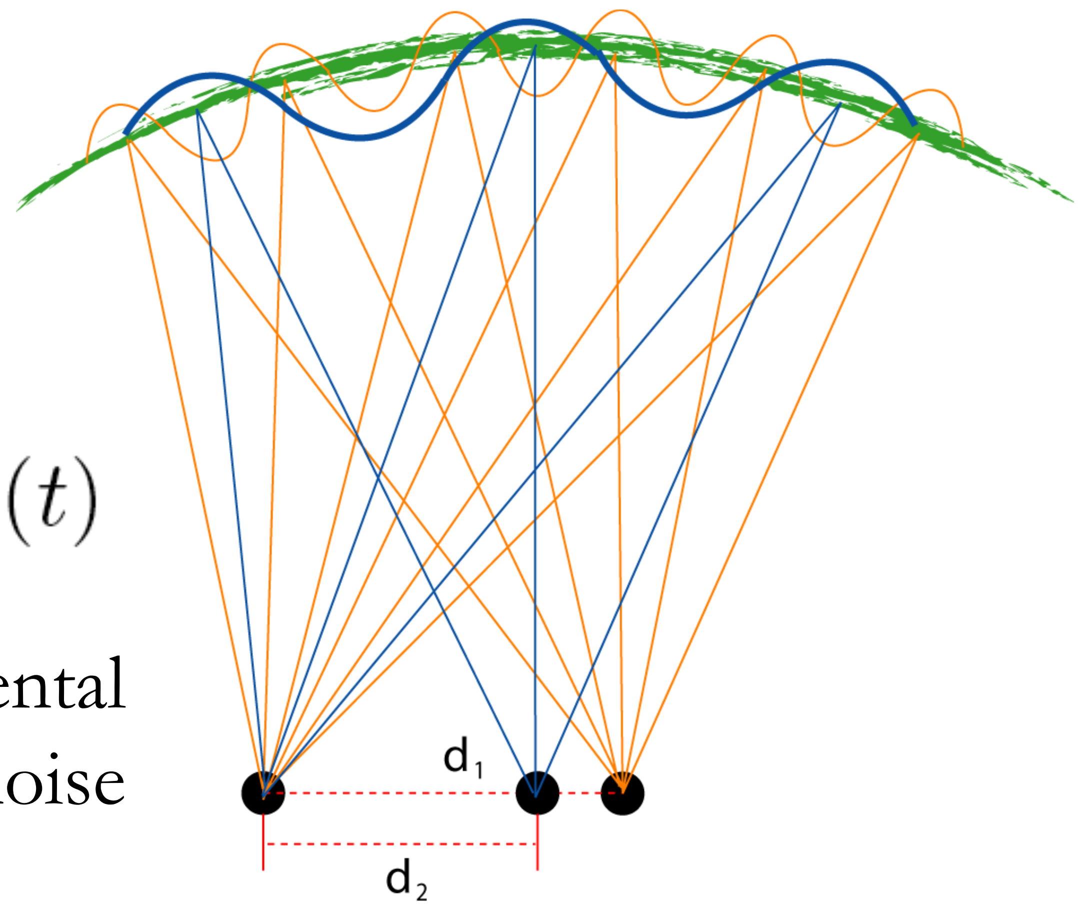
Measured signal

True sky signal

$$s_i(t) = g_i x_i(t) + n_i(t)$$

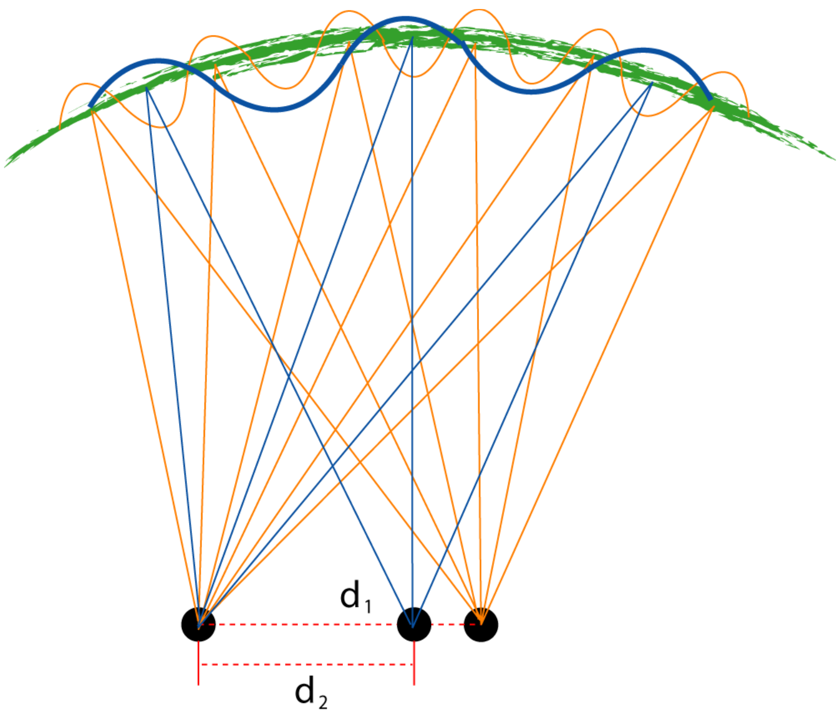
Complex gain: $g_i \equiv e^{\eta_i + i\varphi_i}$

Instrumental noise



21cm tomography requires compact, redundant interferometer arrays

- Traditional radio astronomy:
  - Imaging bright (SNR >> 1) localized sources in a dim background



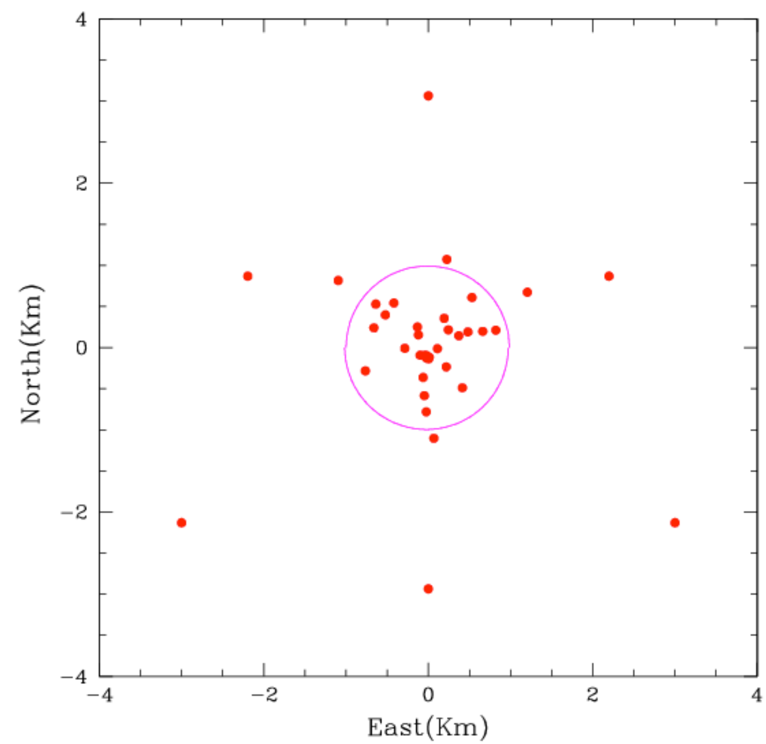
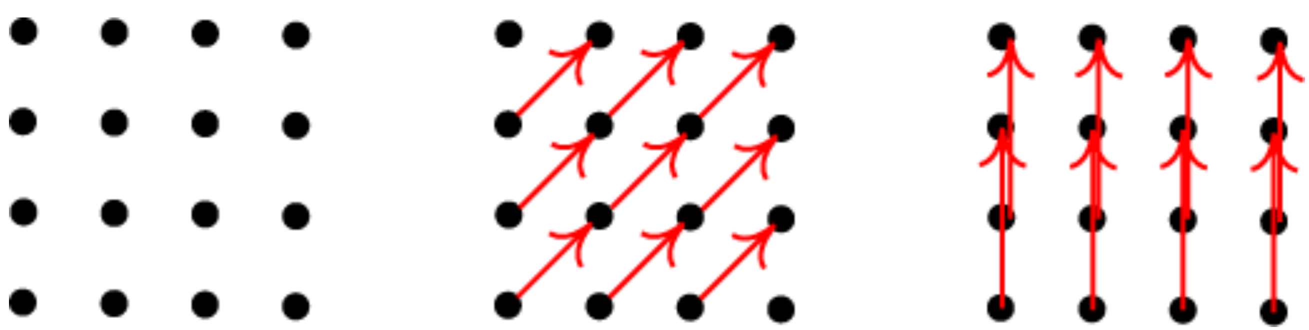


Image credit: ASKAP website

21cm tomography requires compact, redundant interferometer arrays

- Traditional radio astronomy
  – Imaging bright (SNR >> 1) localized sources in a dim background
- 21cm tomography:
  – Measuring dim fluctuations (SNR << 1) over a large area



Unlike traditional interferometers, 21cm tomography experiments have many redundant baselines

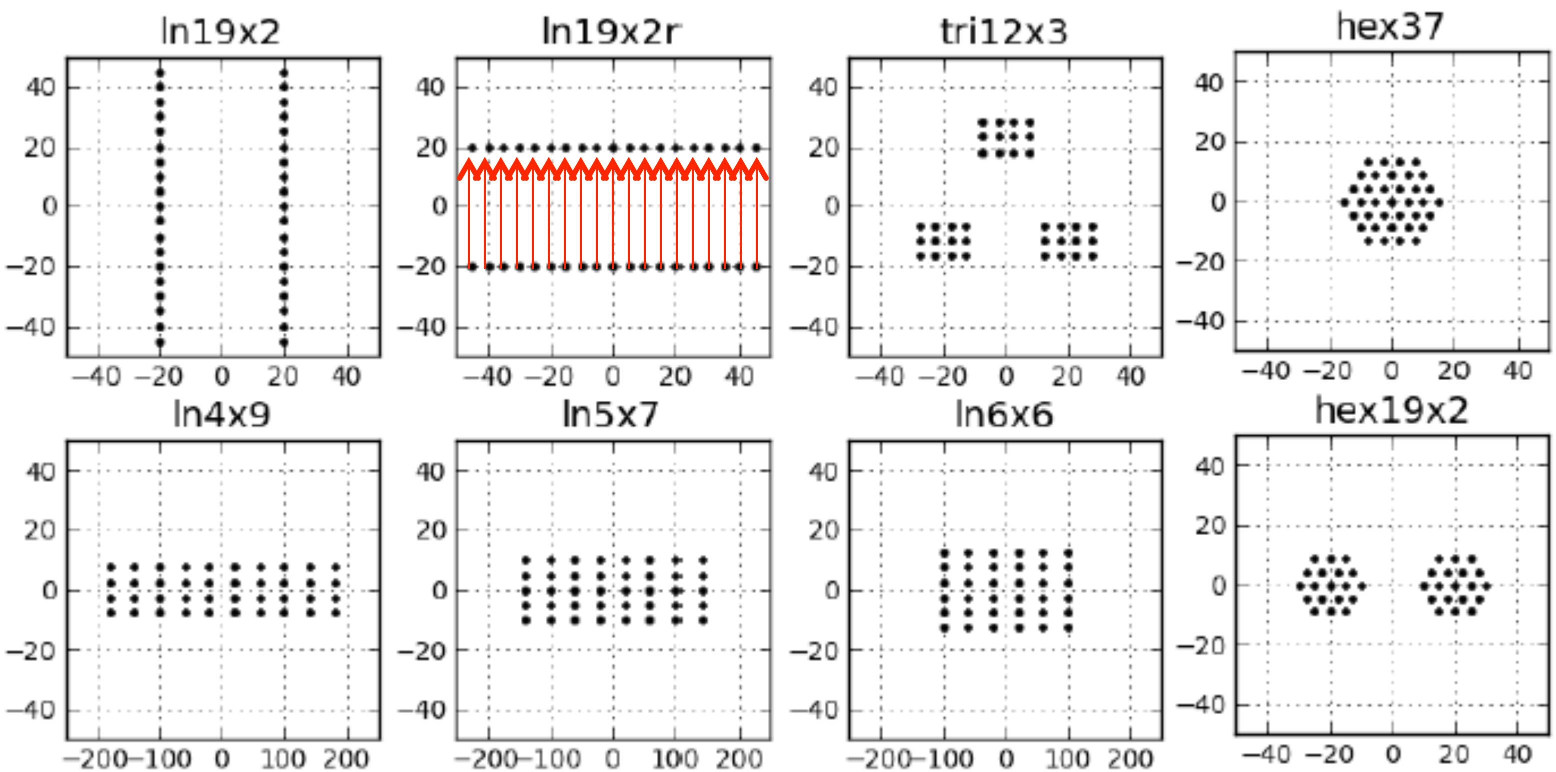
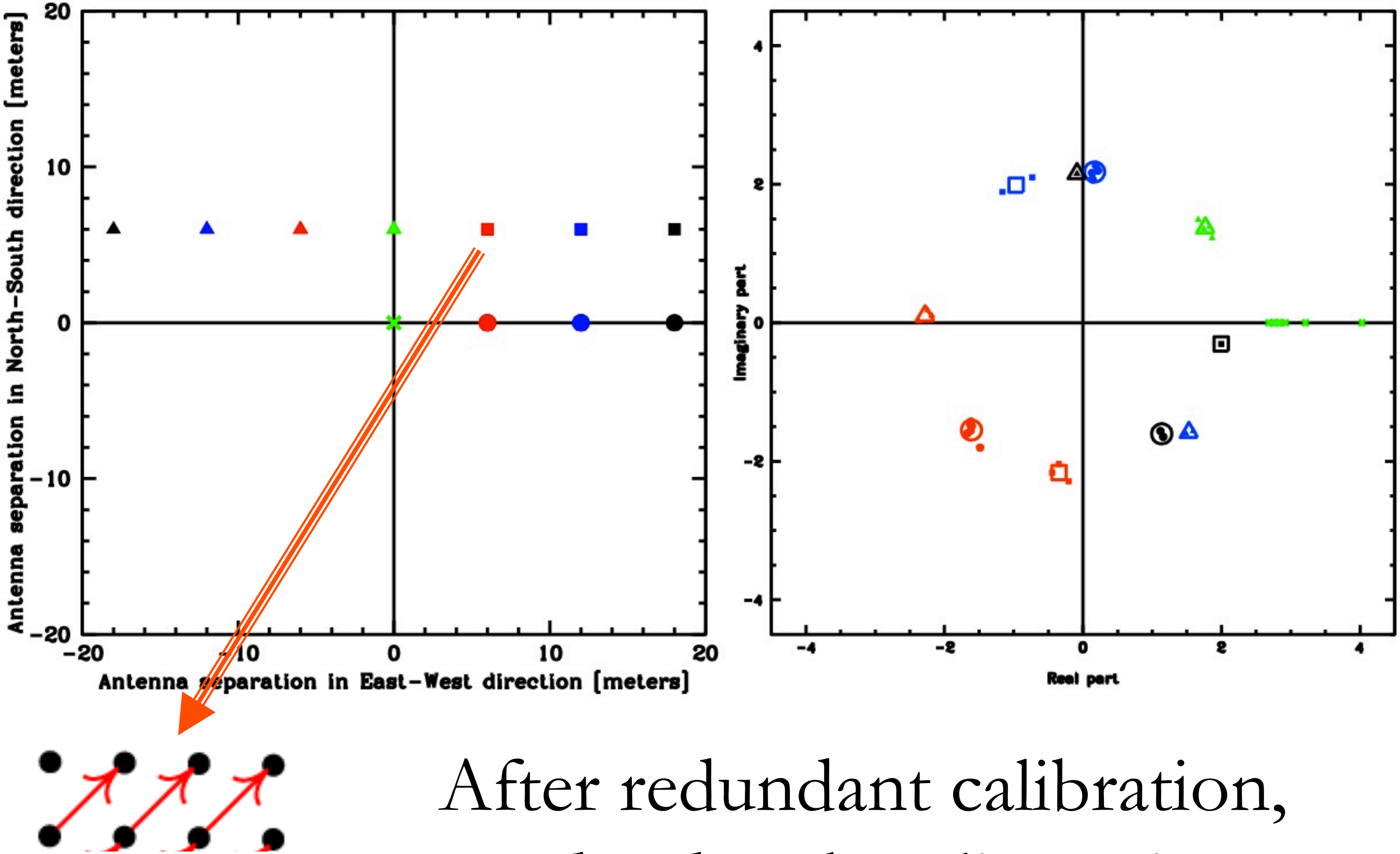
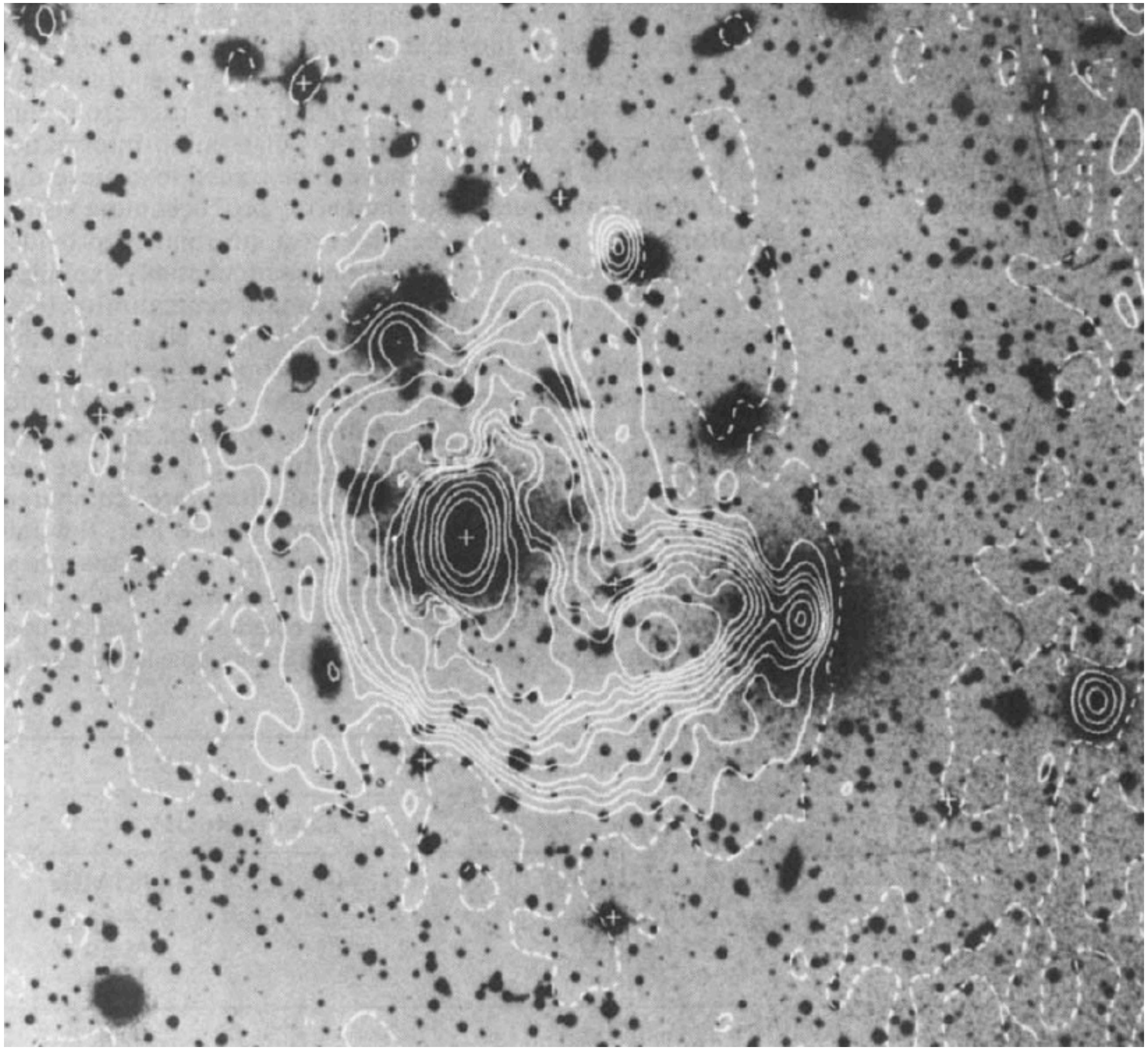


Image credit: Parsons et. al. 2011



After redundant calibration, redundant baselines give identical results

Not (just) a theorist's dream!



Noordam & de Bruyn 1982

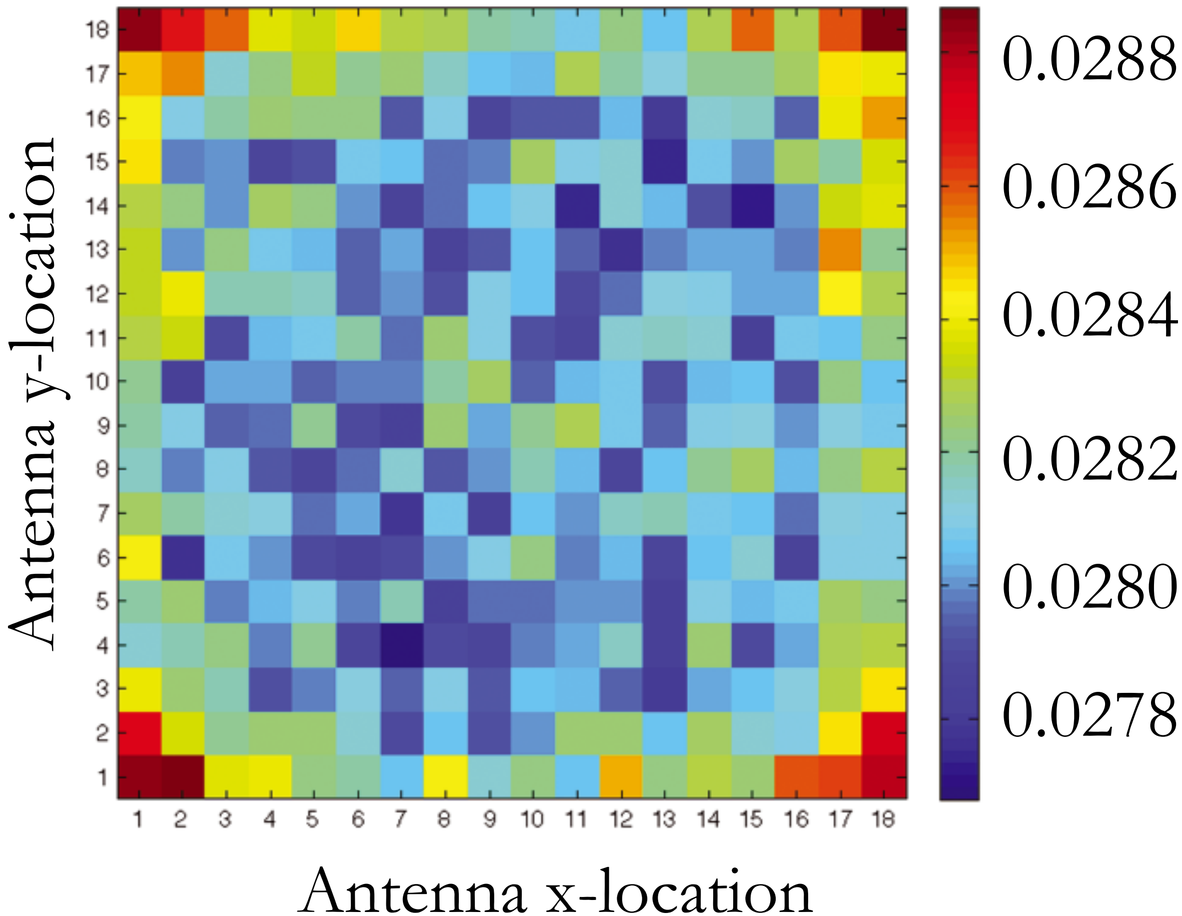
How does this compare to other calibration schemes?

- “Traditional” point source calibration
  - Assumes field of view contains a single point source.
- Self calibration
  - Construct a model of the sky, predict measurements, iterate.
- Redundant calibration
  - Requires a redundant array.
  - Independent of the sky.

Can we do better?

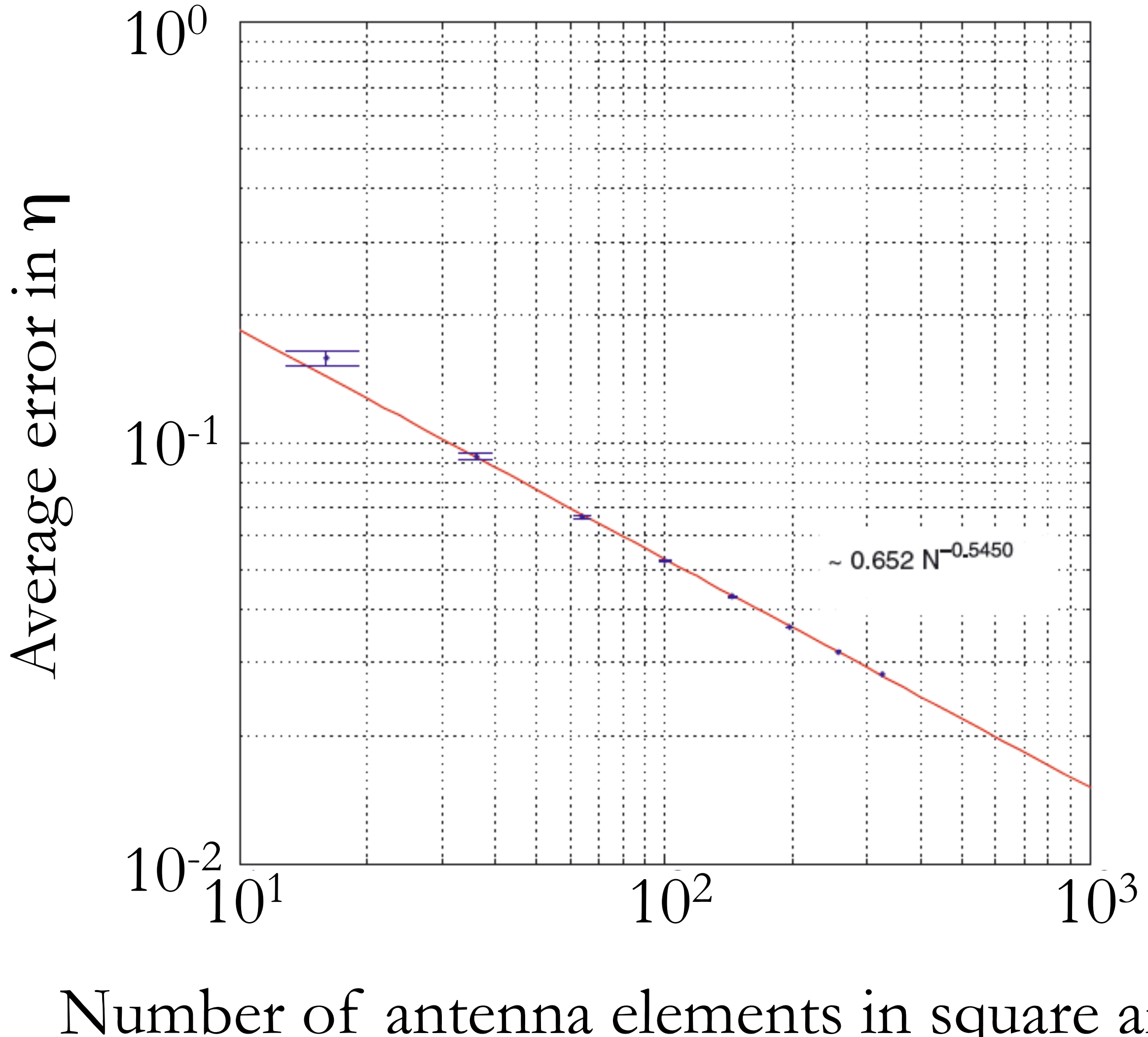
- Better characterization of calibration errors.

Average errors in η for an 18 by 18 square array



$$g_i \equiv e^{\eta_i + i\varphi_i}$$

Average errors in η as a function of array size



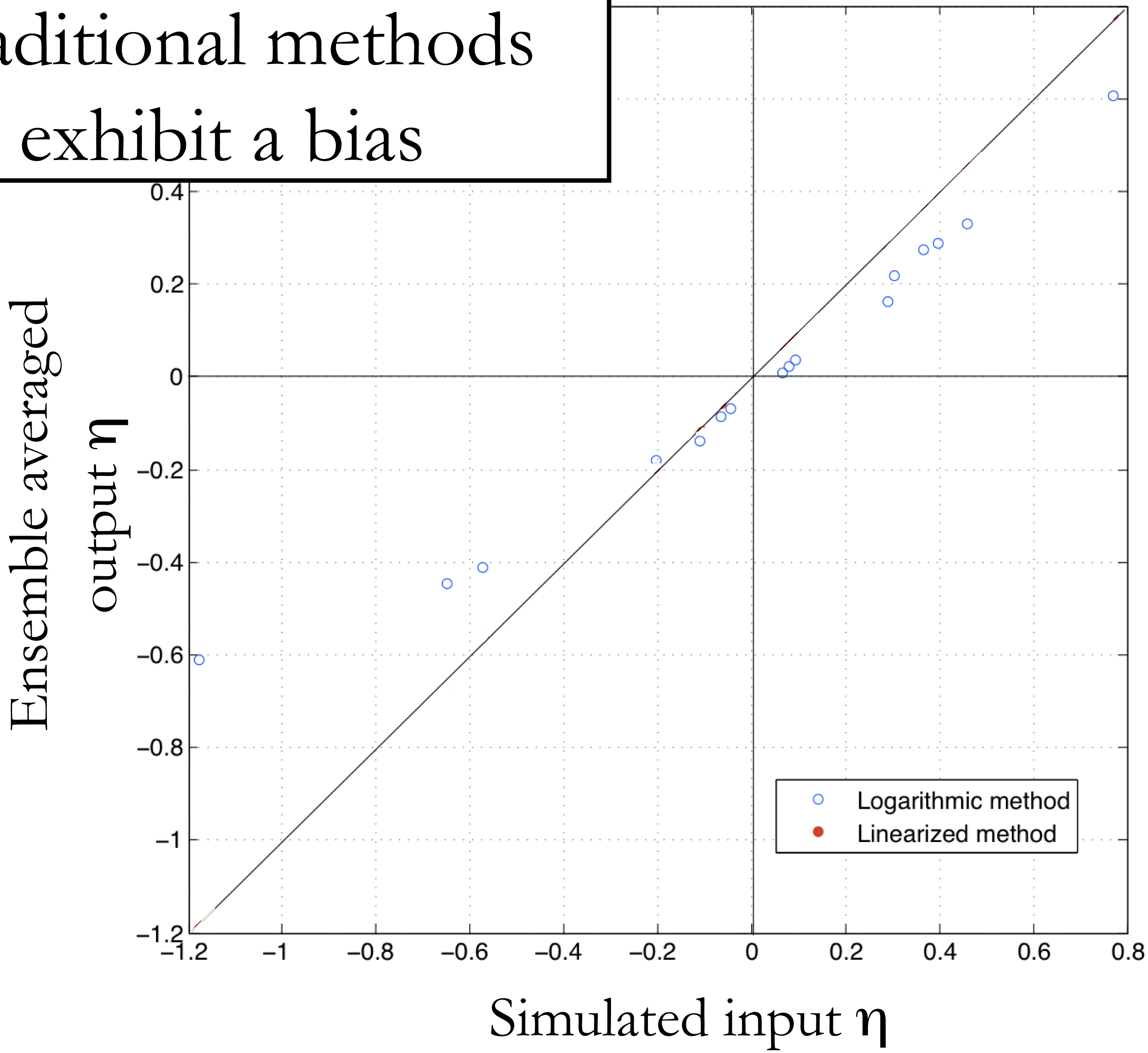
$$g_i \equiv e^{\eta_i + i\varphi_i}$$

Can we do better?

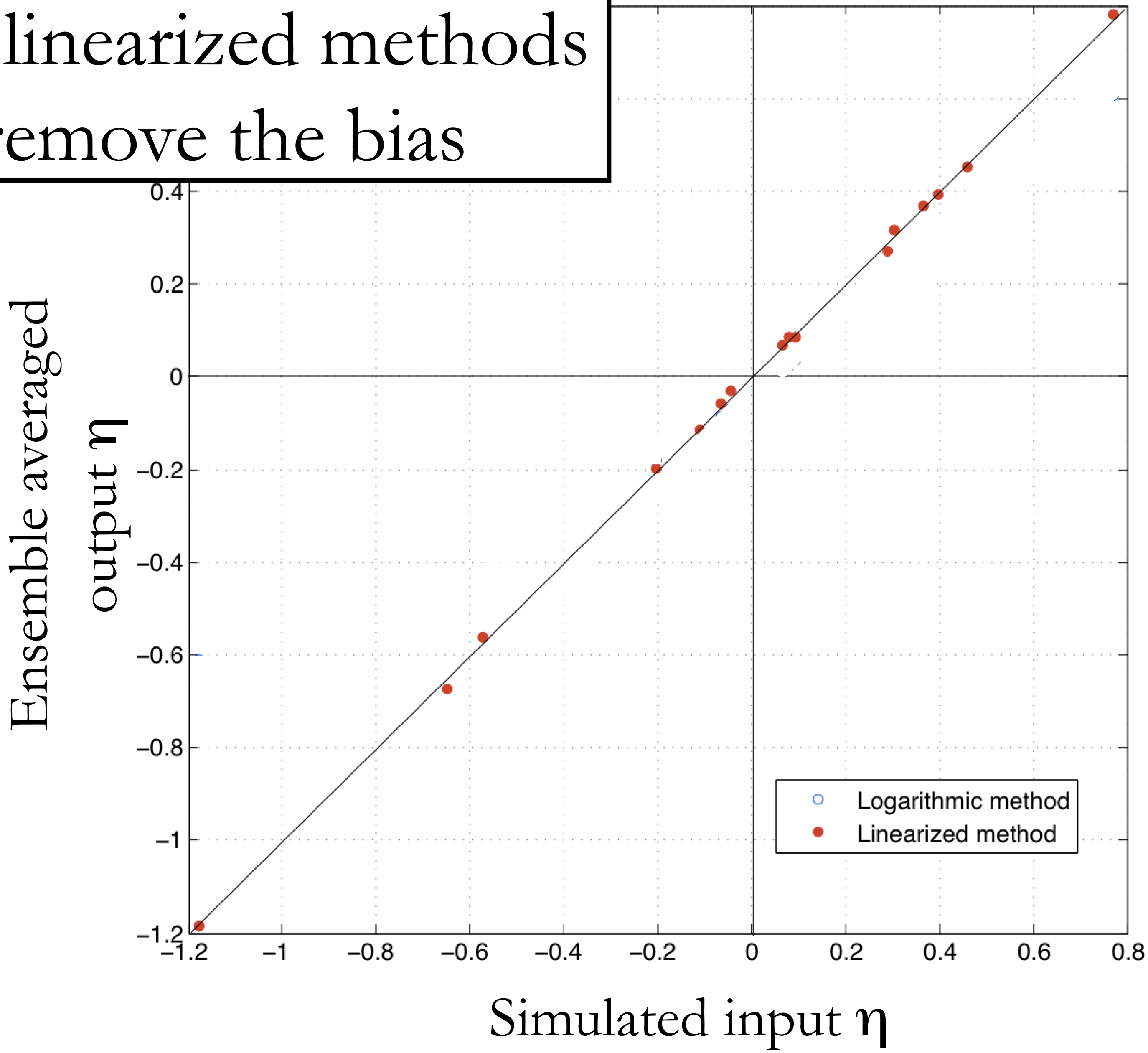
- Better characterization of calibration errors.
- Old, logarithmic version of redundant calibration is biased; new linear version is unbiased.

$$g_i \equiv e^{\eta_i + i\varphi_i}$$

Traditional methods exhibit a bias

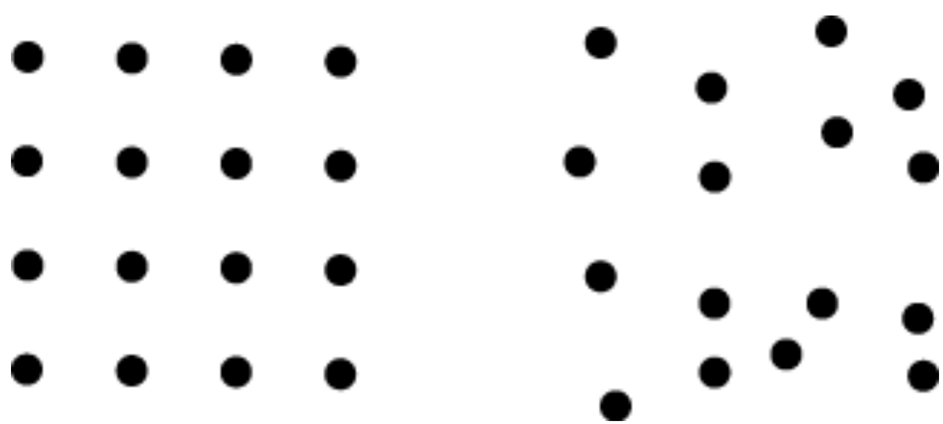


Our linearized methods remove the bias

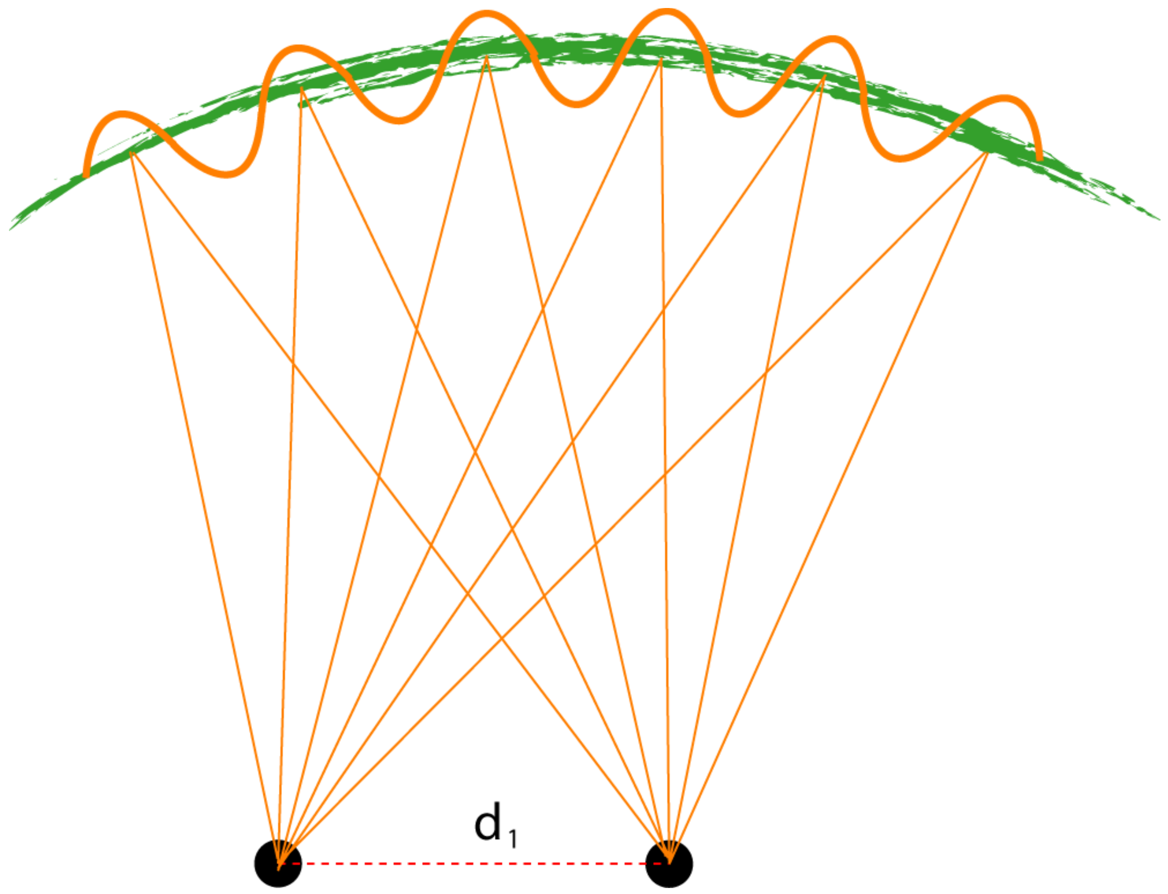


Can we do better?

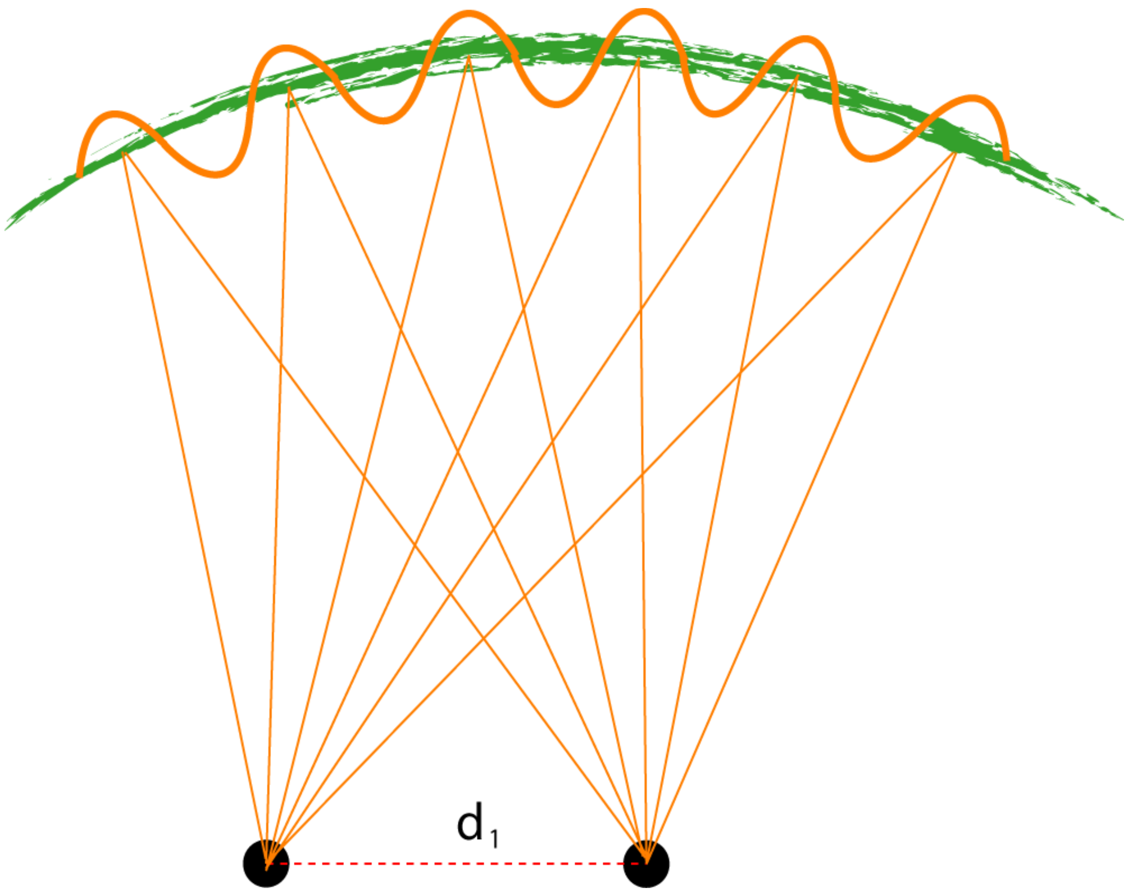
- Better characterization of calibration errors.
- Old, logarithmic version of redundant calibration is biased; new linear version is unbiased.
- Correcting for deviations from perfect redundancy.



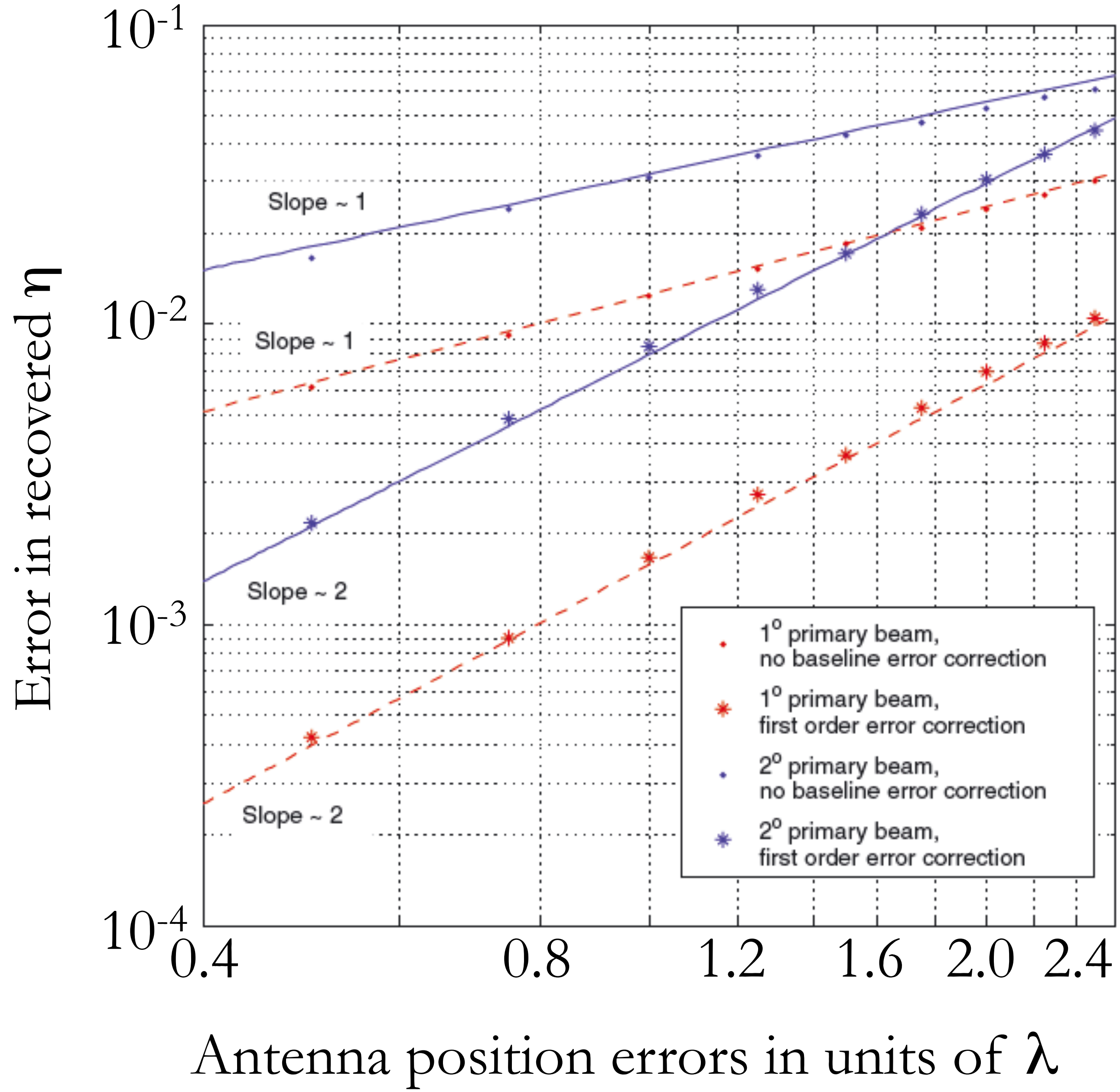
Taylor expand the Fourier sky



Taylor expand the Fourier sky



$$g_i \equiv e^{\eta_i + i\varphi_i}$$



Near-redundancy may be good enough

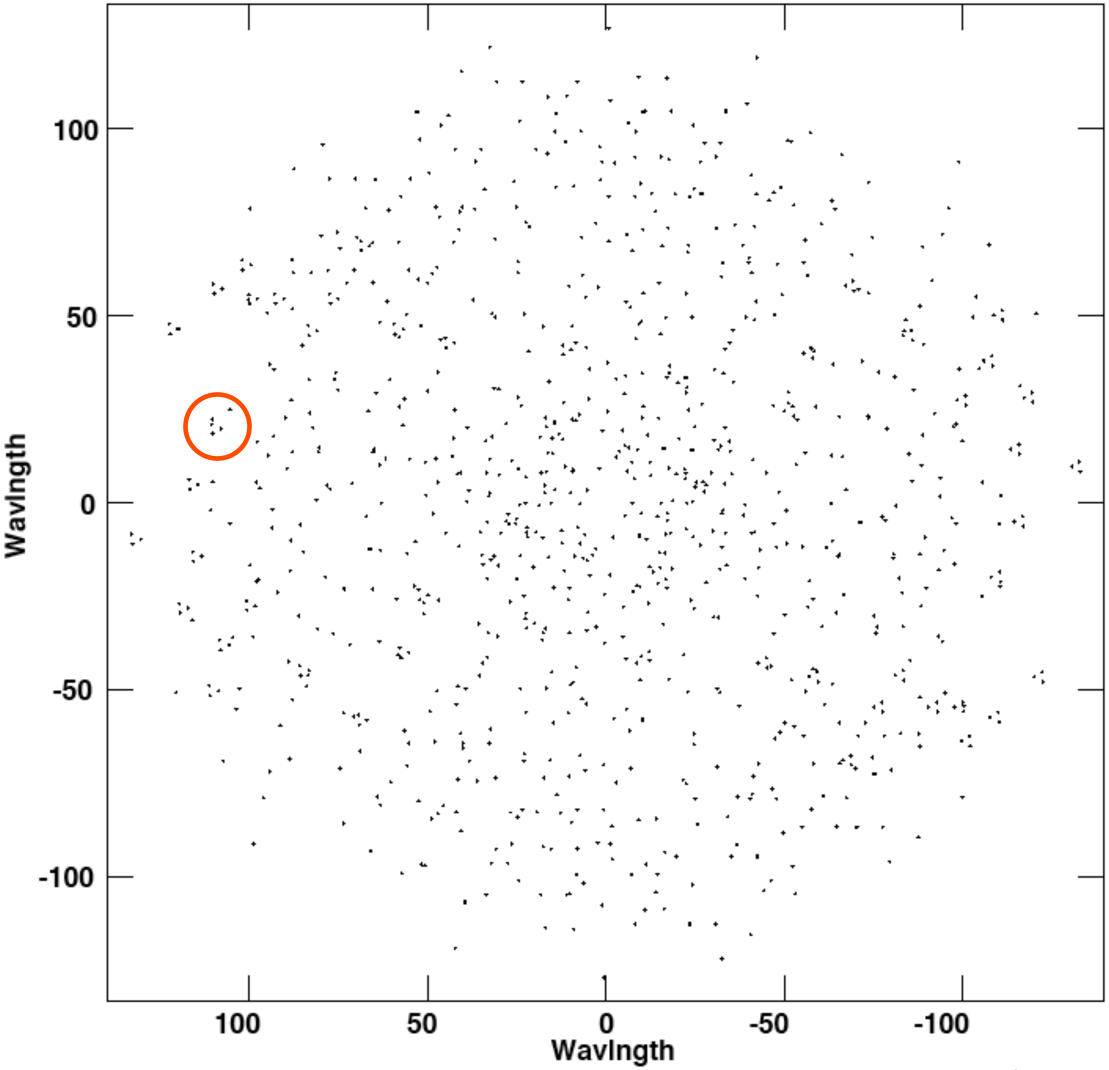
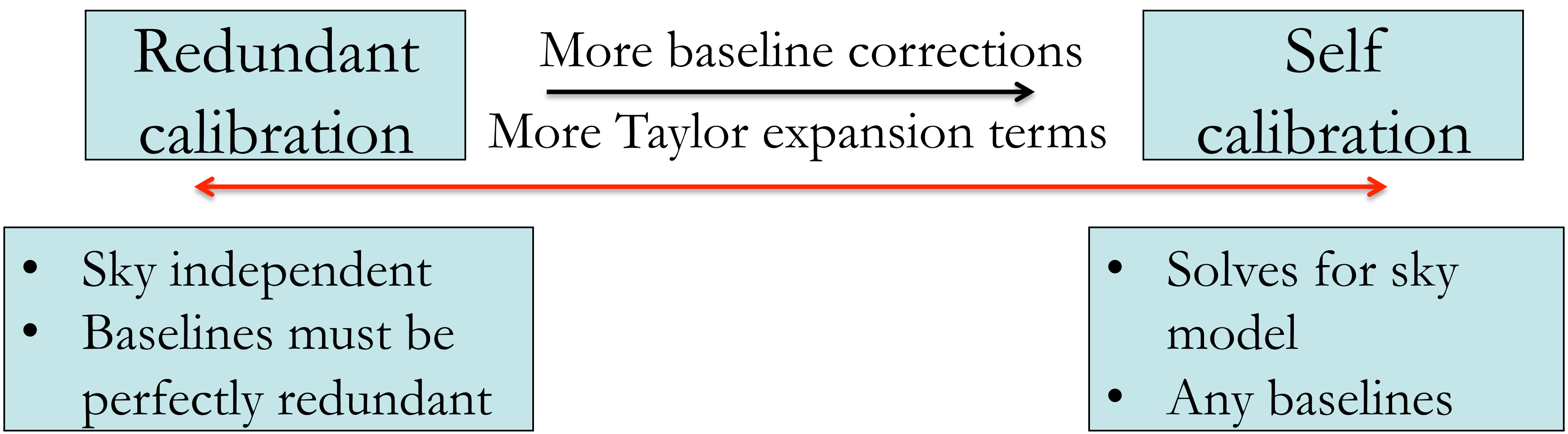


Image Credit: C. Williams

Can we do better?

- Better characterization of calibration errors.
- Old, logarithmic version of redundant calibration is biased; new linear version is unbiased.
- Correcting for deviations from perfect redundancy.
- Self calibration and redundant calibration are special cases that complement each other.



Precision Foreground Subtraction for Precision Cosmology

AL, Tegmark, arXiv:1103.0281, submitted to MNRAS

AL, Tegmark, Phys. Rev. D 83, 103006 (2011)

AL, Tegmark, Bowman, Hewitt, Zaldarriaga, MNRAS 398, 401 (2009)

AL, Tegmark, Zaldarriaga, MNRAS 394, 1575 (2009)

Foreground Modeling

Principal components of the sky

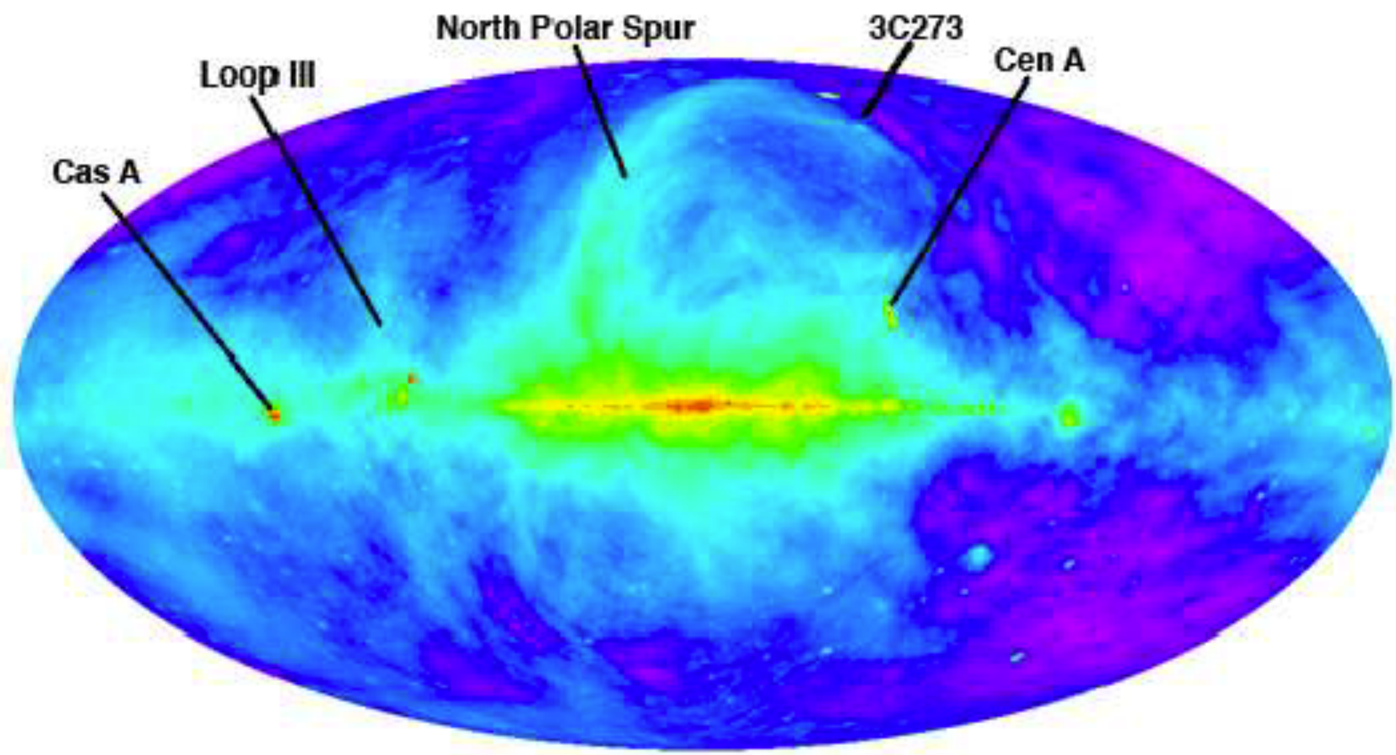
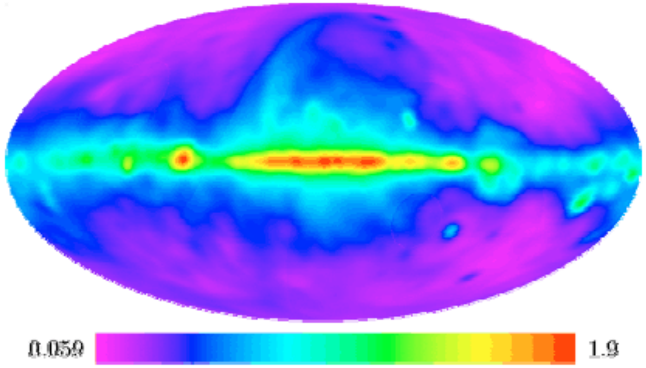


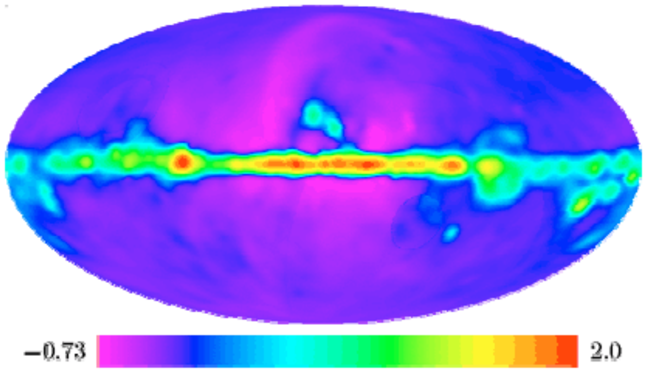
Image credit: de Oliveira-Costa et. al. 2008

Principal components of the sky

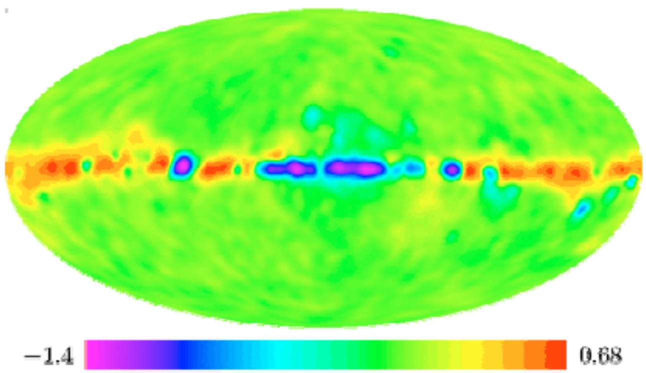
1st principal component



2nd principal component



3rd principal component



Can we do better?

- Understand, using a simple theoretical toy model, why the foregrounds are describable using so few components.

Understanding "why so few" components

- Start with a simple but realistic model.

Parameter	Description	Fiducial Value
B	Source count normalization	$4.0\,\mathrm{mJy}^{-1}\,\mathrm{Sr}^{-1}$
γ	Source count power-law index	1.75
α_{ps}	Point source spectral index	2.5
σ_α	Point source index spread	0.5
A_{sync}	Synchrotron amplitude	335.4 K
α_{sync}	Synchrotron spectral index	2.8
$\Delta\alpha_{sync}$	Synchrotron index coherence	0.1
A_{ff}	Free-free amplitude	33.5 K
α_{ff}	Free-free spectral index	2.15
$\Delta\alpha_{ff}$	Free-free index coherence	0.01

Understanding “why so few” components

- Start with a simple but realistic model.
- Write down covariance function.

$$C(\nu, \nu')$$

Understanding "why so few" components

- Start with a simple but realistic model.
- Write down covariance function.
- Non-dimensionalize to get correlation function.

$$R(\nu, \nu') = \frac{C(\nu, \nu')}{\sigma(\nu)\sigma(\nu')}$$

Understanding "why so few" components

- Start with a simple but realistic model.
- Write down covariance function.
- Non-dimensionalize to get correlation function.
- To a good approximation, correlation function fits the following form with coherence length ν_c=560 MHz!

$$R(\nu,\nu') \approx \exp\left[-\frac{(\nu-\nu')^2}{2\nu_c^2}\right]$$

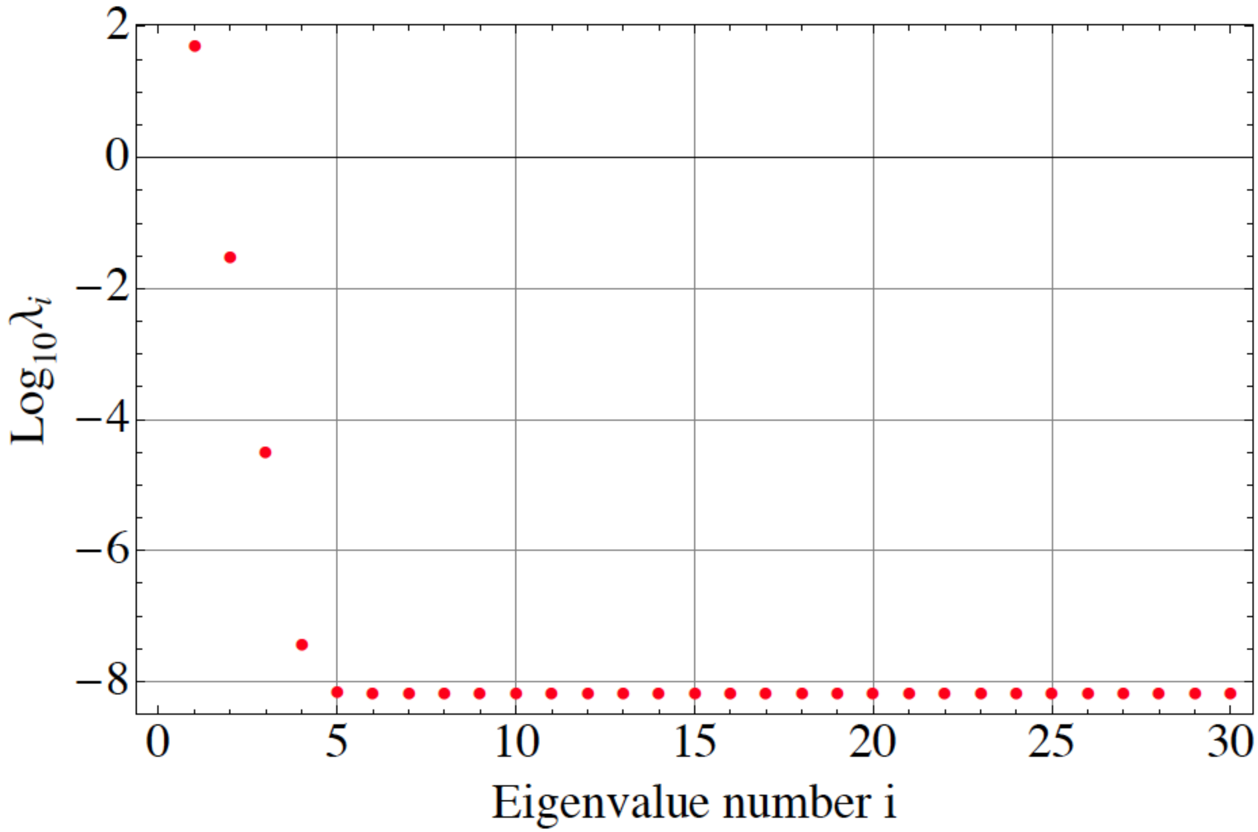
Understanding "why so few" components

- Start with a simple but realistic model.
- Write down covariance function.
- Non-dimensionalize to get correlation function.
- To a good approximation, correlation function fits the following form with coherence length ν_c=560 MHz!
- Find principal components/eigenfunctions:

$$\int R(\nu - \nu')f_n(\nu')d\nu' = \lambda_n f_n(\nu)$$

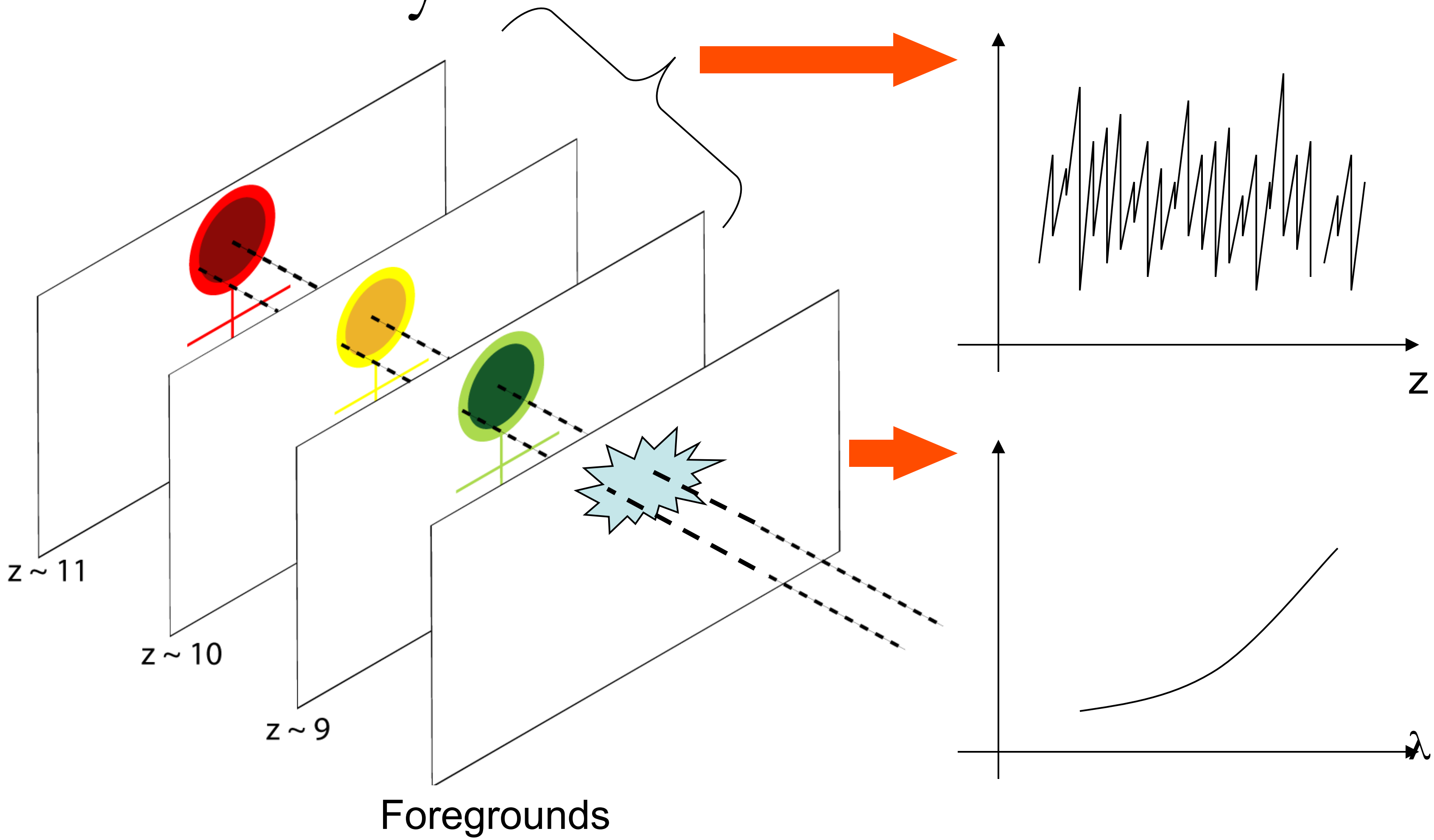
$$\int_{-\infty}^{\infty} \exp\left[-\frac{(\nu-\nu')^2}{2\nu_c}\right] \sin(\gamma_n \nu' + \phi) d\nu' = \lambda_n \sin(\gamma_n \nu + \phi)$$

$$\lambda_n = \sqrt{2\pi\nu_c^2} \exp\left(-2\nu_c^2 \gamma_n^2\right)$$



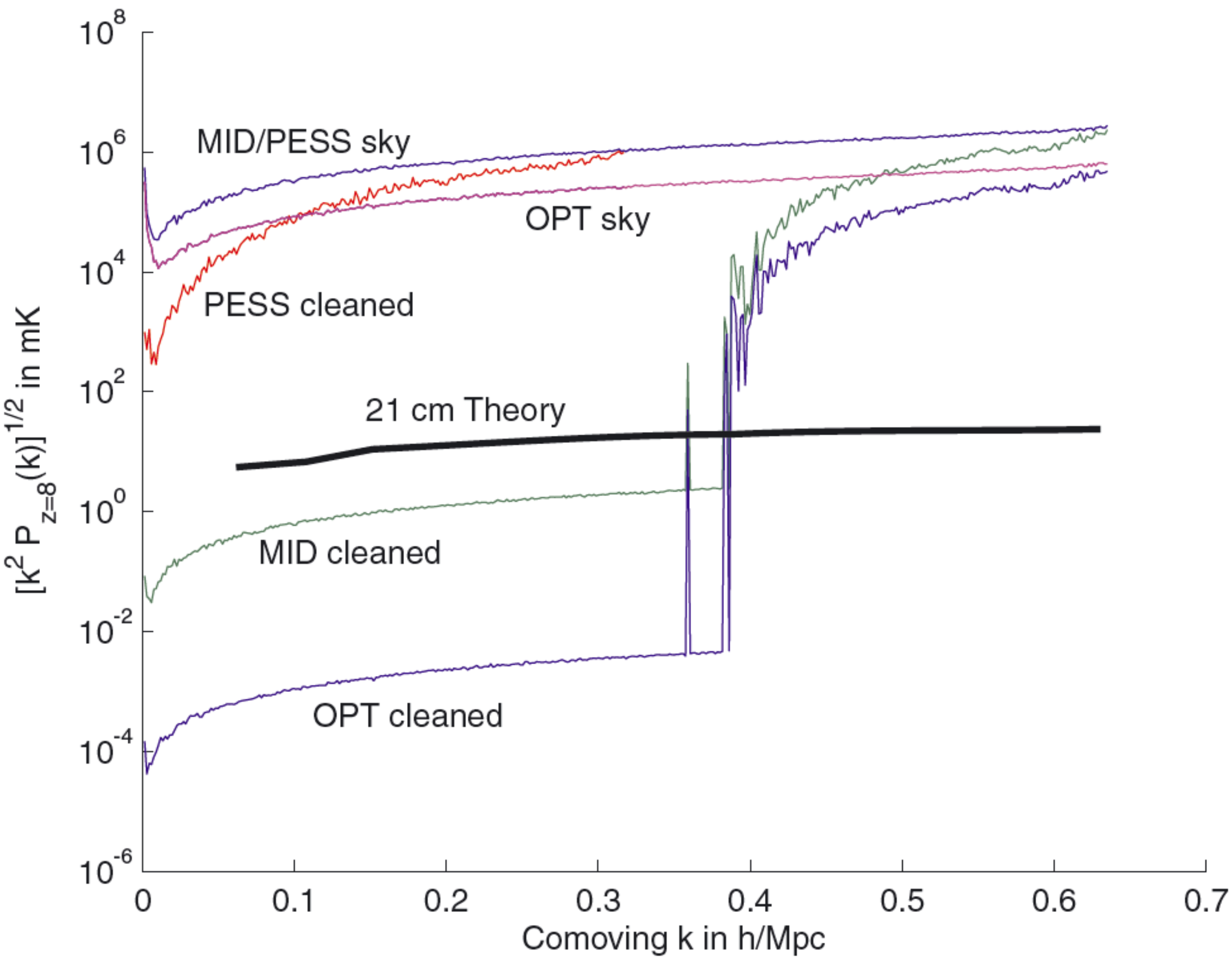
Foreground Subtraction

Method #1: Line-of-Sight Polynomial Subtraction



E.g. Wang et. al. (2006), Bowman et. al. (2009), AL et. al. (2009a,b), Jelic et. al. (2008), Harker et. al. (2009, 2010).

Assumptions		Low-Performance Extreme	Fiducial Model	High-Performance Extreme
Experimental	Tile Arrangement	$\rho(r) \sim r^{-2}$	$\rho(r) \sim r^{-2}$	Monolithic with tiles separated by 40 m
	Rotation synthesis	None	6 hours, continuous	6 hours, continuous
	Noise level	$\sigma_T \sim 1\,\mathrm{mK}$	Noiseless [5]	Noiseless
Analysis	Primary beam width adjustments	None	None	Adjusted to be frequency-independent
	Bright point source flux cut S_{cut}	100 mJy	10 mJy	0.1 mJy
	synthesized beam width adjustments	None	None	Resolutions equalised by extra smoothing
	u-v plane weighting	None (natural)	Uniform	Uniform
	Order of polynomial fit	Constant	Quadratic	Quintic
	Range of polynomial fit	80 MHz	2.4 MHz	2.4 MHz



AL, Tegmark, Zaldarriaga, MNRAS **394**, 1575 (2009)

AL, Tegmark, Bowman, Hewitt, Zaldarriaga, MNRAS **398**, 401 (2009).

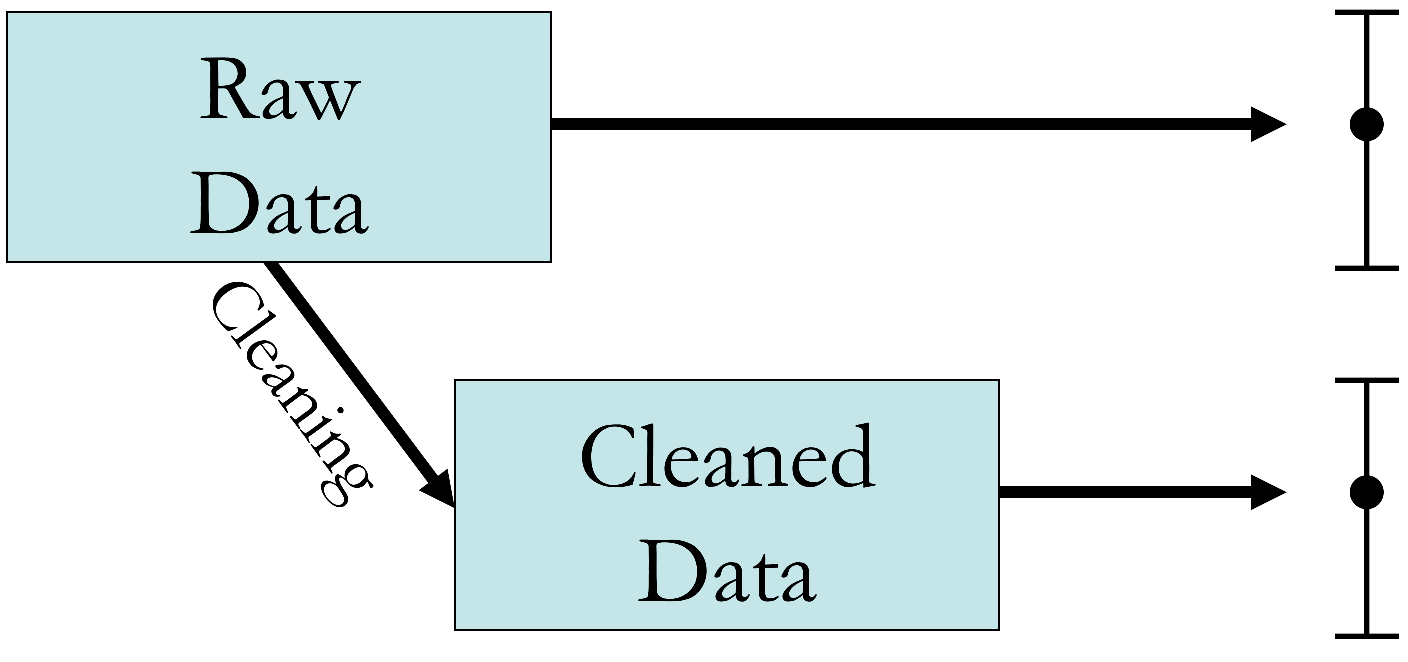
See also: Wang et. al. (2006), Bowman et. al. (2009), Jelic et. al. (2008), Harker et. al. (2009,2010).

Can we do better?

Foreground Subtraction Wish-list

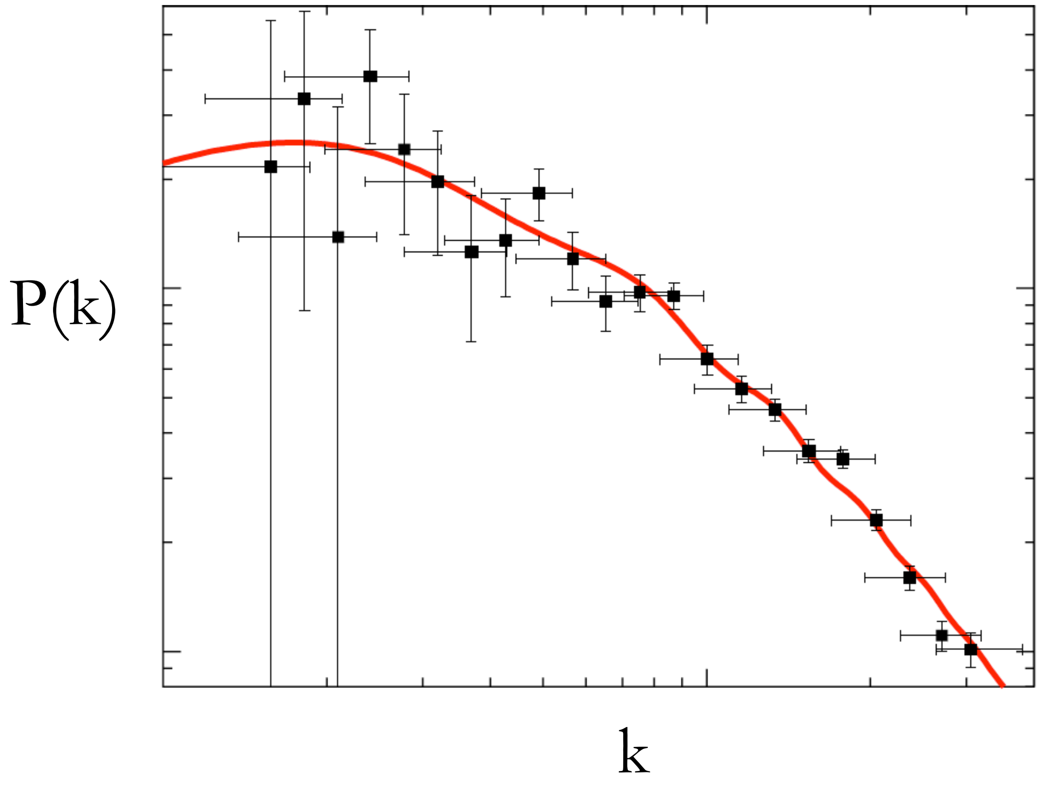
Foreground Subtraction Wish-list

- Lossless



Foreground Subtraction Wish-list

- Lossless
- Small “vertical” error bars



Foreground Subtraction Wish-list

- Lossless
- Small "vertical" error bars
- Small "horizontal" error bars/mode-mixing

Fo… st

- Los…
- Sm…
- Sm…

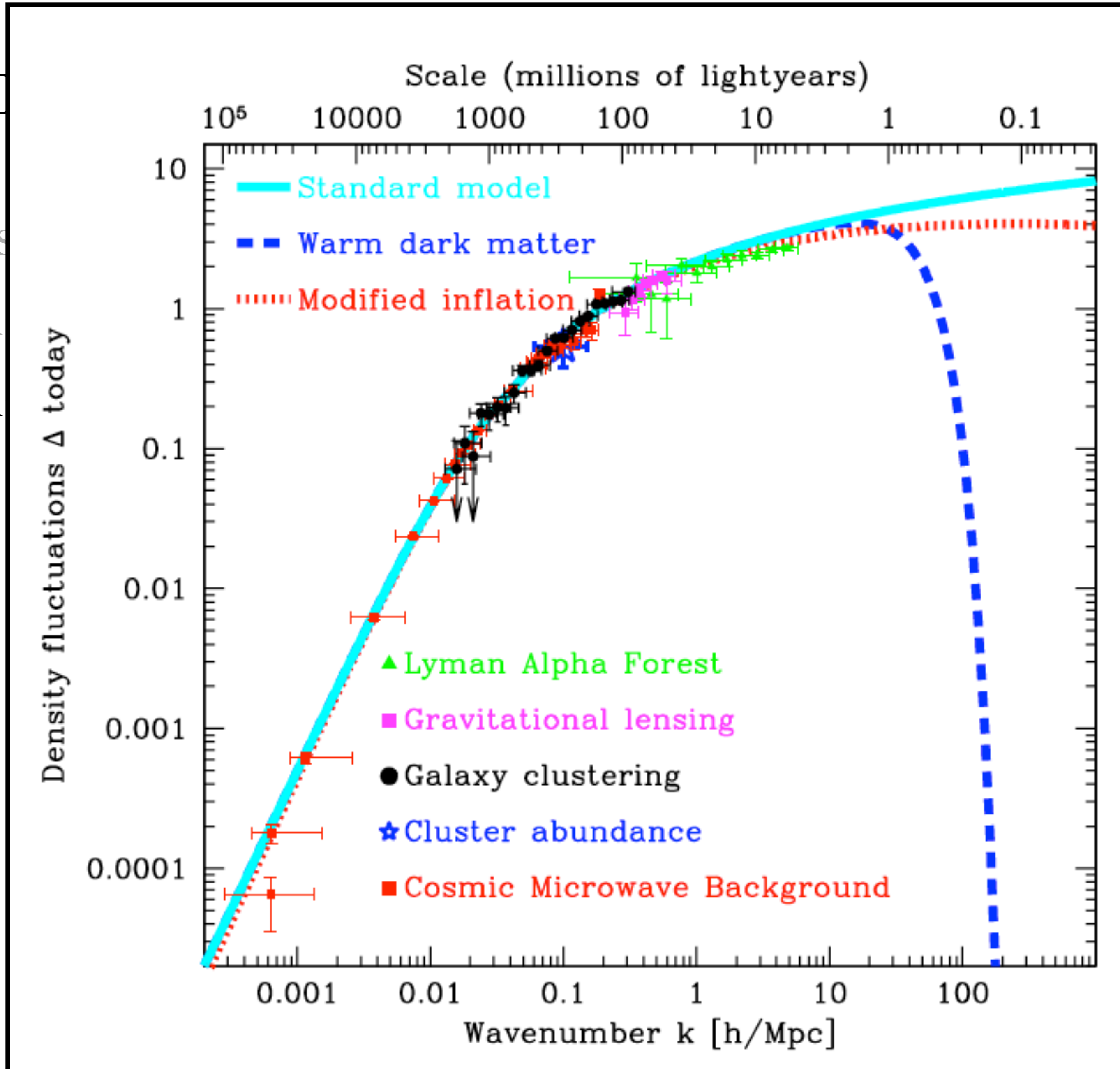


Image credit: Tegmark, Zaldarriaga '09

Foreground Subtraction Wish-list

- Lossl
- Smal
- Smal

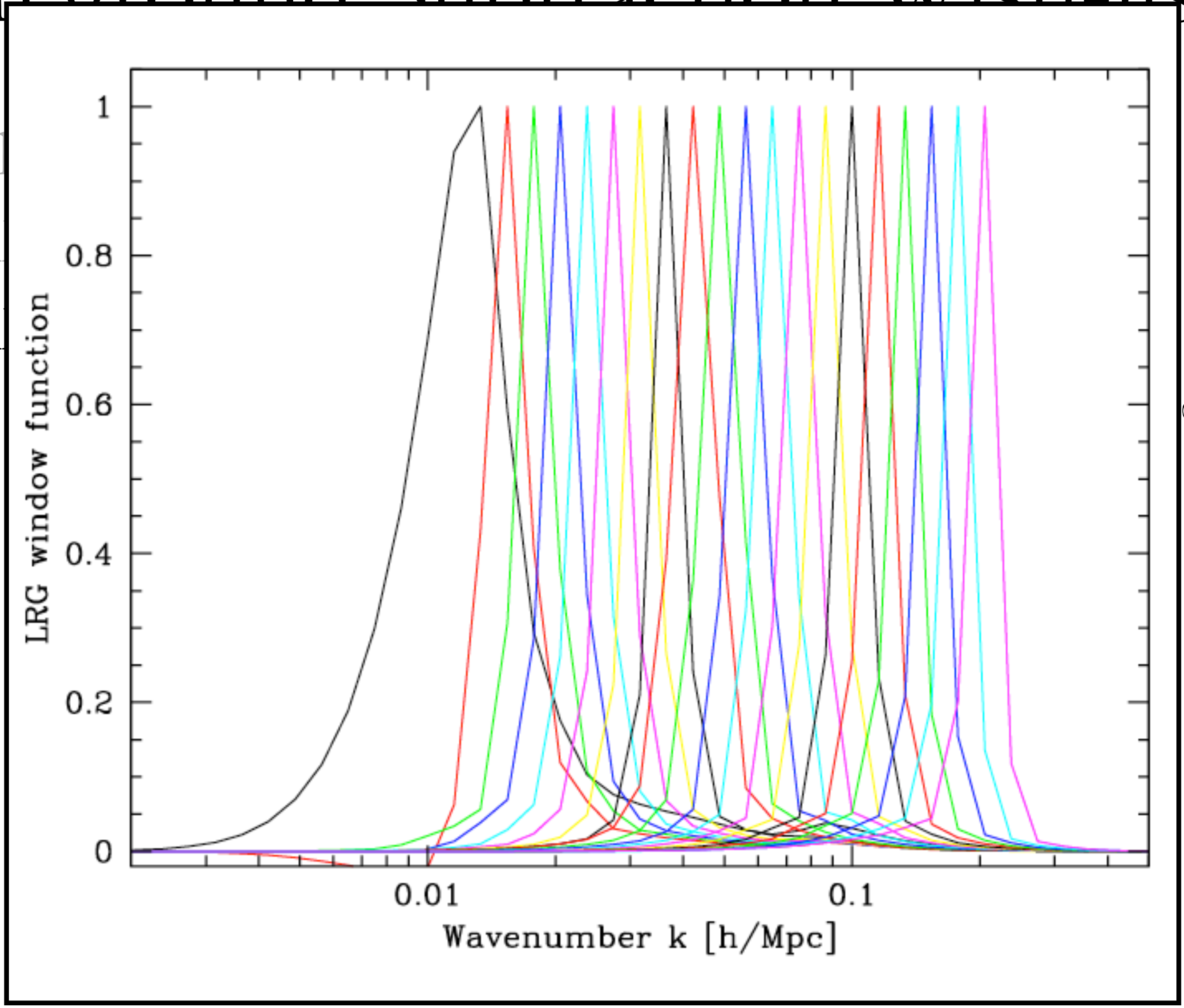
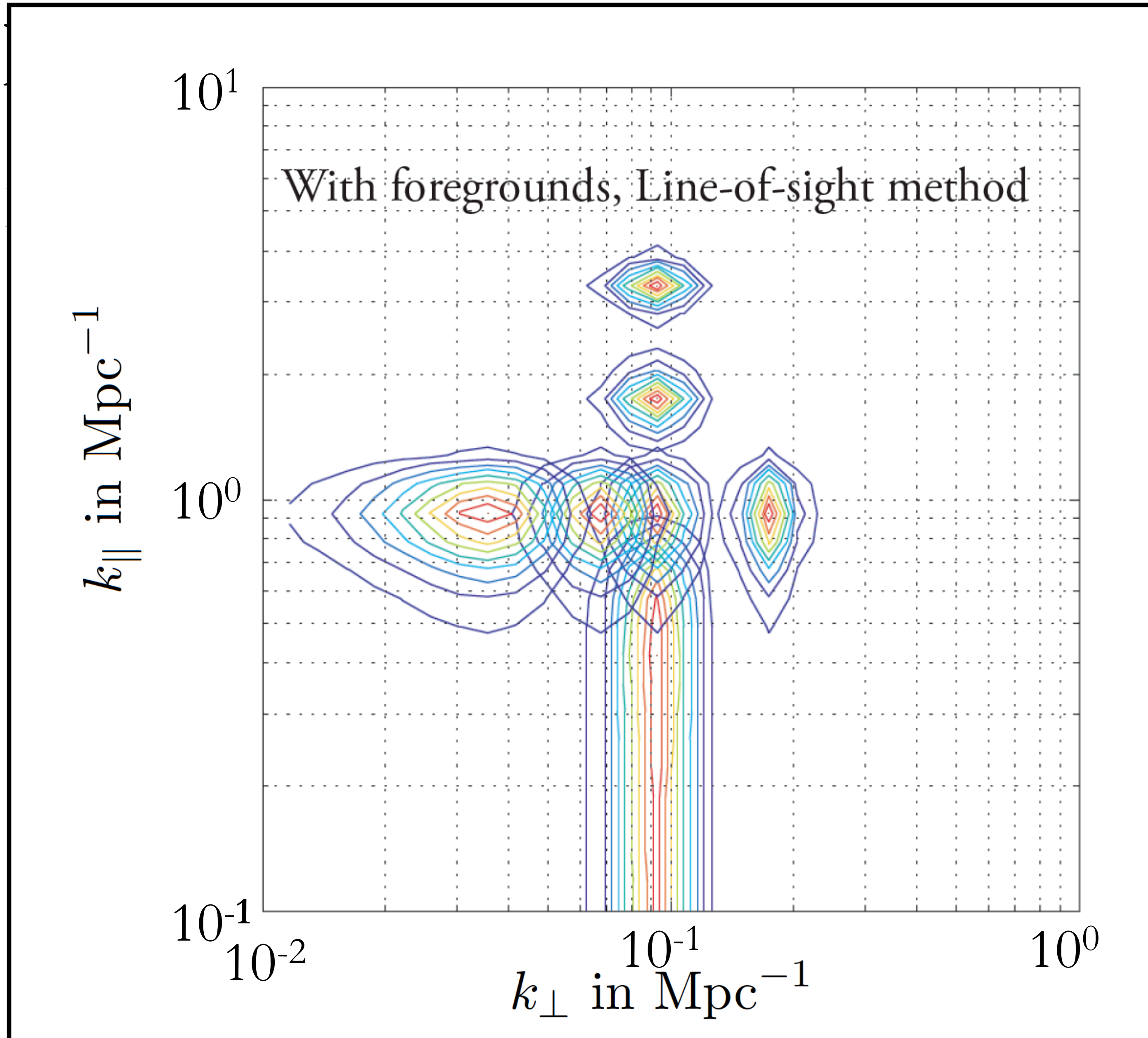


Image credit: SDSS Collaboration

list



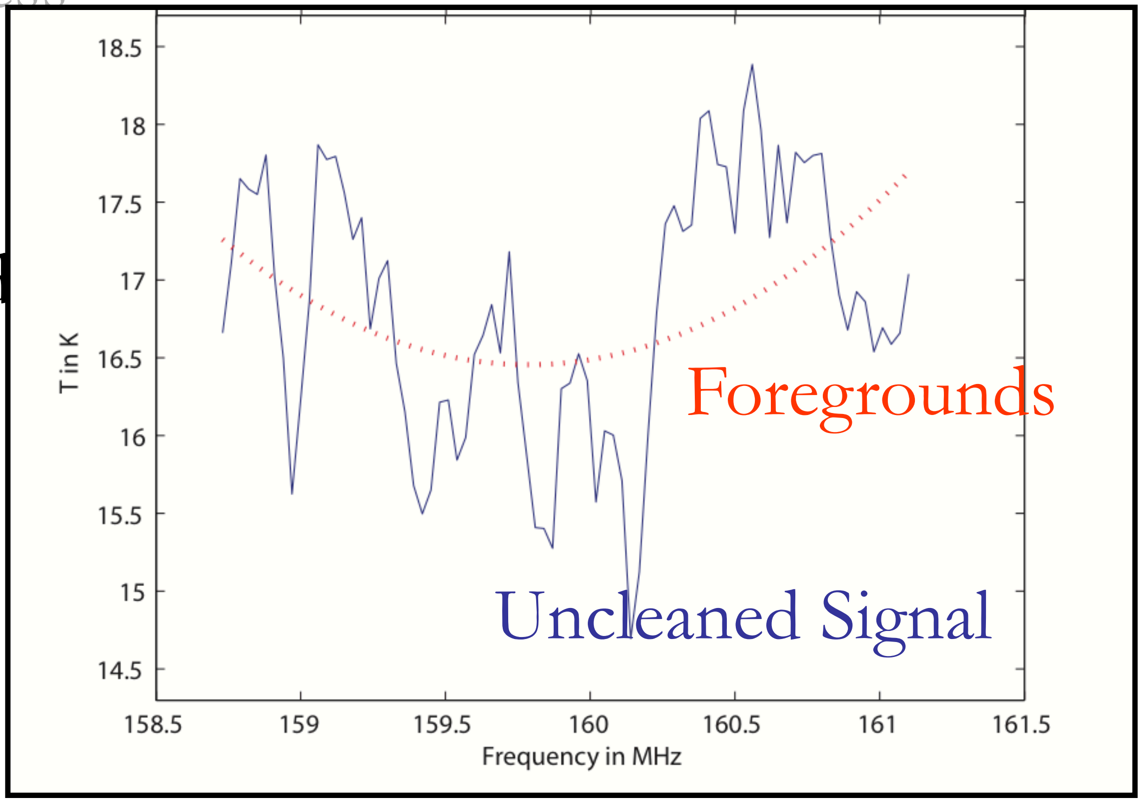
AL, Tegmark, Phys. Rev. D 83, 103006 (2011)

Foreground Subtraction Wish-list

- Lossless
- Small “vertical” error bars
- Small “horizontal” error bars/mode-mixing
- No additive noise/foreground bias

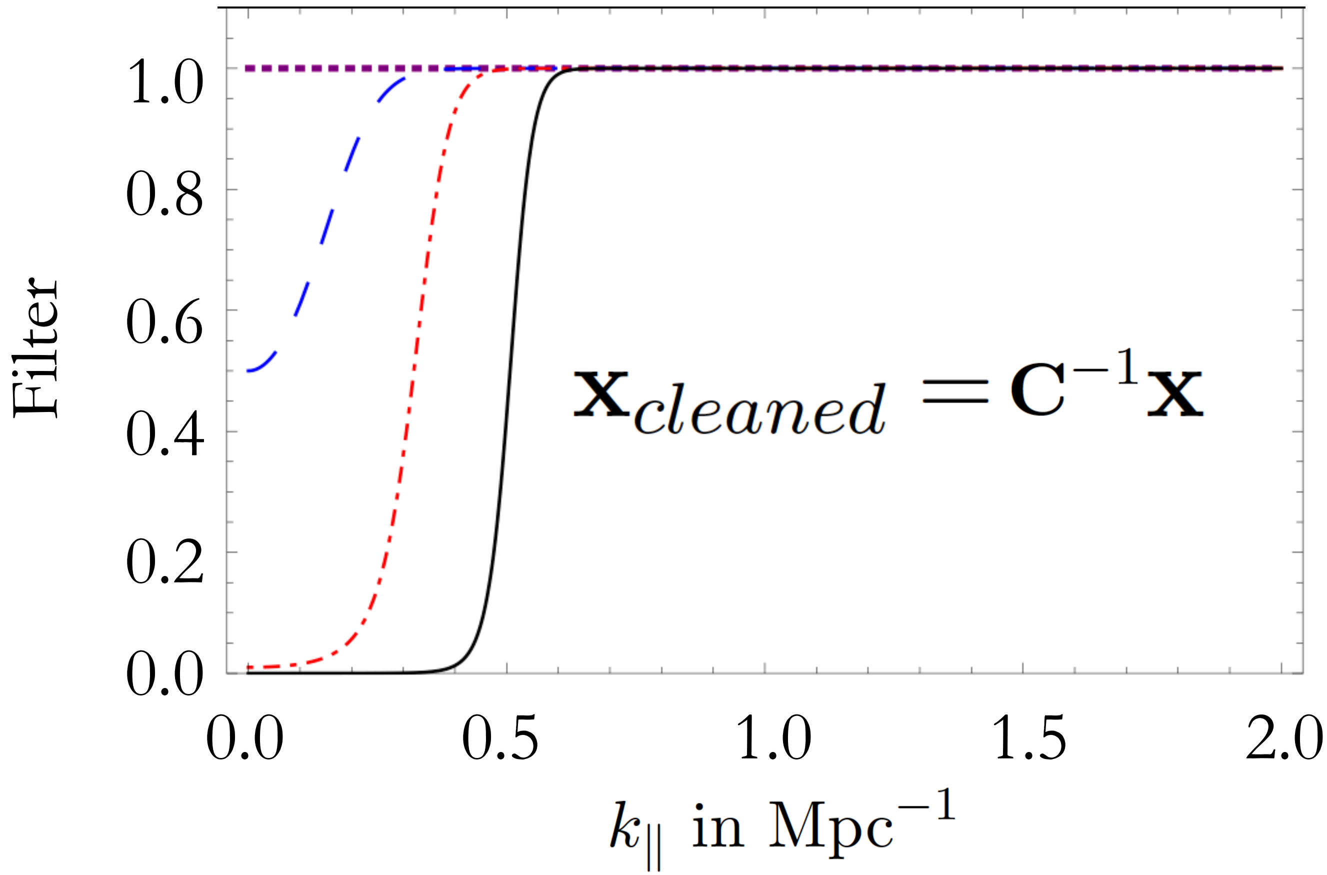
Foreground Subtraction Wish-list

- Lossless
- Small
- Small
- No ad



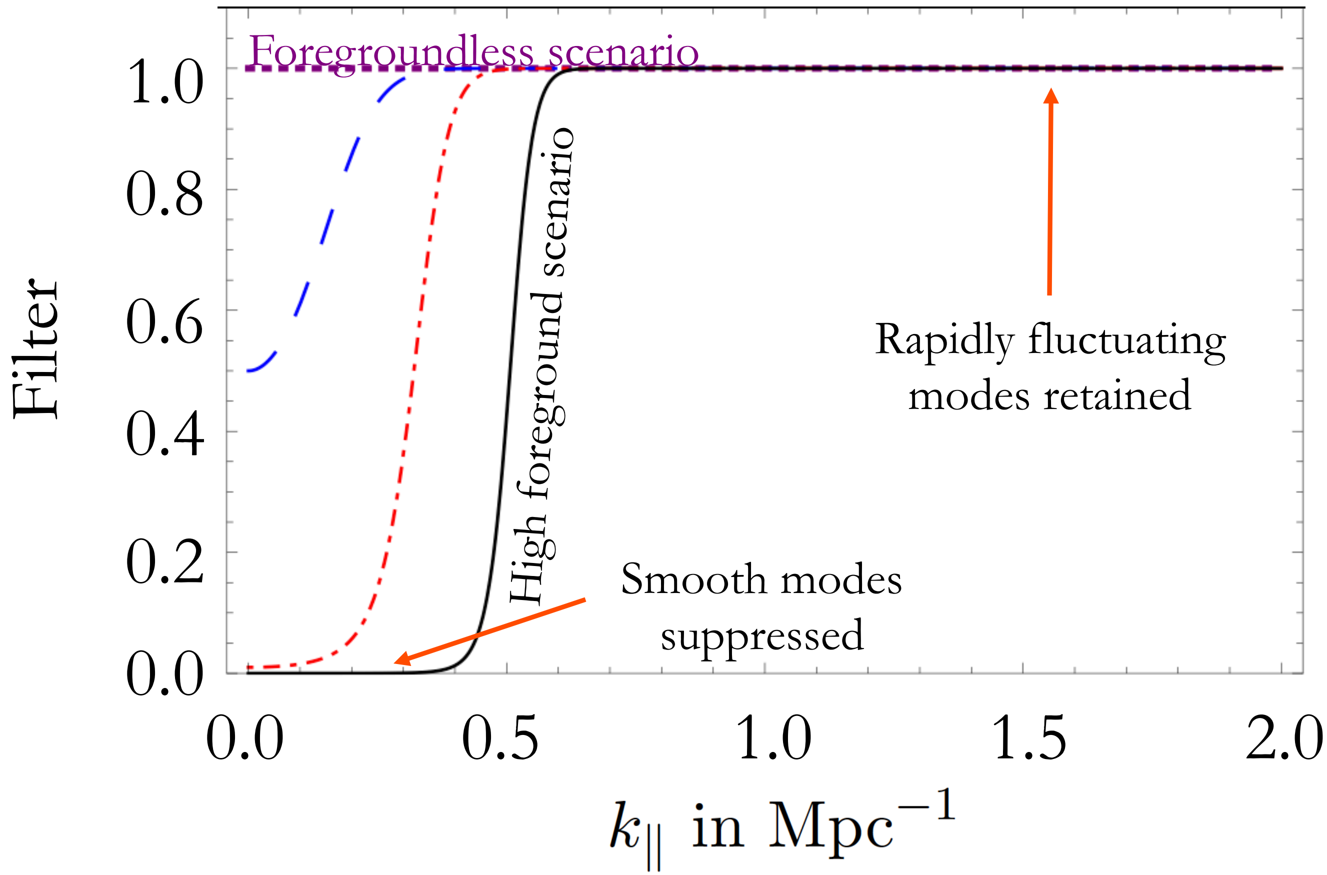
AL, M. Tegmark, M. Zaldarriaga, MNRAS **394**, 1575 (2009)

Method #2: Fourier space filtering/ Inverse variance weighting



For similar methods, see also N. Petrovic & S.P. Oh, MNRAS **413**, 2103 (2011)
G. Paciga et. al., MNRAS **413**, 1174 (2011)

Method #2: Fourier space filtering/ Inverse variance weighting



For similar methods, see also N. Petrovic & S.P. Oh, MNRAS **413**, 2103 (2011)
G. Paciga et. al., MNRAS **413**, 1174 (2011)

Back to our wishlist…

Lossless?

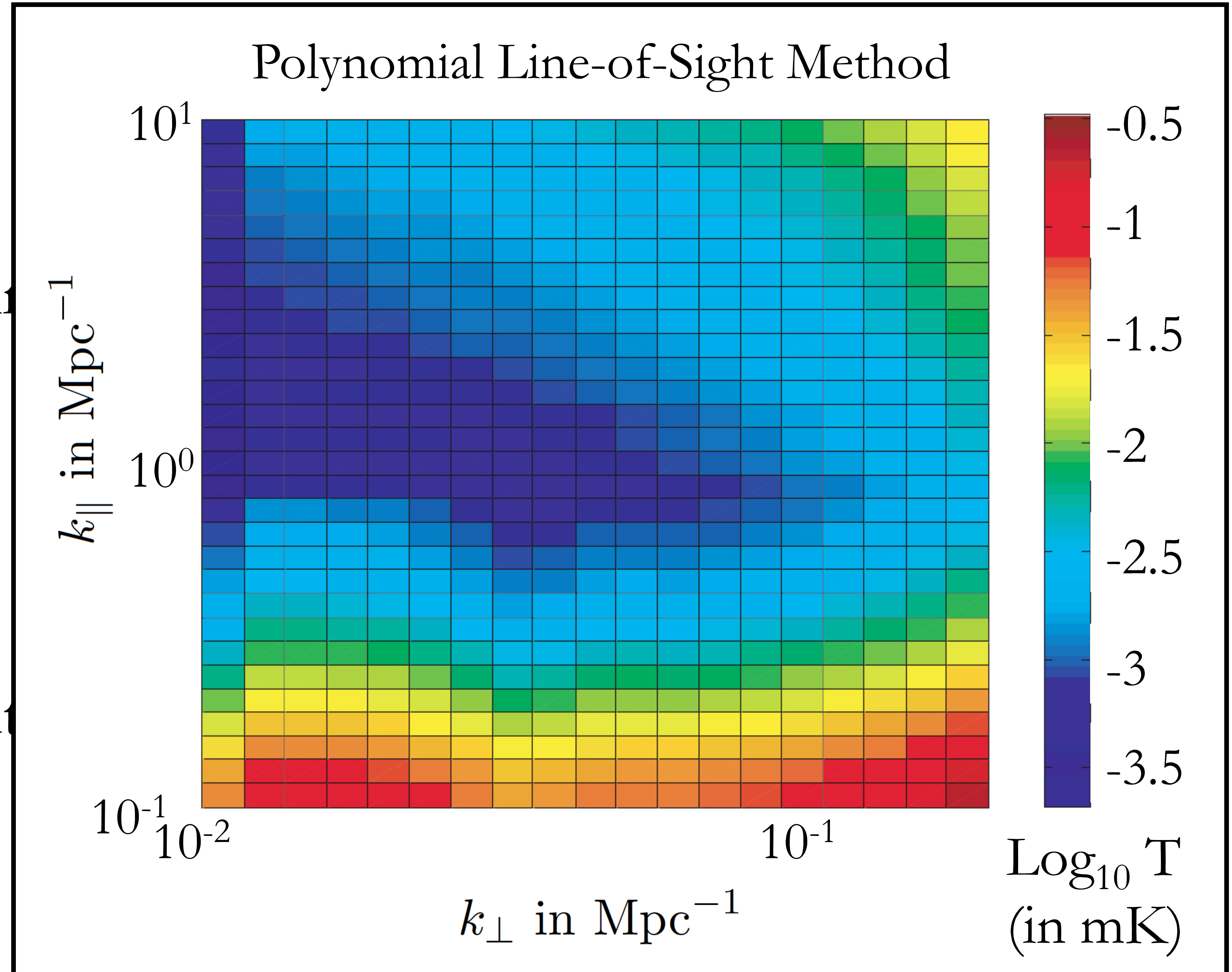
Line-of-Sight Polynomial Subtraction --- **Lossy**

Inverse Variance Subtraction --- **Lossless**

Biased?

Line-of-Sight Polynomial Subtraction --- **Biased in literature, fixable**

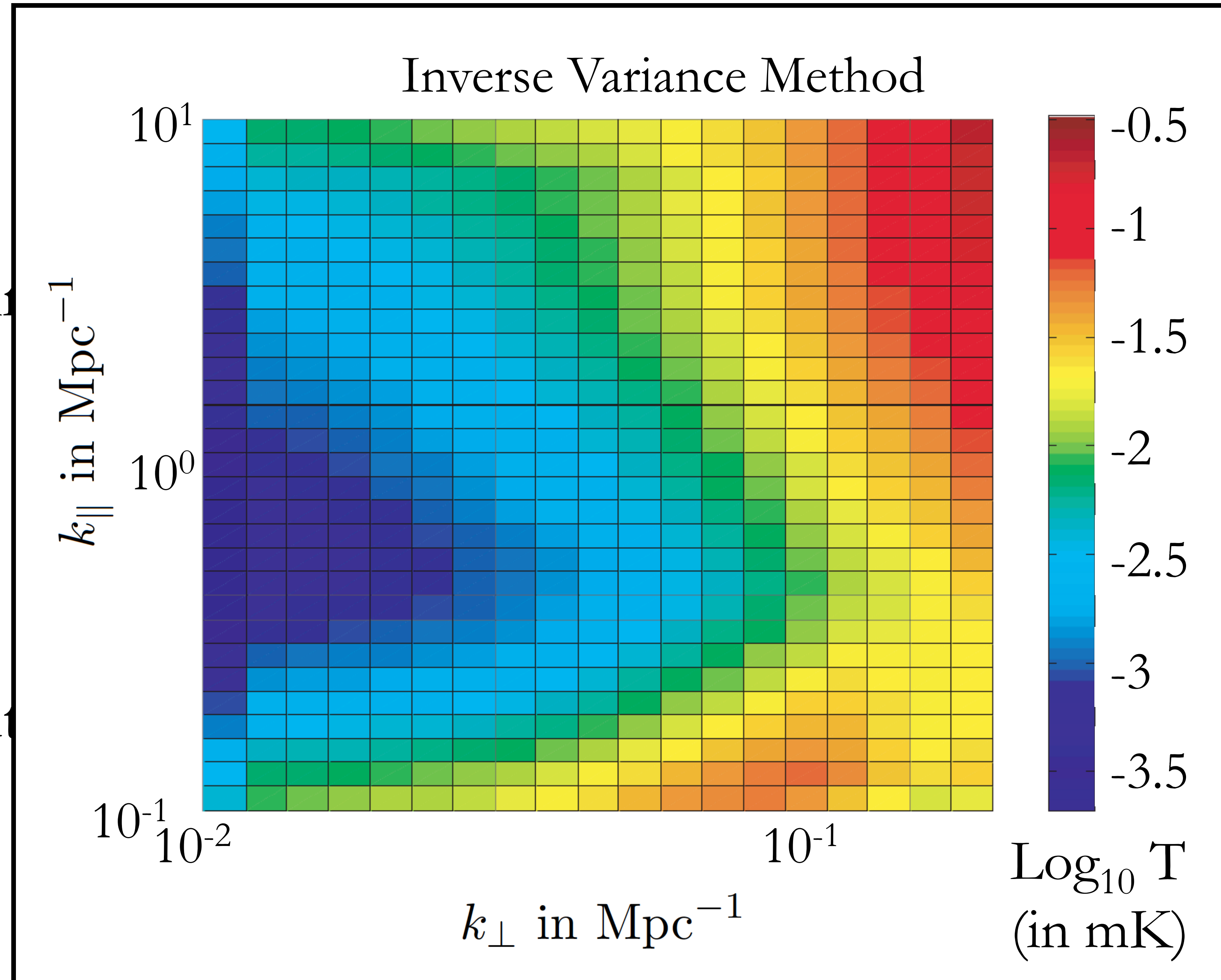
Inverse Variance Subtraction --- **Unbiased**



Polynomial Line-of-Sight Method
10^1
10^0
10^-1
10^-2
10^-1
$k_{||}$ in Mpc^{-1}
$k_\perp$ in Mpc^{-1}
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
Log10 T
(in mK)

Li

But



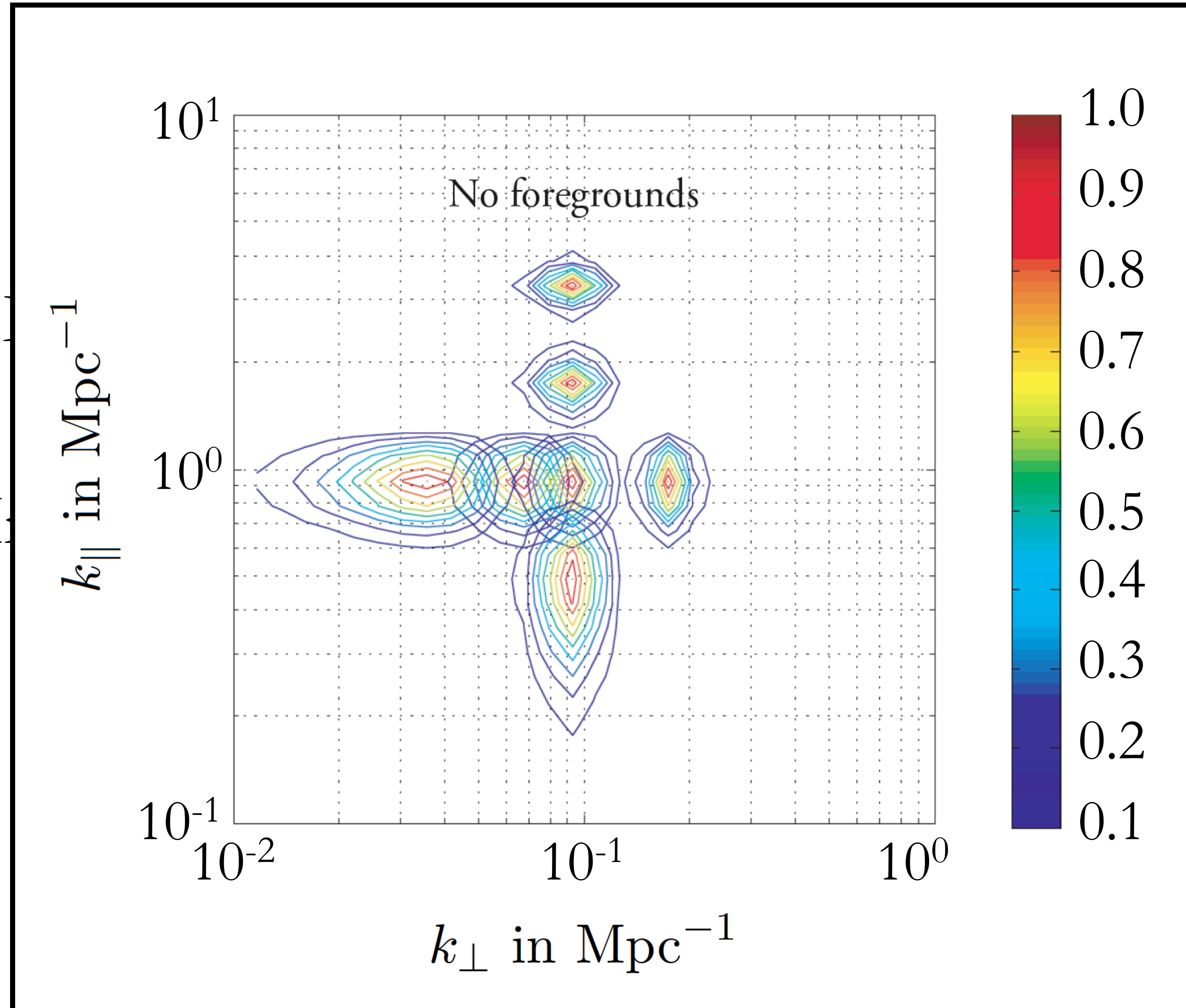
Inverse Variance Method
10^1
10^0
10^{-1}
$k_{||}$ in Mpc^{-1}
10^{-2}
10^{-1}
$k_\perp$ in Mpc^{-1}
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
Log$_{10}$ T
(in mK)

Mode Mixing?

Line-of-Sight Polynomial Subtraction --- **Yes**

Inverse Variance Subtraction --- **Yes, but less.**

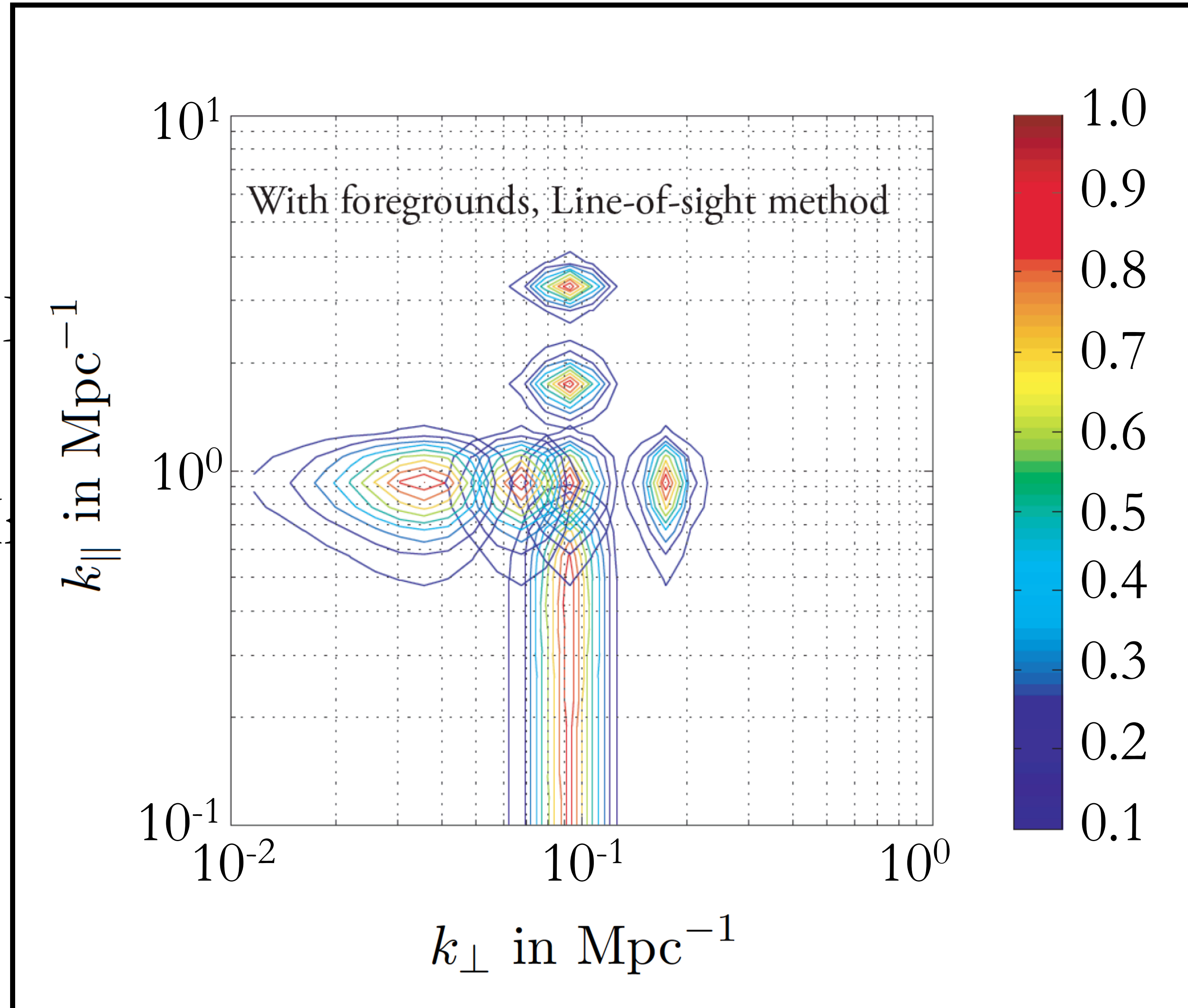
I


No foregrounds
$k_{\|}$ in Mpc^{-1}
$k_{\perp}$ in Mpc^{-1}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-1}
10^{0}
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1

es

ss.

I

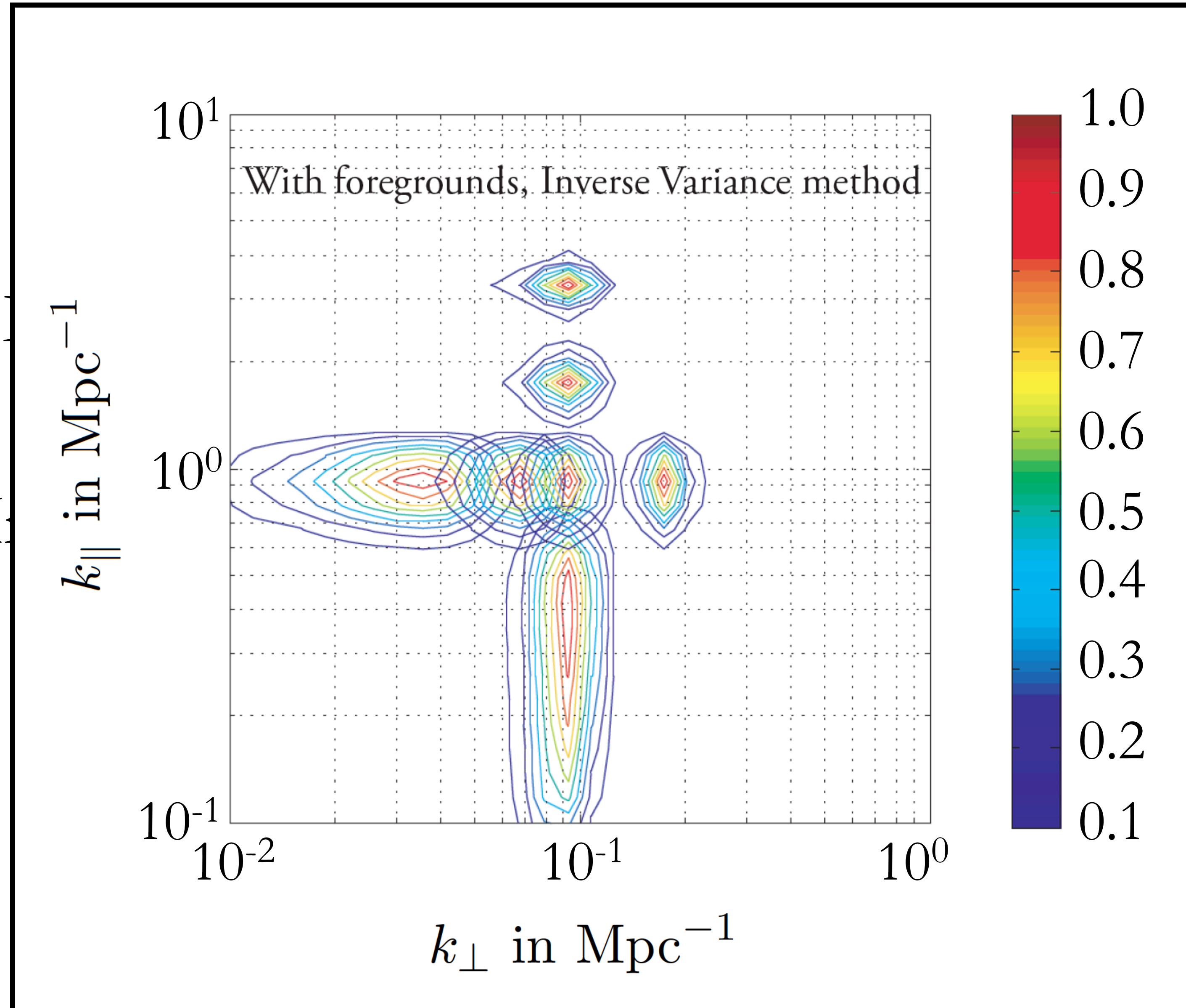


With foregrounds, Line-of-sight method
$k_{||}$ in Mpc^{-1}
$k_{\perp}$ in Mpc^{-1}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-1}
10^{0}
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1

es

ss.

I


With foregrounds, Inverse Variance method
$k_{||}$ in Mpc^{-1}
$k_{\perp}$ in Mpc^{-1}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-1}
10^{0}
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1

es

ss.

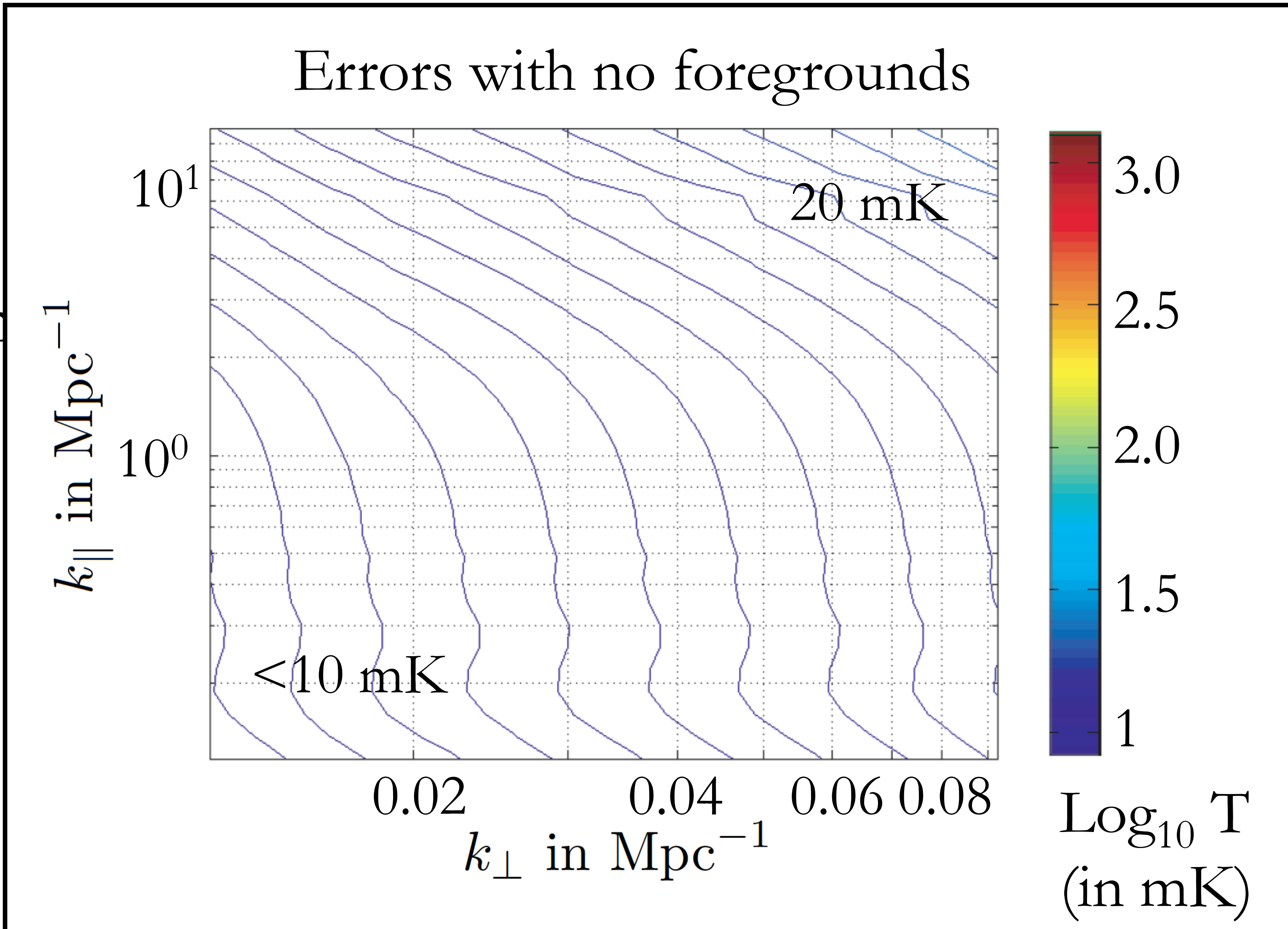
Measurement errors?

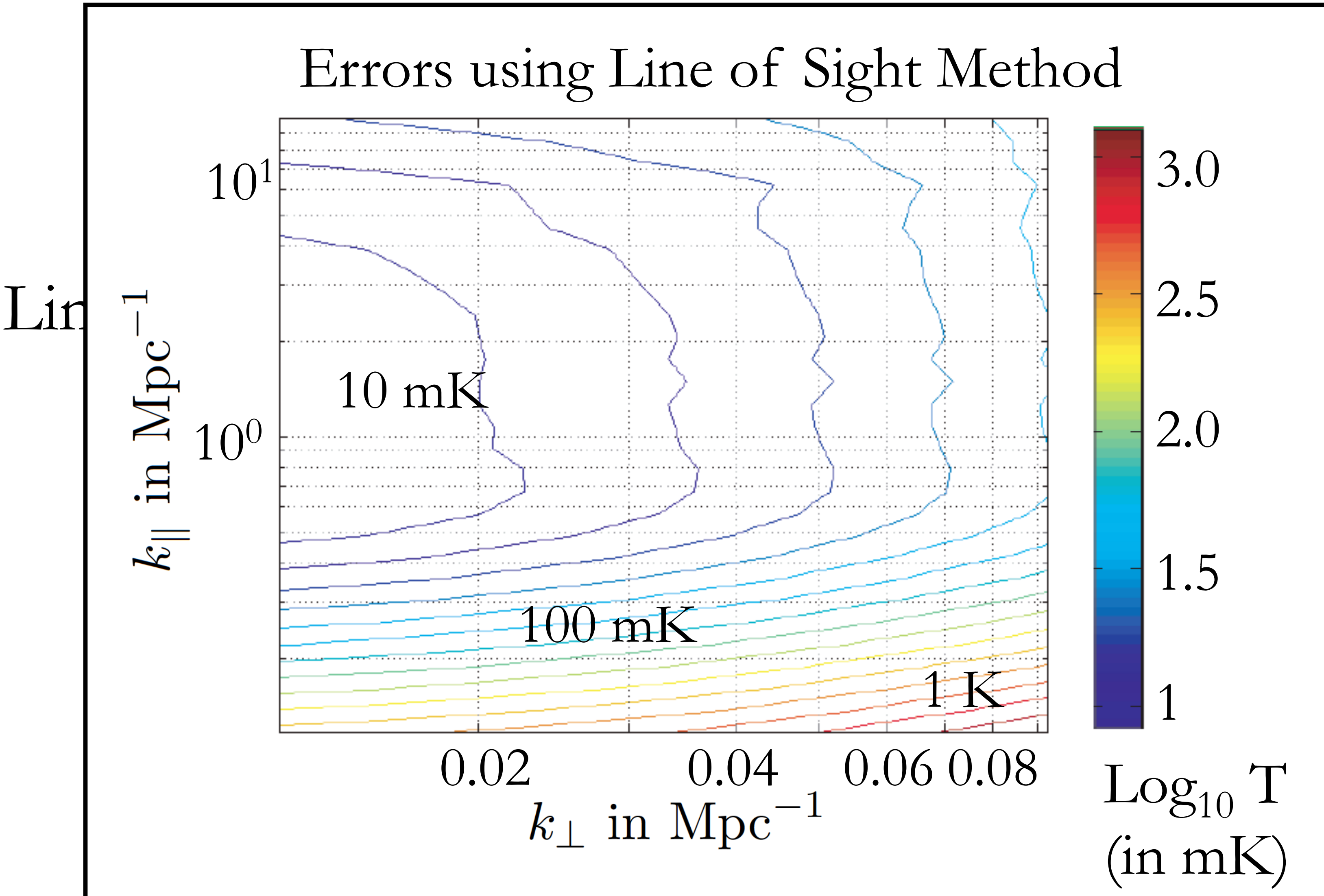
Line-of-Sight Polynomial Subtraction --- **Larger**

Inverse Variance Subtraction --- **Smaller**

Consider errors on the quantity

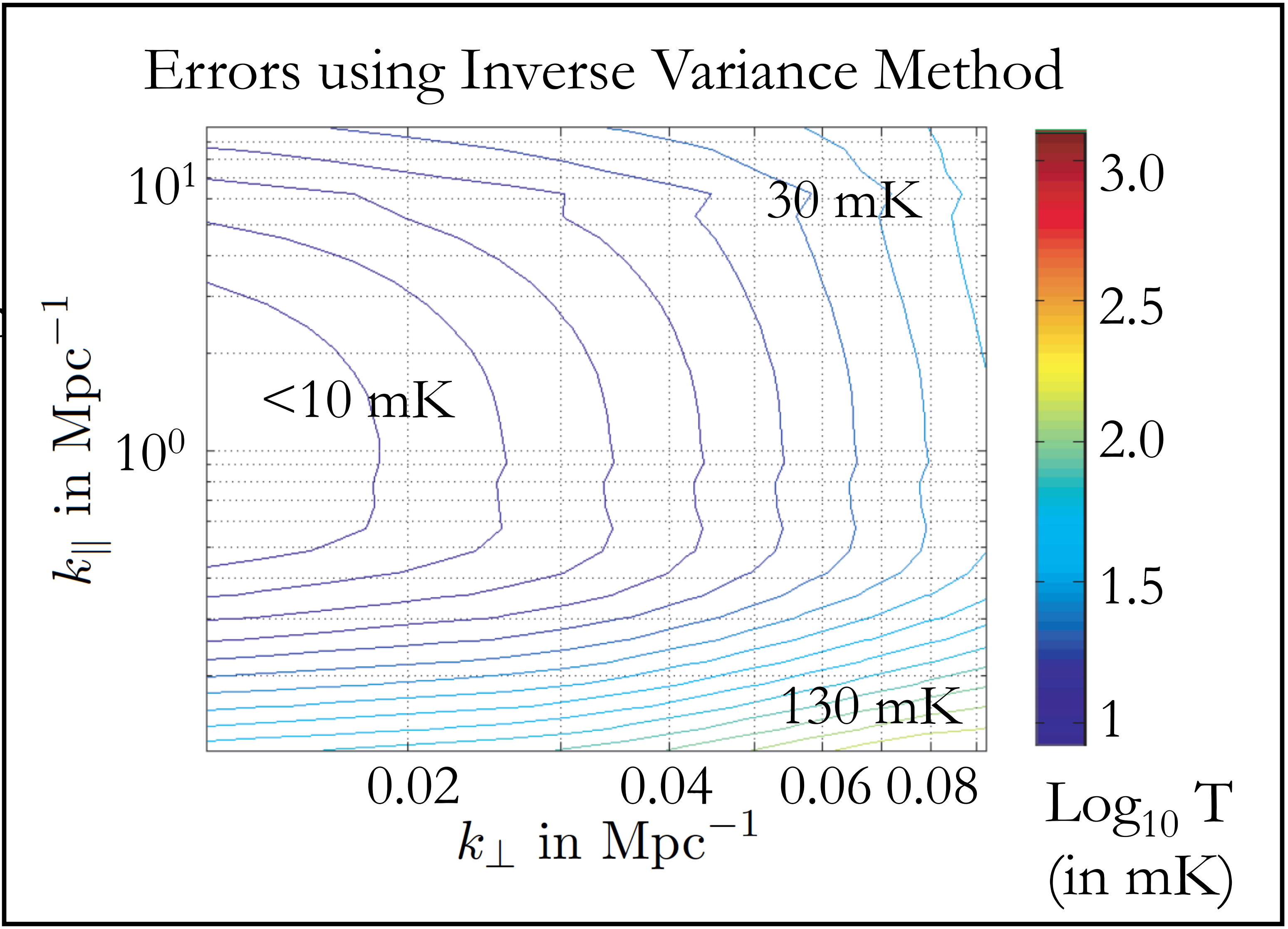
$$\Delta_{21}(k_\perp, k_{||}) = \left[\frac{k_\perp^2 k_{||}}{2\pi^2} P(k_\perp, k_{||})\right]^{\frac{1}{2}}$$

Lin

Errors with no foregrounds
20 mK
<10 mK
10^1
10^0
$k_{||}$ in Mpc^{-1}
0.02
0.04
0.06
0.08
$k_\perp$ in Mpc^{-1}
3.0
2.5
2.0
1.5
1
Log$_{10}$ T
(in mK)



Lin
Errors using Line of Sight Method
10^1
10^0
10 mK
100 mK
1 K
0.02
0.04
0.06
0.08
$k_{\perp}$ in Mpc^{-1}
$k_{||}$ in Mpc^{-1}
3.0
2.5
2.0
1.5
1
Log_{10} T
(in mK)

Lin


Errors using Inverse Variance Method
30 mK
<10 mK
130 mK
10^1
10^0
$k_{||}$ in Mpc^{-1}
0.02
0.04
0.06
0.08
$k_\perp$ in Mpc^{-1}
3.0
2.5
2.0
1.5
1
Log_{10} T
(in mK)

Precision Calibration for Precision Cosmology

- **Redundant calibration**
  - Better characterization of errors, removal of systematic biases, correction for non-exact redundancy.
  - Complements self-calibration

Precision Subtraction for Precision Cosmology

^ Foreground

- **Foreground modeling**
  - Know why foregrounds are describable by ~3 components
- **Foreground subtraction and power spectrum estimation**
  - Inverse variance foreground subtraction is lossless, unbiased, has less mode-mixing, and gives smaller error bars.

And more!